Code No: **R17A0023** 

## MALLA REDDY COLLEGE OF ENGINEERING & TECHNOLOGY (Autonomous Institution – UGC, Govt. of India) II B.Tech I Semester Supplementary Examinations, April 2023 Mathematics-III

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Roll No								

## Time: 3 hours

2

Max. Marks: 70

**Note:** This question paper Consists of 5 Sections. Answer **FIVE** Questions, Choosing ONE Question from each SECTION and each Question carries 14 marks.

## \*\*\* SECTION\_I

1 A Prove that 
$$\int_0^{\pi/2} \frac{d\theta}{\sqrt{\sin\theta}} \times \int_0^{\pi/2} \sqrt{\sin\theta} \, d\theta = \pi.$$
 [7M]

**B** Show that 
$$\Gamma\left(\frac{1}{2}\right)\Gamma(2n) = 2^{2n-1}\Gamma(n)\Gamma\left(n+\frac{1}{2}\right)$$
 [7M]

A (i) Evaluate 
$$\int_0^{\pi/2} \sqrt{\cot\theta} \,\mathrm{d}\theta$$
 [4M]

(ii) Prove that 
$$\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$$
 [3M]

**B** Show that [7M] 
$$\int_{a}^{b} (x-a)^{m} (b-x)^{n} dx = (b-a)^{m+n+1} \beta(m+1,n+1), m > 0, n > 0$$

3 A Prove that 
$$P_n(x) = \frac{1}{2^n n!} \frac{d^n}{dx^n} (x^2 - 1)^n$$
 [7M]

**B** Solve the differential equation [7M]  
$$xy'' + y' + \frac{1}{4}y = 0$$

4 A Prove that [7M]  

$$\int x J_0^2(x) dx \frac{1}{2} x^2 [J_0^2(x) + J_1^2(x)]$$
R Prove that (2n + 1) x P (x) = (n + 1) P (x) + n P (x) + n P (x)
[7M]

OR

**B** Prove that 
$$(2n+1)xP_n(x) = (n+1)P_{n+1}(x) + nP_{n-1}(x)$$
 [7M]

## **SECTION-III**

5 A Show that the function 
$$u = 2 \log(x^2 + y^2)$$
 is harmonic and find its [7M]  
harmonic conjugate.  
**B** E between in  $G$  where  $\int f e^{2z} dx$  is the function of  $f$  [7M]

Evaluate, using Cauchy's integral formula 
$$\oint \frac{e}{(z-1)(z-2)} dz$$
 where C is the circle  $|z| = 3$ .

6 A Evaluate 
$$\int_{(0,1)}^{(1,2)} (x^2 - y) dx + (y^2 + x) dy$$
 along a straight line from (0,1) [7M]  
to (1,2)

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	B	If $f(z) = u + iv$ is an analytic function and $u - v = \frac{\cos x + \sin x - e^{-y}}{2\cos x - e^{y} - e^{-y}}$ .	[7M]
		Find $f(z)$ subject to the condition $f\left(\frac{\pi}{2}\right) = 0$	
7	A	Expand the Laurent's series of $\frac{z^{2-1}}{(z+2)(z+3)}$ about z=0 in the region 2< z <3.	[10M]
	B	Expand log z by Taylor's series about z=1	[4M]
8	A	Find the residues of $\frac{z^2-2z}{(z-1)^2(z^2+1)}$ .	[5M]
	В	Prove that $\int_{-\infty}^{\infty} \frac{x^2 dx}{(x^2 + a^2)(x^2 + b^2)} = \frac{\pi}{a + b}, \ a > 0, b > 0, a \neq b.$ SECTION-V	[9M]
9	A	Find the image of the circle $ z  = 2$ under the transformation $w=3z$	[4M]
	B	Determine the bilinear transformation which maps $z = 0, -1, 2i$ into $w = 5i, \infty, -i/3$	[10M]
10		OR	[ <b>//</b> ]
10	A	Show that the transformation $w = \frac{1}{z}$ transforms the hyperbola $x^2 - y^2 = 1$	
	B	to lemniscate. Let $f(z)$ be analytic function of z in a domain of the z-plane and let	[7M]
		$f'(z) \neq 0$ in D. Then show that $w = f(z)$ is a conformal mapping at all	
		points of D	

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