II B.Tech I Semester Supplementary Examinations, April 2023 Probability Theory and Stochastic Process
(ECE)

| Roll No |  |  |  |  |  |  |  |  |  |  |
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## Time: 3 hours

Max. Marks: 70
Note: This question paper Consists of 5 Sections. Answer FIVE Questions, Choosing ONE Question from each SECTION and each Question carries 14 marks.

## SECTION-I

1 A Two coins are tossed 500 times, and we get:
Two heads: 105 times
One head: 275 times
No head: 120 times
Find the probability of each event to occur.
B One card is drawn from a deck of 52 cards, well-shuffled. Calculate the
OR
$2 \boldsymbol{A}$ Two coins are tossed once. Find a sample space.
$\boldsymbol{B}$ Consider the experiment of rolling a die. Let A be the event 'getting a prime
number', B be the event 'getting an odd number'. Write the sets representing the events.

## SECTION-II

$3 \quad \boldsymbol{A} \quad$ The pdf of a random variable X is given by

$$
f_{X}(x)=\begin{array}{cl}
\frac{1}{b} e^{-\left(\frac{x-a}{b}\right)}, & x \geq a \\
0, & x<a
\end{array}
$$

Demonstrate i) mean value $\mathrm{E}[\mathrm{X}]$ ii) mean square value $\mathrm{E}\left[\mathrm{X}^{2}\right]$

$$
\text { iii) variance } \sigma^{2}
$$

$\boldsymbol{B}$ If the probability density of a random variable is given by

$$
f_{x}(x)=K\left(1-x^{2}\right) ; 0<x<1
$$

Solve the value of $K$ and $F_{X}(x)$.

## OR

$4 \quad \boldsymbol{A} \quad$ Explain the significance of skew $\left(\mu_{3}\right)$ and skewness of single random

B Develop the moment generating function for Gaussian random variable ' X ' and show that its mean is $m_{x}$ and variance is $\sigma_{X}^{2}$

SECTION-III
5 A Make use of the property of a valid joint probability density function(pdf), find a value of the constant ' b ' where the joint pdf $f_{x y}(x, y)=b x y^{2} e^{-2 x y} u(x-2) u(y-1)$ is a valid joint probability density.
$\boldsymbol{B} \quad$ The joint density of two random variables X and Y is
$f_{x y}(x, y)=0.1 \delta(x) \delta(y)+0.12 \delta(x-4) \delta(y)+0.05 \delta(x) \delta(y-1)+$
$0.25 \delta(x-2) \delta(y-1)+0.3 \delta(x-2) \delta(y-3)+0.18 \delta(x-4) \delta(y-3)$
and plot the marginal densities and distributions of X and Y .
OR
$6 \quad \boldsymbol{A} \quad$ Develop the probability density function $\left(f_{w}(w)\right)$ of sum of two statistically independent random variables $(W=X+Y)$.
$\boldsymbol{B}$ Inspect about Joint central moments $\left(\mu_{n k}\right)$ and analyse the various conditions of correlation $\left(R_{X Y}\right)$ of two random variables $\mathrm{X} \& \mathrm{Y}$.

## SECTION-IV

$7 \quad \boldsymbol{A} \quad$ Define Random Process and Explain its classification with neat sketches.
$\boldsymbol{B}$ Show that the random process $\mathrm{X}(\mathrm{t})=\mathrm{A} \cos \left(\omega_{0} t+\theta\right)$ is wide sense
stationary(WSS), if it is assumed that $\mathrm{A} \& \omega_{0}$ are constants and $\theta$ is uniformly distributed random variable on the interval $[0,2 \Pi]$.

## OR

$8 \quad \boldsymbol{A}$ Compare auto-correlation and cross-correlation properties.
$\boldsymbol{B}$ Determine the average power of the random process, $X(t)=A_{0} \cos \left(\omega_{0} t+\theta\right)$,
where $A_{0}, \omega_{0}$ are real constants and $\theta$ is a random variable uniformly distributed on the interval $(0, \pi / 2)$.

## SECTION-V

$9 \quad \boldsymbol{A} \quad$ Analyse the mean square value of the response of an LTI system having white noise at its input.
B $\quad \mathrm{X}(\mathrm{t})$ be the white Gaussian noise with $S_{X X}(\omega)=\frac{N_{0}}{2}$. Assume that $\mathrm{X}(\mathrm{t})$ is input to an LTI system with $h(t)=e^{-t} u(t), \mathrm{Y}(\mathrm{t})$ be the output then Determine the following:
(i) $S_{Y Y}(\omega)$
(ii) $R_{Y Y}(\tau)$
(iii) $E\left[y^{2}(t)\right]$

## OR

$10 \boldsymbol{A}$ Determine the cross-correlation function corresponding to the cross-power
density spectrum $S_{X Y}(\omega)=\frac{8}{(\alpha+j \omega)^{3}}$, where $\alpha>0$ is a constant.
B Find the noise bandwidth of a system having the power transfer function
$|H(\omega)|^{2}=\frac{1}{\left[1+\left(\frac{\omega}{\omega_{0}}\right)^{4}\right]}$ Where $\omega_{0}$ is a real constant and $|\mathrm{H}(0)|=1$.

