UNIT-1

BASICS OF RADAR

- Introduction
- Maximum Unambiguous Range
- Simple form of Radar Equation
- Radar Block Diagram and Operation
- Radar Frequencies and Applications
- Prediction of Range Performance
- Minimum Detectable Signal
- Receiver Noise
- Modified Radar Range Equation

RADAR EQUATION

- SNR
- Envelop Detector
- False Alarm time and Probability
- Integration of Radar Pulses
- Radar Cross Section of Targets (simple targets: sphere and cone sphere)
- Transmitter Power
- PRF and Range Ambiguities
- System Losses (qualitative treatment)
    - Important Formulae
    - Illustrative Problems
    - Questions from Previous Year Examinations

BASICS OF RADAR

Introduction:

Basic principles and features:

- Radar is a contraction of the words **RA**dio **D**etection **A**nd **R**anging. Radar is an electromagnetic system for the detection and location of objects. It operates by transmitting a particular type of waveform, a pulse-modulated sine wave for example, and detects the nature of the echo signal.
- Radar can see through conditions such as darkness, haze, fog, rain, and snow which is not possible for human vision. In addition, radar has the advantage that it can measure the distance or range to the object.
- An elementary form of radar consists of a transmitting antenna emitting electromagnetic Radiation generated by an oscillator of some sort, a receiving antenna, and an energy-detecting device or receiver. A portion of the transmitted signal is intercepted by a reflecting object (target) and is reradiated in all directions. The receiving antenna collects the returned energy and delivers it to a receiver, where it is processed to detect the presence of the target and to extract its location and relative velocity. The distance to the target is determined by measuring the time taken for the radar signal to travel to the target and back. The direction, or angular position, of the target is determined from the direction of arrival of the reflected wave front. The usual method of measuring the direction of arrival is with narrow antenna beams.
- If relative motion exists between target and radar, the shift in the carrier frequency of the reflected wave (Doppler Effect) is a measure of the target's relative (radial) velocity and may be used to distinguish moving targets from stationary objects. In radars which continuously track the movement of a target, a continuous indication of the rate of change of target position is also available.
- It was first developed as a detection device to warn the approach of hostile aircraft and for directing antiaircraft weapons. A well-designed modern radar can extract more information from the target signal than merely range.

Measurement of Range:

- The most common radar waveform is a train of narrow, rectangular-shape pulses modulating a sine wave carrier.
- The distance, or range, to the target is determined by measuring the time T_R taken by the pulse to travel to the target and return.
- Since electromagnetic energy propagates at the speed of light **c (3 x 10^8 m/s)** the range **R** is given by : $R = cT_R / 2$
- The factor 2 appears in the denominator because of the two-way propagation of radar. With the range **R** in kilometers or nautical miles, and T_R in microseconds, the above relation becomes: $R(km) = 0.15 \times T_R\ (\mu S)$ or $R(nmi) = 0.081 \times T_R\ (\mu S)$

Each microsecond of round-trip travel time corresponds to a distance of **0.081** nautical mile, **0.093** statute mile, **150** meters, **164** yards, or **492**feet.

(1 mile = 0.8689 nautical mile or 1.6 km
1 nautical mile = 1.15078 miles or 1.8412 km)

Maximum unambiguous range:

Once the transmitter pulse is emitted by the radar, sufficient time must elapse to allow any echo signals to return and be detected before the next pulse is transmitted. Therefore, the rate at which the pulses may be transmitted is determined by the longest range at which targets are expected. If the pulse repetition frequency is too high, echo signals from some targets might arrive after the transmission of the next pulse, and ambiguities in measuring range might result. Echoes that arrive after the transmission of the next pulse are called second-time-around (or multiple-time-around) echoes. Such an echo would appear to be at a much shorter range than the actual. The range beyond which targets appear as second-time-around echoes (or the farthest target range that can be detected by a Radar without ambiguity) is called the ***maximum unambiguous range*** and is given by: $R_{unambig.} = C/2f_p$
where f_p = pulse repetition frequency, in Hz. A plot of the maximum unambiguous range as a function of pulse repetition frequency is shown in the figure below.

This can also be explained with the following simple relations.

- T_R is the time elapsed between transmission pulse and Echo pulse.
- $T_R = 2R/C$ where R= Range of target
- T_R increases with Range R and in extreme case Echo pulse merges with next Transmitted Pulse. Then T_R becomes equal to T_P Where T_P = Pulse repetition period
- $T_{R\,max} = T_P = 2R_{max}/C$ and so $R_{max} = CT_P/2 = C/2f_P = R_{unambig}$

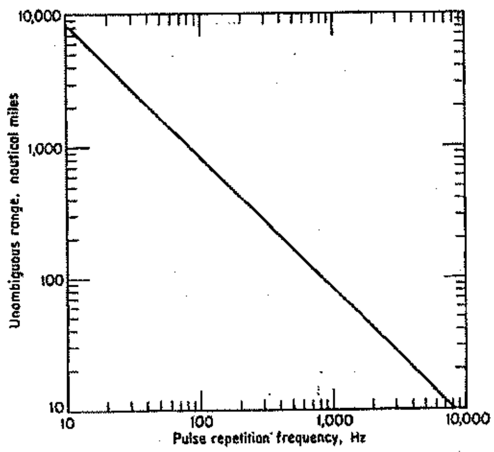


Fig 1.1: Plot of maximum unambiguous range as a function of the pulse repetition frequency.

Simple form of Radar Equation:

The radar equation

- Relates the range of a Radar to the characteristics of the transmitter, receiver, antenna, target, and environment.
- Useful as a means for determining the maximum measurable distance from the radar to the target
- It serves both as a tool for understanding radar operation and as a basis for radar design.

Derivation of the simple form of radar equation:

- If the power of the radar transmitter is denoted by P_t and if an isotropic antenna is used (one which radiates uniformly in all directions) the ***power*** density (watts per unit area) at a distance R from the radar is equal to the transmitter power divided by the surface area $4\pi R^2$ of an imaginary sphere of radius **R** with radar at its centre, or

  Power density from anisotropic antenna = $P_t/4\pi R^2$

- Radars employ directive antennas to direct the radiated power P_t into some particular direction. The **gain G** of an antenna is a measure of the increased power radiated in the direction of the target as compared with the power that would have been radiated from an isotropic antenna. *It may be defined as the ratio of the maximum radiation intensity from the given antenna to the radiation intensity from a lossless, isotropic antenna with the same power input.* (The radiation intensity is the power radiated per unit solid angle in a given direction.)Then the power density at the target from an antenna with a transmitting gain **G** is given by

  Power density from directive antenna = $P_t.G/4\pi R^2$

- The target intercepts a portion of the incident power and reradiates it in various directions. The measure of the amount of incident power intercepted by the target and reradiated back in the direction of the radar is denoted as the radar cross section **σ**, and is defined by the relation

  Power density of echo signal at radar = $(P_t.G/4\pi R^2).(\sigma/4\pi R^2)$

- The radar cross section **σ** has units of area. It is a characteristic of the particular target and is a measure of its size as seen by the radar. The radar antenna captures a portion of the echo power. If the effective area of the receiving antenna is denoted A_e then the power P_r received by the radar is given by

$$P_r = (P_t.G/4\pi R^2).(\sigma/4\pi R^2).A_e$$

$$= (P_t.G.A_e.\sigma)/(4\pi)^2.R^4$$

- The maximum radar range R_{max} is the distance beyond which the target cannot be detected. It occurs when the received echo signal power P_r just equals the minimum detectable signal S_{min}.

Therefore

$$R_{max} = [(Pt.G.A_e.\sigma)/(4\pi)^2.S_{min}]^{1/4} \quad .....(1)$$

This is the fundamental form of the radar equation. Note that the important antenna parameters are the ***transmitting gain*** and the ***receiving effective area***.

Antenna theory gives the relationship between the transmitting gain and the receiving effective area of an antenna as:

$$G = 4\pi A_e/\lambda^2$$

Since radars generally use the same antenna for both transmission and reception, the above relation between gain **G** and affective aperture area A_e can be substituted into the above equation, first for A_e and then for G, to give two other forms of the radar equation.

$$R_{max} = [(Pt.G^2.\lambda^2.\sigma)/(4\pi)^3.S_{min}]^{1/4} \quad ......(2)$$

$$R_{max} = [(Pt.A_e^2.\sigma)/4\pi.\lambda^2.S_{min}]^{1/4} ....(3)$$

These three forms (eqs.1, 2, and 3) illustrate the need to be careful in the interpretation of the radar equation. For example, from Eq. (2) it might be thought that the range of radar varies as $\lambda^{1/2}$, but Eq. (3) indicates a $\lambda^{-1/2}$relationship, and Eq. (1) shows the range to be independent of λ. The correct relationship depends on whether it is assumed the gain is constant or the effective area is constant with wavelength.

Limitations of the simple form of the Limitations of Simple Radar equation:

- Does not adequately describe the performance of practical radar.
- Many important factors that affect range are not explicitly included.
- In practice, the observed maximum radar ranges are usually much smaller than what would be predicted by the above equations, sometimes by as much as a factor of two.

There are many reasons for the failure of the simple radar equation to correlate with actual performance and these will be explained subsequently in the ***modified Radar range equation.***

Radar block diagram and operation:

The operation of a typical pulse radar is described with the help of a simple block diagram shown in the figure below. The transmitter is an oscillator, such as a magnetron, that is "pulsed" (turned on and off) by the modulator to generate a repetitive train of pulses. The magnetron has been the most widely used of the various microwave generators for radar. A typical radar for the detection of aircraft at ranges of 100 or 200 nmi employs a peak power of the order of one megawatt, an average power of several kilowatts, a pulse width of several microseconds, and a

pulse repetition frequency of several hundred pulses per second. The waveform generated by the transmitter travels via a transmission line to the antenna, where it is radiated into space.
A single antenna is generally used for both transmitting and receiving. The receiver must be protected from damage caused by the high power of the transmitter. This is the function of the duplexer. The duplexer also serves to channel the returned echo signals to the receiver and not to the transmitter. The duplexer consists of two gas-discharge devices, one known as a TR (transmit-receive) and the other as ATR (anti-transmit-receive). The TR protects the receiver during transmission and the ATR directs the echo signal to the receiver during reception. Solid-state ferrite circulators and receiver protectors with gas-plasma TR devices and/or diode limiters are also employed as duplexers. The receiver is usually of the super heterodyne type. The first stage normally is a low-noise RF amplifier, such as a parametric amplifier or a low-noise transistor. The mixer and local oscillator (LO) convert the RF signal to an intermediate frequency IF. Typical IF amplifier center frequencies are 30 or 60 MHz and will have a bandwidth of the order of one megahertz.

The IF amplifier should be designed as a ***matched filter*** i.e., *its frequency-response function* ***H(f)*** *should maximize the* ***peak-signal-to-mean-noise-power*** *ratio at the output. This occurs when the magnitude of the frequency-response function* ***|H(f)|*** *is equal to the magnitude of the echo signal spectrum* ***|S(f)|****, and the phase spectrum of the matched filter is the negative of the phase spectrum of the echo signal.* In a radar whose signal waveform approximates a rectangular pulse, the conventional IF filter band pass characteristic approximates a matched filter when the product of the IF bandwidth **B** and the pulse width τ is of the order of unity, that is, $B\tau = 1$.

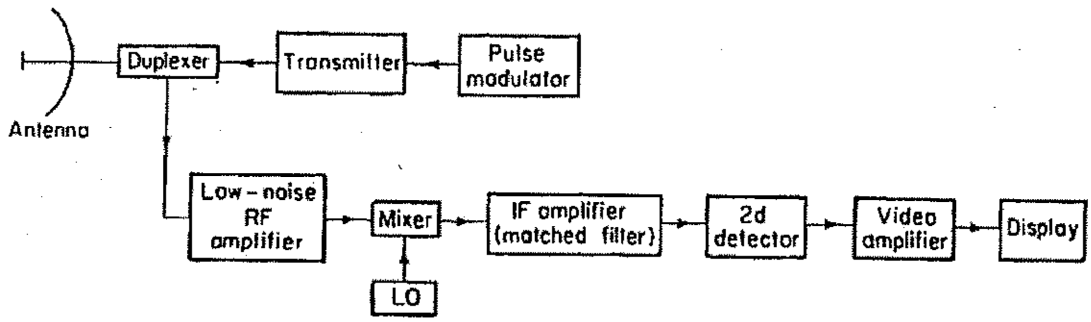


Fig 1.2: Block diagram of a pulse radar.

After maximizing the signal-to-noise ratio in the IF amplifier, the pulse modulation is extracted by the second detector and amplified by the video amplifier to a level where it can be properly displayed, usually on a cathode-ray tube (CRT). Timing signals are also supplied to the indicator to provide the range zero. Angle information is obtained from the pointing direction of the antenna.

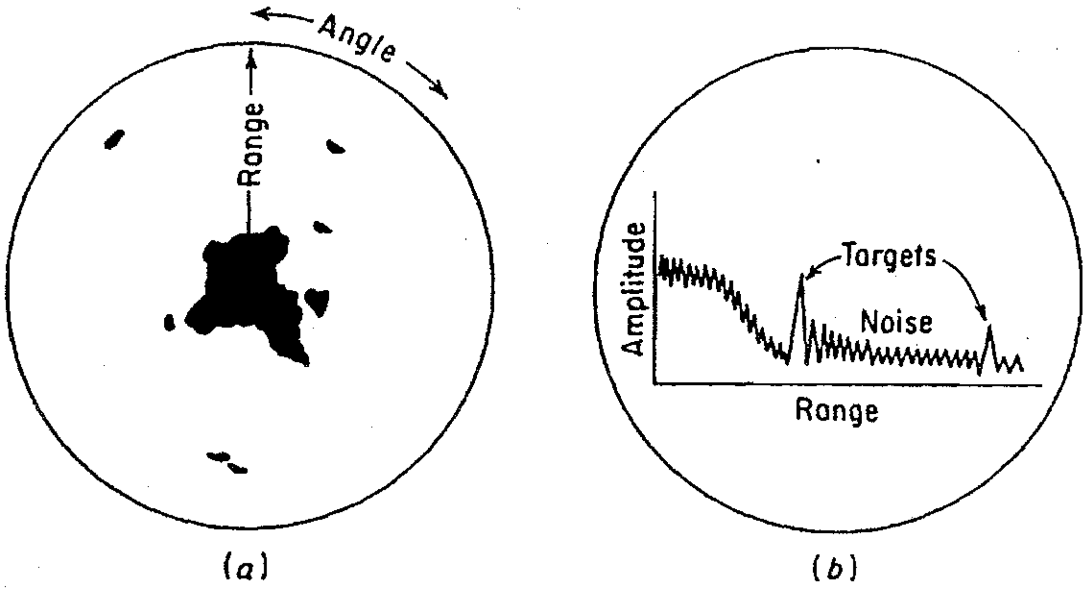


Fig 1.3(a) PPI presentation displaying Range vs. Angle (intensity modulation)
(b) A-scope presentation displaying Amplitude vs. Range (deflection modulation)

The most common form of cathode-ray tube display is the Plan Position Indicator, or **PPI (Fig. a)** which maps in polar coordinates the location of the target in azimuth and range. This is an intensity-modulated display in which the amplitude of the receiver output modulates the electron-beam intensity (z axis) as the electron beam is made to sweep outward from the center of the tube. The beam rotates in angle in response to the antenna position. A ***B-scope*** display is similar to the PPI except that it utilizes rectangular, rather than polar, coordinates to display range vs. angle. Both the B-scope and the PPI, being intensity modulated, have limited dynamic range. Another form of display is the ***A-scope,*** shown in Fig. **b,** which plots target amplitude **(y axis)** vs. range **(x axis),** for some fixed direction. This is a deflection-modulated display. It is more suited for tracking-radar application than for surveillance radar.
A common form of radar antenna is a reflector with a parabolic shape, fed (illuminated) from a point source at its focus. The parabolic reflector focuses the energy into a narrow beam, just as a searchlight or an automobile headlamp. The beam is scanned in space by mechanical pointing of the antenna.

Radar frequencies and applications:

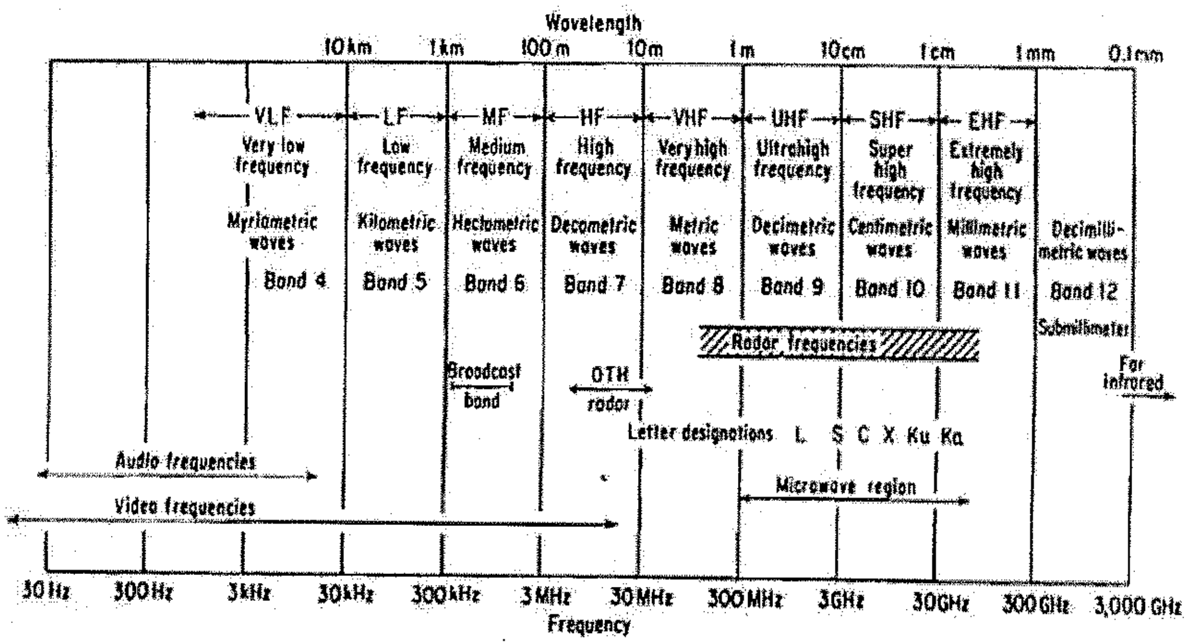
Radar frequencies:

Conventional radars are operated at frequencies extending from about 220 MHz to 35 GHz, a spread of more than seven octaves. These are not necessarily the limits, since radars can be, and have been, operated at frequencies outside either end of this range.

Sky wave HF over-the-horizon (OTH) radar might be at frequencies as low as 4 or 5 MHz, and ground wave HF radars as low as 2 MHz. At the other end of the spectrum, millimeter radars have operated at 94 GHz. Laser radars operate at even higher frequencies.

The place of radar frequencies in the electromagnetic spectrum is shown in the figure below. Some of the nomenclature employed to designate the various frequency regions is also shown in this figure.

ELECROMAGNETIC SPECTRUM



Letter code designation of Radar frequencies:

Early in the development of radar, a letter code such as S, X, L, etc., was employed to designate Radar frequency bands. Although it's original purpose was to guard military secrecy, the designations were maintained, probably out of habit as well as the need for some convenient short nomenclature. This usage has continued and is now an accepted practice of radar engineers. The table below lists the radar-frequency letter-band nomenclature adopted by the **IEEE.** These are related to the specific bands assigned by the International Telecommunications Union for radar. For example, although the nominal frequency range for L band is 1000 to 2000 MHz, a L-band radar is thought of as being confined within the region from 1215 to1400 MHz since that is the extent of the assigned band. Letter-band nomenclature is not a substitute for the actual numerical frequency limits of radars. The specific numerical frequency limits should be used whenever appropriate, but the letter designations of this Table may be used whenever a short notation is desired.

Table 1.1: Standard radar-frequency letter-band nomenclature

Band designation	Nominal frequency range	Specific radiolocation (radar) bands based on ITU assignments for region 2
HF	3-30 MHz	
VHF	30-300 MHz	138-144 MHz 216-225
UHF	300-1000 MHz	420-450 MHz 890-942
L	1000-2000 MHz	1215-1400 MHz
S	2000-4000 MHz	2300-2500 MHz 2700-3700
C	4000-8000 MHz	5250-5925 MHz
X	8000-12,000 MHz	8500-10,680 MHz
K_u	12.0-18 GHz	13.4-14.0 GHz 15.7-17.7
K	18-27 GHz	24.05-24.25 GHz
K_a	27-40 GHz	33.4-36.0 GHz
mm	40-300 GHz	

Applications of Radar:

1. Military Use:

Military is the initial and important user of Radar

(i)Early warning of intruding enemy aircrafts & missile

(ii)Tracking hostile targets and providing location information to Air Defense systems Consisting of Tracking Radars controlling guns and missiles.

(iii)Battle field surveillance

(iv)Information Friend or Foe IFF

(v)Navigation of ships, aircrafts, helicopter etc.

2. Civilian Use:

(i) Air Traffic Control (ATC)

All airports equipped with ATC Radars, for safe landing and take-off and guiding of aircrafts in bad weather and poor visibility conditions.

(ii) Aircraft Navigation

(a) All aircrafts fitted with weather avoidance radars. These Radars give warning information to pilot about storms, snow precipitation etc. lying ahead of aircraft's path.

(b) Radar is used as an altimeter to indicate the height of the aircraft or helicopter.

3. Maritime ship's safety and Navigation:

(i)Radar used to avoid collision of ships during poor visibility conditions (storms, cyclones etc.)

(ii)Guide ships into seaports safely.

4. Meteorological Radar:
Used for weather warnings and forecasting. Provides sufficient advance information to civilian administration for evacuation of population in times cyclones, storms etc.

Prediction of Range Performance:

The simple form of Radar equation derived earlier expresses the maximum radar range R_{max} in terms of radar and target parameters:

$$R_{max} = [(P_t . G . A_e . \sigma) / (4\pi)^2 . S_{min}]^{1/4}$$

Where P_t = transmitted power, watts
G = antenna gain
A_e = antenna effective aperture, m^2
Σ = radar cross section, m^2
S_{min} = minimum detectable signal, watts

All the parameters are to some extent under the control of the radar designer, except for the target cross section **σ.** The radar equation states that if long ranges are desired,

- The transmitted power must be large,
- The radiated energy must be concentrated into a narrow beam (high transmitting antenna gain),
- The received echo energy must be collected with a large antenna aperture (also synonymous with high gain) and
- The receiver must be sensitive to weak signals.

In practice, however, the simple radar equation does not predict the range performance of actual radar equipment to a satisfactory degree of accuracy. The predicted values of radar range are usually optimistic. In some cases, the actual range might be only half of that is predicted.
Part of this discrepancy is due to

- The failure of the above equation to explicitly include the various losses that can occur throughout the system or
- The loss in performance usually experienced when electronic equipment is operated in the field rather than under laboratory-type conditions and
- Another important factor i.e the statistical or unpredictable nature of several of the parameters in the radar equation.

The minimum detectable signal S_{min} and the target cross section **σ** are both statistical in nature and must be expressed in statistical terms.

- Other statistical factors which do not appear explicitly in the simple radar equation but which have an effect on the radar performance are the meteorological conditions along the propagation path and the performance of the radar operator, if one is employed.

The statistical nature of these several parameters does not allow the maximum radar range to be described by a single number. Its specification must include a statement of the probability that the radar will detect a certain type of target at a particular range.

- *Hence in order to cover these aspects, the simple radar equation will be **modified** to include most of the important factors that influence radar range performance.*

Minimum detectable signal:

- The ability of a radar receiver to detect a weak echo signal is limited by the noise energy that occupies the same portion of the frequency spectrum as does the signal energy and accompanies the signal.
- The weakest signal the receiver can detect is called the ***minimum detectable signal.*** It is difficult to define ***minimum detectable signal*** (MDS) because of its statistical nature and because the criterion for deciding whether a target is present or not is not too well defined.
- Detection is normally based on establishing a threshold level at the output of the receiver (as shown by the dotted line in the figure below.)Whenever Rx output signal which is a mixture of echo and noise crosses this threshold, then it is detected as a target. This is called ***threshold detection.***
- Consider the output of a typical radar receiver as a function of time as shown in the figure below which typically represents one sweep of the video output displayed on an A-scope.

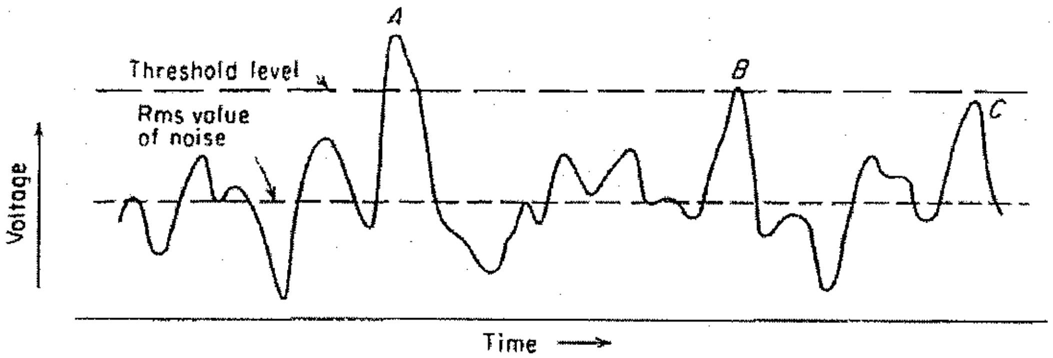


Fig 1.4: Typical envelope of the radar receiver output as a function of time. *A,* B, and *C* are three targets representing signal plus noise. A and B are valid detections, but C is a missed detection.

- The envelope has a fluctuating appearance due to the random nature of noise and consists of three targets A, B and C of different signal amplitudes.
- The voltage envelope shown above is assumed to be from a matched-filter receiver. Receiver echoes no more look like rectangular pulses. They appear as triangular as shown in the figure above.
- If the threshold level were set sufficiently high, the envelope would not generally exceed the threshold if noise alone were present, but would exceed it if a strong signal were present. If the signal were small, however, it would be more difficult to recognize its

presence. The threshold level must be low if weak signals are to be detected, but it cannot be so low that noise peaks cross the threshold and give a false indication of the presence of targets.

- The signal at **A** is large which has much larger amplitude than the noise. Hence target detection is possible without any difficulty and ambiguity.
- Next consider the two signals at **B** and **C**, representing target echoes of equal amplitude. The noise voltage accompanying the signal at **B** is large enough so that the combination of signal plus noise exceeds the threshold and target detection is still possible.
- But for the target **C** the noise is not as large and the resultant signal plus noise does not cross the threshold and hence target is not detected.
- ***Threshold Level setting:*** Weak signals such as **C** would not be lost if the threshold level were lower. But too low a threshold increases the likelihood that noise alone will rise above the threshold and is taken as target. Such an occurrence is called a ***false alarm.*** Therefore, if the threshold is set too low, false target indications are obtained, but if it is set too high, targets might be missed. The selection of the proper threshold level is a compromise that depends upon how important it is if a mistake is made either by
  1. Failing to recognize a signal that is present (***probability of a miss***) or by
  2. Falsely indicating the presence of a signal when it does not exist (***probability of a false alarm***)
- The signal-to noise ratio necessary to provide adequate detection is one of the important parameters that must be determined in order to compute ***the minimum detectable signal.***
- Although the detection decision is usually based on measurements at the video output, it is easier to consider maximizing the signal-to-noise ratio at the output of the IF amplifier rather than in the video. The receiver may be considered linear up to the output of the IF. It is shown that maximizing the signal-to-noise ratio at the output of the IF is equivalent to maximizing the video output. The advantage of considering the signal-to-noise ratio at the IF is that the assumption of linearity may be made. It is also assumed that the IF filter characteristic approximates the matched filter, so that the output signal-to-noise ratio is maximized.

Receiver noise:

- Noise is unwanted electromagnetic energy which interferes with the ability of the receiver to detect the wanted signal thus limiting the receiver sensitivity.

It may originate within the receiver itself, or it may enter via the receiving antenna along with the desired signal. If the radar were to operate in a perfectly noise-free environment so that no external sources of noise accompanied the desired signal, and if the receiver itself were so perfect that it did not generate any excess noise, ***there would still exist an unavoidable component of noise generated by the thermal motion of the conduction electrons in the ohmic portions of the receiver input stages. This is called thermal noise, or Johnson's noise, and is directly proportional to the temperature of the ohmic portions of the circuit and the receiver band***

width. The available noise power generated by a receiver of bandwidth B_n(in hertz) at a temperature **T** (degrees Kelvin) is given by : ***Available thermal-noise power*** = kTB_n
where k = Boltzmann's constant =**1.38 x 10^{-23}J/deg.** If the temperature T is taken to be290 K, which corresponds approximately to room temperature (62^{0}F), the factor **kT** is **4 x 10^{-21}W/Hz** of bandwidth. If the receiver circuitry were at some other temperature, the thermal-noise power would be correspondingly different.

- A receiver with a reactance input such as a parametric amplifier need not have any significant ohmic loss. The limitation in this case is the thermal noise seen by the antenna and the ohmic losses in the transmission line.
- For radar receivers of the super heterodyne type (the type of receiver used for most radar applications), the receiver bandwidth is approximately that of the **intermediate-frequency** stages. It should be cautioned that the bandwidth B_n mentioned above is not the 3-dB, or half-power, bandwidth commonly employed by electronic engineers. It is an integrated bandwidth and is given by:

$$B_n = \frac{\int_{-\infty}^{\infty} |H(f)|^2 \, df}{|H(f_0)|^2}$$

where **H (f)** = frequency-response characteristic of IF amplifier (filter) and f_0 = frequency of maximum response (usually occurs at mid band).

- ***The bandwidth B_n is called the noise bandwidth and is the bandwidth of an equivalent rectangular filter whose noise-power output is same as the filter with characteristic H(f).*** It is not theoretically same as the 3-dB bandwidth. The 3-dB bandwidth is widely used since it is easy to measure. The measurement of noise bandwidth however involves a complete knowledge of the response characteristic **H(f).** The frequency-response characteristics of many practical radar receivers are such that the 3 dB and the noise bandwidths do not differ appreciably. Therefore the 3-dB bandwidth may be used in many cases as an approximation to the noise bandwidth.
- The noise power in practical receivers is often greater than can be accounted for by thermal noise alone and is due to mechanisms other than the thermal agitation of the conduction electrons. The exact origin of the extra noise components is not important except to know that it exists. Whether the noise is generated by a thermal mechanism or by some other mechanism the total noise at the output of the receiver may be considered to be equal to the thermal-noise power obtained from an "ideal"receiver multiplied by a factor called the ***noise figure.***
- The noise figure F_n of a receiver is defined by the equation:

  $$F_n = N_o / kT_o B_n G_a$$

  = (Noise output of practical receiver)/ (Noise output of ideal receiver at std. temp T_o)
  Where N_o = noise output from receiver, and G_a = available gain. The standard temperature T_o is taken to be 290 **K,** according to the Institute of Electrical and Electronics Engineers definition. The noise N_o is measured over the linear portion of the receiver input-output characteristic, usually at the output of the IF amplifier before the nonlinear second detector. The receiver bandwidth B_n is that of the IF amplifier in most receivers. The

available gain G_a is the ratio of the signal out S_o to the signal in S_i and $kT_o B_n$ is the input noise N_i in an ideal receiver. The above equation may be rewritten as:

$$F_n = \frac{S_i/N_i}{S_o/N_o}$$

Therefore, the ***noise figure*** may be interpreted, as a measure of the degradation of signal-to noise-ratio as the signal passes through the receiver.

Modified radar equation:

Rearranging the above two equations for F_n , the input signal may be expressed as

$$S_i = \frac{kT_0 B_n F_n S_o}{N_o}$$

If the minimum detectable signal S_{min} is that value of S_i corresponding to the minimum ratio of output (IF) signal-to-noise ratio $(S_o/N_o)_{min}$ necessary for detection, then

$$S_{min} = kT_0 B_n F_n \left(\frac{S_o}{N_o}\right)_{min}$$

Substituting this expression for S_{min} into the earlier basic Radar equation results in the following form of the **modified radar equation:**

$$R^4_{max} = \frac{P_t G A_e \sigma}{(4\pi)^2 k T_0 B_n F_n (S_o/N_o)_{min}} \qquad \text{.........(4)}$$

RADAR EQUATION

For the study of SNR, false alarm probability and probability of detection etc an introduction to the basics of probability and probability density functions is necessary and they are briefly explained here.

Introduction to Probability-density functions:

Probability is a measure of the likelihood of occurrence of an event. The scale of probability ranges from **0 to 1.** An event which is certain is assigned the probability **1.** An impossible event is assigned the probability **0.** The intermediate probabilities are assigned so that the more likely an event, the greater is its probability.

One of the more useful concepts of probability theory needed to analyze the detection of signals in noise is the ***probability-density function***. Consider the variable **x** as representing atypical measured value of a random process such as a noise voltage or current. Imagine each **x** to define a point on a straight line corresponding to the distance from a fixed reference point. The distance of **x** from the reference point might represent the value of the noise current or the noise voltage. Divide the line into small equal segments of length **Δx** and count the number of times that **x** falls in each interval. The probability-density function **p(x)** is then defined as

$$p(x) = \lim_{\substack{\Delta x \to 0 \\ N \to \infty}} \frac{(\text{number of values in range } \Delta x \text{ at } x)/\Delta x}{\text{total number of values} = N}$$

Then, the probability that a particular measured value lies within the infinitesimal width ***dx*** centered at **x** is simply **p(x) dx.** The probability that the value of **x** lies within the finite range from x_1 **to** x_2 is found by integrating **p(x)** over the range of interest, or

$$\text{Probability } (x_1 < x < x_2) = \int_{x_1}^{x_2} p(x)\, dx$$

By definition, the probability-density function is positive. Since every measurement must yield some value, the integral of the probability density over all values of x must be equal to unity. That is,

$$\int_{-\infty}^{\infty} p(x)\, dx = 1 \qquad \text{............(5)}$$

The average value of a variable function **Ø(x)**that is described by the probability-density function **p(x)** is,

$$\langle \phi(x) \rangle_{av} = \int_{-\infty}^{\infty} \phi(x) p(x)\, dx$$

This follows from the definition of an average value and the probability-density function. The mean or average value of **x** is

$$\langle x \rangle_{av} = m_1 = \int_{-\infty}^{\infty} x p(x)\, dx$$

and the mean square value is

$$\langle x^2 \rangle_{av} = m_2 = \int_{-\infty}^{\infty} x^2 p(x)\, dx$$

The quantities m_1 and m_2 are sometimes called the first and second moments of the random variable **x.** **If x** represents an electric voltage or currentm_1 is the d-c component. It is the value read by a direct-current voltmeter or ammeter. The mean square value $m_2$of the current when multiplied by the resistance gives the mean power. The mean square value of voltage times the conductance is also the mean power. The variance is defined as

$$\mu_2 = \sigma^2 = \langle (x - m_1)^2 \rangle_{av} = \int_{-\infty}^{\infty} (x - m_1)^2 p(x)\, dx = m_2 - m_1^2 = \langle x^2 \rangle_{av} - \langle x \rangle_{av}^2$$

The variance is the mean square deviation of **x** about its mean and is sometimes called the second central moment. If the random variable is a noise current, the product of the variance **σ** and resistance gives the mean power of the a-c component. The square root of the variance **σ** is called the ***standard deviation*** and is the ***root-mean-square (rms)*** value of the a-c component.

We shall explain briefly two PDFs that are required for our study of Signal to Noise Ratio.(SNR) :

1.) The ***Gaussian or normal probability density*** is one of the most important in noise theory, since many sources of noise such as thermal noise or shot noise may be represented by Gaussian statistics. Also, a Gaussian representation is often more convenient to manipulate mathematically. The Gaussian density function shown in the fig 1.5(a) below has a bell-shaped appearance and is defined by:

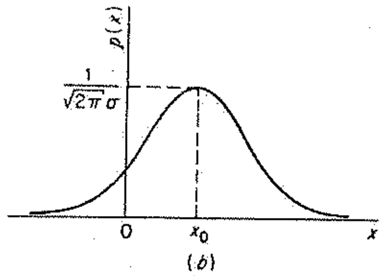


Fig. 1.5(a) Gaussian or normal probability density function

$$p(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp \frac{-(x - x_0)^2}{2\sigma^2}$$

where **exp [ ]** is the exponential function and the parameters have been adjusted to satisfy the normalizing condition of Eq. 5. It can be shown that

$$m_1 = \int_{-\infty}^{\infty} x p(x)\, dx = x_0 \qquad m_2 = \int_{-\infty}^{\infty} x^2 p(x)\, dx = x_0^2 + \sigma^2 \qquad \mu_2 = m_2 - m_1^2 = \sigma^2$$

The probability density of the sum of a large number of independently distributed quantities approaches the Gaussian probability-density function no matter what the individual distributions may **be,** provided that the contribution of any one quantity is not comparable with the resultant of all others. This is the ***central limit theorem.*** Another property of the Gaussian distribution is that no matter how large a value **x** we may choose, there is always some finite probability of finding a greater value. If the noise at the input of the threshold detector were truly Gaussian, then no matter how high the threshold were set, there would always be a chance that it would be exceeded by noise and appear as a false alarm. However, the probability diminishes rapidly with increasing **x,** and for all practical purposes the probability of obtaining an exceedingly high value of **x** , is negligibly small.

2.) The ***Rayleigh probability density function*** is also of special interest to the radar systems engineer. It describes the envelop of the noise output from a narrow band filter (such as the IF filter in super heterodyne receiver).The cross-section fluctuations of certain types of complex radar targets and many kinds of clutter and weather echoes. The Rayleigh density function is shown in the fig.1.5 (b) below and is given by :

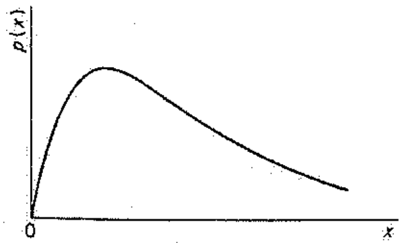


Fig 1.5 (b): Rayleigh probability density function

$$p(x) = \frac{2x}{\langle x^2 \rangle_{av}} \exp\left(-\frac{x^2}{\langle x^2 \rangle_{av}}\right) \qquad x \geq 0$$

The parameter x might represent a voltage and $\langle x^2 \rangle_{AV}$ the mean, or average, value of the voltage squared.

Signal to Noise Ratio (SNR):

The results of statistical noise theory will be applied to obtain:

- ***The signal-to-noise ratio at the output of the IF amplifier necessary to achieve a specified probability of detection without exceeding a specified probability of false alarm.***

The output signal-to-noise ratio thus obtained is substituted into the final modified radar equation, we have obtained earlier.

The details of system that is considered:

- IF amplifier with bandwidth B_{IF} followed by a second detector and a video amplifier with bandwidth B_V as shown in the figure below.
- The second detector and video amplifier are assumed to form an envelope detector, that is, one which rejects the carrier frequency but passes the modulation envelope.
- To extract the modulation envelope, the video bandwidth must be wide enough to pass the low-frequency components generated by the second detector, but not so wide as to pass the high-frequency components at or near the intermediate frequency.
- The video bandwidth B_V must be greater than $B_{IF}/2$ in order to pass all the video modulation.

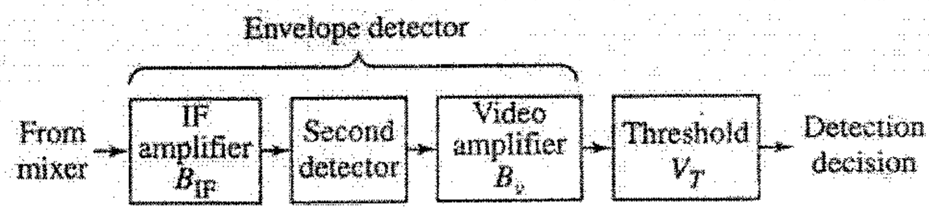


Figure 1.6: Envelope detector.

Step 1: To determine the Probability of false alarm when noise alone is assumed to be present as input to the receiver:

The noise entering the IF filter (the terms filter and amplifier are used interchangeably) is assumed to be Gaussian, with probability-density function given by

$$p(v) = \frac{1}{\sqrt{2\pi\psi_0}} \exp \frac{-v^2}{2\psi_0}$$

Where:

- **p(v) dv** is the probability of finding the noise voltage ***v*** between the values of **v** and **v + dv**
- ***ψ_0*** is the variance, or mean-square value of the noise voltage, and the mean value of **v** is taken to be zero.

(Compare this with the Standard Probability density function of Gaussian noise

$$p(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp \frac{-(x - x_0)^2}{2\sigma^2}$$

*With σ^2 replaced by ψ_0 and **$(x—x_0)$** replaced by **v** with mean value of zero)*

If Gaussian noise were passed through a narrowband IF filter whose Bandwidth is small compared with its mid band frequency-the probability density of the envelope of the noise voltage output is shown by **Rice** to be of the form of **Rayleigh** probability-density function

$$p(R) = \frac{R}{\psi_0} \exp\left(-\frac{R^2}{2\psi_0}\right) \qquad \text{..........(6)}$$

where ***R is the amplitude of the envelope of the filter output***. The probability that the envelope of the noise voltage will lie between the values of V_1 and V_2 is

$$\text{Probability } (V_1 < R < V_2) = \int_{V_1}^{V_2} \frac{R}{\psi_0} \exp\left(-\frac{R^2}{2\psi_0}\right) dR$$

The probability that the noise voltage envelope will exceed the voltage threshold V_T is

$$\text{Probability } (V_T < R < \infty) = \int_{V_T}^{\infty} \frac{R}{\psi_0} \exp\left(-\frac{R^2}{2\psi_0}\right) dR$$

$$= \exp\left(-\frac{V_T^2}{2\psi_0}\right) = P_{fa} \qquad \text{...........(7)}$$

Whenever the voltage envelope exceeds the threshold V_T, a target is considered to have been detected. Since the probability of a false alarm is the probability that noise will cross the threshold, the above equation gives the probability of a false alarm, denoted by P_{fa}.

The probability of false alarm as given above by itself does not indicate that Radar is troubled by the false indications of Target. The time between the false alarms T_{FA} is a better measure of the effect of Noise on the Radar performance. (Explained with reference to the figure below)

The average time interval between crossings of the threshold by noise alone is defined as the ***false-alarm time*** T_{FA}

$$T_{fa} = \lim_{N \to \infty} \frac{1}{N} \sum_{k=1}^{N} T_k$$

where T_K is the time between crossings of the threshold V_T by the noise envelope, when the slope of the crossing is positive.

The false-alarm probability may also be defined as the ratio of the duration of time the envelope is actually above the threshold to the total time it ***could have been*** above the threshold, i.e.

$$P_{fa} = \frac{\sum_{k=1}^{N} t_k}{\sum_{k=1}^{N} T_k} = \frac{\langle t_k \rangle_{av}}{\langle T_k \rangle_{av}} = \frac{1}{T_{fa} B}$$

.... **(8)**

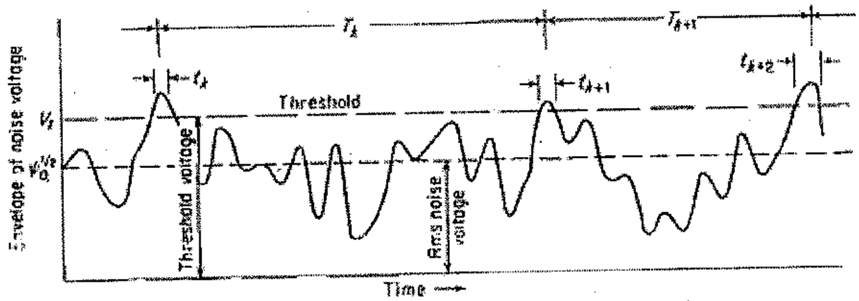


Fig 1.7: Envelope of receiver output illustrating false alarms due to noise.

where t_K and T_K are shown in the Figure above. ***The average duration of a noise pulse is approximately the reciprocal of the bandwidth B,*** which in the case of the envelope detector is B_{IF}. Equating eqs.**7** and **8** we get

$$T_{fa} = \frac{1}{B_{IF}} \exp \frac{V_T^2}{2\psi_0}$$

..........(9)

A plot of the above equation is shown in the figure below with **($V_T^2/2\ \psi_0$)** as the abscissa.

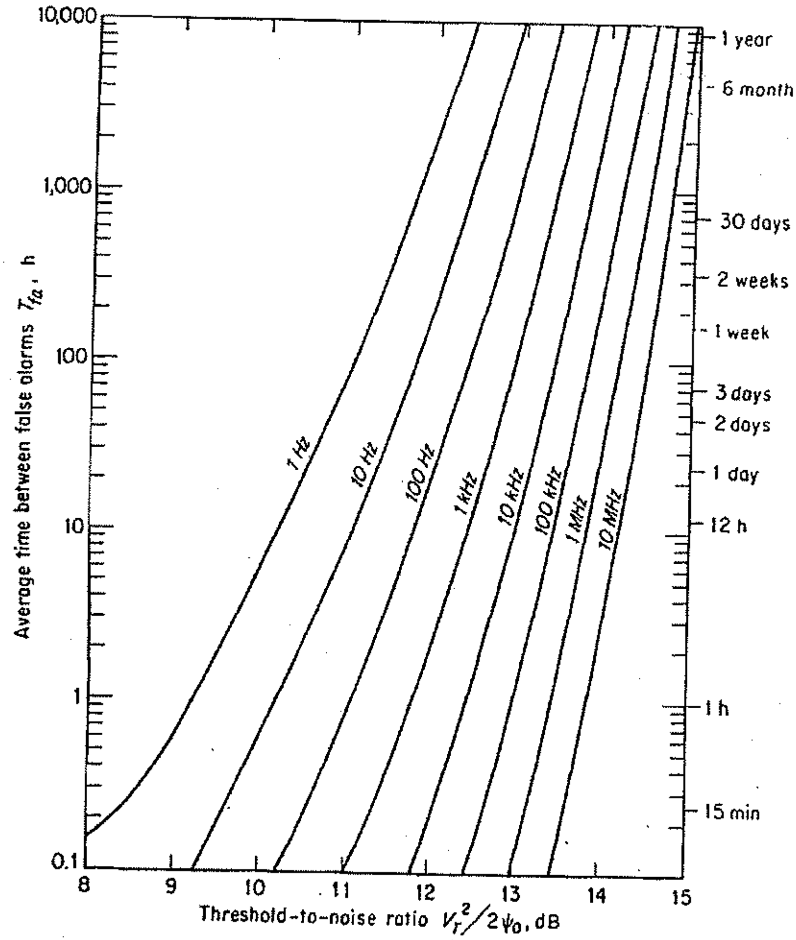


Fig. 1.8: Average time between false alarms as a function of the threshold level V_T and the receiver Band width B. ψ_0 is the mean square noise voltage

Step 2 :

To determine Probability of detection when a sine wave signal is present along with noise:

Thus far, a receiver with only a noise input was discussed. Next, consider a sine-wave signal of amplitude **A** to be present along with noise at the input to the IF filters. The frequency of the signal is the same as the IF mid band frequency f_{IF}. The output of the envelope detector has a probability-density function given by

$$p_s(R) = \frac{R}{\psi_0} \exp\left(-\frac{R^2 + A^2}{2\psi_0}\right) I_0\left(\frac{RA}{\psi_0}\right) \quad \text{.............(10)}$$

where $I_0(Z)$ is the modified Bessel function of zero order and argument **Z.**

When the signal is absent, ***A*** = 0 and the above equation for **PDF** for signal plus noise reduces to the probability-density function for noise alone. This Equation is sometimes called the **Rice** probability-density function.

The probability that the signal will be detected (which is the ***probability of detection)*** is the same as the probability that the envelope **R** will exceed the predetermined threshold V_T. The probability of detection P_d is therefore:

$$P_d = \int_{V_T}^{\infty} p_s(R)\, dR = \int_{V_T}^{\infty} \frac{R}{\psi_0} \exp\left(-\frac{R^2 + A^2}{2\psi_0}\right) I_0\left(\frac{RA}{\psi_0}\right) dR \quad \text{..........(11)}$$

When the expression of **PDF** for **$P_s(R)$[Eq. 10]**is substituted in the above equation for probability of detection **[eqn.11]** it cannot be evaluated by simple means, and numerical & empirical techniques or a series approximation must be used.

The expression for P_d given by equation **(11)** after series expansion is a function of the signal amplitude A, threshold voltage V_T ,and mean noise power Ψ_0 . In Radar systems analysis, it is more convenient to use Signal to Noise power ratio (S/N) rather than signal to noise voltage ratio $A/\Psi_0^{½}$.These are related by:

$A/\Psi_0^{½}$ = Signal Amplitude/rms noise voltage = √2 (rms signal voltage)/rms noise voltage

= (2.Signal Power/Noise power)$^{1/2}$ = (2.S/N)$^{1/2}$

The probability of detection P_d can then be expressed in terms of S/N , and Threshold- Noise ratio $V_T^2/2\Psi_0$.The probability of false alarm is also a function of $V_T^2/2\Psi_0$ as given by : **P_{FA} = Exp($-V_T^2/2\psi_0$).**

The two expressions for P_d and P_{FA} can now be combined by eliminating the Threshold- Noise ratio **$V_T^2/2\Psi_0$** that is common in both expressions so as to get a single expression relating the probability of detection P_d ,Probability of false alarm P_{FA} and signal to Noise ratio S/N. The result is plotted in the figure below.

A much easier empirical formula developed by ***Albersheim*** for the relation ship between **S/N,P_{FA}** and **P_d** is also given below :

S/N = A + 0.12AB + 1.7 B

Where A = ln [0.62/ P_{FA}] and B = ln [P_d /(1- P_d)]

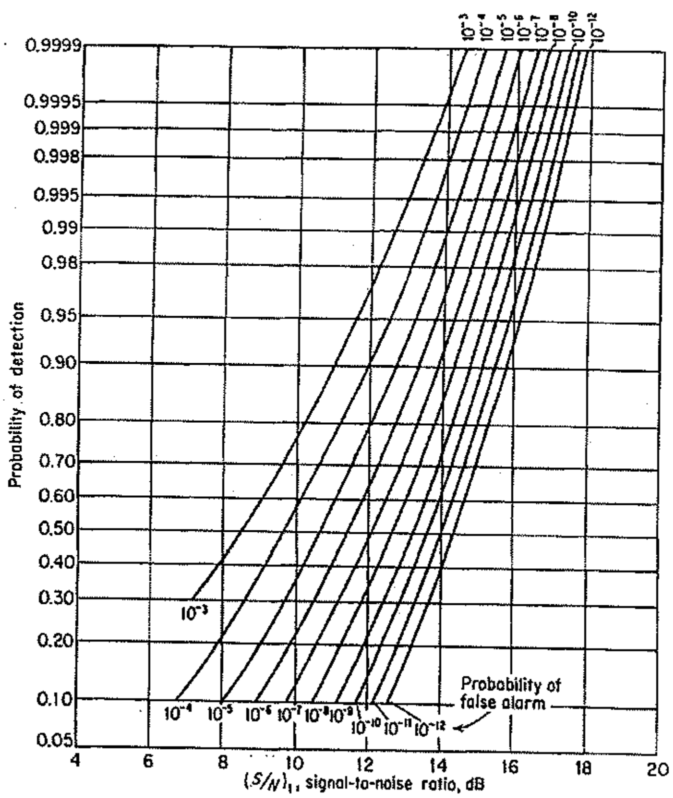


Fig. 1.9: Probability of detection for a sine wave in noise as a function of the signal-to-noise (power) ratio and the probability of false alarm

System design sequence:

- Both the false-alarm time T_{FA} and the detection probability P_d are specified by the system requirements.
- The radar designer computes the probability of the false alarm using the above T_{fa} & the relation

$$P_{fa} = 1/T_{fa} . B$$

- Then from the figure above or using the ***Albersheim's*** empirical equation given above the required signal-to-noise ratio to achieve the above P_{fa} & P_d is determined.

For example, suppose that the desired false-alarm time was **15 min** and the IF bandwidth was **1** MHz. This gives a false-alarm probability of **1.11×10^{-9}**. Figure above indicates that a signal-to-noise ratio of **13.1** dB is required to yield a **0.50** probability of detection, **14.7 dB** for **0.90**, and **16.5 dB** for **0.999**.

This is the signal-to-noise ratio that is to be used in the final modified Radar Equation we have obtained earlier.

$$R^4_{max} = \frac{P_t G A_e \sigma}{(4\pi)^2 k T_0 B_n F_n (S_o/N_o)_{min}}$$

Integration of Radar Pulses:

The relation between the signal to noise ratio, the probability of detection and the probability of false alarm as shown in the figure or as obtained using the ***Albersheim's*** empirical equation applies for a single pulse only. However, many pulses are usually returned from any target on each radar scan and can be used to improve detection. The number of pulses n_B returned from a point target as the radar antenna scans through its beam width is

$$n_B = \theta_B \cdot f_P / \theta'_S = \theta_B \cdot f_P / 6\,\omega_m$$

where θ_B = antenna beam width, deg
f_P = pulse repetition frequency, Hz
θ'_S = antenna scanning rate, deg/s
ω_m = antenna scan rate, rpm

The process of summing all the radar echo pulses for the purpose of improving detection is called integration.

Integration may be accomplished in the radar receiver either before the second detector (in the IF) or after the second detector (in the video).

- Integration before the detector is called pre detection or coherent integration. In this the phase of the echo signal is to be preserved if full benefit is to be obtained from the summing process
- Integration after the detector is called post detection or non coherent integration. In this phase information is destroyed by the second detector. Hence post detection integration is not concerned with preserving RF phase. Due to this simplicity it is easier to implement in most applications, but is not as efficient as pre detection integration.

If ***n*** pulses, all of the same signal-to-noise ratio, were integrated by an ideal pre detection integrator, the resultant or integrated signal-to-noise (power) ratio would be exactly **n** times that of a single pulse. If the same ***n*** pulses were integrated by an ideal post detection device, the resultant signal-to-noise ratio would be less than **n** times that of a single pulse. ***This loss in integration efficiency is caused by the nonlinear action of the second detector, which converts some of the signal energy to noise energy in the rectification process.***

Due to its simplicity, Post detection integration is preferred many a times even though the integrated signal-to-noise ratio may not be as high as that of Pre-detection. An alert, trained operator viewing a properly designed cathode-ray tube display is a close approximation to the theoretical post detection integrator.

The efficiency of post detection integration relative to ideal pre-detection integration has been computed by ***Marcum*** when all pulses are of equal amplitude. The integration efficiency may be defined as follows:

$$E_i(n) = \frac{(S/N)_1}{n(S/N)_n}$$

where n = number of pulses integrated

$(S/N)_1$ = value of signal-to-noise ratio of a single pulse required to produce a given probability of detection(for **n = 1**)

$(S/N)_n$ = value of signal-to-noise ratio per pulse required to produce the same probability of detection when **n** pulses (of equal amplitude) are integrated

The improvement in the signal-to-noise ratio when n pulses are integrated post detection is $n.E_i(n)$ and is the integration-improvement factor. ***It may also be thought of as the effective number of pulses integrated by the post detection integrator.*** The improvement with ideal pre detection integration would be equal to **n**. Integration loss in decibels is defined as $L_i(n) = 10 \log [1/E_i(n)]$.
The integration-improvement factor (or the integration loss) is not a sensitive function of either the probability of detection or the probability of false alarm.

The radar equation with n pulses integrated can be written

$$R_{max}^4 = \frac{P_t G A_e \sigma}{(4\pi)^2 k T_0 B_n F_n (S/N)_n}$$

where the parameters are the same as in the earlier Radar equation except that $(S/N)_n$, is the signal-to-noise ratio of one of the **n** equal pulses that are integrated to produce the required probability of detection for a specified probability of false alarm. Substituting the equation for integration efficiency

$$E_i(n) = \frac{(S/N)_1}{n(S/N)_n}$$

into the above Radar equation gives the final modified Radar equation including integration efficiency.

$$R_{max}^4 = \frac{P_t G A_e \sigma n E_i(n)}{(4\pi)^2 k T_0 B_n F_n (S/N)_1}$$

Radar Cross Section of Targets:

The radar cross section of a target is the (fictional) area intercepting that amount of power which when scattered equally in all directions, produces an echo at the radar equal to that from the target. Or in other terms

$$\sigma = \frac{\text{power reflected toward source/unit solid angle}}{\text{incident power density}/4\pi}$$

$$= \lim_{R \to \infty} 4\pi R^2 \left| \frac{E_r}{E_i} \right|^2$$

Where **R** = distance between radar and target
E_r = strength of reflected field at radar
E_i = strength of incident field at target

This equation is equivalent to the radar range equation of Sec. 1.2. For most common types of radar targets such as aircraft, ships, and terrain, the radar cross section does not necessarily bear a simple relationship to the physical area, except that the larger the target size, the larger will be the cross section.

Scattering and ***diffraction:*** are variations of the same physical process. When an object scatters an electromagnetic wave, the scattered field is defined as the difference between the total field in the presence of the object and the field that would exist if the object were absent (but with the sources unchanged). On the other hand, the diffracted field is the total field in the presence of the object. With radar backscatter, the two fields are the same, and one may talk about scattering and diffraction interchangeably.

Radar cross section of a simple sphere: is shown in the figure below as a function of its circumference measured in wavelengths.(**$2\pi a/\lambda$** where ***a*** is the radius of the sphere and **λ** is the wavelength). The plot consists of three regions.

1. Rayleigh Region:

- The region where the size of the sphere is small compared with the wavelength (**$2\pi a/\lambda \ll 1$**) is called the **Rayleigh** region.
- Named after Lord Rayleigh who, in the early**1870** first studied scattering by small particles. Lord Rayleigh was interested in the scattering of light by microscopic particles, rather than in radar. His work preceded the original electromagnetic echo experiments of Hertz by about fifteen years.
- The Rayleigh scattering region is of interest to the radar engineer because the cross sections of raindrops and other meteorological particles fall within this region at the usual radar frequencies. Since the cross section of objects within the Rayleigh region varies as **λ^{-4}**, rain and clouds are essentially invisible to radars which operate at relatively long wavelengths (low frequencies). The usual radar targets are much larger than raindrops or cloud particles, and lowering the radar frequency to the point where rain or cloud echoes are negligibly small will not seriously reduce the cross section of the larger desired targets. On the other hand, if it were desired to actually observe, rather than eliminate, raindrop echoes, as in meteorological or weather-observing radar, the higher radar frequencies would be preferred.

2. Optical region:

- It is at the other extreme from the **Rayleigh** region where the dimensions of the sphere are large compared with the wavelength (**$2\pi a/\lambda \gg 1$**). For large **$2\pi a/\lambda$**, the radar cross section approaches the optical cross section **πa^2**.

3. Mie or Resonance region:

- Between the optical and the Rayleigh region is the ***Mie,*** or resonance, region. The cross section is oscillatory with frequency within this region. The maximum value is 5.6 dB greater than the optical value, while the value of the first null is 5.5 dB below the optical value. (The theoretical values of the maxima and minima may vary according to the method of calculation employed.

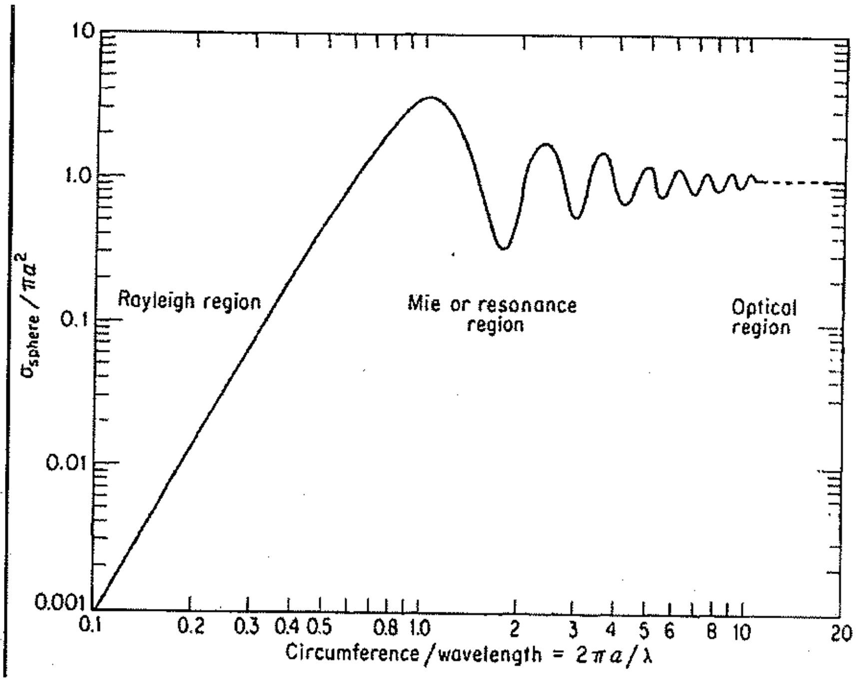


Figure 1.10: Radar cross section of the sphere. a = radius; λ= wavelength.

Since the sphere is a sphere no matter from what aspect it is viewed, its cross section will not be aspect-sensitive. The cross section of other objects, however, will depend upon the direction as viewed by the radar. (Aspect angle)

Radar cross section of a cone-sphere:

- An interesting radar scattering object is the cone-sphere, a cone whose base is capped with a sphere such that the first derivatives of the contours of the cone and sphere are equal at the joint. Figure below is a plot of the nose-on radar cross section. The next Figure is a plot as a function of aspect angle. The cross section of the cone-sphere from the vicinity of the nose-on direction is quite low.
- Scattering from any object occurs from discontinuities. The discontinuities, and hence the backscattering, of the cone-sphere are from the tip and from the join between the cone and the sphere.
- The nose-on radar cross section is small and decreases as the square of the wavelength. The cross section is small over a relatively large angular region. **A large specular** *(having qualities of a mirror)* return is obtained when the cone-sphere is viewed at near perpendicular incidence to the cone surface, i.e., when **θ**= 90 - **α**, where **α**= cone half angle. From the rear half of the cone-sphere, the radar cross section is approximately that of the sphere.

- The nose-on cross section of the cone-sphere varies, but its maximum value is approximately **$0.4\lambda^2$** and its minimum is **$0.01\lambda^2$** for a wide range of half-angles for frequencies above the Rayleigh region. The null spacing is also relatively insensitive to the cone half-angle.

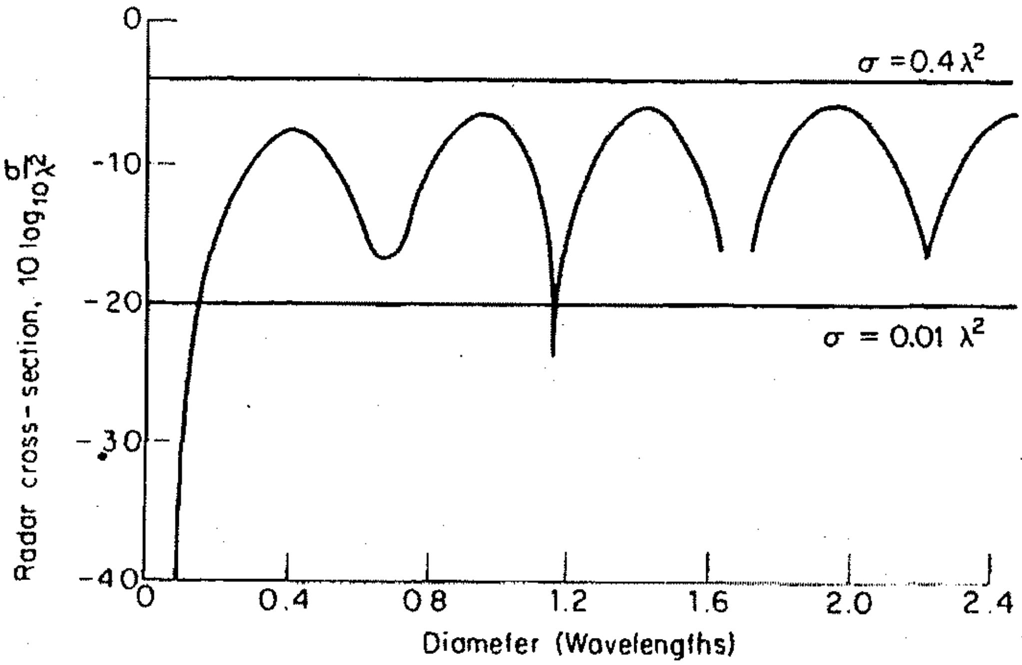


Figure 1.11: Radar cross section of a cone sphere with 15^0 half angle as a function of the diameter in Wave lengths.

- In order to realize in practice the very low theoretical values of the radar cross section for a cone sphere, the tip of the cone must be sharp and not rounded, the surface must be smooth (roughness small compared to a wavelength), the join between the cone and the sphere must have a continuous first derivative, and there must be no holes, windows, or protuberances on the surface.

- Shaping of the target, as with the cone-sphere, is a good method for reducing the radar cross section. Materials such as carbon-fiber composites, which are sometimes used in aerospace applications, can further reduce the radar cross section of targets as compared with that produced by highly reflecting metallic materials.

Transmitter Power:

The peak power: The power P_t in the radar equation is called the ***peak*** power. This is not the instantaneous peak power of a sine wave. It is the power averaged over that carrier-frequency cycle which occurs at the maximum power of the pulse.

The average radar power P_{av}: It is is defined as the average transmitter power over the pulse-repetition period. If the transmitted waveform is a train of rectangular pulses of width τ and pulse-repetition period $T_p = 1/f_p$, then the average power is related to the peak power by

$$P_{av} = \frac{P_t \tau}{T_p} = P_t \tau f_p$$

Duty cycle: The ratio P_{av}/P_t, τ/T_p, or $\tau.f_p$ is called the duty cycle of the radar. A pulse radar for detection of aircraft might have typically a duty cycle of 0.001, while a CW radar which transmits continuously has a duty cycle of unity.

Writing the radar equation in terms of the average power rather than the peak power, we get

$$R_{max}^4 = \frac{P_{av} G A_e \sigma n E_i(n)}{(4\pi)^2 k T_0 F_n (B_n \tau)(S/N)_1 f_p}$$

The bandwidth and the pulse width are grouped together since the product of the two is usually of the order of unity in most pulse-radar applications.

Pulse Repetition Frequencies and Range Ambiguities:

- The pulse repetition frequency (**prf**) is determined primarily by the maximum range at which targets are expected. If the **prf** is made too high, the likelihood of obtaining target echoes from the wrong pulse transmission is increased. Echo signals received after an interval exceeding the pulse-repetition period are called ***multiple time around echoes.***
- Consider the three targets labeled ***A, B,*** and **C** in the figure below. Target **A** is located within the maximum unambiguous range R_{unamb} [= $C.T_P/2$] of the radar, target **B** is at a distance greater than R_{unamb} but less than $2R_{unamb}$ and the target **C** is greater than $2R_{unamb}$ but less than $3R_{unamb}$ The appearance of the three targets on an A-scope is shown in the figure below. The multiple-time-around echoes on the A-scope cannot be distinguished from proper target echoes actually within the maximum unambiguous range. Only the range measured for target ***A*** is correct; those for **B** and **C** are not.
- One method of distinguishing multiple-time-around echoes from unambiguous echoes is to operate with a varying pulse repetition frequency. The echo signal from an unambiguous range target will appear at the same place on the A-scope on each sweep no matter whether the **prf** is modulated or not. However, echoes from multiple-time-around targets will be spread over a finite range as shown in the figure below. The **prf** may be changed continuously within prescribed limits, or it may be changed discretely among several predetermined values. The number of separate pulse repetition frequencies will depend upon the degree of the multiple time targets. Second-time targets need only two separate repetition frequencies in order to be resolved.

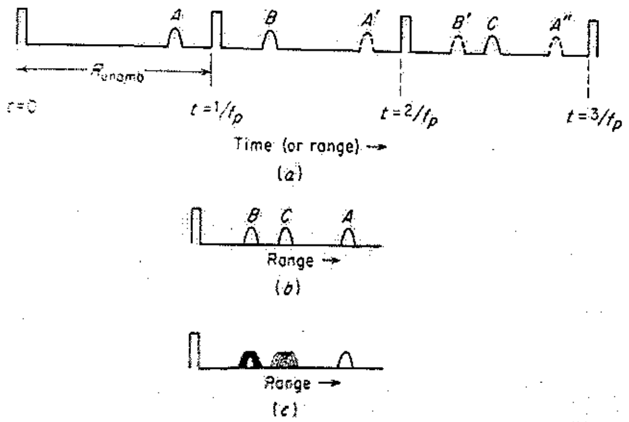


Fig. 1.12: Multiple-time-around echoes that give rise to ambiguities in range. (a) Three targets A, B and C, where A is within R_{unamb}, and B and Care multiple-time-around targets(b)the appearance of the three targets on the A-scope (c) appearance of the three targets on the A-scope with a changing prf.

System Losses:

- The losses in a radar system reduce the signal-to-noise ratio at the receiver output. They are two kinds, predictable with certain precision beforehand and unpredictable. The antenna beam-shape loss, collapsing loss, and losses in the microwave plumbing are examples of losses which are predictable if the system configuration is known. These losses are real and cannot be ignored.
- Losses not readily subject to calculation and which are less predictable include those due to field degradation and to operator fatigue or lack of operator motivation. They are subject to considerable variation and uncertainty.
- Although the loss associated with any one factor may be small, there are many possible loss mechanisms in a complete radar system, and their sum total will be significant.
- In this section, loss (number greater than unity) and efficiency (number less than unity)are used interchangeably. One is simply the reciprocal of the other.

Plumbing loss: This is loss in the transmission lines which connect the transmitter output to the antenna.(cables and waveguides) The losses in decibels per100ft for radar transmission lines are shown in the figure below. At the lower radar frequencies the transmission line introduces little loss, unless its length is exceptionally long. At higher radar frequencies, loss/attenuation will not be small and has to be taken into account.

Connector losses: In addition to the losses in the transmission line itself, an additional loss occurs at each connection or bends in the line and at the antenna **rotary joint** if used. Connector losses are usually small, but if the connection is poorly made, it can contribute significant attenuation. Since the same transmission line is generally used for both receiving and transmission, the loss to be inserted in the radar equation is ***twice*** the one-way loss.

Duplexer loss: The signal suffers attenuation as it passes through the duplexer. Generally, the greater the isolation required from the duplexer on transmission, the larger will be the insertion loss. Insertion loss means the loss introduced when the component is inserted into the transmission line. The precise value of the insertion loss depends to a large extent on the particular design. For a typical duplexer it might be of the order of 1 dB.

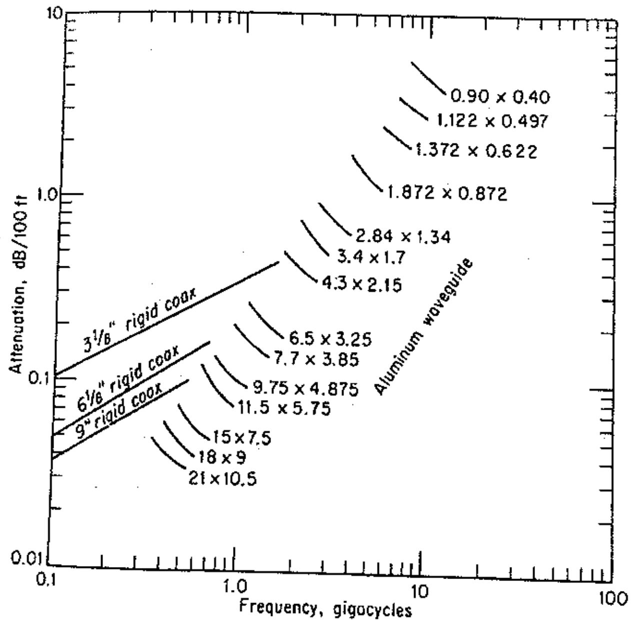


Fig.1.13: Theoretical (one-way) attenuation of RF transmission lines. Waveguide sizes are inches and are the inside dimensions. ***(Data, from Armed Services Index of R.F. Transmission Lines and Fittings, ASESA. 49-B)***

In S-band (3000 MHz) radar, for example, the typical plumbing losses will be as follows:

100 ft of RG-113/U A1 waveguide transmission line (two-way):1.0 dB	
Loss due to poor connections (estimate):	*0.5 dB*
Rotary-joint loss:	*0.4 dB*
Duplexer loss:	*1.5 dB*
Total plumbing loss:	*3.4 dB*

Beam-shape loss: The antenna gain that appears in the radar equation was assumed to be a constant equal to the maximum value. But in reality the train of pulses returned from a target with scanning radar is modulated in amplitude by the shape of the antenna beam. To properly take into account the pulse-train modulation caused by the beam shape, the computations of the probability of detection (as explained earlier) would have to be performed assuming a modulated train of pulses rather than constant-amplitude pulses. But since this computation is difficult, in this text, this approach is not used. Instead a

beam-shape loss is added to the radar equation and a maximum gain is employed in the radar equation rather than a gain that changes pulse to pulse. This is a simpler, though less accurate method.
Scanning loss: When the antenna scans rapidly enough that the gain on transmit is not the same as the gain on receive. An additional loss has to be computed, called the scanning loss. The technique for computing scanning loss is similar in principle to that for computing beam-shape loss. Scanning loss is important for rapid-scan antennas or for very long range radars such as those designed to view extraterrestrial objects.

Limiting loss: Limiting in the radar receiver lowers the probability of detection. Although a well-designed and engineered receiver will not limit the received signal under normal circumstances, intensity modulated **CRT** displays such as the PPI or the B-scope have limited dynamic range and may limit. Some receivers, however, might employ limiting for some special purpose, like pulse compression .Limiting results in a loss of only a fraction of a decibel for a large number of pulses integrated, provided the limiting ratio (ratio of video limit level to rms noise level) is as large as2 or 3.

Collapsing loss: If the radar were to integrate additional noise samples along with the wanted Signal-to-noise pulses, the added noise results in degradation called the ***collapsing loss.*** It can occur:

- In displays which collapse the range information, such as the C-scope which displays elevation vs. azimuth angle, the echo signal from a particular range interval must compete in a collapsed-range C-scope display, not only with the noise energy contained within that range interval, but with the noise energy from all other range intervals at the same elevation and azimuth.
- In some 3D radars (range, azimuth, and elevation) that display the outputs at all elevations on a single PPI (range, azimuth) display, the collapsing of the 3D radar information on to a 2D display results in a loss.

The mathematical derivation of the collapsing loss, assuming a square-law detector carried out by Marcum shows that the integration of ***m*** noise pulses along with **n** signal-plus-noise pulses with signal-to-noise ratio per pulse $(S/N)_n$, is equivalent to the integration of **m+n** signal-to-noise pulses each with signal-to-noise ratio $n(S/N)_n/(m+n)$. The collapsing loss in this case is equal to the ratio of the integration loss L_i for **(m+n)** pulses to the integration loss for **n** pulses, or

$$L_i(m, n) = \frac{L_i(m+n)}{L_i(n)}$$

For example, assume that 10 signal-plus-noise pulses are integrated along with 30 noise pulses and that $P_d = 0.90$ and $n_f = 10^8$. From the published data on Integration loss we have $L_i(40)$ is **3.5 dB** and $L_i(10)$ is **1.7 dB**, so that the collapsing loss is **1.8dB**. It is also possible to account for the collapsing loss by substituting into the radar equation the parameter $E_i(m+n)$ for $E_i(n)$, since $E_i(n) = 1/L_i(n)$.

Non ideal equipment: The ***transmitter power*** in the radar equation was assumed to be the specified output power (either peak or average). However, all transmitting tubes are not uniform in quality, and even any individual tube cannot be expected to remain at the same level of performance throughout its useful life. Also, the power is usually not uniform over the operating band of frequencies. Thus, for one reason or another, the transmitted power may be other than the design value. To allow for this variation, a loss factor has to be introduced. This factor can vary with the application, but in the absence of a correct number, a loss of about 2 dB might be used as an approximation.
Variations in the ***receiver noise figure*** over the operating frequency band also are to be expected. Thus, if the best noise figure over the band is used in the radar equation, a loss factor has to be introduced to account for its poorer value elsewhere in the frequency band.

If the receiver is not the exact matched filter for the transmitted waveform, a loss in Signal-to-noise ratio will occur.
A typical value of loss for anon-matched receiver might be about **1 db.** Because of the exponential relation between the false-alarm time and the threshold level a slight change in the threshold can cause a significant change in the false alarm time. In practice, therefore, it may be necessary to set the threshold level slightly higher than calculated so as to insure a tolerable false alarm time in the event of circuit instabilities. This increase in the threshold is equivalent to a loss.

Operator loss: An alert, motivated, and well-trained operator performs as described by theory. However, when distracted, tired, overloaded, or not properly trained, operator performance will decrease. The resulting loss in system performance is called operator loss. There is little guidance available on how to account for the performance of an operator. Hence normally it is better to take steps to avoid loss due to operator performance rather than tolerate it by including it as a loss factor in the radar equation.
Field degradation: When a radar system is operated under laboratory conditions by engineering personnel and experienced technicians, the inclusion of the above losses into the radar equation gives a realistic description of the performance of the radar under normal conditions (ignoring anomalous propagation effects). However, when a radar is operated under field conditions the performance usually deteriorates even more than that can be accounted for by the above losses, especially when the equipment is operated and maintained by inexperienced or unmotivated personnel. It may even apply, to some extent, to equipment operated by professional engineers under adverse field conditions. *Factors which contribute to field degradation are poor tuning, weak tubes, water in the transmission lines, incorrect mixer-crystal current, deterioration of receiver noise figure, poor TR tube recovery, loose cable connections etc.*
To minimize field degradation Radars should be designed with built-in automatic performance-monitoring equipment. Careful observation of performance-monitoring instruments and timely preventative maintenance will keep the radar performance up to design level. Radar characteristics that might be monitored include transmitter power, receiver noise figure, the spectrum and/or shape of the transmitted pulse, and the decay time of the TR tube.
A good estimate of the field degradation is difficult to obtain since it cannot be predicted and is dependent upon the particular radar design and the conditions under which it is operating. A degradation of **3** dB is sometimes assumed when no other information is available.

Other loss factors:

- A Radar designed to discriminate between moving targets and stationary objects (MTI radar) introduces an additional loss. The MTI discrimination technique results in complete loss of sensitivity for certain values of target velocity relative to the radar. These are called ***blind speeds.***
- In a radar with overlapping range gates, the gates may be wider than optimum for practical reasons. The additional noise introduced by the non optimum gate width will result in some degradation.
- The straddling loss accounts for the loss in signal-to-noise ratio for targets not at the center of the range gate or at the center of the filter in a multiple-filter-bank processor.
- Another factor that has a profound effect on the radar range performance is the propagation medium.

There are many causes of loss and inefficiency in a Radar. Although each of them may be small, the sum total can result in a significant reduction in radar performance. It is important to understand the origins of these losses, not only for better predictions of radar range, but also for the purpose of keeping them to a minimum by careful radar design

Important formulae:

- **Range of a Radar $R = cT_R / 2$**
- **Maximum unambiguous Range $R_{unambig.} = C/2f_p$**
- **Basic Radar equation : $R_{max} = [(P_t \cdot G \cdot A_e \cdot \sigma) / (4\pi)^2 \cdot S_{min}]^{1/4}$**
- **Gain of an antenna $G = 4\pi A_e / \lambda^2$**
- **Noise figure of a receiver $F_n = N_o / kT_o B_n G_a$**
  Also given by

$$F_n = \frac{S_i/N_i}{S_o/N_o}$$

- **If the minimum detectable signal S_{min} is that value of S_i corresponding to the minimum ratio of output (IF) signal-to-noise ratio $(S_o/N_o)_{min}$ necessary for detection, then**

$$S_{min} = kT_0 B_n F_n \left(\frac{S_o}{N_o}\right)_{min}$$

- **And modified Maximum possible range in terms of the IF amplifier output signal to noise ratio and noise figure**

$$R_{max}^4 = \frac{P_t G A_e \sigma}{(4\pi)^2 k T_0 B_n F_n (S_o/N_o)_{min}}$$

- **Relationships between Probability of false alarm, Probability of detection and S/N :**
  - **$P_{fa} = 1/T_{fa} \cdot B$**
  - **$P_{FA} = \text{Exp}(-V_T^2/2\psi_0)$.**
  - $$T_{fa} = \frac{1}{B_{IF}} \exp \frac{V_T^2}{2\psi_0}$$
  - ***Albersheim*** empirical relationship between **S/N, P_{FA}** and **P_d**:

S/N = A + 0.12AB + 1.7 B

Where $A = \ln[0.62/P_{FA}]$ and $B = \ln[P_d/(1-P_d)]$

- **The radar equation with *n* pulses integrated :**

$$R_{max}^4 = \frac{P_t G A_e \sigma}{(4\pi)^2 k T_0 B_n F_n (S/N)_n}$$

- **Integration efficiency :**

$$E_i(n) = \frac{(S/N)_1}{n(S/N)_n}$$

- **Final Radar equation including Integration efficiency with n pulses integrated:**

$$R_{max}^4 = \frac{P_t G A_e \sigma n E_i(n)}{(4\pi)^2 k T_0 B_n F_n (S/N)_1}$$

Illustrative Problems:

Example 1: A certain Radar has PRF of 1250 pulses per second. What is the maximum unambiguous range? Max. Unambiguous Range is given by

$$R_{unambig.} = C/2f_p$$

$$R_{unambig.} = 3x10^8/2x1250 \text{ mtrs} = 120X10^3 \text{mts} = 120\text{Kms}$$

Example 2: A shipboard radar has 0.9 micro sec transmitted pulse width. Two small boats in the same direction are separated in range by 150 mts. Will the radar detect the two boats as two targets?

***Radar Range Resolution: The range resolution of a Radar is its ability to distinguish two closely spaced targets along the same line of sight (LOS). The Range resolution is a function of the pulse length, where the pulse length L_P= c x τ/2*(Two way range corresponding to the pulse width)**

Radar Range resolution $= 3x10^8 x0.9x10^{-6}/2 = 135$ mtrs.

Since the boats are at 150 Mts. apart, and the range Resolution is 135mtrs., the radar can detect the 2 boats as 2 separate targets.

Example 3: A Pulse Radar transmits a peak power of 1 Mega Watt. It has a PRT equal to 1000 micro sec and the transmitted pulse width is 1 micro sec. Calculate (i)Maximum unambiguous range (ii) Average Power (iii)Duty Cycle (iv) Energy transmitted & (v)Bandwidth

(i) Maximum unambiguous range = $c.T_P/2 = 3x10^8 x1000x10^{-6}/2 = 150x10^3$ mtrs = 150 Kms

(ii) Average Power = $P_P \times \tau / T_P = 1x10^6\ 1x10^{-6}/1000x10^{-6} = 1000$ watts = 1kw

(iii)Duty Cycle = $\tau / T_P = 1x10^{-6}/1000x10^{-6} = 0.001$

(iv) Energy transmitted = $P_P \times \tau$ (Peak power xTime) = $1x10^6 x1x10^{-6} = 1$ Joule

(v)Bandwidth = $1/\tau = 1/10^{-6} =$ 1Mhz

Example 4: The Bandwidth of I.F. Amplifier in a Radar Receiver is 1 MHZ. If the Threshold to noise ratio is 12.8 dB determine the False Alarm Time.

T_{fa} = False Alarm Time $T_{fa} = [1/B_{IF}]\ \text{Exp}\,V_T^2/2\psi_0$ where $B_{IF} = 1X\,10^6$ HZ

Threshold to Noise Ratio = 12.8 dB

i.e. $10\ \text{Log}_{10}[V_T^2/2\psi_0] = 12.8\text{db}$

$\therefore V_T^2/2\psi_0 = \text{Antilog}_{10}\,[12.8/10] = 19.05$

$\therefore T_{fa} = 1/(1X\,10^6)\ \text{Exp}\ 19.05 = 187633284/10^6 = 187.6$ Seconds

Example 5: The probability density of the envelope of the noise voltage output is given by the Rayleigh probability-density function

$$p(R) = \frac{R}{\psi_0} \exp\left(-\frac{R^2}{2\psi_0}\right)$$

where R is the amplitude of the envelope of the filter outputfor $R \geq 0$. If P_{fa} needed is $\leq 10^{-5}$. Determine the Threshold Level.

The probability of false alarm P_{FA} in terms of the threshold voltage level is given by :

$$P_{FA} = \text{Exp}(-V_T^2/2\psi_0) = 10^{-5}$$

Taking logarithms on both the sides we get

$-5\ \text{Log}_e 10 = (-V_T^2/2\psi_0)$

$5 \times 2.3026 = (V_T^2/2\psi_0)$

$V_T^2 = 11.5 \times 2\ \psi_0$

$V_T = \sqrt{23} \times \sqrt{\psi_0} = 4.8 \times \sqrt{\psi_0}$

Example 6: The bandwidth of an IF amplifier is 1 MHz and the average false-alarm time that could be tolerated is 15 min. Find the probability of a false alarm.
The relationship between average false-alarm time T_{FA}, probability of a false alarm P_{FA} and the IF bandwidth B is given by :

$$P_{fa} = 1/T_{fa}\,.\,B$$

Substituting $B = 1$ MHz ie 10^6 and T_{fa}=15 mnts. i.e. 900 secs. we get PFA=1.11×10^{-9}

Example 7: What is the ratio of threshold voltage to the rms value of the noise voltage necessary to achieve this false-alarm time?
This is found out using the relationship $P_{FA} = \text{Exp}\,(-V_T^2/2\psi_0)$
from which the ratio of Threshold voltage to rms value of the noise voltage is given by

$$V_T/\sqrt{\psi_0} = \sqrt{2 \ln (1/P_{fa})} = \sqrt{2 \ln 9X10^8} = 6.45 = 16.2\text{dB}$$

Example 8: Typical parameters for a ground-based search radar are : 1. Pulse repetition frequency :300 Hz, 2. Beam width : 1.5^0, and 3. Antenna scan rate: 5 rpm (30^0/s). Find out the number of pulses returned from a point target on each scan.
Solution : The number of pulses retuned from a point target on each scann_B is given by:

$$n_B = \theta_B \,.\, f_P / \theta'_S = \theta_B \,.\, f_P / 6\,\omega_m$$

Substituting the above values we get : $n_B = 1.5 \times 300 / 30 = 15$

Questions from Previous Year Examinations:

1.(a) Derive Radar range equation in terms of MDS (minimum detectable signal) (b) What is maximum unambiguous range? How is it related with PRF?

2.(a) Explain the various system losses in a Radar (b) The bandwidth of The IF amplifier in a Radar is 1 Mhz and the threshold noise ratio is 13 db. Determine the false alarm time.

3.(a) Explain the basic principles of Radar and discuss about various parameters which improve the performance of the Radar (b) Discuss about Radar frequencies and list out the Applications of Radars.

4.(a) In a Radar receiver the mean noise voltage is 80 mv and the IF BW is 1 Mhz. If the tolerable false alarm time is 25 mnts., calculate the threshold voltage level and the probability of false alarm. (b) Bring out the advantages of Integration of Radar pulses.

5 (a) Discuss about the factors that influence the prediction of Radar range. (b) Define noise bandwidth of a radar receiver. How does it differ from 3-dB band width? Obtain the expression for minimum detectable signal in terms of noise bandwidth, noise figure and other relevant parameters. [8+8]

6. (a) Write the simplified version of radar range equation and explain how this equation does not adequately describe the performance of practical radar? (b) What are the specific bands assigned by the ITU for the Radar? What the corresponding frequencies? [8+8]

7. (a) Explain how the Radar is used to measure the range of a target? (b) Draw the block diagram of the pulse radar and explain the function of each block. [8+8]

8. (a) A low power, short range radar is solid-state throughout, including a low-noise RF amplifier which gives it an overall noise figure of 4.77dB. If the antenna diameter is 1m, the IF bandwidth is 500kHz, the operating frequency is 8 GHz and the radar set is supposed to be capable of detecting targets of 5m^2 cross sectional area at a maximum distance of 12 km, what must be the peak transmitted pulse power? (b) The average false alarm time is a more

significant parameter than the false alarm probability. Give the reasons. (c) Why post detection integration is not as efficient as pre-detection integration of radar pulses? [8+4+4]

9. (a) Explain the functioning and characteristics of PPI display and A-Scope. [8]

10. (a) Explain how the Radar is used to measure the direction and position of target? (b) What are the peak power and duty cycle of a radar whose average transmitter power is 200W, pulse width of 1μs and a pulse repetition frequency of 1000Hz? [8+8]

11. (a) Explain how a threshold level is selected in threshold detection? (b) How to find the number of pulses that returned from a point target as the radar antenna scans through its beam width? (c) Why most of the radar receivers are considered as envelop detectors while calculating the SNR? [6+4+6]

12. (a) Obtain the SNR at the output of IF amplifier of Radar Receiver for a specified probability of detection without exceeding a specified probability of false alarm. (b) Explain how system losses will affect on the Radar Range? [8+8]

13. (a) What are the different range of frequencies that a radar can operate and give their applications? (b) What are the basic functions of radar? In indicating the position of a target, what is the difference between azimuth and elevation? [8+8]

14. (a) Describe how pulse repetition frequency of a Radar system controls the range of it's detection? (b) Explain how the Transmitted power affects the range. [8+8]

15. (a) Draw the block diagram of a pulsed radar and explain it's operation. (b) What are the desirable pulse characteristics and the factors that govern them in a Radar system? [10+6]

16. (a) Explain the radar cross section of the sphere. (b) Discuss in brief about pulse repetition frequency and range ambiguities.

17. (a) Define Range resolution and explain the parameters which affect the range resolution. (b)Distinguish between Monostatic and Bistatic Radars (c) Explain RCS of target. [6+5+4]

UNIT-2

CW AND FREQUENCY MODULATED RADAR

- Doppler effect
- CW radar block diagram
- Isolation between Transmitter and receiver
- Nonzero IF receiver
- Receiver Bandwidth requirements
- Applications of CW Radar
- Illustrative problems

FM-CW RADAR

- Introduction
- Range and Doppler Measurement
- Block Diagram and characteristics (Approaching and Receding targets)
- FM-CW Altimeter
- Multiple frequency CW Radar

  - Important Formulae
  - Illustrative Problems
  - Questions from Previous Year Examinations

CW AND FREQUENCY MODULATED RADAR

Doppler Effect:

A technique for separating the received signal from the transmitted signal when there is relative motion between radar and target is based on recognizing the change in the echo-signal frequency caused by the Doppler effect.

It is well known in the fields of optics and acoustics that if either the source of oscillation or the observer of the oscillation is in motion, an apparent shift in frequency will result. This is the ***Doppler effect*** and is the basis of CW radar. If **R** is the distance from the radar to target, the total number of wavelengths **λ** contained in the two-way path between the radar and the target are ***2R/λ***.The distance **R** and the wavelength ***λ*** are assumed to be measured in the same units.

Since one wavelength corresponds to an phase angle excursion of **2π**radians, the total phase angle excursion **Ø** made by the electromagnetic wave during its transit to and from the target is **4πR/λ** radians. If the target is in motion, **R** and the phase **Ø** are continually changing. A change in **Ø** with respect to time is equal to frequency. This is the Doppler angular frequency ω_d and is given by:

$$\omega_d = 2\pi f_d = d\varnothing/dt = d(4\pi R/\lambda)/dt = (4\pi/\lambda) . dR/dt = (4\pi/\lambda). V_r = 4\pi . V_r / \lambda$$

where f_d is the Doppler frequency shift in Hz, and V_r = relative velocity of the target with respect to the Radar. The Doppler frequency shift f_d is given by

$$f_d = 2V_r / \lambda = 2V_r f_0 / c$$

where f_0 is the transmitted frequency and **c** is the velocity of propagation of the electromagnetic waves (same as that of light) = 3×10^8m/s. If f_d is in hertz. V_r in knots, and **λ** in meters then the Doppler frequency f_d is given by

$$f_d = 1.03 V_r / \lambda$$

A plot of this equation is shown in the figure below

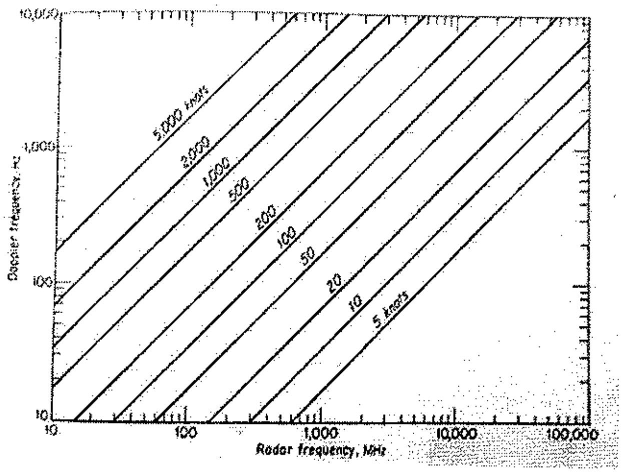


Figure: Doppler frequency $f_d$as a function of radar frequency and target relative velocity.

The relative velocity may be written as **V_r= V .cos θ** where **V** is the target speed and **θ** is angle made by the target trajectory and the line joining radar and target. When **θ=0** the Doppler frequency is maximum. The Doppler is zero when the trajectory is perpendicular to the radar line of sight **($\theta = 90^0$).**

The CW radar is of interest not only because of its many applications, but its study also serves as a means for better understanding the nature and use of the Doppler information contained in the echo signal, whether in a CW or a pulse radar (MTI) application. In addition to allowing the received signal to be separated from the transmitted signal, the CW radar provides a measurement of relative velocity which may be used to distinguish moving targets from stationary objects or clutter.

CW radar:

Consider the simple CW radar as illustrated by the block diagram of Figure below. The transmitter generates a continuous (unmodulated) oscillation of frequency **fo**, which is radiated by the antenna. A portion of the radiated energy is intercepted by the target and is scattered, some of it in the direction of the radar, where it is collected by the receiving antenna. If the target is in motion with a velocity **V_r** relative to the radar, the received signal will be shifted in frequency from the transmitted frequency **f_o** by an amount **+/-f_d** as given by the equation : **$f_d = 2V_r / \lambda = 2 V_r f_0 / c$** . The plus sign associated with the Doppler frequency applies if the distance between target and radar is decreasing (approaching target) that is, when the received signal frequency is greater than the transmitted signal frequency. The minus sign applies if the distance is increasing (receding target). The received echo signal at a frequency **f_o+/- f_d** enters the radar via the antenna and is heterodyned in the detector (mixer) with a portion of the transmitter signal **f_o** to produce a Doppler beat note of frequency **f_d**. The sign of **f_d** is lost in this process.

The purpose of the Doppler amplifier is to eliminate echoes from stationary targets and to amplify the Doppler echo signal to a level where it can operate an indicating device. It's frequency response characteristic is shown in the figure (b) below. The low-frequency cutoff must be high enough to reject the d-c component caused by stationary targets, but yet it must be low enough to pass the smallest Doppler frequency expected. Sometimes both conditions cannot be met simultaneously and a compromise is necessary. The upper cutoff frequency is selected to pass the highest Doppler frequency expected.

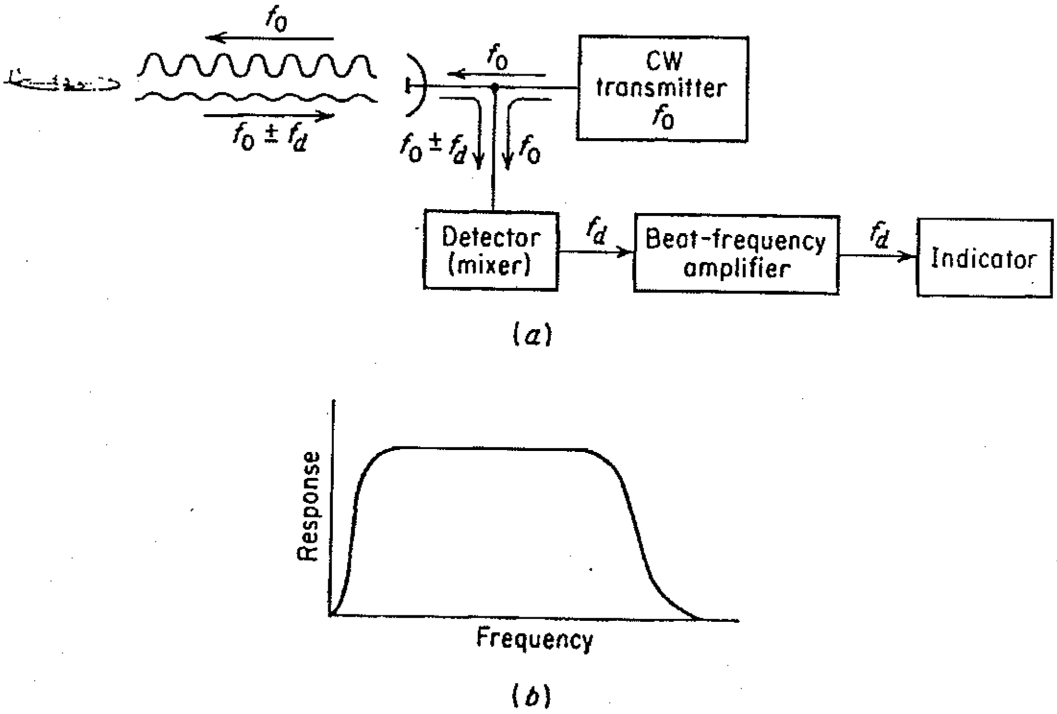


Figure : Simple CW radar block diagram (b) response characteristic of beat-frequency amplifier

Isolation between transmitter and receiver:

Isolation between transmitter and receiver is an important aspect to be studied and addressed in simple CW radars where a single antenna serves the purpose of both transmission and reception as described above. The related important aspects are explained below.

- In principle, a single antenna may be employed since the necessary isolation between the transmitted and the received signals is achieved via separation in frequency as a result of the Doppler Effect. In practice, it is not possible to eliminate completely the transmitter leakage. However, transmitter leakage is neither always undesirable. A moderate amount of leakage entering the receiver along with the echo signal supplies the reference necessary for the detection of the Doppler frequency shift. If a leakage signal of sufficient magnitude were not present, a sample of the transmitted signal has to be deliberately introduced into the receiver to provide the necessary reference frequency.
- There are two practical effects which limit the amount of transmitter leakage power which can be tolerated at the receiver. These are:
  (1) The maximum amount of power the receiver input circuitry can withstand before it is physically damaged or its sensitivity reduced (burnout) and

(2) The amount of transmitter noise due to hum, microphonics, stray pick-up &instability which enters the receiver from the transmitter and affects the receiver sensitivity.

Hence additional isolation is usually required between the transmitter and the receiver if the sensitivity is not to be degraded either by burnout or by excessive noise. The amount of isolation required depends on the transmitter power and the accompanying transmitter noise as well as the ruggedness and the sensitivity of the receiver. For example, if the safe value of power which might be applied to a receiver is 10 mW and if the transmitter power is 1 kW, the isolation between transmitter and receiver must be at least 50 dB.

- The amount of isolation needed in a long-range CW radar is more often determined by the noise that accompanies the transmitter leakage signal rather than by any damage caused by high power. For example, suppose the isolation between the transmitter and receiver is such that **10** mW of leakage signal appeared at the receiver. If the minimum detectable signal is 10^{-13}watt (100 dB below 1mW), the transmitter noise must be at least **110** dB (preferably 120 or 130 dB) below the transmitted carrier.
- The transmitter noise of concern in Doppler radar includes those noise components that lie within the same frequency range as the Doppler frequencies. If complete elimination of the direct leakage signal at the receiver could be achieved, it might not entirely solve the isolation problem since echoes from nearby fixed targets (clutter) can also contain the noise components of the transmitted signal.
- The receiver of a pulsed radar is isolated and protected from the damaging effects of the transmitted pulse by the duplexer, which short-circuits the receiver input during the transmission period. Turning off the receiver during transmission with a duplexer is not possible in a CW radar since the transmitter is operated continuously.
- In CW Radars Isolation between transmitter and receiver might be obtained with a single antenna by using a hybrid junction, circulator, turnstile junction, or with separate polarizations. Separate antennas for transmitting and receiving might also be used.
  - The amount of isolation which can be readily achieved between the arms of practical hybrid junctions such as the magic-T, rat race, or short-slot coupler is of the order of 20 to 30 dB. In some instances, when extreme precision is exercised, an isolation of perhaps 60 dB or more might be achieved. But one limitation of the hybrid junction is the 6-dB loss in overall performance which results from the inherent waste of half the transmitted power and half the received signal power. Both the loss in performance and the difficulty in obtaining large isolations have limited the application of the hybrid junction to short-range radars.
  - Ferrite isolation devices such as the circulator do not suffer the 6-dB loss inherent in the hybrid junction. Practical devices have isolation of the order of 20 to 50 dB. Turnstile junctions achieve isolations as high as 40 to 60 dB.
  - The use of orthogonal polarizations for transmitting and receiving is limited to short range radars because of the relatively small amount of isolation that can be obtained.
- An important factor which limits the use of isolation devices with a common antenna is the reflections produced in the transmission line by the antenna. The reflection

coefficient from a mismatched antenna with a voltage standing-wave ratio **σ** is | =(σ- l) / (σ+1). Therefore, if an isolation of **20 dB** is to be obtained, the VSWR must be less than **1.22.** If **40 dB** of isolation is required, the VSWR must be less than **1.02.**

- The largest isolations are obtained with two antennas: one for transmission, the other for reception-physically separated from one another. Isolations of the order of 80 dB or more are possible with high-gain antennas. The more directive the antenna beam and the greater the spacing between antennas, the greater will be the isolation. A common radome enclosing the two antennas should be avoided since it limits the amount of isolation that can be achieved.
- Additional isolation can be obtained by properly introducing a controlled sample of the transmitted signal directly into the receiver. The phase and amplitude of this "buck-off" signal are adjusted to cancel the portion of the transmitter signal that leaks into the receiver. An additional 10 dB of isolation might be obtained.
- The transmitter signal is never a pure CW waveform. Minute variations in amplitude (AM) and phase (FM) can result in sideband components that fall within the Doppler frequency band. These can generate false targets or mask the desired signals. Therefore, both AM and FM modulations can result in undesired sidebands. AM sidebands are typically 120 dB below the carrier, as measured in a 1 kHz band, and are relatively constant across the usual Doppler spectrum of interest. The normal antenna isolation plus "feed through nulling" usually reduces the AM components below receiver noise in moderate power radars. FM sidebands are usually significantly greater than AM, but decrease with increasing offset from the carrier. These can be avoided by stabilizing the output frequency of the CW transmitter and by feeding back the extracted FM noise components so as to reduce the original frequency deviation.

Intermediate-frequency receiver:

Limitation of Zero IF receiver:

The receiver in the simple CW radar *shown earlier* is in some respects analogous to a super heterodyne receiver. Receivers of this type are called homodyne receivers, or super heterodyne receivers with zero IF. The function of the local oscillator is replaced by the leakage signal from the transmitter. Such a receiver is simpler than the one with a more conventional intermediate frequency since no IF amplifier or local oscillator is required. However, this simpler receiver is not very sensitive because of increased noise at the lower intermediate frequencies caused by flicker effect. Flicker-effect noise occurs in semiconductor devices such as diode detectors and cathodes of vacuum tubes. The noise power produced by the flicker effect varies as $1/f^{\alpha}$ where α is approximately unity. This is in contrast to shot noise or thermal noise, which is independent of frequency. Thus, at the lower range of frequencies (audio or video region), where the Doppler frequencies usually are found, and the detector of the CW receiver can introduce a considerable amount of flicker noise, resulting in reduced receiver sensitivity. For short-range, low-power, applications this decrease in sensitivity might be tolerated since it can be compensated by a modest increase in antenna aperture and/or additional transmitter power.

But for maximum efficiency with CW radar, the reduction insensitivity caused by the simple Doppler receiver with zero IF cannot be tolerated.

Non zero IF Receiver:

The effects of flicker noise are overcome in the normal super heterodyne receiver by using an intermediate frequency high enough to make the flicker noise small compared with the normal receiver noise. This results from the inverse frequency dependence of flicker noise. Figure below shows the block diagram of a CW radar whose receiver operates with a nonzero IF. Separate antennas are shown for transmission and reception. Instead of the usual local oscillator found in the conventional super heterodyne receiver, the local oscillator (or reference signal) is derived in the receiver from a portion of the transmitted signal mixed with a locally generated signal of frequency equal to that of the receiver IF. Since the output of the mixer consists of two sidebands on either side of the carrier plus higher harmonics, a narrowband filter selects one of the sidebands as the reference signal. The improvement in receiver sensitivity with an intermediate-frequency super heterodyne might be as much as 30 dB over the simple zero IF receiver discussed earlier.

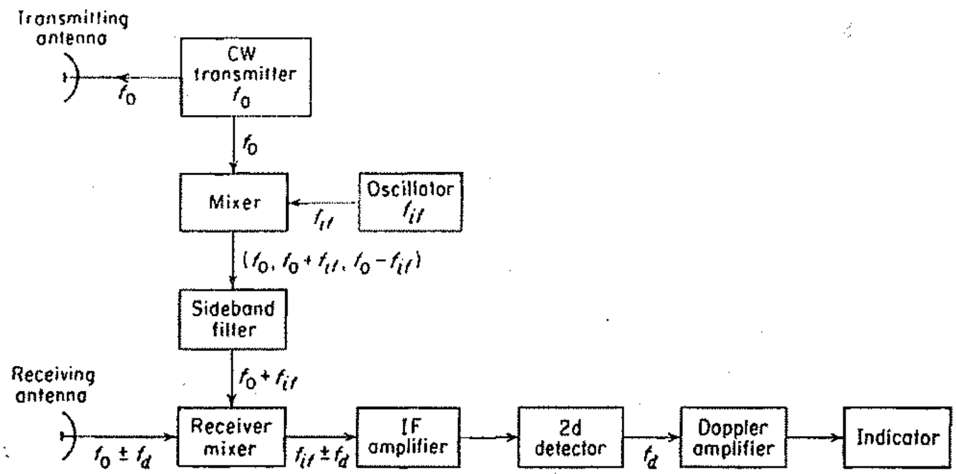


Figure: Block diagram of a CW Doppler radar with nonzero IF receiver, also called sideband super heterodyne Receiver.

Receiver bandwidth requirements:

One of the requirements of the Doppler-frequency amplifier in the simple CW radar (Zero IF) or the IF amplifier of the sideband super heterodyne (Non Zero IF) is that it has to be wide enough to pass the expected range of Doppler frequencies. In most cases of practical interest the expected range of Doppler frequencies will be much wider than the frequency spectrum occupied by the signal energy. Consequently, the use of a wideband amplifier covering the expected Doppler range will result in an increase in noise and a lowering of the receiver sensitivity. If the frequency of the Doppler-shifted echo signal were known beforehand, narrowband filter-that is just wide enough to reduce the excess noise without eliminating a significant amount of signal energy might be used. If the waveforms of the echo signal are known, as well as its carrier frequency, a matched filter could also be considered.

Several factors tend to spread the CW signal energy over a finite frequency band. These must be known if an approximation to the bandwidth required for the narrowband Doppler filter is to be obtained.

If the received waveform were a sine wave of infinite duration, its frequency spectrum would be a delta function as shown in the figure (a) below and the receiver bandwidth would be infinitesimal. But a sine wave of infinite duration and an infinitesimal bandwidth cannot occur in nature. The more normal situation is an echo signal which is a sine wave of finite duration. The frequency spectrum of a finite-duration sine wave has a shape of the form **$[\sin\pi(f-f_0)\delta]/\pi(f-f_0)]$** where **$f_0$** and **$\delta$** are the frequency and duration of the sine wave, respectively, and **f** is the frequency variable over which the spectrum is plotted (Fig b).

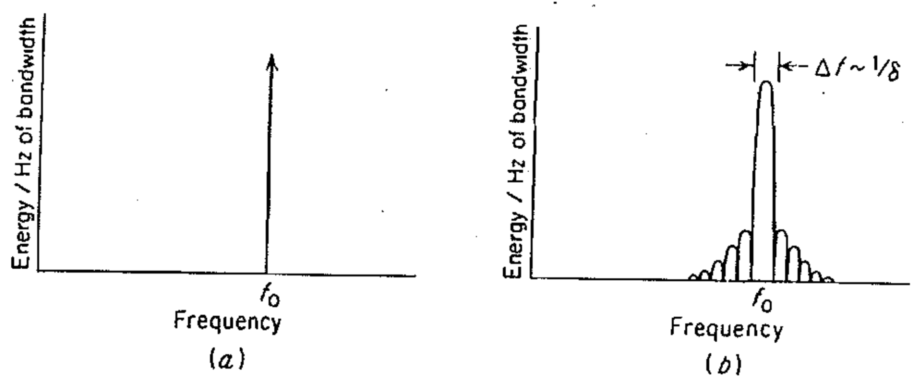


Figure: Frequency spectrum of CW oscillation of (a) infinite duration and (b) finite duration

Note that this is the same as the spectrum of a pulse of sine wave, the only difference being the relative value of the duration **δ.** In many instances, the echo is not a pure sine wave of finite duration but is perturbed by fluctuations in cross section, target accelerations, scanning fluctuations, etc., which tend to broaden the bandwidth still further. Some of these spectrum broadening-effects are considered below.

Causes for Spectrum broadening:

- **Spread due to finite time on target:** Assume a CW radar with an antenna beam width of θ_B deg. scanning at the rate of θ'_s deg/s. The time on target (duration of the received signal) is **$\delta = \theta_B/\theta_s$** sec. Thus, the signal is of finite duration and the bandwidth of the receiver must be of the order of the reciprocal of the time on target **(θ'_s/θ_B).** Although this is not an exact relation, it is a good enough approximation for purposes of the present discussion. If the antenna beam width is 2^0 and the scanning rate is 36^0/s (6 rpm), the spread in the spectrum of the received signal due to the finite time on target would be equal to 18 Hz, independent of the transmitted frequency.

- In addition to the spread of the received signal spectrum caused by the finite time on target, the spectrum gets widened due to target cross section fluctuations. The fluctuations widen the spectrum by modulating the echo signal. The echo signal from a propeller-driven aircraft can also contain modulation components at a frequency proportional to the propeller rotation. The frequency range of propeller modulation depends upon the shaft-rotation speed and the number of propeller blades. It is usually in the vicinity of 50 to 60 Hz for World War 2 aircraft engines. This could be a potential source of difficulty in a **CW** radar since it might mask the target's Doppler signal or it might cause an erroneous measurement of Doppler frequency. In some instances, propeller modulation can be of advantage. It might permit the detection of propeller-driven aircraft passing on a tangential trajectory, even though the Doppler frequency shift is zero.
- The rotating blades of a helicopter and the compressor stages of a jet engine can also result in a modulation of the echo and a widening of the spectrum that can degrade the performance of a **CW** Doppler radar.

- If the target's relative velocity is not constant, a further widening of the received signal spectrum occurs. If $\mathbf{a_r}$ is the acceleration of the target with respect to the radar, the signal will occupy a bandwidth

$$\Delta f_d = \left(\frac{2a_r}{\lambda}\right)^{1/2}$$

If, for example, $\mathbf{a_r}$ is twice the acceleration due to gravity, the receiver bandwidth is approximately 20 Hz when the Radar wavelength is 10 cm.

When the Doppler-shifted echo signal is known to lie somewhere within a relatively wideband of frequencies, a bank of narrowband filters as shown below spaced throughout the frequency range permits a measurement of frequency and improves the signal-to-noise ratio.

- The bandwidth of each individual filter should be wide enough to accept the signal energy, but not so wide as to introduce more noise. The center frequencies of the filters are staggered to cover the entire range of Doppler frequencies.

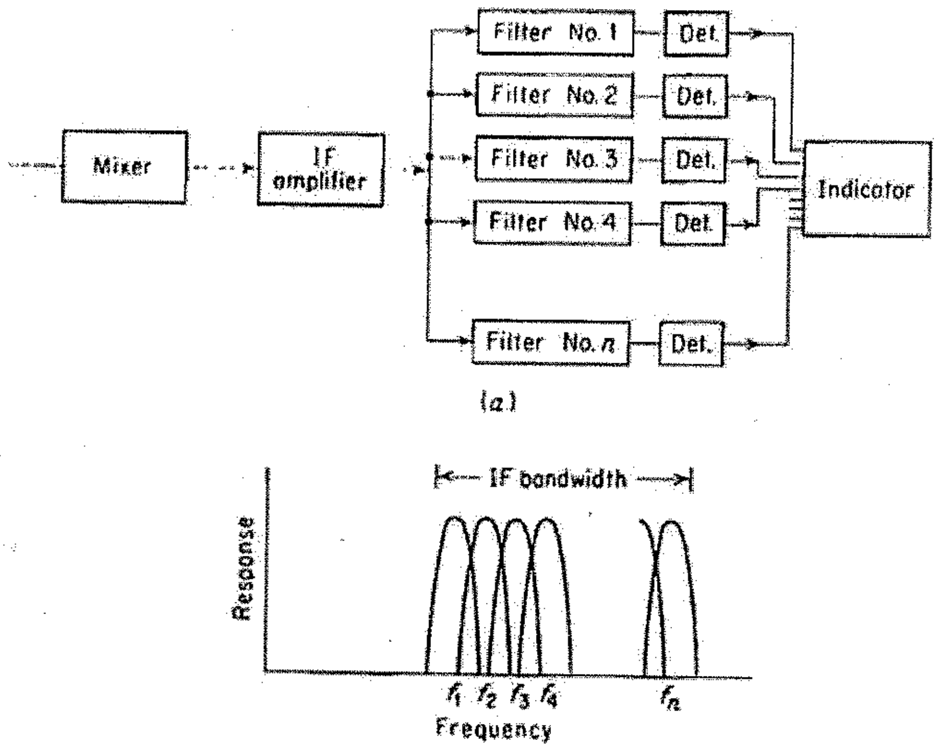


Figure: (a) Block diagram of IF Doppler filter bank (b) frequency-response characteristic of Doppler filter bank.

- A bank of narrowband filters may be used after the detector in the video of the simple CW radar instead of in the IF. The improvement in signal-to-noise ratio with a video filter bank is not as good as can be obtained with an IF filter bank, but the ability to measure the magnitude of Doppler frequency is still preserved. Because of fold over, a frequency which lies to one side of the IF carrier appears, after detection, at the same video frequency as one which lies an equal amount on the other side of the IF. Therefore the sign of the Doppler shift is lost with a video filter bank, and it cannot be directly determined whether the Doppler frequency corresponds to an approaching or to a receding target. (The sign of the Doppler maybe determined in the video by other means.) One advantage of the fold over in the video is that only half the number of filters are required than in the IF filter bank.
- The equivalent of a bank of contiguous bandpass filters may also be obtained by converting the analog IF or video signal to a set of sampled, quantized signals which are processed with digital circuitry by using Fast Fourier Transform algorithm.
- A bank of overlapping Doppler filters, whether in the IF or video, increases the complexity of the receiver. When the system requirements permit a time sharing of the Doppler frequency range, the bank of Doppler filters may be replaced by a single

narrowband tunable filter which searches in frequency over the band of expected Doppler frequencies until a signal is found.

Sign of the radial velocity. In some applications of CW radar it is of interest to know whether the target is approaching or receding. This can be determined with separate filters located on either side of the intermediate frequency. If the echo-signal frequency lies below the carrier, the target is receding. If the echo frequency is greater than the carrier, the target is approaching.

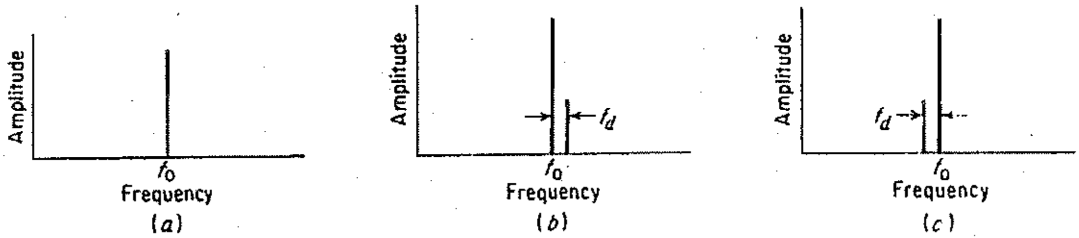


Figure: Spectra of received signals. (a) No Doppler shift, no relative target motion; (b) approaching target; (c) receding target.

Although the Doppler-frequency spectrum "folds over" in the video because of the action of the detector, it is possible to determine its sign from a technique available in single-sideband communications. If the transmitter signal is given by: $E_t = E_0 \cos\omega_0 t$

The echo signal from a moving target will be:

$$Er = k_1 . E_o \cos [(\omega_o +/- \omega_d)t) + \emptyset]$$

where E_o = amplitude of transmitter signal

k_1 = a constant determined from the radar equation

ω_o = angular frequency of transmitter, rad/s

ω_d = Doppler angular frequency shift

$\emptyset$ = a constant phase shift, which depends upon range of initial detection

The sign of the Doppler frequency, and therefore the direction of target motion, may be found by splitting the received signal into two channels as shown in the figure below. In channel **A** the signal is processed as in the simple CW radar. The received signal and a portion of the transmitter are heterodyned in the detector (mixer) to yield a difference signal

$$E_A = K_2 . E_0 . \cos (+/- \omega_d t + \emptyset)$$

The other channel is similar, except that a 90^0 phase delay is introduced in the reference signal. The output of the channel **B** mixer is

$$E_B = K_2 . E_0 . \cos (+/- \omega_d t + \emptyset + \pi/2)$$

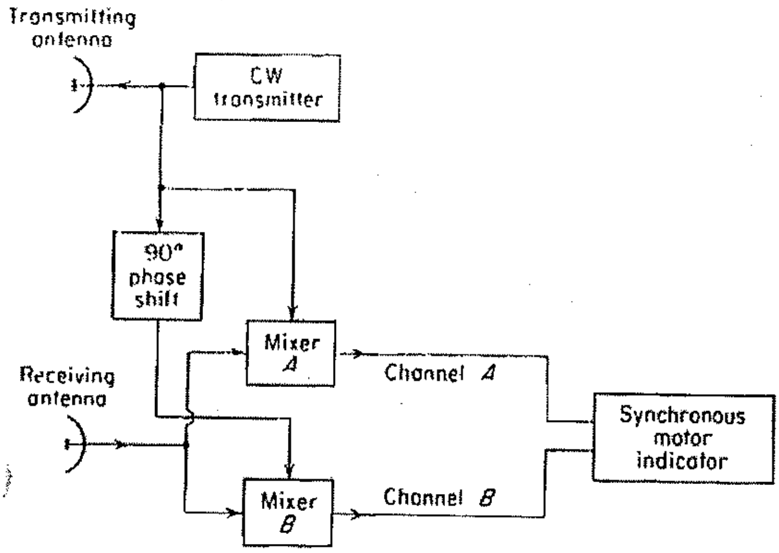


Figure: Measurement of Doppler direction using synchronous, two-phase motor.

If the target is approaching (positive Doppler) the outputs from the two channels are

$$E_A(+) = K_2.E_0.\cos(\omega_d t + \phi) \quad \text{and} \quad E_B(+) = K_2.E_0.\cos(\omega_d t + \phi + \pi/2)$$

On the other hand, if the target is receding (negative Doppler)

$$E_A(-) = K_2.E_0.\cos(\omega_d t - \phi) \quad \text{and} \quad E_B(-) = K_2.E_0.\cos(\omega_d t - \phi - \pi/2)$$

[Using the trigonometric relation $\cos(-\omega_d) = \cos\omega_d$*]*

The sign of ω_d and the direction of the target's motion may be determined according to whether the output of channel **B** leads or lags the output of channel **A.** One method of determining the relative phase relationship between the two channels is to apply the outputs to a synchronous two-phase motor. The direction of motor rotation is an indication of the direction of the target motion. Electronic methods may be used instead of a synchronous motor to sense the relative phase of the two channels.

Applications of CW radar:

- Measurement of the relative velocity of a moving target, as in the police speed monitor or in the rate-of-climb meter for vertical-take-off aircraft.
- Control of traffic lights, regulation of tollbooths, vehicle counting.
- As a sensor in antilock braking systems, and for collision avoidance.

- In railways, as a speedometer to replace the conventional axle-driven tachometer. In such an application it would be unaffected by errors caused by wheel slip on accelerating or wheel slide when braking.
- Monitoring the docking speed of large ships.
- Measurement of the velocity of missiles, ammunition, and baseballs.

The principal advantage of CW Doppler radar over the other (non radar) methods of measuring speed is that there need not be any physical contact with the object whose speed is being measured. In industry this is used to measure turbine-blade vibration, the peripheral speed of grinding wheels, and the monitoring of vibrations in the cables of suspension bridges.

- Most of the above applications can be satisfied with a simple, solid-state CW source with powers in tens of milli watts
- High-power CW radars for the detection of aircraft and other targets have been developed and have been used in such systems as the Hawk missile systems. (Shown below)
- The difficulty of eliminating the leakage of the transmitter signals into the receiver has limited the utility of unmodulated CW radar for many long-range applications.
- The CW radar, when used for short or moderate ranges, is characterized by simpler equipment than a pulse radar. The amount of power that .can be used with a CW radar is dependent on the isolation that can be achieved between the transmitter and receiver since the transmitter noise that finds its way into the receiver limits the receiver sensitivity. (The pulse radar has no similar limitation to its maximum range because the transmitter is not operative when the receiver is turned on.)
- Major disadvantage of the simple CW radar is its inability to obtain a measurement of range. This limitation can be overcome by modulating the CW carrier, as in the frequency-modulated radar.
- Some anti-air-warfare guided missile systems employ semi active homing guidance in which a receiver in the missile receives energy from the target, the energy having been transmitted from an "illuminator" external to the missile. The illuminator will be at the launch platform. CW illumination has been used in many successful systems. An example is the Hawk tracking illuminator shown in the figure below. It is tracking radar as well as an illuminator since it must be able to follow the target as it travels through space.

CW radar allows operation in the presence of clutter and has been well suited for low altitude missile defense systems. A block diagram of a CW tracking illuminator is shown in the figure above. Note that following the wide-band Doppler amplifier is a speed ***gate,*** which is a narrow-band tracking filter that acquires the targets Doppler and tracks its changing Doppler frequency shift.

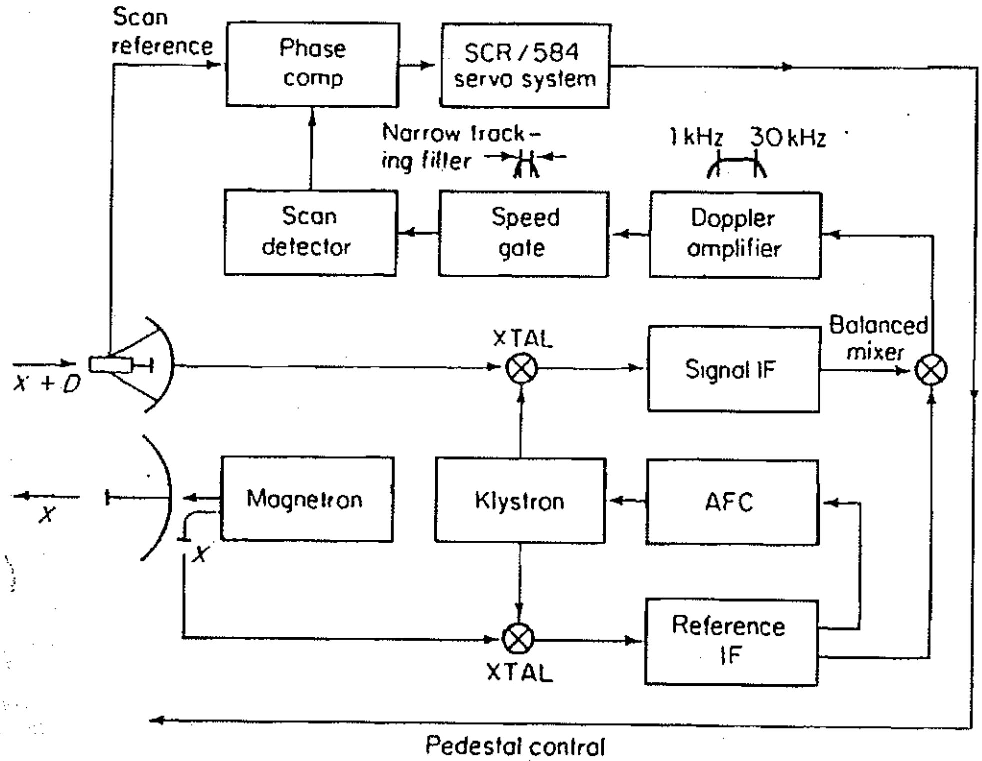


Figure: Block diagram of a CW tracking-illuminator

FM-CW RADAR

Introduction:

The inability of the simple CW radar to measure range is mainly due to the lack of a Timing mark. The timing mark permits the time of transmission and the time of return to be recognized but it increases the spectrum of the transmitted waveform. The sharper or more distinct the mark, the more accurate the measurement of the transit time. But the more distinct the timing mark, the broader will be the transmitted spectrum. This follows from the properties of the Fourier transform. Therefore a finite spectrum of necessity must be transmitted if transit time or range is to be measured. The spectrum of a CW transmission can be broadened by the application of a modulation - amplitude, frequency, or phase. An example of an amplitude modulation is the pulse radar. A widely used technique to insert a timing mark is to frequency-modulate the carrier. The timing mark is the changing frequency. The transit time is proportional to the difference in frequency between the transmitter signal and the echo signal. The greater the transmitter frequency deviation in a given time interval, the more accurate is the measurement of the transit time but the transmitted spectrum also becomes larger.

Range and Doppler measurement:

In the frequency-modulated CW radar (abbreviated FM-CW), the transmitter frequency is changed as a function of time in a known manner. Assume that the transmitter frequency increases linearly with time, as shown by the solid line in the figure below.

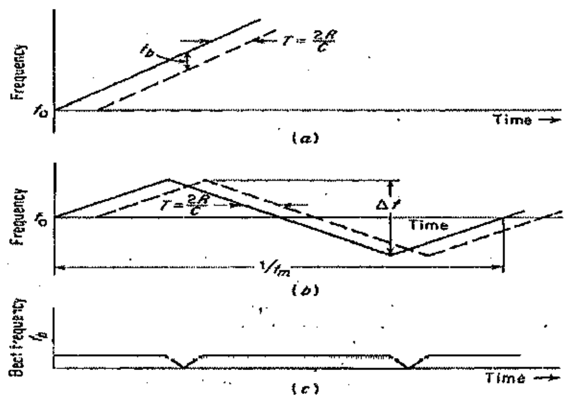


Figure: Frequency-time relation-ships in FM-CW radar. Solid curve represents transmitted signal; dashed curve represents echo. (a) Linear frequency modulation (b) triangular frequency modulation(c) beat note of (b).

If there is a reflecting object at a distance **R**, the echo signal will return after a time **T** = **2R/c**. The dashed line in the figure represents the echo signal. When the echo signal is heterodyned with a portion of the transmitter signal in a nonlinear element such as a diode, a beat note f_b will be produced. If there is no Doppler frequency shift, the beat note (difference frequency) is a measure of the target's range and $f_b = f_r$ where f_r, is the beat frequency only due to the target's range. If the rate of change of the carrier frequency is **f_0(dot)** then the beat frequency is given by:

$$f_r = \dot{f}_0 T = \frac{2R}{c} \dot{f}_0$$

In any practical CW radar, the frequency cannot be continually changed in one direction only. Periodicity in the modulation is necessary, as in the triangular frequency-modulation waveform shown in fig.b. The modulation need not necessarily be triangular. It can be saw tooth, sinusoidal, or some other shape. The resulting beat frequency as a function of time is shown in fig.c for triangular modulation. The beat note is of constant frequency except at the turn-around region. If a frequency change of **Δf** is modulated at a rate f_m, then the beat frequency is

$$f_r = (2R/c).2f_m.\Delta f = 4Rf_m.\Delta f / c$$

$$\text{Or } R = c f_r \ / \ 4 f_m . \Delta f \qquad \text{.......[Eq.1]}$$

Thus the measurement of the beat frequency determines the range **R**.

A block diagram illustrating the principle of the FM-CW radar is shown in the figure below. A portion of the transmitter signal acts as the reference signal required to produce the beat frequency. It is introduced directly into the receiver via a cable or other direct connection.

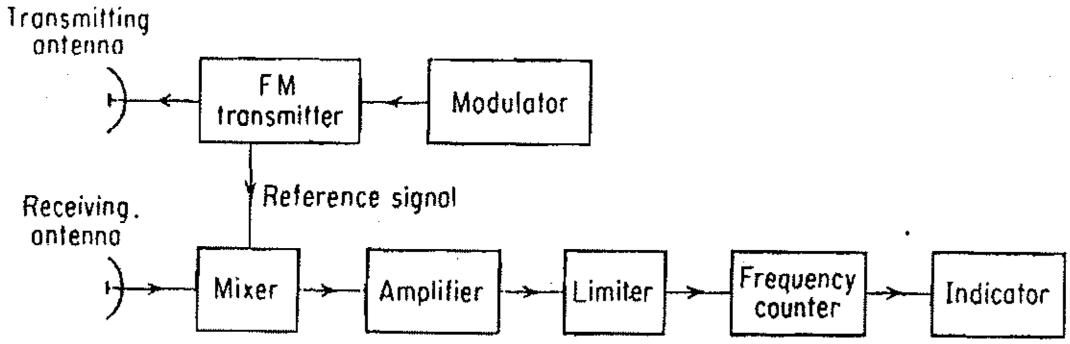


Figure: Block diagram of FM-CW radar

Ideally the isolation between transmitting and receiving antennas is made sufficiently large so as to reduce to a negligible level the transmitter leakage signal which arrives at the receiver via the coupling between antennas. The beat frequency is amplified and limited to remove any amplitude fluctuations. The frequency of the amplitude-limited beat note is measured with a cycle-counting frequency meter calibrated in distance.

In the above, the target was assumed to be stationary. If this assumption is not applicable, a Doppler frequency shift will be superimposed on the FM range beat note and an erroneous range measurement results. The Doppler frequency shift causes the frequency-time plot of the echo signal to be shifted up or down as shown in the figure (a). On one portion of the frequency-modulation cycle the beat frequency (fig. b) is increased by the Doppler shift, while on the other portion, it is decreased.

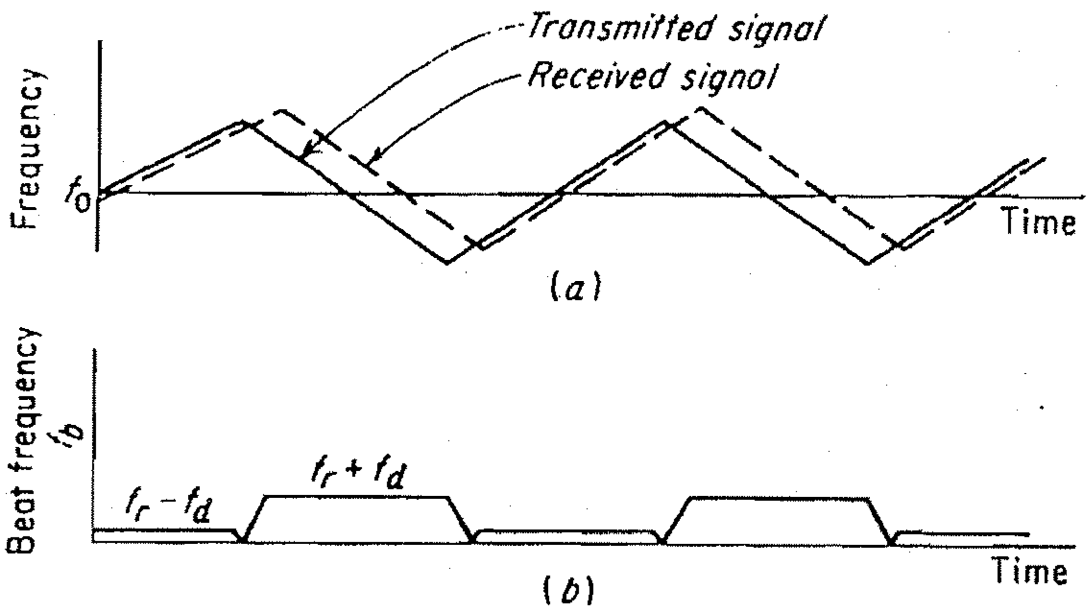


Figure: Frequency-time relationships in FM-CW radar when the received signal is shifted in frequency by the Doppler effect (a) Transmitted (solid curve) and echo (dashed curve) (b) beat frequency

If for example, the target is approaching the radar, the beat frequency f_b(up) produced during the increasing or up portion of the FM cycle will be the difference between the beat frequency due to the range f_r and the Doppler frequency shift f_d. Similarly, on the decreasing portion, the beat frequency f_b (down) is the sum of the two.

$$f_b(\text{up}) = f_r - f_d$$

$$f_b(\text{down}) = f_r + f_d$$

The range frequency f_r may be extracted by measuring the average beat frequency; that is, ½[f_b(up)+ f_b (down)] = f_r. If f_b(up) and f_b(down) are measured separately, for example, by switching a frequency counter every half modulation cycle, one-half the difference between the frequencies will yield the Doppler frequency. This assumes $f_r > f_d$. If, on the other hand, $f_r < f_d$, such as might occur with a high-speed target at short range, the roles of the averaging and the difference-frequency measurements are reversed; the averaging meter will measure Doppler velocities, and the difference meter measures range.

If the FM-CW radar is used for single targets only, such as in the radio altimeter, it is not necessary to employ a linear modulation waveform. This is certainly advantageous since a sinusoidal or almost sinusoidal frequency modulation is easier to obtain with practical equipment than are linear modulations. The beat frequency obtained with sinusoidal modulation is not constant over the modulation cycle as it is with linear modulation. However, it may be shown that the ***average*** beat frequency measured over a modulation cycle, when substituted into Eq. (1) yields the correct value of target range. If the target is in motion and the beat signal contains a component due to the -Doppler frequency shift, the range frequency can be extracted, as before, if the average frequency is measured. To extract the Doppler frequency, the modulation waveform must have equal upsweep and down sweep time intervals.

FM-CW Altimeter:

The FM-CW radar principle is used in the aircraft radio altimeter to measure height above the surface of the earth. The large backscatter cross section and the relatively short ranges required of altimeters permit low transmitter power and low antenna gain. Since the relative motion between the aircraft and ground is small, the effect of the Doppler frequency shift also may usually be neglected.
The band from **4.2 to 4.4GHz** is reserved for radio altimeters, although they have in the past operated at UHF. The transmitter power is relatively low and can be obtained from a CW Magnetron, a backward-wave oscillator, or a reflex klystron, but now they have been replaced by the solid state transmitter.
The altimeter can employ a simple homodyne receiver, but for better sensitivity and stability the super heterodyne is preferred whenever its more complex construction can be tolerated. The block diagram of the FM-CW radar with a sideband super heterodyne receiver is shown in the figure below.

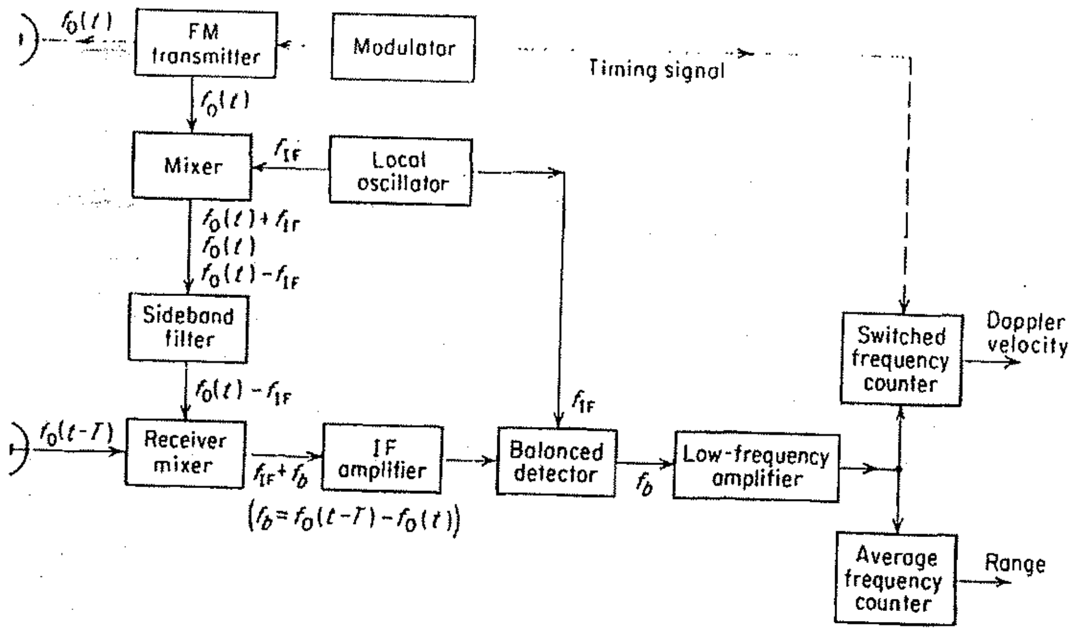


Figure: Block diagram of a FM-CW radar using sideband super heterodyne receiver

A portion of the frequency-modulated transmitted signal is applied to a mixer along with the oscillator signal. The selection of the local-oscillator frequency is a bit different from that in the usual super heterodyne receiver. The local-oscillator frequency f_{IF} is the same as the intermediate frequency used in

the receiver, whereas in the conventional super heterodyne the LO frequency is of the same order of frequency as the RF signal.
The output of the mixer consists of the varying transmitter frequency $f_o(t)$ plus two sideband frequencies, one on either side of $f_o(t)$ and separated from $f_o(t)$by the local-oscillator frequency f_{IF}. The filter selects the lower sideband, $f_o(t) - f_{IF}$ and rejects the carrier and the upper sideband. The side band that is passed by the filter is modulated in the same fashion as the transmitted signal. The sideband filter must have sufficient bandwidth to pass the modulation, but not the carrier or other sideband. The filtered sideband serves the function of the local oscillator.
When an echo signal is present, the output of the receiver mixer is an IF signal of frequency $(f_{IF}+f_b)$ where $f_b$is composed of the range frequency f_r and the Doppler velocity frequency f_d. The IF signal is amplified and applied to the balanced detector along with the local-oscillator signal f_{IF}. The output of the detector contains the beat frequency (range frequency and the Doppler velocity frequency), which is amplified to a level where it can actuate the frequency-measuring circuits.
In the above figure, the output of the low-frequency amplifier is divided into two channels: one feeds an average-frequency counter to determine the range, and the other feeds a switched frequency counter to determine the Doppler velocity (assuming $f_r>f_d$). Only the averaging frequency counter need be used in an altimeter application, since the rate of change of altitude is usually small.
A target at short range will generally result in a strong signal at low frequency, while one at long range will result in a weak signal at high frequency. Therefore the frequency characteristic of the low frequency amplifier in the FM-CW radar may be shaped to provide attenuation at the low frequencies corresponding to short ranges and large echo signals. Less attenuation is applied to the higher frequencies, where the echo signals are weaker.

Multiple-frequency CW Radar:

Although it was indicated earlier that CW radar can not measure range, it is possible under some circumstances to do so by measuring the phase of the echo signal relative to the phase of the transmitted signal. Consider a CW radar radiating a single-frequency sine wave of the form $\sin 2\pi f_o t$(The amplitude of the signal is taken to be unity since it does not influence the result) the signal travels to the target at a range **R** and returns to the radar after a time **T = 2R/c** where **c** is the velocity of propagation. The echo signal received at the radar is $\sin [2\pi f_o(t - T)]$. If the transmitted and received signals are compared in a phase detector, the output is proportional to the phase difference between the two and is given by :

$$\Delta\phi = 2\pi f_0 T = 4\pi f_o R/c.$$

The phase difference may therefore be used as a measure of the range, or

$$R = \frac{c\,\Delta\phi}{4\pi f_0} = \frac{\lambda}{4\pi}\Delta\phi \quad \text{............ [Eq. 2]}$$

However, the measurement of the phase difference $\Delta\phi$ is unambiguous only if $\Delta\phi$ does not exceed 2πradians. Substituting $\Delta\phi = 2\pi$ into the above equation (Eq.1) gives the maximum unambiguous range as $\lambda/2$.At radar frequencies this unambiguous range is much too small to be of any practical interest.

Unambiguous range may be extended considerably by utilizing two separate CW signals differing slightly in frequency. The unambiguous range in this case corresponds to half wavelength at the difference frequency.

The transmitted waveform is assumed to consist of two continuous sine waves of frequency f_1 *and* f_2 separated by an amount **Δf.** For convenience, the amplitudes of all signals are set equal to unity. The voltage waveforms of the two components of the transmitted signal v_{1T} *and* v_{2T} may be written as

$$v_{1T} = \sin(2\pi f_1 t + \phi_1)$$

$$v_{2T} = \sin(2\pi f_2 t + \phi_2)$$

where **Ø1 and Ø2** are arbitrary (constant) phase angles. The echo signal is shifted in frequency by the Doppler Effect. The form of the Doppler-shifted signals corresponding to the two frequencies f_1 *and* f_2 are:

$$v_{1R} = \sin\left[2\pi(f_1 \pm f_{d1})t - \frac{4\pi f_1 R_0}{c} + \phi_1\right]$$

$$v_{2R} = \sin\left[2\pi(f_2 \pm f_{d2})t - \frac{4\pi f_2 R_0}{c} + \phi_2\right]$$

Where R_o = range to target at a particular time $t = t_o$(range that would be measured if target were not moving)

f_{d1} = Doppler frequency shift associated with frequency f_1

f_{d2} = Doppler frequency shift associated with frequency f_2

Since the two RF frequencies $f_1$and $f_2$are approximately the same (that is $f_2=f_1+ \Delta f$, where $\Delta f \ll f_1$)the Doppler frequency shifts $f_{d1}$and f_{d2} can be assumed to be equal to each other.
Therefore we may write$f_{d1}= f_{d2}= f_d$
The receiver separates the two components of the echo signal and heterodynes each received signal component with the corresponding transmitted waveform and extracts the two Doppler-frequency components given below:

$$v_{1D} = \sin\left(\pm 2\pi f_d t - \frac{4\pi f_1 R_0}{c}\right)$$

$$v_{2D} = \sin\left(\pm 2\pi f_d t - \frac{4\pi f_2 R_0}{c}\right)$$

The phase difference between these two components is

$$\Delta\phi = \frac{4\pi(f_2 - f_1)R_0}{c} = \frac{4\pi\ \Delta f R_0}{c}$$

Hence

$$R_0 = \frac{c\ \Delta\phi}{4\pi\ \Delta f}$$

which is same as that of **Eq..2,** with **Δf** substituted in place of f_o.

Important aspects of Multi Frequency Radar:

- The two frequencies of the two-frequency radar were described as being transmitted simultaneously. They may also be transmitted sequentially in some applications by rapidly switching a single RF source.
- A large difference in frequency between the two transmitted signals improves the accuracy of the range measurement since large **Δf** means a proportionately large change in **ΔØ** for a given range. However, there is a limit to the value of **Δf** since **ΔØ** cannot be greater than **2π** radians if the range is to remain unambiguous. The maximum unambiguous range R_{unamb} is

$$R_{\mathbf{unamb}} = \frac{c}{2\,\Delta f}$$

Therefore **Δf** must be less than $c/2R_{unamb}$.Note that when **Δf** is replaced by the pulse repetition rate, the above equation gives the maximum unambiguous range of a pulse radar.

- A qualitative explanation of the operation of the two-frequency radar may be had by considering both carrier frequencies to be in phase at zero range. As they progress outward from the radar, the relative phase between the two increases because of their difference infrequency. This phase difference may be used as a measure of the elapsed time. When the two signals slip in phase by one cycle, the measurement of phase, and hence range, becomes ambiguous.
- The two-frequency CW radar is essentially a single-target radar since only one phase difference can be measured at a time. If more than one target is present, the echo signal becomes complicated and the meaning of the phase measurement becomes doubtful.
- The theoretical rms range error with which range can be measured with the two-frequency CW radar was estimated to be

$$\delta R = \frac{c}{4\pi\ \Delta f\,(2E/N_0)^{1/2}}$$

Where ***E*** = energy contained in received signal and ***No*** = noise power per hertz of bandwidth.
The above Equation indicates that the greater the separation **Δf** between the two frequencies, the lesser will be the **rms** error.

- However if the frequency difference **Δf** increases unambiguous Range decreases. The selection of **Δf** represents a compromise between the requirements of accuracy and ambiguity. Both accurate and unambiguous range measurements can be made by transmitting three or more frequencies instead of just two.

For example, if the three frequencies f_1,f_2 and $f_3$are such that $f_3 - f_1 = k(f_2 - f_1)$ where k is a factor of the order of 10 or 20, the pair of frequencies f_3, f_1(with greater Δf)gives an ambiguous but accurate range measurement while the pair of frequencies f_2, f_1(with lesser Δf) resolve the ambiguities in the measurement of Range. Likewise, further accuracy improvement with out reducing the ambiguous range can be obtained by adding more frequencies. As more frequencies are added the spectrum and target resolution approach that obtained with a pulse or an **FM-CW** waveform

Important Formulae:

- **Relation between Relative velocity V_r and Doppler frequency f_d :** $f_d = 2V_r / \lambda = 2V_r f_0 / c$
- **Relation between reflection coefficient and VSWR σ :** $| = (\sigma - l) / (\sigma + 1)$.
- **Change in Doppler frequency due to target's acceleration:**

$$\Delta f_d = \left(\frac{2a_r}{\lambda}\right)^{1/2}$$

- **In a FM CW Radar:**
  - **Target's Range velocity f_r is given by (Assuming there is no Doppler shift):**

    $$f_r = 4Rf_m.\Delta f / c$$

    Where f_m = modulating frequency and Δf = frequency swing
  - **Target's Range velocity f_r and Doppler frequency f_d are given by (with Doppler shift for Approaching target):**

    $f_r = ½[f_b(up)+ f_b (down)]$ and $f_d = ½[f_b (down) - f_b(up)]$

    where :

$$f_b(\text{up}) = f_r - f_d$$
$$f_b(\text{down}) = f_r + f_d$$

Illustrative problems:

Example1: Determine the Range and Doppler velocity of an approaching target using a triangular modulation FMCW Radar. Given : Beat frequency f_b(up) = 15KHz and f_b(down) = 25KHz , modulating frequency : 1MHz, Δf : 1KHz and Operating frequency : 3Ghz

Solution:

We know $f_r = ½[f_b(up)+ f_b (down)] = ½(15+25) = 20$ Khz

$f_d = ½[f_b (down) - f_b(up)] = ½(25-15) = 5$ Khz

The Range **R** in terms of f_r , f_m and Δf is given by : **R** = $c f_r / 4f_m.\Delta f$

$= (3x10^8)20x10^3 / 4(1x10^6x1x10^3)$ mtrs = 1500 mtrs = 1.5 Kms

Example 2: What should be the VSWR of a mismatched antenna if an isolation of 20 dB is to be obtained between the receiver and the transmitter in a CW Radar using a common antenna .

Solution : Isolation 20 db corresponds to a reflection coefficient of 0.1

[Since 20 log $_{10}$(1/) = 20 , log $_{10}$(1/) = 1 , 1/ = 10 and 0.1]

From the Relation between reflection coefficient and VSWR **σ : | = (σ- l) / (σ+1)** we can get

σ = (+) / (1-) and using the value of the reflection coefficient of 0.1 in this relation we get

VSWR = (1+0.1)/(1-0.1) = 1.1/0.9 = 1.22

Questions from Previous Year Examinations:

1.The transmitter power is 1 KW and safe value of power which might be applied to a receiver is 10mW. Find the isolation between transmitter and receiver in dB. Suggest the appropriate isolator.

2.(a) What is the Doppler effect? What are some of the ways in which it manifests itself? What are its radar applications?

(b) what is the relation between bandwidth and the acceleration of the target with respect to radar?

3.(a) How to find the target speed from Doppler frequency?

(b) Write the applications of CW Radar.

(c) What are the factors that limit the amount of isolation between Transmitter and Receiver of CW Radar? [4+6+6]

4.(a) Explain the operation of the two frequency CW Radar.

(b) How to select the difference between the two transmitted signals of CW radar? [8+8]

5.(a) With the help of a suitable block diagram explain the operation of a CW Radar with non zero IF amplifier

(b) list down and explain the applications of CW radar

6.(a) Draw the block diagram of a FMCW Radar using side band super heterodyne receiver and explain it's operation.

(b) With a transmit (CW) frequency of 5GHz, calculate the Doppler frequency seen by a Stationary Radar when the target radial velocity is 100 km/h (62.5 mph)? [10+6]

UNIT- 3
MTI AND PULSE DOPPLER RADAR

- **Introduction**
- **Principle of Operation**
- **MTI Radar with Power Amplifier Transmitter and Power Oscillator Transmitter**
- **Delay Line Cancellers- Filter Characteristics**
- **Blind Speeds**
- **Double Cancellations**
- **Staggered PRFs**
- **Range Gated Doppler Filters**
- **MTI Radar Parameters**
- **Limitations to MTI Performance**
- **MTI vs. Pulse Doppler Radar**
    - **Previous Years' Examination Questions**

MTI AND PULSE DOPPLER RADAR

Introduction:

The Doppler frequency shift [$f_d = 2V_r / \lambda$] produced by a moving target may be used in a pulse radar just as in the CW radar, to determine the relative velocity of a target or to separate desired moving targets from undesired stationary objects (clutter). Although there are applications of pulse radar where a determination of the target's relative velocity is made from the Doppler frequency shift, the use of Doppler to separate small moving targets in the presence of large clutter has been of greater interest. Such a pulse radar that utilizes the Doppler frequency shift as a means of discriminating moving targets from fixed targets is called a **MTI** (moving target indication) or a **pulse Doppler** radar. The two are based on the same physical principle, but in practice there are differences between **MTI** and **Pulse Doppler** radar.

- The MTI radar, usually operates with ambiguous Doppler measurement (so-called **blind speeds)** but with unambiguous range measurement (no second-time around echoes).
- A pulse Doppler radar operates with ambiguous range measurement but with unambiguous Doppler measurement. Its pulse repetition frequency is usually high enough to operate with unambiguous Doppler (no Blind speeds) but at the expense of range ambiguities.

The discussion in this chapter mostly is based on the **MTI Radar,** but much of what applies to MTI can be extended to **Pulse Doppler Radar** as well.

Salient Features of MTI:

- MTI is a necessity in high-quality air-surveillance radars that operate in the presence of clutter.
- Its design is more challenging than that of a simple pulse radar or a simple CW radar.
- A **MTI** capability adds to a radar's cost and complexity and often system designers must accept compromises they might not wish to.
- The basic MTI concepts were introduced during World War 2, and most of the signal processing theory on which **MTI** (and **pulse Doppler**) radar depends was formulated during the mid-1950s.
- However, the implementation of theory to practice was speeded up only subsequently after the availability of the necessary signal-processing technology.
- It took almost twenty years for the full capabilities offered by MTI signal-processing theory to be converted into practical and economical Radar equipment. The chief factor that made this possible was the development of reliable, small, and inexpensive digital processing hardware.

Principle of operation:

A simple CW radar studied earlier is shown in Fig.1 (a). In principle, the CW radar may be converted into a pulse radar as shown in Fig.1 (b) by providing a power amplifier and a modulator to turn the amplifier on and off for the purpose of generating pulses. The chief difference between the pulse radar of Fig. 1(b) and the one studied earlier is that a small portion of the CW oscillator power that generates the transmitted pulses is diverted to the receiver to take the place of the local oscillator. However, this CW signal does more than function as a replacement for the local oscillator. It acts as the **coherent reference** needed to detect the Doppler frequency shift. By **coherent** it means that the phase of the transmitted signal is preserved in the reference signal. The reference signal is the distinguishing feature of **coherent MTI radar.**

If the CW oscillator voltage is represented as **$A_1 \sin 2\pi f_t t$** where **A_1** = amplitude and f_t the carrier frequency

- Then the reference signal is: **$V_{ref} = A_2 \sin 2\pi f_t t$** .... **(1)**
- And the Doppler-shifted echo-signal voltage is

$$V_{echo} = A_3 \sin\left[2\pi(f_t \pm f_d)t - \frac{4\pi f_t R_0}{c}\right] \qquad \text{..........(2)}$$

Where **A_2** = amplitude of reference signal
A_3 = amplitude of signal received from a target at a range R_0
f_d = Doppler frequency shift
t = time
c = velocity of propagation

- The reference signal and the target echo signal are heterodyned in the mixer stage of the receiver. Only the low-frequency (difference-frequency) component from the mixer is of interest and is a voltage given by:

$$V_{diff} = A_4 \sin\left(2\pi f_d t - \frac{4\pi f_t R_0}{c}\right) \qquad \text{............(3)}$$

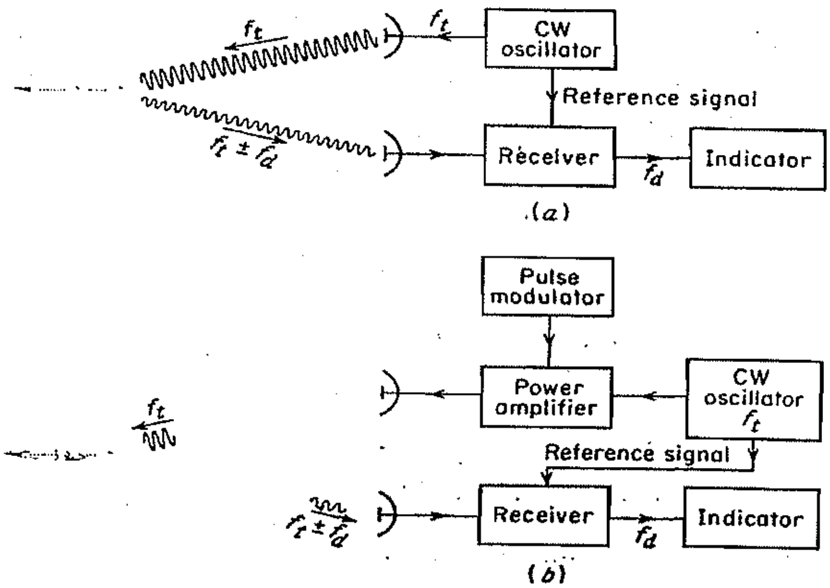


Figure 1: (a) Simple CW Radar (b) Pulse Radar using Doppler Information

Note that the equations **(1) to (3)** above represent sine wave carriers upon which the pulse modulation is imposed. The difference frequency is equal to the Doppler frequency **f_d**. For stationary targets the Doppler frequency shift will be zero and hence **V_{diff}** will not vary with time and may take on any constant value from **+ A_4 to $-A_4$** including zero. However, when the target is in motion relative to the radar **f_d** has a value other than zero and the voltage corresponding to the difference frequency from the mixer **[Eq. (3)]** will be a function of time.

- An example of the output from the mixer when the Doppler frequency **f_d** is large compared with the reciprocal of the pulse width is shown in **Fig.2 (b).** The Doppler signal may be readily discerned from the information contained in a single pulse.
- If, on the other hand **f_d** is small compared with the reciprocal of the pulse duration, the pulses will be modulated with an amplitude given by Eq. **(4.3) [Fig. 2(c)]** and many pulses will be needed to extract the Doppler information.
- The case illustrated in **Fig. 2(c)** is more typical of aircraft-detection radar, while the waveform of **Fig. 2(b)** might be more applicable to a radar used for the detection of extraterrestrial targets such as ballistic missiles or satellites.
- Ambiguities in the measurement of Doppler frequency can occur in the case of the discontinuous measurement of **Fig. 2(c)** but not when the measurement is made on the basis of a single pulse.
- The video signals shown in Fig.2are called **bipolar,** since they contain both positive and negative amplitudes.

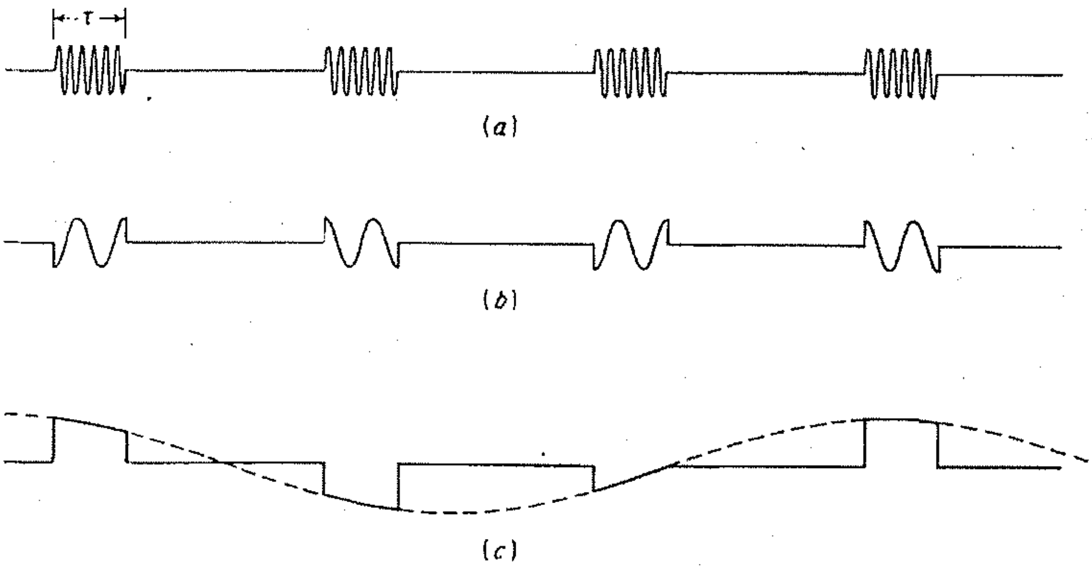


Figure 2 (a) RF echo pulse train (b) video pulse train for Doppler frequency f_d>l/τ(c) video pulse train for Doppler frequency f_d<l/τ .

Moving targets may be distinguished from stationary targets by observing the video output on an A-scope (amplitude vs. range). A single sweep on an A-scope might appear as in **Fig. 3 (a)** shown below. This sweep shows several fixed targets and two moving targets indicated by the two arrows. On the basis of a single sweep, moving targets cannot be distinguished from fixed targets.

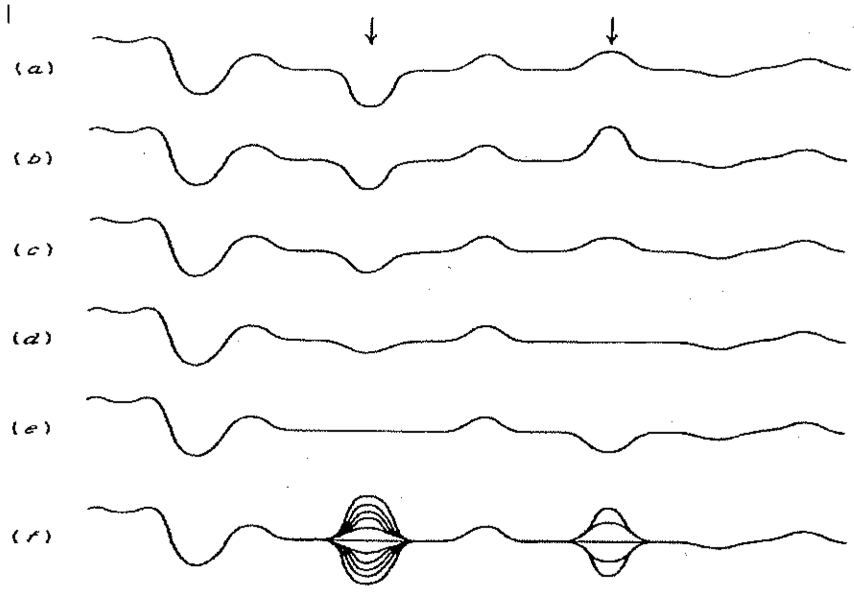


Figure 3 (a-e) Successive sweeps of a MTI radar A-scope display (echo amplitude as a function of time) (f) superposition of many sweeps: arrows indicate position of moving targets.

Successive A-scope sweeps (pulse-repetition intervals) are shown in Fig. 3 (a) to (e). Echoes from fixed targets remain constant throughout, but echoes from moving targets vary in amplitude from sweep to sweep at a rate corresponding to the Doppler frequency. The superposition of the successive A-scope sweeps is shown in Fig. 3(f). The moving targets produce, with time, a" **butterfly**" effect on the A-scope.

Concept of delay-line canceller:

Although the butterfly effect is suitable for recognizing moving targets on an A-scope, it is not appropriate for display on the PPI. One method commonly employed to extract Doppler information in a form suitable for display on the PPI scope is with a delay-line canceller as shown in the Fig. 4 below.

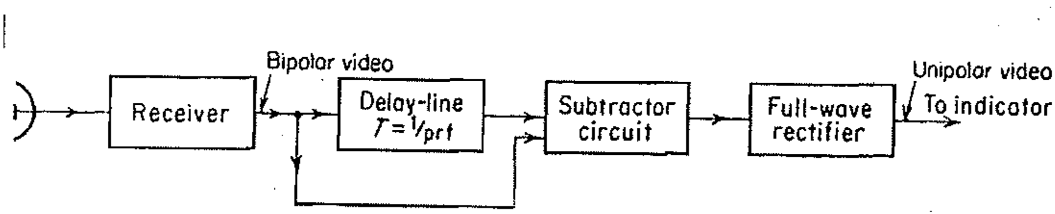


Figure 4: MTI Receiver with delay-line canceller

The delay-line canceller acts as a filter to eliminate the d-c component of fixed targets and to pass the a-c components of moving targets. The video portion of the receiver is divided into two channels. One is a normal video channel. In the other, the video signal experiences a time delay equal to one pulse-repetition period (equal to the reciprocal of the pulse repetition frequency). The outputs from the two channels are subtracted from one another. The fixed targets with unchanging amplitudes from pulse to pulse are canceled on subtraction. However, the amplitudes of the moving-target echoes are not constant from pulse to pulse and subtraction results in an uncanceled residue. The output of the subtraction circuit is a bipolar video just as was the input. Before bipolar video can intensity-modulate a PPI display it must be converted into unipotential voltages (unipolar video) by a full-wave rectifier.

MTI Radar with Power Amplifier Transmitter:

The simple MTI radar shown in **Fig. 1(b)** is not the most typical. The block diagram of a more common MTI radar employing a power amplifier is shown in the **Fig. 5 below.** The significant difference between this MTI configuration and that of **Fig. 1(b)** is the manner in which the reference signal is generated. In **Fig. 5**, the coherent reference is supplied by an oscillator called the **coho**, which stands for coherent oscillator. The **coho** is a stable oscillator whose frequency is the same as the intermediate frequency used in the receiver. In addition to providing the reference signal, the output of the **coho** is also mixed with the local-oscillator frequency f_l.The local oscillator also must be a stable oscillator and is called **stalo,** for stable local oscillator. The RF echo signal is heterodyned with the **stalo** signal to produce the IF just as in the conventional super heterodyne receiver. The **stalo, coho** and the mixer in which they are mixed are called Receiver- Exciter because of the dual role they serve both the receiver and the transmitter.

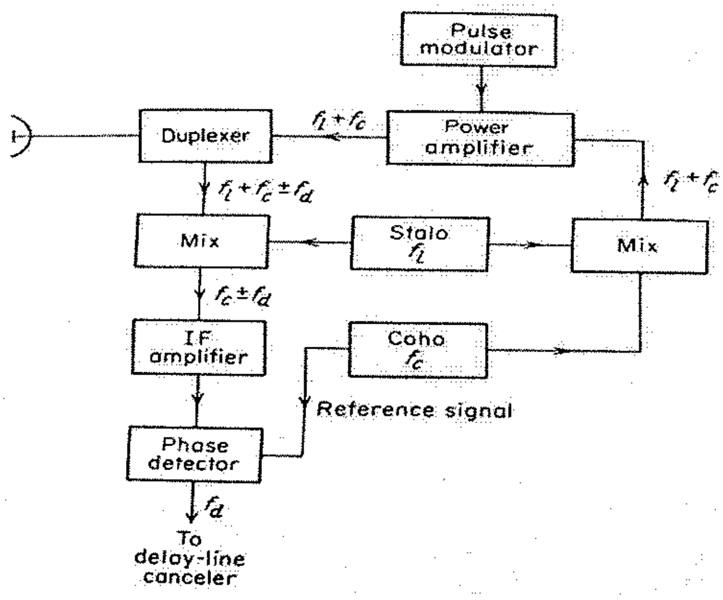


Figure 5: Block diagram of MTI radar with power-amplifier transmitter.

The characteristic feature of coherent MTI radar is that the transmitted signal must be coherent (in phase) with the reference signal in the receiver. This is accomplished in the radar system shown in **Fig. 5** by generating the transmitted signal from the **coho** reference signal. The function of the **stalo** is to provide the necessary frequency translation from the **IF** to the transmitted **(RF)** frequency. Although the phase of the **stalo** influences the phase of the transmitted signal, any **stalo** phase shift is canceled on reception because the **stalo** that generates the transmitted signal also acts as the local oscillator in the receiver. The reference signal from the **coho** and the IF echo signal are both fed into a mixer called the **Phase detector.** The phase detector differs from the normal amplitude detector since its output is proportional to the phase difference between the two input signals.

Any one of a number of transmitting-tube types might be used as the power amplifier. These include the triode, tetrode, klystron, traveling-wave tube, and the crossed-field amplifier.

A transmitter which consists of a stable low-power oscillator followed by a power amplifier is sometimes called **MOPA**, which stands for **M**aster-**O**scillator **P**ower **A**mplifier.

MTI radar with power-oscillator transmitter:

Before the development of the klystron amplifier, the only high-power transmitter available at microwave frequencies for radar application was the magnetron oscillator. In an oscillator, the phase of the RF bears no relationship from pulse to pulse. For this reason, the reference signal cannot be generated by a continuously running oscillator. However, a coherent reference signal may be readily obtained with the power oscillator by readjusting the phase of the **coho** at the beginning of each sweep according to the phase of the transmitted pulse. The phase of the **coho** is locked to the phase of the transmitted pulse each time a pulse is generated.

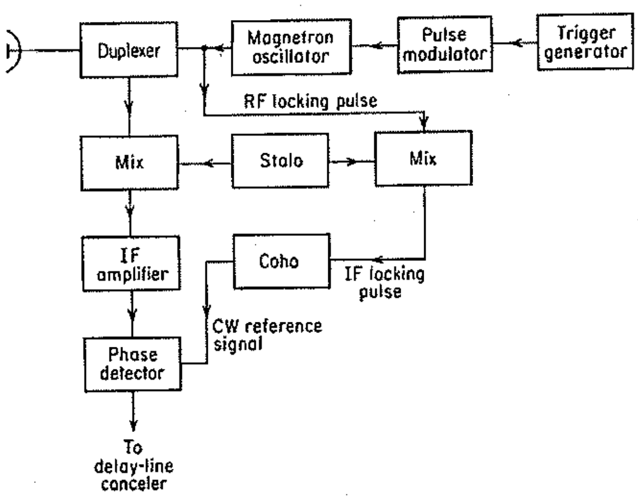


Figure 6: Block diagram of MTI radar with power-oscillator transmitter

Delay Line Cancellers:

The simple MTI delay-line canceller showed in **Fig. 4** is an example of a time-domain filter. The capability of this device depends on the quality of the medium used in the delay line. The Pulse modulator delay line must introduce a time delay equal to the pulse repetition interval. For typical ground-based air surveillance radars this will be several milliseconds. Delay times of this magnitude cannot be achieved with practical electromagnetic transmission lines. By converting the electromagnetic signal to an acoustic signal, it is possible to utilize delay lines of a reasonable physical length since the velocity of propagation of acoustic waves is about 10^{-5} that of electromagnetic waves. After the necessary delay is introduced by the acoustic line, the signal is converted back to an electromagnetic signal for further processing. The early acoustic delay lines developed during World War 2 used liquid delay lines filled with either water or mercury. Liquid delay lines were large and inconvenient to use. They were replaced in themid-1950s by the solid fused-quartz delay line that used multiple internal reflections to obtain a compact device. These analog acoustic delay lines were, in turn replaced in the early 1970s by storage devices based on digital computer technology. The use of digital delay lines requires that the output of the MTI receiver phase-detector be quantized into a sequence of digital words. The compactness and convenience of digital processing allows the implementation of more complex delay-line cancellers with filter characteristics not practical with analog methods. One of the advantages of a time-domain delay-line canceller as compared to the more conventional frequency-domain filter is that a single network operates at all ranges and does not require a separate filter for each range resolution cell. Frequency-domain Doppler filter banks are of interest in some forms of MTI and Pulse-Doppler radar.

Filter Characteristics of the Delay Line Canceller:

The delay-line canceller acts as a filter which rejects the d-c component of clutter. Because of its periodic nature, the filter also rejects energy in the vicinity of the pulse repetition frequency and its harmonics. The video signal of Eq. **(3)** received from a particular target at a range R_0 is

$$V_1 = k \sin (2\pi f_d t - \emptyset_0) \quad \text{.................. (4)}$$

where $\emptyset_0$ = phase shift and **k** = amplitude of video signal. The signal from the previous transmission, which is delayed by a time **T** = pulse repetition interval, is

$$V_2 = k \sin [2\pi f_d(t - T) - \emptyset_0] \quad \text{.................. (5)}$$

Everything else is assumed to remain essentially constant over the interval **T** so that **k** is the same for both pulses. The output from the subtractor is

$$V = V_1 - V_2 = 2k \sin \pi f_d T \cos [2 \pi f_d(t - T/2) - \emptyset o] \quad \text{.................. (6)}$$

It is assumed that the gain through the delay-line canceller is unity. The output from the canceller Eq. (6) consists of a cosine wave at the Doppler frequency & with an amplitude **$2k \sin \pi f_d T$**: Thus, the amplitude of the canceled video output is a function of the Doppler frequency shift and the pulse-repetition interval, or prf. The magnitude of the relative frequency-response of the delay-line canceller [ratio of the amplitude of the output from the delay-line canceller, **$2k \sin (\pi f_d T)$** to the amplitude of the normal radar video **k**] **is** shown in the Fig. 7 below.

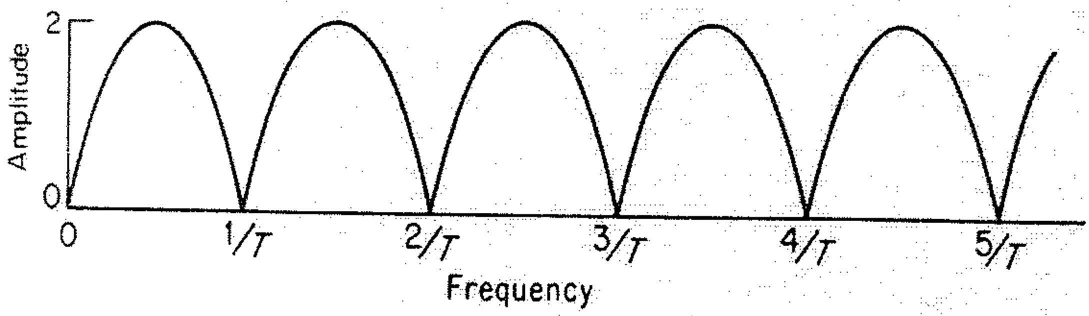


Figure (7): Frequency response of the single delay-line canceller: T = delay time =1/f_P.

Blind speeds:

The response of the single-delay-line canceller will be zero whenever the argument **($\pi f_d T$)** in the amplitude factor of **Eq. (6)** is 0, π, 2π, . .., etc., or when

$$f_d = \frac{n}{T} = nf_p \quad \text{................ (7)}$$

where n = 0, 1, 2, . . . , and f_p= pulse repetition frequency. The delay-line canceller not only eliminates the d-c component caused by clutter (**n = 0)**, but unfortunately it also rejects any moving target whose Doppler frequency happens to be the same as the **PRF** or a multiple thereof. Those relative target velocities which result in zero MTI response are called **blind speeds** and are given by

$$v_n = \frac{n\lambda}{2T} = \frac{n\lambda f_p}{2} \qquad n = 1, 2, 3, \ldots \quad \text{.................. (8)}$$

where v_n is the n^{th} blind speed.

The blind speeds are one of the limitations of pulse MTI radar which do not occur with CW radar. They are present in pulse radar because Doppler is measured by discrete samples -(pulses) at the prf rather than continuously. If the first blind speed is to be greater than the maximum radial velocity expected from the target, the product, **λf_P** must be large. Thus, the MTI radar must operate at long wavelengths (low frequencies) or with high pulse repetition frequencies, or both. Unfortunately, there are usually constraints other than blind speeds which determine the wavelength and the pulse repetition frequency. Therefore, blind speeds might not be easy to avoid. Low radar frequencies have the disadvantage that antenna beam widths, for a given-size antenna, are wider than at the higher frequencies and would not be satisfactory in applications where angular accuracy or angular resolution is important. The pulse repetition frequency cannot always be varied over wide limits since it is primarily determined by the unambiguous range requirement. In Fig.8, the first blind speed **v_1**, is plotted as a function of the maximum unambiguous range **(R_{unamb}= cT/2)**, with radar frequency as the parameter. If the first blind speed were 600 knots, the maximum unambiguous range would be 130 nautical miles at a frequency of 300 MHz (UHF), 13 nautical miles at **3000** MHz (**S** band), and 4 nautical miles at 10,000 **MHz** (X band). Since commercial jet aircraft have speeds of the order of 600knots, and military aircraft even higher, blind speeds in the MTI radar can be a serious limitation.

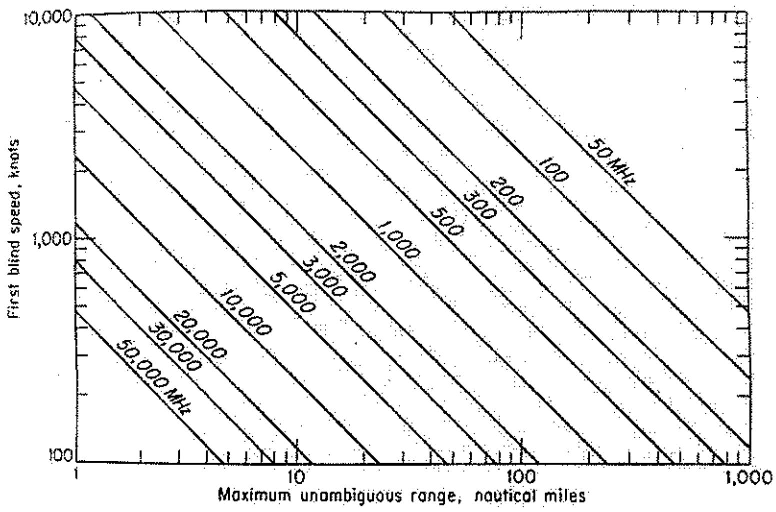


Figure (8): Plot of MTI radar first blind speed as a function of maximum unambiguous range.

Double cancellation:

The frequency response of a single-delay-line canceller (Fig. **7)** does not always have as broad a clutter-rejection null as might be desired in the vicinity of d-c. The clutter-rejection notches may be widened by passing the output of the delay-line canceller through a second delay-line canceller as shown in Fig. **9 below.** The output of the two single-delay line cancellers in cascade is the square of that from a single canceller. Thus the frequency response is($4 \sin^2 \pi f_d T$). The configuration of Fig. **9** is called a double-delay-line canceller, or simply a ***double canceller.*** The relative response of the double canceller compared with that of a single-delay-line canceller is shown in Fig. **10.** The finite width of the clutter spectrum is also shown (hatched) in this figure so as to illustrate the additional cancellation of clutter offered by the double canceller.

The two-delay-line configuration of Fig.**9 (b)** has the same frequency-response characteristic as the double-delay-line canceller. The operation of the device is as follows. **A** signal **f (t)** is inserted into the adder along with the signal from the preceding pulse period, with its amplitude weighted by the factor -2, plus the signal from the previous two pulse periods. The output of the adder is therefore

$$f(t) - 2f(t + T) + f(t + 2T)$$

which is the same as the output from the double-delay-line canceller.

$$f(t) - f(t + T) - f(t + T) + f(t + 2T)$$

This configuration is commonly called the three-pulse canceller.

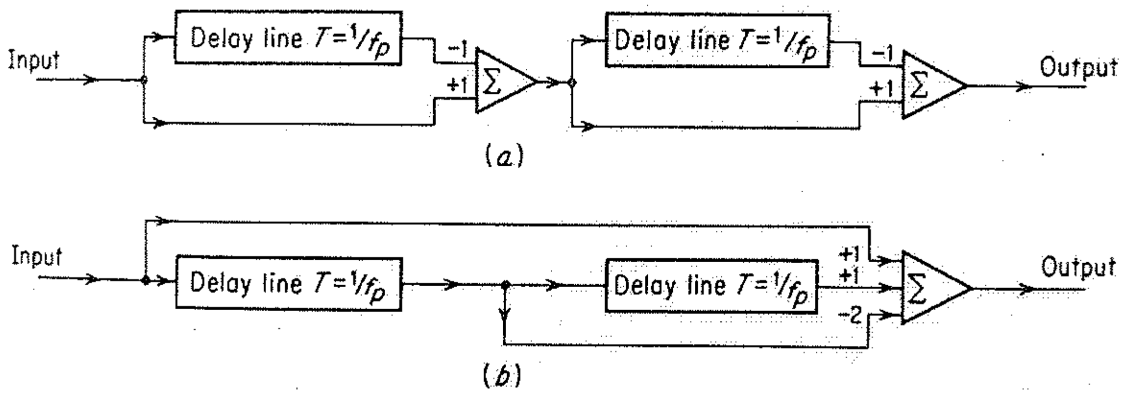


Figure 9 : (a) Double-delay-line canceller (b)three-pulse canceller.

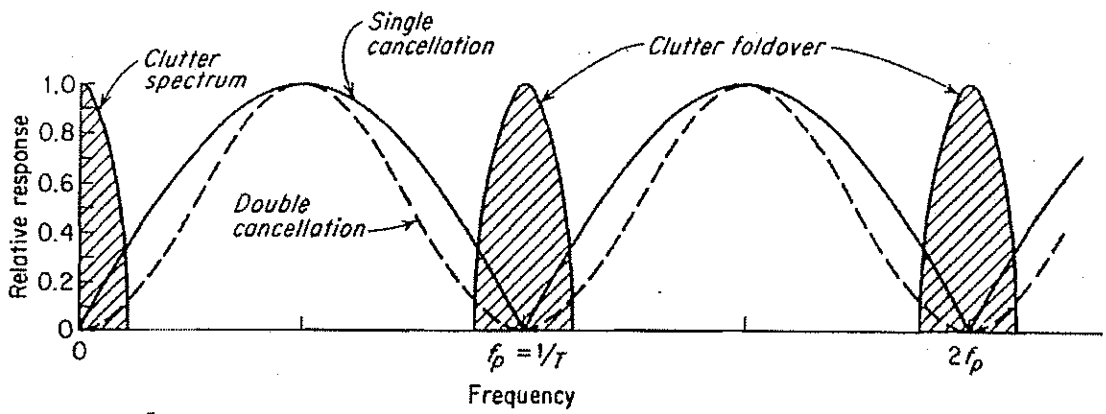


Figure (10): Relative frequency response of the single-delay-line canceller (solid curve) and the double delay-line canceller (dashed curve). Shaded area represents clutter spectrum.

Multiple or staggered Pulse Repetition Frequencies:

The use of more than one pulse repetition frequency offers additional flexibility in the design of MTI Doppler filters. It not only reduces the effect of the blind speeds of Eq. **8,** but it also allows a sharper low-frequency cutoff in the frequency response than might be obtained with a cascade of single-delay-line cancellers with **$\sin^n \pi f_d T$** response. The blind speeds of two independent radars operating at the same frequency will be different if their pulse repetition frequencies are different. Therefore, if one radar were "blind "to moving targets, it would be unlikely that the other radar would be" blind" also.

Instead of using two separate radars, the same result can be obtained with one radar which time-shares its pulse repetition frequency between two or more different values (multiple PRF's). The pulse repetition frequency might be switched every other scan or every time the antenna is scanned a half beam width, or the period might be alternated on every other pulse. When the switching is pulse to pulse, it is known as a **staggered PRF**. An example of the composite (average) response of a MTI radar operating with two separate pulse repetition frequencies on a time-shared basis is shown in **Fig.11.** The pulse repetition frequencies are in the ratio of **5**:4. Note that the first blind speed of the composite response is increased several times over what it would be for a radar operating on only a single pulse repetition frequency. Zero response occurs only when the blind speeds of each prf coincide. In the example of Fig.11, the blind speeds are coincident for **$4/T_1$= $5/T_2$**.Although the first blind speed may be extended by using more than one **PRF**, regions of low sensitivity might appear within the composite passband.The closer the ratio **T_1 : T_2** approaches unity, the greater will be the value of the first blind speed. However, the first null in the vicinity of **f_d** = **$1/T_1$**becomes deeper. Thus, the choice of **T_1/T_2** is a compromise between the value of the first blind speed and the depth of the nulls within the filter pass band.

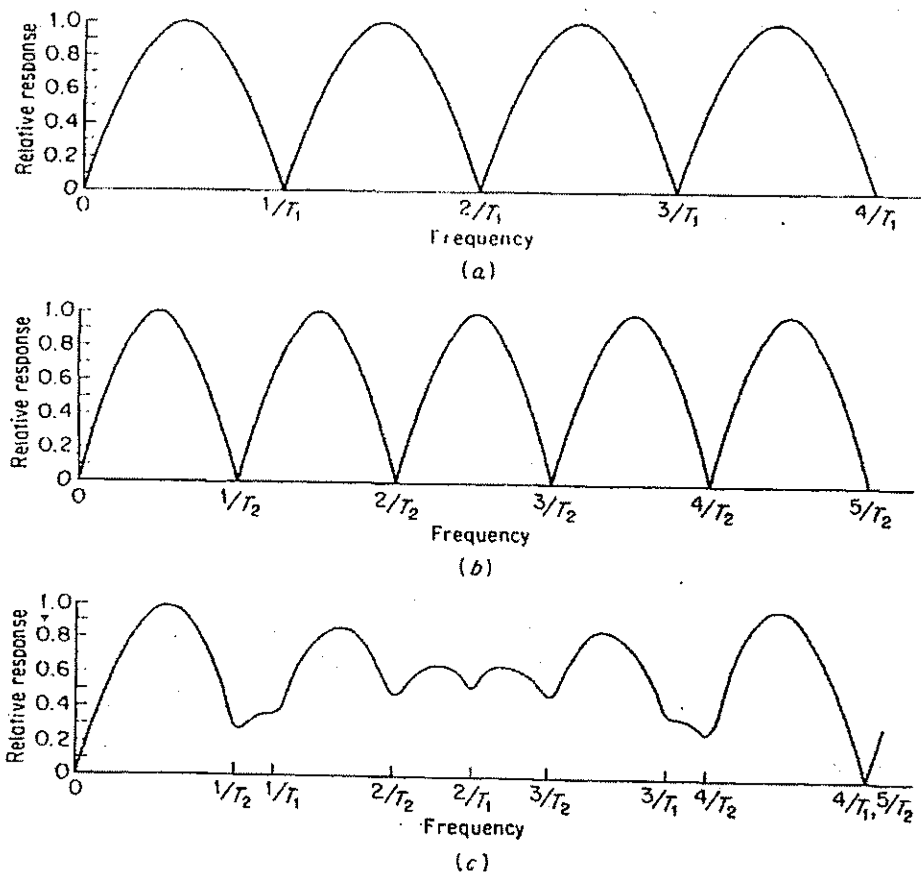


Figure 11 (a) Frequency-response of a single-delay-line canceller for f_p= $1/T_1$ (b) same for f_p= l/T_2 (c) Composite response with T_1/T_2= 4/5.

The depth of the nulls can be reduced and the first blind speeds increased by operating with more than two inter pulse periods. Figure 12 below shows the response of a five-pulse stagger (four periods) that might be used with a long-range air traffic control radar. In this example the periods are in the ratio 25 : 30 : 27 : 31 and the first blind speed is **28.25** times that of a constant **prf** waveform with the same average period. If the periods of the staggered waveforms have the relationship $n_1/T_1 = n_2/T_2 = \ldots\ldots = n_N/T_N$, where $n_1, n_2, \ldots, n_N$ are integers, and if v_B is equal to the first blind speed of a non-staggered waveform with a constant period equal to the average period $T_{av} = (T_1 + T_2 + \ldots T_N)/N$ then the first blind speed v_1 is given by :

$$\frac{v_1}{v_B} = \frac{n_1 + n_2 + \cdots + n_N}{N}$$

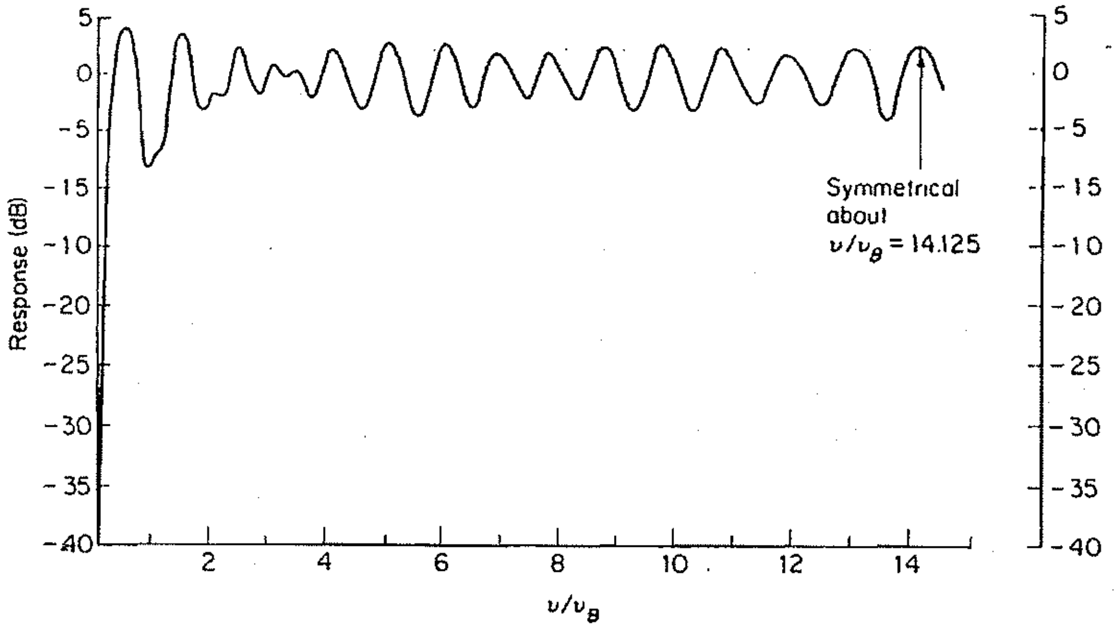


Figure 12: Frequency response of a five-pulse (four-period) stagger.

Range-gated Doppler filters:

The delay-line canceller, which can be considered as a time-domain filter, is widely used in MTI radar to separate moving targets from stationary clutter. It is also possible to employ the more common frequency-domain band pass filters of conventional design in MTI radar to separate the Doppler-frequency-shifted targets. The filter configuration however would be more complex, than the single, narrow-band pass filter. A narrowband filter with a pass band designed to pass the Doppler frequency components of moving targets will "ring" when excited by the usual short radar pulse. That is, its pass band is much narrower than the reciprocal of the input pulse width so that the output will be of much greater duration than the input. The narrowband filter "smears" the input pulse since the impulse response is approximately the reciprocal of the filter bandwidth. This smearing destroys the range resolution. If more than one target is present they cannot be resolved. Even if only one target is present, the noise from the other range cells that do not contain the target will interfere with the desired target signal. The result is a reduction in sensitivity due to a collapsing loss.

The loss of the range information and the collapsing loss may be eliminated by first quantizing the range (time) into small intervals. This process is called ***range gating.*** The width of the range gates depends upon the range accuracy desired and the complexity which can be tolerated, but they are usually of the order of the pulse width. Range resolution is established by gating. Once the radar return is quantized into range intervals, the output from each gate may be applied to a narrowband filter since the pulse shape need no longer be preserved for range resolution. A collapsing loss does not take place since noise from the other range intervals is excluded.

A block diagram of the video of an MTI radar with multiple range gates followed by clutter-rejection filters is shown in Fig. 13 below. The output of the phase detector is sampled sequentially by the range gates. Each range gate opens in sequence just long enough to sample the voltage of the video waveform corresponding to a different range interval in space. The range gate acts as a switch or a gate which opens and closes at the proper time. The range gates are activated once each pulse-repetition interval. The output for a stationary target is a series of pulses of constant amplitude. An echo from a moving target produces a series of pulses which vary in amplitude according to the Doppler frequency. The output of the range gates is stretched in a circuit called the ***boxcar generator,*** or ***sample-and-hold*** circuit, whose purpose is to aid in the filtering and detection process by emphasizing the fundamental of the modulation frequency and eliminating harmonics of the pulse repetition frequency. The clutter rejection filter is a band pass filter whose bandwidth depends upon the extent of the expected clutter spectrum.

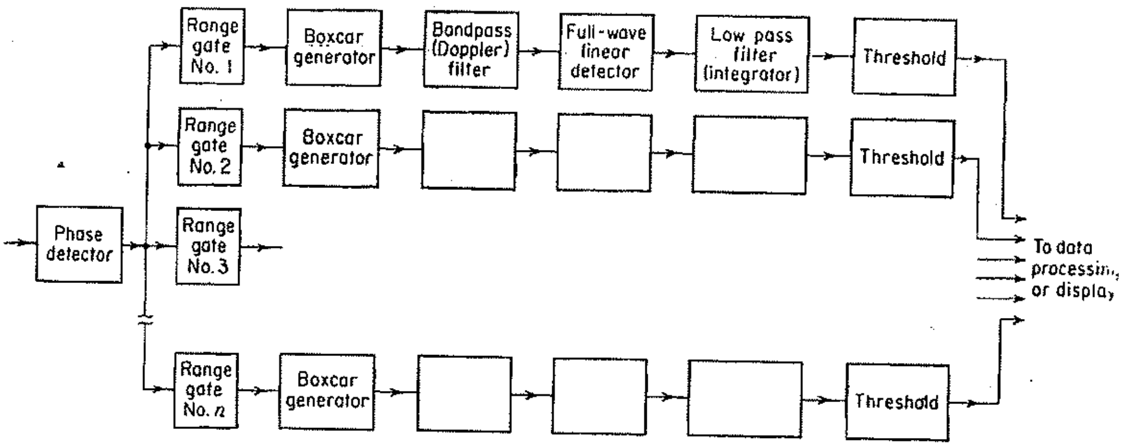


Figure 13: Block diagram of MTI radar using range gates and filters.

Following the Doppler filter is a full-wave linear detector and an integrator (a low-pass filter). The purpose of the detector is to convert the bipolar video to unipolar video. The output of the integrator is applied to a threshold-detection circuit. Only those signals which cross the threshold are reported as targets. Following the threshold detector, the outputs from each of the range channels must be properly combined for display on the PPI or A-scope or for any other appropriate indicating or data-processing device. The CRT display from this type of MTI radar appears "cleaner" than the display from a normal MTI radar, not only because of better clutter rejection, but also because the threshold device eliminates many of the unwanted false alarms due to noise. The frequency-response characteristic of the range-gated MTI appears as in Fig. 14. The shape of the rejection band is determined primarily by the shape of the band pass filter of Fig. 13.

The band pass filter can be designed with a variable low-frequency cutoff that can be selected to conform to the prevailing clutter conditions. The selection of the lower cutoff might be at the option of the operator or it can be done adaptively. A variable lower cutoff might be advantageous when the width of the clutter spectrum changes with time as when the radar receives unwanted echoes from birds. A relatively wide notch at zero frequency is needed to remove moving birds. If the notch were set wide enough to remove the birds, it might be wider than necessary for ordinary clutter and desired targets might be removed. Since the appearance of birds varies with the time of day and the season, it is important that the width of the notch be controlled according to the local conditions.

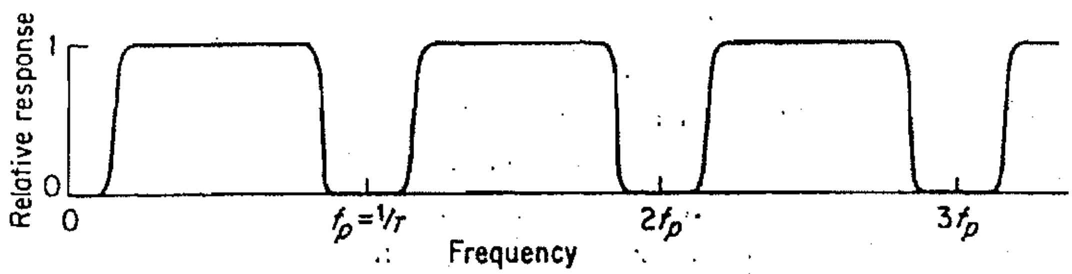


Figure 14: Frequency-response characteristic of an MTI using range gates and filters.

MTI radar using range gates and filters is usually more complex than an MTI with a single-delay-line canceller. The additional complexity is justified in those applications where good MTI performance and the flexibility of the range gates and filter MTI are desired. The better MTI performance results from the better match between the clutter filter characteristic and the clutter spectrum.

Limitations to MTI Performance:

The improvement in signal-to-clutter ratio of an MTI is affected by factors other than the design of the Doppler signal processor such as:

- Instabilities of the transmitter and receiver
- physical motions of the clutter
- Finite time on target (or scanning modulation)
- And limiting in the receiver

Before discussing these limitations, we shall study the related definitions.

Definitions related to MTI Performance:

MTI improvement factor: The signal-to-clutter ratio at the output of the MTI system divided by the signal-to-clutter ratio at the input, averaged uniformly over all target radial velocities of interest.

Sub clutter visibility: The ratio by which the target echo power may be weaker than the coincident clutter echo power and still be detected with specified detection and false alarm probabilities. All target radial velocities are assumed equally likely. A sub clutter visibility of, for example, 30 dB implies that a moving target can be detected in the presence of clutter even though the clutter echo power is 1000 times the target echo power. Two radars with the same sub clutter visibility might not have the same ability to detect targets in clutter if the resolution cell of one is greater than the other and accepts a greater clutter signal power. i.e., both radars might reduce the clutter power equally, but one starts with greater clutter power because its resolution cell is greater and "sees" more clutter targets.

Clutter visibility factor: The signal-to-clutter ratio, after cancellation or Doppler filtering that provides stated probabilities of detection and false alarm.

Clutter attenuation. The ratio of clutter power at the canceller input to the clutter residue at the output, normalized to the attenuation of a single pulse passing through the unprocessed channel of the canceller. (The ***clutter residue*** is the clutter power remaining at the output of a MTI system.)

The improvement factor (I): Is equal to the sub clutter visibility **(SCV)** times the clutter visibility factor **(V_{OC})**. In decibels, **I(dB) = SCV(dB) + Voc(dB)**. When the MTI is limited by noise like system instabilities, the clutter visibility factor should be chosen as is the signal to noise ratio as defined in Radar Equation.

Limitations:

Equipment instabilities: Pulse-to-pulse changes in the amplitude, frequency, or phase of the transmitter signal, changes in the ***Stalo*** or ***Coho*** oscillators in the receiver, jitter in the timing of the pulse transmission, variations in the time delay through the delay lines, and changes in the pulse width can cause the apparent frequency spectrum from perfectly stationary clutter to broaden and thereby lower the improvement factor of an MTI radar. The stability of the equipment in MTI radar must be considerably better than that of an ordinary radar. It can limit the performance of MTI radar if sufficient care is not taken in design, construction, and maintenance.

Internal fluctuation of clutter: Although clutter targets such as buildings, water towers, bare hills or mountains produce echo signals that are constant in both phase and amplitude as a function of time, there are many types of clutter that cannot be considered as absolutely stationary. Echoes from trees, vegetation, sea, rain, and chaff fluctuate with time, and these fluctuations can limit the performance of MTI radar. Because of its varied nature, it is difficult to describe precisely the clutter echo signal. However, for purposes of analysis, most fluctuating clutter targets may be represented by a model consisting of many independent scatterers located within the resolution cell of the radar. The echo at the radar receiver is the vector sum of the echo signals received from each of the individual scatterers i.e., the relative phase as well as the amplitude from each scatterer influences the resultant composite signal. If the individual scatters remain fixed from pulse to pulse, the resultant echo signal will also remain fixed. But any motion of the scatterers relative to the radar will result in different phase relationships at the radar receiver. Hence the phase and amplitude of the new resultant echo signal will differ pulse to pulse.

Antenna scanning modulation: As the antenna scans by a target, it observes the target for a finite time equal to : $t_o = n_B/f_P = \theta_B / \theta'_S$ where n_B = number of hits received, f_p = pulse repetition frequency, θ_B = antenna beam width and θ'_S = antenna scanning rate. The received pulse train of finite duration t_o has a frequency spectrum (which can be found by taking the Fourier transform of the waveform) whose width is proportional to $1/t_o$. Therefore, even if the clutter were perfectly stationary, there will still be a finite width to the clutter spectrum because of the finite time on target. If the clutter spectrum is too wide because the observation time is too short, it will affect the improvement factor. This limitation has sometimes been called ***scanning fluctuations or scanning modulation.***

Limiting in MTI Radar: A limiter is usually employed in the IF amplifier just before the MTI processor to prevent the residue from large clutter echoes from saturating the display. Ideally a MTI radar should reduce the clutter to a level comparable to receiver noise.
However, when the ***MTI improvement factor*** is not great enough to reduce the clutter sufficiently, the clutter residue will appear on the display and prevent the detection of aircraft targets whose cross sections are larger than the clutter residue. This condition may be prevented by setting the limit level L, relative to the noise N, equal to the MTI improvement factor I; or L/N = I. If the limit level relative to noise is set higher than the improvement factor. Clutter residue obscures part of the display. If it is set too low, there may be a "black hole" effect on the display. The limiter provides a constant false alarm rate (CFAR) and is essential to usable MTI Performance.
Unfortunately, nonlinear devices such as limiters have side-effects that can degrade performance. Limiters cause the spectrum of strong clutter to spread into the canceller pass- band, and result in the generation of additional residue that can significantly degrade MTI performance as compared with a perfect linear system.

Pulse Doppler Radar Vs MTI:

A Pulse radar that extracts the Doppler frequency shift for the purpose of detecting moving targets in the presence of clutter is either a **MTI Radar** or a **Pulse Doppler Radar.** The distinction between them is based on the fact that in a sampled measurement system like a pulse Radar, ambiguities arise in measuring both the Doppler frequency (relative velocity) and the Range (time delay). Range ambiguities are avoided with a **low** sampling rate (low pulse repetition frequency), and Doppler frequency ambiguities are avoided with a **high** sampling rate. However, in most radar applications the sampling rate, or pulse repetition frequency, cannot be selected to avoid both types of measurement ambiguities. Therefore, a compromise must be made and the nature of the compromise generally determines whether the radar is called an **MTI** or a **Pulse Doppler Radar**.

- MTI usually refers to a Radar in which the pulse repetition frequency is chosen low enough to avoid ambiguities in range (no multiple-time-around echoes) but with the consequence that the frequency measurement is ambiguous and results in blind speeds, **Eq. (8)**.
- The pulse Doppler radar, on the other hand, has a high pulse repetition frequency that avoids blind speeds, but it experiences ambiguities in range. It performs Doppler filtering on a single spectral line of the pulse spectrum.
- A radar which employs multiple pulse repetition frequencies to avoid blind speeds is usually classed as an MTI if it's average **PRF** would cause blind speeds. The justification for this

definition is that the technology and design philosophy of a multiple **PRF** radar are more like that of an MTI than a pulse Doppler radar.

- The pulse Doppler radar is more likely to use range-gated Doppler filter-banks than delay-line cancellers. Also, a power amplifier such as a klystron is more likely to be used than a power oscillator like the magnetron. A pulse Doppler radar operates at a higher duty cycle than does an MTI. Although it is difficult to generalize, the MTI radar seems to be the more widely used of the two, but pulse Doppler radar is usually more capable of reducing clutter.

Previous Years' Examination Questions:

1. Explain the following limitations of MTI radar.
   (a) Equipment instabilities.
   (b) Scanning modulation.
   (c) Internal fluctuation of clutter.
2. (a) Explain the function of time domain filter in a MTI Radar with an example.
   (b) A MTI radar operates at 10GHz with a PRF of 300 pps. Calculate the lowest blind speed?
3. (a) An MTI radar is operated at 9GHz with a PRF of 3000 pps. Calculate the first two lowest blind speeds for this radar. Derive the formula used.
   (b) Discuss the limitations of non-coherent MTI Radar systems. [12+4]
4. (a) Write the description of Range gated Doppler filters.
   (b) Explain the operation of MTI radar with 2 pulse repetition frequencies. [8+8]
5. (a) Draw and explain the frequency response characteristics of a MTI using Range gates and Filters.
   (b) A MTI Radar operates at frequency of 6Ghz with a PRF of 800 PPS . Calculate the lowest blind speeds of this Radar.
6. (a) Compare and contrast the situations with a Power amplifier and Power oscillator in the transmitter of a MTI system.
   (b) Calculate the blind speed for a Radar with the following specifications: Wave length: 0.1 Mtr. and PRF : 200 Hz
7. (a) Describe Range gated Doppler filters.
   (b) Differentiate blind phases from blind speeds.
   (c) Discuss the application of electrostatic storage tubes in MTI radar. [6+5+5]
8. (a) Briefly explain about range – gated Doppler filters.

   (b) Describe the importance of double cancellation.
9. (a) Compare MTI Radar with Pulse Doppler radar

   (b) Explain the function of a single delay line canceller and derive an expression for the frequency response function.
10. (a) What is an MTI Radar and how does it operate.
    (b) Define blind speed. A MTI radar operates at 5 Ghz with a PRF of 100PPS. Find the three lowest blind speeds of this Radar. Explain the importance of Staggered PRF. (8+7)