R-22

MRCET CAMPUS

MALLA REDDY COLLEGE OF ENGINEERING & TECHNOLOGY

(AUTONOMOUS INSTITUTION – UGC, GOVT. OF INDIA)



DEPARTMENT of AERONAUTICAL ENGINEERING

AIRCRAFT STABILITY AND CONTROL

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Department of ANE

AIRCRAFT STABILITY AND CONTROL

DIGITAL NOTES B.TECH (R-22 Regulation)(III YEAR – I SEM) (2025-2026)

DEPARTMENT OF AERONAUTCAL ENGINEERING

MALLA REDDY COLLEGE OF ENGINEERING &TECHNOLOGY

(Autonomous Institution – UGC, Govt. of India)

Recognized under 2(f) and 12 (B) of UGC ACT 1956

(Affiliated to JNTUH, Hyderabad, Approved by AICTE - Accredited by NBA & NAAC – 'A' Grade - ISO 9001:2015 Certified)

Maisammaguda, Dhulapally (Post Via. Hakimpet), Secunderabad – 500100, Telangana State, India

DEPARTMENT OF AERONAUTICAL ENGINEERING

Vision

- Department of Aeronautical Engineering aims to be indispensable source in Aeronautical Engineering which has a zeal to provide the value driven platform for the students to acquire knowledge and empower themselves to shoulder higher responsibility in building a strong nation.

Mission

- The primary mission of the department is to promote engineering education and research. To strive consistently to provide quality education, keeping in pace with time and technology. Department passions to integrate the intellectual, spiritual, ethical, and social development of the students for shaping them into dynamic engineers.

QUALITY POLICY

- Impart up-to date knowledge to the students in Aeronautical area to make them quality engineers. Make the students experience the applications on quality equipment and tools. Provide systems, resources, and training opportunities to achieve continuous improvement. Maintain global standards in education, training, and services.

PROGRAM OUTCOMES
(PO's)

Engineering Graduates will be able to:

- Engineering knowledge: Apply the knowledge of mathematics, science, engineering fundamentals, and an engineering specialization to the solution of complex engineering problems.
- Problem analysis: Identify, formulate, review research literature, and analyze complex engineering problems reaching substantiated conclusions using first principles of mathematics, natural sciences, and engineering sciences.
- Design / development of solutions: Design solutions for complex engineering problems and design system components or processes that meet the specified needs with appropriate consideration for the public health and safety, and the cultural, societal and environmental considerations.
- Conduct investigations of complex problems: Use research-based knowledge and research methods including design of experiments, analysis and interpretation of data, and synthesis of the information to provide valid conclusions.
- Modern tool usage: Create, select, and apply appropriate techniques, resources, and modern engineering and IT tools including prediction and modeling to complex engineering activities with an understanding of the limitations.
- The engineer and society: Apply reasoning informed by the contextual knowledge to assess societal, health, safety, legal and cultural issues and the consequent responsibilities relevant to the professional engineering practice.
- Environment and sustainability: Understand the impact of the professional engineering solutions in societal and environmental contexts, and demonstrate the knowledge of, and need for sustainable development.
- Ethics: Apply ethical principles and commit to professional ethics and

Responsibilities and norms of the engineering practice. Individual and team work: Function effectively as an individual, and as a member or leader in diverse teams, and in multidisciplinary settings.

- Communication: Communicate effectively on complex engineering activities with the engineering community and with society at large, such as, being able to comprehend and write effective reports and design documentation, make effective presentations, and give and receive clear instructions.
- Project management and finance: Demonstrate knowledge and understanding of the engineering and management principles and apply these to one's own work, as a member and leader in a team, to manage projects and in multi disciplinary environments.
- Life- long learning: Recognize the need for, and have the preparation and ability to engage in independent and life-long learning in the broadest context of technological change.

PROGRAM EDUCATIONAL OBJECTIVES – Aeronautical Engineering

- PEO1 (PROFESSIONALISM & CITIZENSHIP): To create and sustain a community of learning in which students acquire knowledge and learn to apply it professionally with due consideration for ethical, ecological and economic issues.

- PEO2 (TECHNICAL ACCOMPLISHMENTS): To provide knowledge based services to satisfy the needs of society and the industry by providing hands on experience in various technologies in core field.

- PEO3 (INVENTION, INNOVATION AND CREATIVITY): To make the students to design, experiment, analyze, and interpret in the core field with the help of other multi disciplinary concepts wherever applicable.

- PEO4 (PROFESSIONAL DEVELOPMENT): To educate the students to disseminate research findings with good soft skills and become a successful entrepreneur.

- PEO5 (HUMAN RESOURCE DEVELOPMENT): To graduate the students in building national capabilities in technology, education and research

PROGRAM SPECIFIC OUTCOMES – Aeronautical Engineering

- To mould students to become a professional with all necessary skills, personality and sound knowledge in basic and advance technological areas.

- To promote understanding of concepts and develop ability in design manufacture and maintenance of aircraft, aerospace vehicles and associated equipment and develop application capability of the concepts sciences to engineering design and processes.

- Understanding the current scenario in the field of aeronautics and acquire ability to apply knowledge of engineering, science and mathematics to design and conduct experiments in the field of Aeronautical Engineering.

- 4. To develop leadership skills in our students necessary to shape the social, intellectual, business and technical worlds.

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3	**UNIT 3: Lateral ,Directional & Dynamic Stability-Response To Control**	
4	**UNIT 4: Control System Modeling and Feedback Control**	
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MALLA REDDY COLLEGE OF ENGINEERING AND TECHNOLOGY

III Year B.Tech. ANE- I Sem **L/T/P/C**
2/1/-/3

(R22A2113) Aircraft Stability and Control

Objectives:

1. To understand the concepts of stability and control of aircraft.
2. To familiarize with various Aircraft motions and related stability
3. Analyze the different type of modes in longitudinal, lateral and directional motion of aircraft, and recovery from those modes.
4. Understand Method of mathematical modeling of physical systems
5. Review Performance analysis and design of aircraft control system

UNIT – I: Stability, Control Requirements &Static Longitudinal Stability and Control: Degree of freedom of a system -Static and dynamic stability -Purpose of controls -Inherently and marginally stable airplanes. **Stick Fixed**-Basic equilibrium equation-stability criterion-contribution of wing, tail, fuselage-the most aft center of gravity, Power effects-elevator power, stick fixed neutral point-**stick free stability**-Hinge moment coefficient-stick free neutral points-maneuvers-stick force gradients-stick force per'g'-Aerodynamic balancing.

UNIT – II: **Aircraft Equations of motion- Perturbed Motion- Linearized, Decoupled Equations**: Aircraft Equations of Motion (EOM), Aircraft Position and Orientation, Stability-Frame andBody-Frame, Euler's Equations, Small Disturbance Theory and Linearization of EOM, Decoupling into longitudinal and lateral-directional motions- conditions for validity- role of symmetry.

UNIT – III: **Lateral ,Directional & Dynamic Stability-Response To Control**: Lateral and directionalstability-definition, static directional stability rudder fixed , directional control, stick – free directional stability, dihedral effect and lateral control, estimation of airplane dihedral effect, lateral control introduction ,estimation of lateral control power, Adverse yaw , aileron control forces.

Solutions to the stability quartic of the linearized equations of motion, the principal mode-phugoid, short period, Dutch roll and spiral modes-further approximations, restricted degrees of motion-solutions, response to controls, auto rotation and spin.

UNIT – IV: **Control System Modeling and Feedback Control**

Basic components of control system, open loop system, closed loop system, Types. Reduction of block diagrams - rules and conventions. Stability analysis- Routh Hurwitz, Bode Plot, Polar plot, - determination of gain margin and phase margin.

UNIT - V: Design of Aircraft Controller, Stability and Control Augmentation & Auto Pilots

Design of Stability Augmentation System (SAS) using displacement & rate feed-back, Control augmentation system, Full authority fly-by-wire control, need for automatic control. Auto pilots- purpose, functioning, displacement auto pilot, pitch, yaw, bank, altitude and velocity hold auto pilot.

Text Books:

1. Yechout, T. R. et al., Introduction to Aircraft Flight Mechanics, AIAA education Series,2003, ISBN 1-56347-577-4.
2. Airplane performance stability and control by Courtland D.Perkins ,Robert E.Hage Johnwiley& sons.

Reference Books:

1. Etkin, B. and Reid, L. D., Dynamics of Flight, 3rd Edition. John Wiley, 1998, ISBN 0-47103418-5.
2. Schmidt, L. V., Introduction to Aircraft Flight Dynamics, AIAA Education Series, 1998,ISBN A-56347-226-0.
3. McCormick, B. W., Aerodynamics, Aeronautics and Flight Mechanics, 2nd Edition., WileyIndia, 1995, ISBN 978-]
4. Nelson, R. C., Flight Stability and Automatic Control, 2nd Edition., Tata Mc Graw Hill,2007, ISBN 0-07-066110-3.

Outcomes:

1. An understanding of the static stability of aircraft and describe stick fixed and stick freeneutral point
2. Can able to linearize equations of motion
3. Able to understand the dihedral effect and adverse yaw and understands about thelongitudinal modes and lateral –directional modes
4. The student should be able to model and analyze control system components.
5. Identify different methods toanalyze the plot the stability and margins

Code No: R20A2112 **R20**

MALLA REDDY COLLEGE OF ENGINEERING & TECHNOLOGY

(Autonomous Institution – UGC, Govt. of India)

III B.Tech II Semester Regular Examinations, May 2023

Aircraft Stability and Control

(AE)

Roll No										

Time: 3 hours **Max. Marks: 70**

Note: This question paper Consists of 5 Sections. Answer **FIVE** Questions, Choosing ONE Question from each SECTION and each Question carries 14 marks.

***

SECTION-I

1 Drive an expression for the tail contribution to the pitching moment of an aircraft (assume i_{t1} is the wing setting angle and i_{t2} is the tail setting angle). **[14M]**

OR

2 Derive the equations of motion of a rigid body. State the underlying law of physics. Use the suitable diagram labeling the forces and moments on an airplane. **[14M]**

SECTION-II

3 a) Define static and dynamic stability **[7M]**
b) Explain about inherently and marginally stable airplanes. **[7M]**

OR

4 Define sideslip. Explain the criterion for lateral static stability. **[14M]**

SECTION-III

5 Explain the directional stability contribution of vertical tail with equations. **[14M]**

OR

6 a) Explain about lateral and directional stability. **[7M]**
b) Explain in detail dihedral effect and adverse yaw **[7M]**

SECTION-IV

7 What are the basic components of control system? Write short notes about it. **[14M]**

OR

8 List and briefly about rules and conventions in *Control System Modelling and Feedback Control.* **[14M]**

SECTION-V

9 Explain briefly about
(i) Full authority fly-by-wire control, **[7M]**
(ii) Need for *automatic control*. **[7M]**

OR

10 Explain briefly about the purpose, functioning of auto pilot. **[14M]**

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R18

Code No: R18A2112

MALLA REDDY COLLEGE OF ENGINEERING & TECHNOLOGY

(Autonomous Institution – UGC, Govt. of India)

III B.Tech I Semester Supplementary Examinations, June 2022

Aircraft Stability and Control

(AE)

Roll No										

Time: 3 hours **Max. Marks: 70**

Answer Any **Five** Questions
All Questions carries equal marks.

***

1		Explain the wing contribution towards static longitudinal stability.	**[14M]**
2		Derive an expression for stick fixed neutral point of an airplane.	**[14M]**
3		How Earth Axis to Body Axis transformation takes place? Show Euler's angles and explain.	**[14M]**
4		Explain about small perturbation approach. How it is useful in linearization of EOM?	**[14M]**
5		Explain the phenomenon of Dutch roll, using all the aerodynamic coefficients and aircraft parameters influencing the motion.	**[14M]**
6		Discuss qualitatively, various ways of improving the lateral and directional stability of an aircraft.	**[14M]**
7	i.	Explain closed loop control systems with three examples.	**[7M]**
	ii.	What are the Basic components of control system?	**[7M]**
8		Explain the concept of Autopilot in detail.	**[14M]**

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R18

Code No: R18A2112

MALLA REDDY COLLEGE OF ENGINEERING & TECHNOLOGY

(Autonomous Institution – UGC, Govt. of India)

III B.Tech I Semester Supplementary Examinations, July/August 2021

Aircraft Stability and Control

(AE)

Roll No										

Time: 3 hours **Max. Marks: 70**

Answer Any **Five** Questions
All Questions carries equal marks.

***

1 Explain about Static Margin and derive an expression of $\frac{dCm}{dCL}$ for stick fixed condition with neat sketches. **[14M]**

2 What are the purpose for which the maneuver performance of an aircraft to be limited by the structural strength of the airframe? Explain about the determination of stick-free neutral point by flight test. **[14M]**

3 Explain about aircraft axis systems in detail with neat sketches. **[14M]**

4 Discuss the dynamic stability aspects of an aircraft considering its linearized longitudinal equations of motion being analyzed under three degrees of freedom system. **[14M]**

5 An airplane is in phugoid mode. Assume that the damping of the oscillation is so small that it can be neglected. The altitude of aircraft is oscillating between 1035 m &1000 m. The speed is observed to be 48 m/s at the altitude 1035 m. Estimate the speed at 1000 m altitude. **[14M]**

6 Justify the statement that in longitudinal Dynamic stability- "The sideways velocity and the angular velocities in roll and yaw are always zero". **[14M]**

7 a) Explain open loop and closed loop control systems with examples. **[7M]**
b) Explain about determination of gain margin and phase margin **[7M]**

8 Explain in details about the Design of Stability Augmentation System (SAS) using displacement & rate feed-back. Draw neat sketch of it. **[14M]**

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PRINCIPLES OF STABILITY AND CONTROL

INTRODUCTION

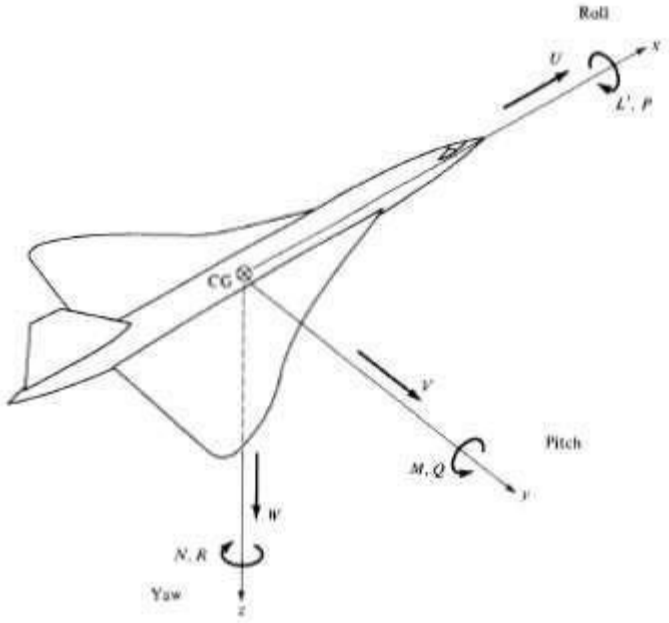


Figure 7.2 Definition of the airplane's axes along with the translational and rotational motion along and about these axes.

Airplane performance, as discussed in Chap. 6, is governed by forces (along and perpendicular to the flight path), with the translational motion of the airplane as a response to these forces. In contrast, airplane stability and control, discussed in the present chapter, are governed by moments about the center of gravity, with the rotational motion of the airplane as a response to these moments. Therefore, moments and rotational motion are the main focus of this chapter.

Consider an airplane in flight, as sketched in Figure 7.2. The center of gravity (the point through which the weight of the complete airplane effectively acts) is denoted as cg. The xyz orthogonal axis system is fixed relative to the airplane; the x axis is along the fuselage, the y axis is along the wingspan perpendicular to the x axis, and the z axis is directed downward, perpendicular to the xy plane. The origin is at the center of gravity. The translational motion of the airplane is given by the velocity components U, V, and W along the x, y, and z directions, respectively. (Note that the resultant freestream velocity V_∞ is the vector sum of U, V, and W.) The rotational motion is given by the angular velocity components P, Q, and R about the x, y, z axes, respectively. These rotational velocities are due to the moments L', M, and N about the x, y, and z axes, respectively. (The prime is put over the symbol "L" so that the reader avoids confusing it with lift.) Rotational motion about the x axis is called *roll*; L' and P are the *rolling*

moment and velocity, respectively. Rotational motion about the y axis is called *pitch*; M and Q are the *pitching* moment and velocity, respectively. Rotational motion about the z axis is called *yaw*; N and R are the *yawing* moment and velocity, respectively.

There are three basic controls on an airplane—the ailerons, elevator, and rudder—which are designed to change and control the moments about the x, y, and z axes. These control surfaces are shown in Figure 7.3; they are flaplike surfaces that can be deflected back and forth at the command of the pilot. The ailerons are mounted at the trailing edge of the wing, near the wingtips. The elevators are located on the horizontal stabilizer. In some modern aircraft, the complete horizontal stabilizer is rotated instead of just the elevator (so-called flying tails). The rudder is located on the vertical stabilizer, at the trailing edge. Just as in the case of wing flaps discussed in Sec. 5.17, a downward deflection of the control surface will increase the lift of the wing or tail. In turn, the moments will be changed, as sketched in Figure 7.4. Consider Figure 7.4*a*. One aileron is deflected up, the other down, creating a differential lifting force on the wings, thus contributing to the rolling moment L'. In Figure 7.4*b*, the elevator is deflected

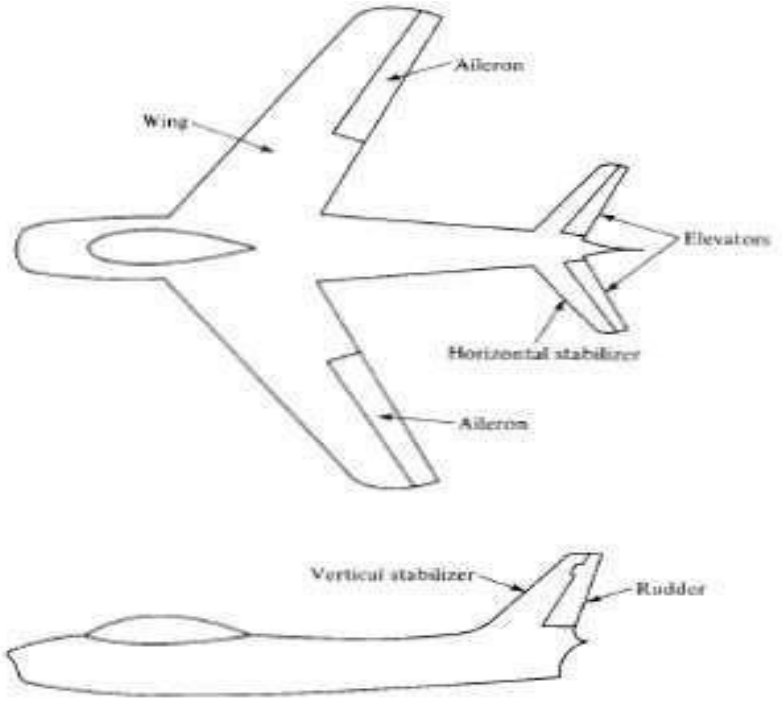


Figure 7.3 Some airplane nomenclature.

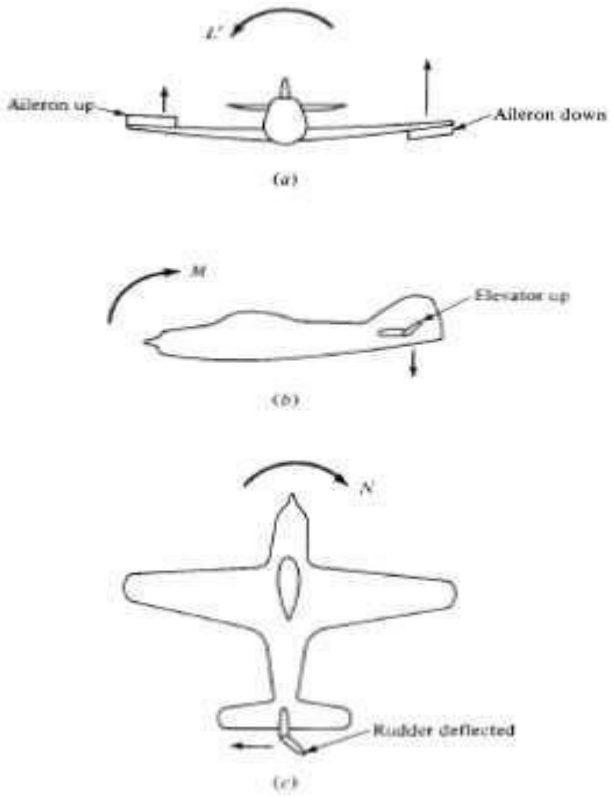


Figure 7.4 Effect of control deflections on roll, pitch, and yaw. (*a*) Effect of aileron deflection; lateral control. (*b*) Effect of elevator deflection; longitudinal control. (*c*) Effect of rudder deflection; directional control.

upward, creating a negative lift at the tail, thus contributing to the pitching moment M. In Figure 7.4*c*, the rudder is deflected to the right, creating a leftward aerodynamic force on the tail, thus contributing to the yawing moment N.

Rolling (about the x axis) is also called *lateral motion*. Referring to Figure 7.4*a*, we see that ailerons control roll; hence they are known as *lateral controls*. Pitching (about the y axis) is also called *longitudinal motion*. In Figure 7.4*b*, we see that elevators control pitch; hence they are known as *longitudinal controls*. Yawing (about the z axis) is also called *directional motion*. Figure 7.4*c* shows that the rudder controls yaw; hence it is known as the *directional control*.

All the above definitions and concepts are part of the basic language of airplane stability and control; they should be studied carefully. Also, in the process, the following question becomes apparent: what is meant by the words "stability and control" themselves? This question is answered in the next section.

There are two types of stability: static and dynamic. They can be visualized as follows.

A Static Stability

Consider a marble on a curved surface, such as a bowl. Imagine that the bowl is upright and the marble is resting inside, as shown in Figure 7.5*a*. The marble is stationary; it is in a state of *equilibrium*, which means that the moments acting on the marble are zero. If the marble is now disturbed (moved to one side, as shown by the dotted circle in Figure 7.5*a*) and then released, it will roll back toward the bottom of the bowl, i.e., toward its original equilibrium position. Such a system is *statically* stable. In general, we can state that

> *If the forces and moments on the body caused by a disturbance tend initially to return the body toward its equilibrium position, the body is statically stable. The body has positive static stability.*

Now, imagine the bowl is upside-down, with the marble at the crest, as shown in Figure 7.5*b*. If the marble is placed precisely at the crest, the moments will be zero and the marble will be in equilibrium. However, if the marble is now

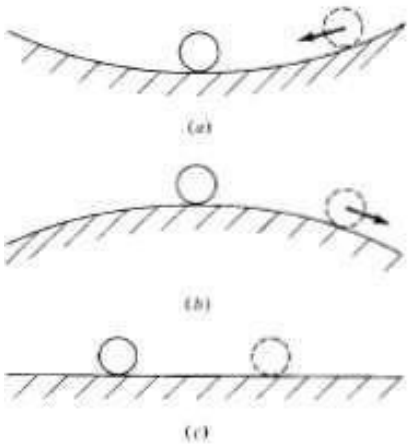


Figure 7.5 Illustration of static stability. (*a*) Statically stable system. (*b*) Statically unstable system. (*c*) Statically neutral system.

disturbed (as shown by the dotted circle in Figure 7.5*b*), it will tend to roll down the side, away from its equilibrium position. Such a system is *statically unstable*. In general, we can state that

> *If the forces and moments are such that the body continues to move* away *from its equilibrium position after being disturbed, the body is* statically unstable. *The body has* negative *static stability.*

Finally, imagine the marble on a flat horizontal surface, as shown in Figure 7.5*c*. Its moments are zero; it is in equilibrium. If the marble is now disturbed to another location, the moments will still be zero, and it will still be in equilibrium. Such a system is *neutrally stable*. This situation is rare in flight vehicles, and we will not be concerned with it here.

Emphasis is made that static stability (or the lack of it) deals with the *initial* tendency of a vehicle to return to equilibrium (or to diverge from equilibrium) after being disturbed. It says nothing about whether it ever reaches its equilibrium position, nor how it gets there. Such matters are the realm of dynamic stability, as follows.

B Dynamic Stability

Dynamic stability deals with the *time history* of the vehicle's motion after it initially responds to its static stability. For example, consider an airplane flying at an angle of attack α_e such that its moments about the center of gravity are zero. The airplane is therefore in equilibrium at α_e; in this situation, it is *trimmed*, and α_e is called the trim angle of attack. Now assume that the airplane is disturbed (say, by encountering a wind gust) to a new angle of attack α as shown in Figure 7.6. The airplane has been pitched through a *displacement* $\alpha - \alpha_e$. Now, let us observe the subsequent pitching motion after the airplane has been disturbed by the gust. We can describe this motion by plotting the instantaneous displacement vs. time, as shown in Figure 7.7. Here $\alpha - \alpha_e$ is given as a function of time t. At $t = 0$, the displacement is equal to that produced by the gust. If the airplane is

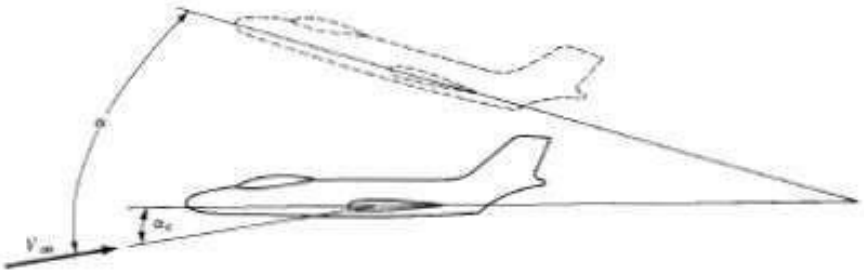


Figure 7.6 Disturbance from the equilibrium angle of attack.

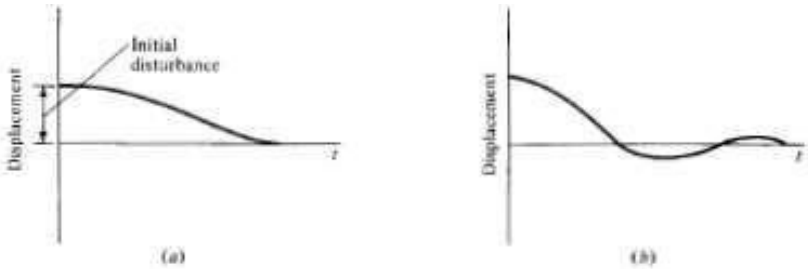


Figure 7.7 Examples of dynamic stability. (*a*) Aperiodic; (*b*) Damped oscillations.

statically stable, it will *initially* tend to move back toward its equilibrium position, that is, $\alpha - \alpha_e$ will initially decrease. Over a lapse of time, the vehicle may monotonically "home-in" to its equilibrium position, as shown in Figure 7.7*a*. Such motion is called aperiodic. Alternately, it may first overshoot the equilibrium position and approach α_e after a series of oscillations with decreasing amplitude, as shown in Figure 7.7*b*. Such motion is described as damped oscillations. In both situations, Figures 7.7*a* and 7.7*b*, the airplane eventually returns to its equilibrium position after some interval of time. These two situations are examples of *dynamic stability* in an airplane. Thus, we can state that

> *A body is dynamically stable if, out of its own accord, it eventually returns to and remains at its equilibrium position over a period of time.*

On the other hand, after initially responding to its static stability, the airplane may oscillate with increasing amplitude, as shown in Figure 7.8. Here, the equilibrium position is never maintained for any period of time; the airplane in this case is *dynamically unstable* (even though it is statically stable). Also, it is theoretically possible for the airplane to pitch back and forth with constant-amplitude oscillations. This is an example of a *dynamically neutral* body; such a case is of little practical interest here.

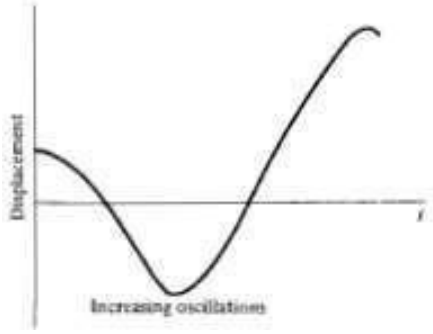


Figure 7.8 An example of dynamic instability.

C Control

The conventional control surfaces (elevators, ailerons, and rudder) on an airplane were discussed in Sec. 7.1 and sketched in Figures 7.3 and 7.4. Their function is usually (1) to change the airplane from one equilibrium position to another and (2) to produce nonequilibrium accelerated motions such as maneuvers. The study of the *deflections* of the ailerons, elevators, and rudder necessary to make the airplane do what we want and of the amount of *force* that must be exerted by the pilot (or the hydraulic boost system) to deflect these controls is part of a discipline called "airplane control," to be discussed later in this chapter.

CRITERIA FOR LONGITUDINAL STATIC STABILITY

Static stability and control about all three axes shown in Figure 7.2 is usually a necessity in the design of conventional airplanes. However, a complete description of all three types—lateral, longitudinal, and directional static stability and control (see Figure 7.4)—is beyond the scope of this book. Rather, the intent here is to provide only the flavor of stability and control concepts, and to this end only the airplane's longitudinal motion (pitching motion about the y axis) will be considered. This pitching motion is illustrated in Figure 7.4*b*. It takes place in the plane of symmetry of the airplane. Longitudinal stability is also the most important static stability mode; in airplane design, wind-tunnel testing, and flight research, it usually earns more attention than lateral or directional stability.

Consider a rigid airplane with fixed controls, e.g., the elevator in some fixed position. Assume the airplane has been tested in a wind tunnel or free flight and that its variation of M_{cg} with angle of attack has been measured. This variation is illustrated in Figure 7.12, where $C_{M,cg}$ is sketched versus α_a. For many conventional airplanes, the curve is nearly linear, as shown in Figure 7.12. The value of $C_{M,cg}$ at zero lift (where $\alpha_a = 0$) is denoted by $C_{M,0}$. The value of α_a where

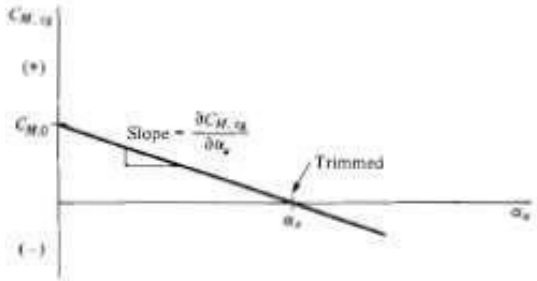


Figure 7.12 Moment coefficient curve with a negative slope.

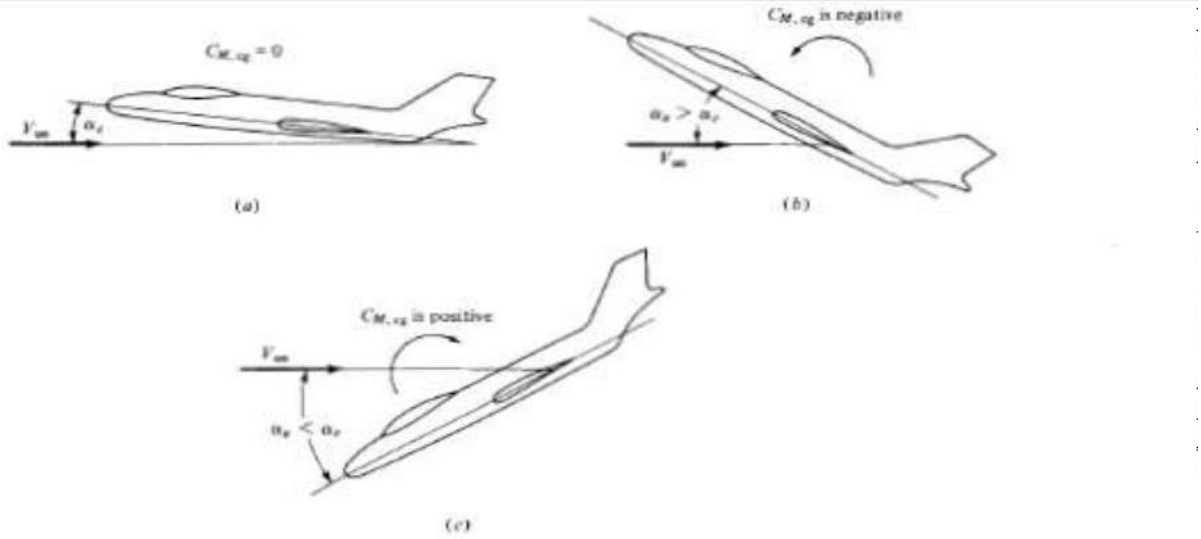


Figure 7.13 Illustration of static stability: (*a*) Equilibrium position (trimmed). (*b*) Pitched upward by disturbance. (*c*) Pitched downward by disturbance. In both *b* and *c*, the airplane has the initial tendency to return to its equilibrium position.

$M_{cg} = 0$ is denoted by α_e; as stated in Sec. 7.3, this is the equilibrium, or trim, angle of attack.

Consider the airplane in steady, equilibrium flight at its trim angle of attack α_e as shown in Figure 7.13*a*. Suddenly, the airplane is disturbed by hitting a wind gust, and the angle of attack is momentarily changed. There are two possibilities: an increase or a decrease in α_a. If the airplane is pitched upward, as shown in Figure 7.13*b*, then $\alpha_a > \alpha_e$. From Figure 7.12, if $\alpha_a > \alpha_e$, the moment about the center of gravity is negative. As discussed in Sec. 5.4, a negative moment (by convention) is counterclockwise, tending to pitch the nose downward. Hence, in Figure 7.13*b*, the airplane will initially tend to move back toward its equilibrium position after being disturbed. On the other hand, if the plane is pitched downward by the gust, as shown in Figure 7.13*c*, then $\alpha_a < \alpha_e$. From Figure 7.12, the resulting moment about the center of gravity will be positive (clockwise) and will tend to pitch the nose upward. Thus, again we have the situation where the airplane will initially tend to move back toward its equilibrium position after being disturbed. From Sec. 7.2, this is precisely the definition of static stability. Therefore, we conclude that an airplane which has a $C_{M,cg}$ versus α_a variation like that shown in Figure 7.12 is *statically stable*. Note from Figure 7.12 that $C_{M,0}$ is positive and that the slope of the curve, $\partial C_{M,cg}/\partial \alpha_a$, is negative. Here, the partial derivative, defined in Sec. 7.2 D, is used for the slope of the moment coefficient curve. This is because (as we shall see) $C_{M,cg}$ depends on a number of

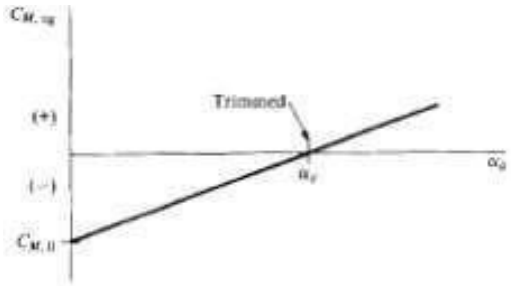


Figure 7.14 Moment coefficient curve with a positive slope.

other variables in addition to α_a, and therefore it is mathematically proper to use $\partial C_{M,cg}/\partial \alpha_a$ rather than $dC_{M,cg}/d\alpha_a$ to represent the slope of the line in Figure 7.12. As defined in Sec. 7.2 D, $\partial C_{M,cg}/\partial \alpha_a$ symbolizes the instantaneous rate of change of $C_{M,cg}$ with respect to α_a, with all other variables held constant.

Consider now a different airplane, with a measured $C_{M,cg}$ variation as shown in Figure 7.14. Imagine the airplane is flying at its trim angle of attack α_e as shown in Figure 7.15*a*. If it is disturbed by a gust, pitching the nose upward, as shown in Figure 7.15*b*, then $\alpha_a > \alpha_e$. From Figure 7.14 this results in a positive (clockwise) moment, which tends to pitch the nose even further away from its

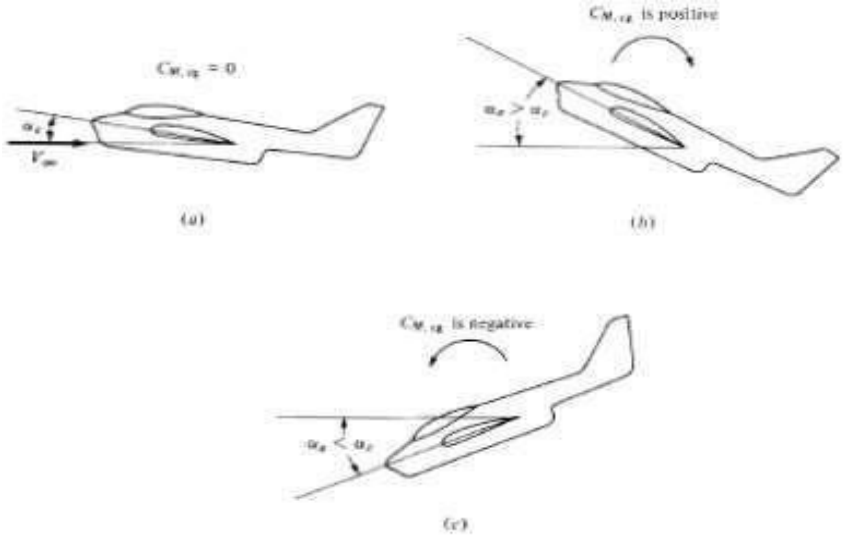


Figure 7.15 Illustration of static instability. (*a*) Equilibrium position (trimmed). (*b*) Pitched upward by disturbance. (*c*) Pitched downward by disturbance. In both *b* and *c*, the airplane has the initial tendency to diverge farther away from its equilibrium position.

equilibrium position. Similarly, if the gust pitches the nose downward (Figure 7.15*c*), a negative (counterclockwise) moment results, which also tends to pitch the nose further away from its equilibrium position. Therefore, because the airplane always tends to diverge from equilibrium when disturbed, it is *statically unstable*. Note from Figure 7.14 that $C_{M,0}$ is negative and $\partial C_{M,cg}/\partial \alpha_a$ is positive for this airplane.

For both airplanes, Figures 7.12 and 7.14 show a positive value of α_e. Recall from Figure 6.4 that an airplane moves through a range of angle of attack as it flies through its velocity range from V_{stall} (where α_a is the largest) to V_{max} (where α_a is the smallest). The value of α_e must fall within this flight range of angle of attack, or else the airplane cannot be trimmed for steady flight. (Remember that we are assuming a fixed elevator position: we are discussing "stick-fixed" stability.) When α_e does fall within this range, the airplane is *longitudinally balanced*.

From the above considerations, we conclude the following.

The necessary criteria for longitudinal balance and static stability are:

1. $C_{M,0}$ *must be positive.*
2. $\partial C_{M,cg}/\partial \alpha_a$ *must be negative.*

That is, the $C_{M,cg}$ *curve must look like Figure 7.12.*

Some frequently used notations

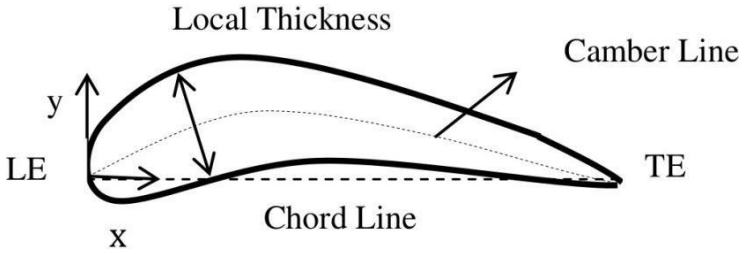


Figure1: Schematic diagram of a cambered wing

- **CamberLine**isthelocusofpointsmidwaybetweentheupperandlowersurface of an airfoil section as measured perpendicular to itself.
- **Leading Edge** (LE) is the most forward point on the camber line.
- **Trailing Edge** (TE)is the most rearward point on the camber line.
- **ChordLine**isastraightlineconnectingtheleadingedgeandthetrailingedge.
- **Maximum Camber** is the maximum distance between the chord line and the camber line as measured perpendicular to the chord line.
- **Local Thickness:** at any point along the chord line is the distance between the upper and lower surface as measured perpendicular to the camber line.
- **Maximum Thickness:** is the maximum distance between the upper and lower surfaces as measured perpendicular to the camber line.
- *D*:Drag parallel to *V* (opposite to motion)
- *L*:Lift perpendicular to *V*
- *A*:Axial Force parallel to chord
- *N*:Normal force perpendicular to chord

- M:PitchingMoment(positive nose up)

$$D,L,A,N \rightarrow C_D \quad ,C_L \ ,C_A ,C_N \rightarrow \frac{F}{\frac{1}{2}\rho V^2 S_{ref}}$$

$$m \rightarrow C_m \quad \rightarrow \frac{M}{\frac{1}{2}\rho V^2 S_{ref} c}$$

Where,

C_D, C_L, C_A, C_N are force coefficients

C_m is pitching moment coefficient.

Important Relations

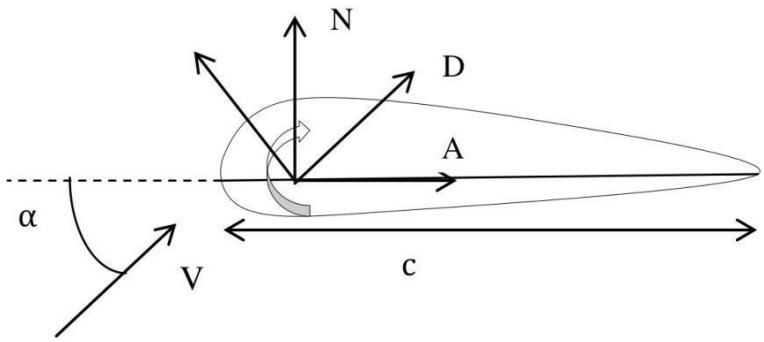


Figure2: Axial force coefficient

$$C_L = C_N \cos\alpha - C_A \sin\alpha$$

$$C_D = C_A \cos\alpha + C_N \sin\alpha$$

- Similarly,

$$C_N = C_L \cos\alpha + C_D \sin\alpha$$

$$C_A = C_D \cos\alpha - C_L \sin\alpha$$

- Since $C_D C_L$ so, axial force coefficient C_A is often negative

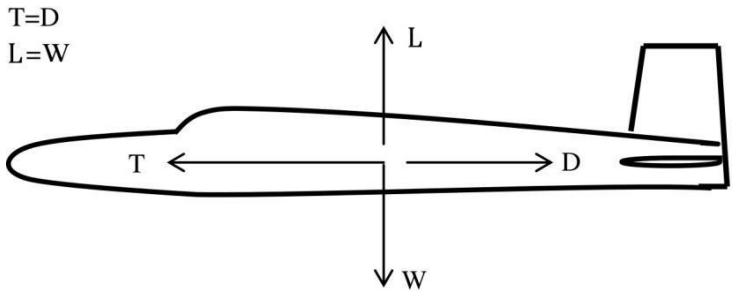


Figure3: Schematic diagram of Static equilibrium

Trim-A pilot's perspective

- When the controls are set so that the resultant forces and the moments about the center of gravity are all zero, the aircraft is said to be in Trim, which simply means static equilibrium.
- Airplane in cruise is a typical example of an airplane in static equilibrium.

Aircraft control surfaces

Primary control surfaces are:

- **Ailerons:** To control the rolling moment
- **Elevator:** To control the pitching moment
- **Rudder:** To control the yawing moment

These control surfaces provide two functions:

1. The control surfaces must be able to maintain static trim over the entire range of airspeed and altitude for which the aircraft is able to fly. This includes being able to trim the aircraft against any asymmetric thrust force.
2. The control surfaces must provide sufficient moment necessary to maneuver the aircraft in range of speed and altitudes.

Degree of Freedom: Rigid body: Airplane

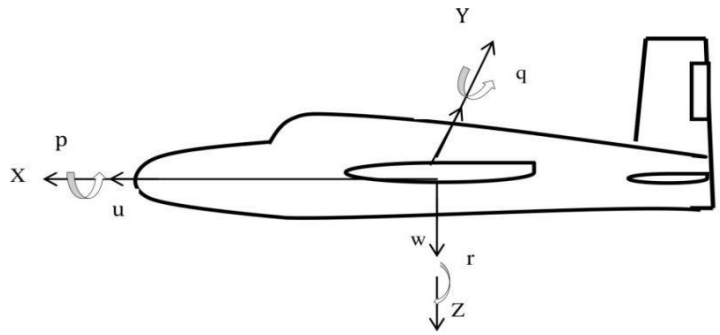


Figure4: Degree of freedom

Arigid air plane in free flight has six degrees of freedom:

- Three translational (*u*,*v*,*w*)
- Three rotational (*p*,*q*,*r*)

Important to Understand

- Foranairplanetobeinfullystabletrim,therecanbenoinstabilityinanyofthe six degrees of freedom .Hence,
- Small translational disturbances in axial, normal or side slip velocity must all result in a return to the original trimmed equilibrium condition.
- Similarly, rotational disturbances in roll, pitch and yaw must all result in a return to the original equilibrium attitude.

It was pointed out in Chapter 1 that the problems of airplane longitudinal equilibrium, static stability, and control can be developed from the very important static equation of moments about the Y axis through the airplane's center of gravity. As these problems are probably the most important in design of the airplane for adequate stability and control characteristics, considerable emphasis is placed on them in this section. A study of the static equation of moments about the Y axis is a very complex one when the effects of power, component interference, and free controls are taken into account. However, the equations can be handled even in their most complicated forms if use is made of experimental evidence gained in many wind-tunnel and flight tests.

The longitudinal stability theory will first be developed for the simplified case of an airplane in gliding flight with controls locked (stick-fixed) and propellers windmilling. The theories thus obtained will be expanded later to account for the effects of power and free controls.

The study of the longitudinal equilibrium and static stability of the airplane, then, requires an investigation into the moments about the airplane's Y axis through the c.g. and their variation with the airplane's lift coefficient. Equilibrium demands that the summation of these moments equal zero, and static stability demands that a diving moment accompany an increase in lift coefficient and a stalling moment accompany a decrease in lift coefficient from equilibrium.

Throughout this section, it will be assumed that any wing or tail surface can be represented by a mean aerodynamic chord, the forces and moments on which represent all the forces and moments operating on the surface. It will also be assumed that there exists an aerodynamic center on this mean aerodynamic chord about which the wing pitching moment coefficient is invariant with lift coefficient. The forces and moments acting on any wing or tail surface can be repre-

sented by a lift and drag force acting at the aerodynamic center, together with a pitching moment about the aerodynamic center whose coefficient is invariant with lift coefficient. The aerodynamic center of any airfoil section is located very close to the 25 per cent chord point. The aerodynamic center of a typical airfoil section is shown in Figure 5–1. Methods for predicting the mean aerodynamic chord and the aerodynamic center location for the three-dimensional wing planform are discussed in Section 2–2.

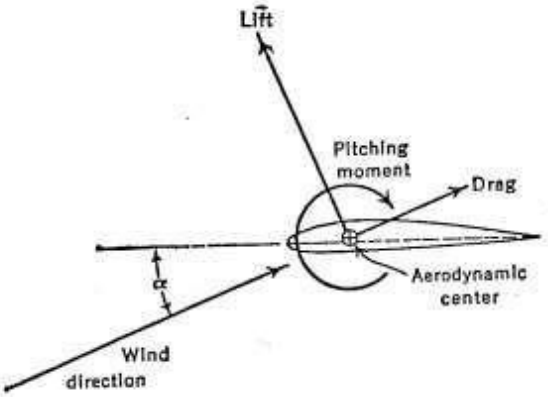


FIGURE 5–1. Aerodynamic forces on an airfoil section.

The forces and moments operating on a normal airplane configuration are shown below in detail for gliding flight with propeller off. The angles i_w and i_t are the wing and tail incidence with respect to the airplane reference line, and ϵ is the average angle of downwash behind the wing at the horizontal tail. All angles and derivatives with respect to angles will be given in degrees to escape confusion.

The lift and drag are by definition always perpendicular and parallel to the wind, respectively. It is therefore inconvenient to use these forces to obtain moments, for their arms to the center of gravity vary with angle of attack. For this reason all forces are resolved into normal and chordwise forces whose axes remain fixed with the airplane and whose arms are therefore constant.

Resolving the wing forces perpendicular and parallel to the airplane reference:

$$\begin{aligned} N &= L\cos(\alpha - i_w) + D\sin(\alpha - i_w) \\ C &= D\cos(\alpha - i_w) - L\sin(\alpha - i_w) \end{aligned} \tag{5–1}$$

Resolving the tail force along the same axes as shown in Figure 5–2, the following summation of moments about the airplane's c.g. is obtained:

$$M_{cg} = Nx_a + Cz_a + M_{ac} + M_{Fus} + M_{Nac} + M_{act} + C_t h_t - N_t l_t \tag{5–2}$$

For equilibrium it is necessary for M_{cg} to equal zero. It has been found convenient to place this equation in coefficient form by dividing

through by qS_wc, where q is the dynamic pressure in pounds per square foot, S_w the wing area in square feet, and c the mean aerodynamic chord in feet. The ratio q_t/q is called the tail efficiency, η_t, and for power-off flight is less than unity, because of the loss of energy as the air interacts with parts of the wing wake and fuselage boundary layer.

$$C_{m_{cg}} = C_N \frac{x_a}{c} + C_c \frac{z_a}{c} + C_{m_{ac}} + C_{m_{\substack{Fus \\ Nac}}} + C_{m_{act}} \frac{S_t}{S_w} \frac{c_t}{c} \eta_t + C_{c_t} \frac{S_t}{S_w} \frac{h_t}{c} \eta_t - C_{N_t} \frac{S_t}{S_w} \frac{l_t}{c} \eta_t \quad (5\text{–}3)$$

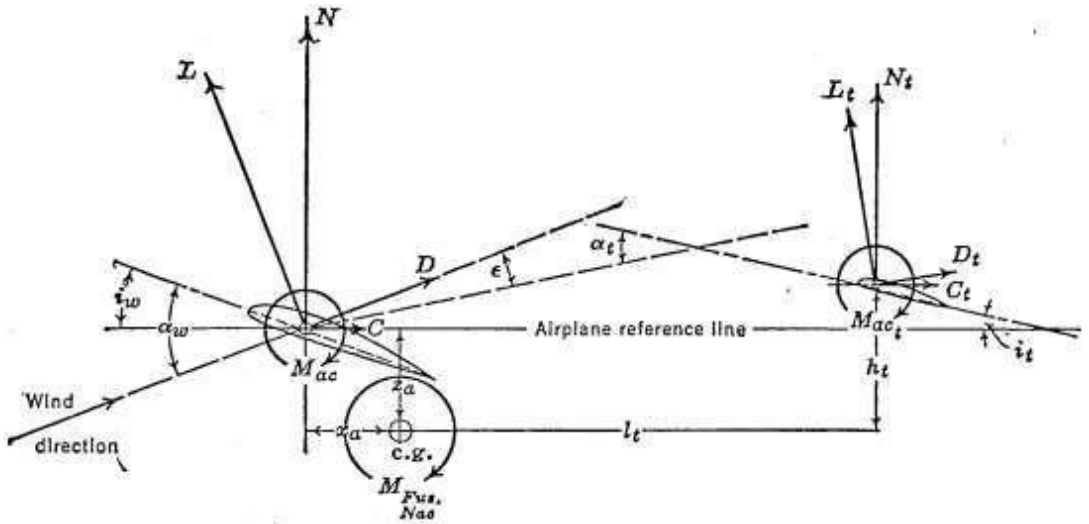


FIGURE 5–2. Forces and moments in plane of symmetry.

The fifth and sixth terms of equation (5–3) have been found to be negligible when compared with the other terms, and will therefore be eliminated from further consideration. This leaves

$$C_{m_{cg}} = C_N \frac{x_a}{c} + C_c \frac{z_a}{c} + C_{m_{ac}} + C_{m_{\substack{Fus \\ Nac}}} - C_{N_t} \frac{S_t}{S_w} \frac{l_t}{c} \eta_t \quad (5\text{–}4)$$

This is the equilibrium equation in pitch which must sum up to $C_{m_{cg}} = 0$ for equilibrium in a given flight condition. The pitching moment coefficient will be shown to be a function of the lift coefficient, and the slope of the curve of pitching moment coefficient plotted against lift coefficient is used for evaluating the static stability of the airplane in pitch. Typical plots of this curve are shown in Figure 5–3.

For the airplane to be stable the slope of the C_m versus C_L curve must be negative. If, as is shown in Figure 5–3, the airplane is in equilibrium at $C_L = 1.0$ ($C_{m_{cg}} = 0$), an increase in airplane lift coefficient

be negative. If, as is shown in Figure 5–3, the airplane is in equilibrium at $C_L = 1.0$ ($C_{m_{cg}} = 0$), an increase in airplane lift coefficient

along curve A will create a nose-down pitching moment tending to reduce the lift coefficient back to equilibrium, whereas a decrease in lift coefficient will create a nose-up moment tending to increase the lift coefficient. A positive slope of this curve, as shown by curve B, will result in static instability, for, although in equilibrium at the same lift coefficient, an increase in airplane lift coefficient will create nose-up moments tending to increase the lift coefficient still further.

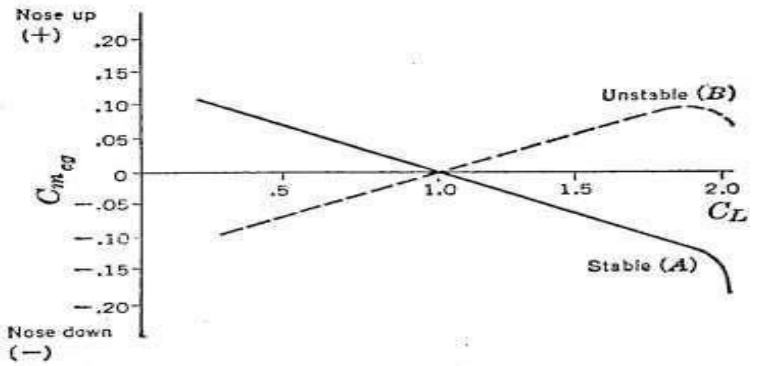


FIGURE 5–3. Typical pitching moment curves.

The slope of this curve of C_m versus C_L is given by the derivative dC_m/dC_L, and a negative sign for this derivative is required for static longitudinal stability. The slope of the pitching moment curve dC_m/dC_L can be obtained analytically by differentiating equation (5–4) with respect to C_L:

$$\frac{dC_m}{dC_L} = \underbrace{\frac{dC_N}{dC_L}\frac{x_a}{c} + \frac{dC_c}{dC_L}\frac{z_a}{c} + \frac{dC_{mac}}{dC_L}}_{\text{Contr. of wing}} + \underbrace{\left(\frac{dC_m}{dC_L}\right)_{\substack{Fus\\Nac}}}_{\substack{\text{Contr. of}\\\text{fuselage and}\\\text{nacelles}}} - \underbrace{\frac{dC_{N_t}}{dC_L}\frac{S_t}{S_w}\frac{l_t}{c}\eta_t}_{\substack{\text{Contr. of}\\\text{horizontal tail}}} \qquad (5\text{–}5)$$

The contribution of the various parts of the airplane to the total airplane stability can be broken down as shown in equation (5–5). These various contributions will be studied separately.

5–2 Wing Contribution

The wing contribution to the static longitudinal stability of the airplane can be analyzed by studying the first three terms of equation (5–5). In order to express these in terms of the familiar lift coefficient,

airplane can be analyzed by studying the first three terms of equation (5–5). In order to express these in terms of the familiar lift coefficient,

use can be made of equations (5–1) as follows:

$$C_N = C_L \cos(\alpha - i_w) + C_D \sin(\alpha - i_w)$$
$$C_c = C_D \cos(\alpha - i_w) - C_L \sin(\alpha - i_w) \tag{5–6}$$

The slopes dC_N/dC_L and dC_c/dC_L can be obtained by differentiating the above equations with respect to C_L:

$$\frac{dC_N}{dC_L} = \cos(\alpha - i_w) - C_L \sin(\alpha - i_w)\frac{d\alpha}{dC_L} + \frac{dC_D}{dC_L}\sin(\alpha - i_w) + C_D \cos(\alpha - i_w)\frac{d\alpha}{dC_L} \tag{5–7}$$

$$\frac{dC_c}{dC_L} = \frac{dC_D}{dC_L}\cos(\alpha - i_w) - C_D \sin(\alpha - i_w)\frac{d\alpha}{dC_L} - C_L \cos(\alpha - i_w)\frac{d\alpha}{dC_L} - \sin(\alpha - i_w)$$

As the angle of attack is usually a small angle, it is permissible to make approximations to the terms $\sin(\alpha - i_w)$ and $\cos(\alpha - i_w)$. Making these assumptions and noting that the drag coefficient is very small with respect to unity, one can reduce (5–7) to the following form:

$$\frac{dC_N}{dC_L} = 1$$
$$\frac{dC_c}{dC_L} = \frac{dC_D}{dC_L} - C_L\frac{d\alpha}{dC_L} = C_L\left(\frac{2}{\pi eA} - \frac{.035}{(dC_L/d\alpha)^\circ}\right) \tag{5–8}$$

with $(dC_L/d\alpha)^\circ$, per degree.

The derivative $dC_{mac}/dC_L = 0$ by definition of the aerodynamic center.

With these substitutions the wing contribution of equation (5–5) becomes

$$\left(\frac{dC_m}{dC_L}\right)_{Wing} = \frac{x_a}{c} + C_L\left(\frac{2}{\pi eA} - \frac{.035}{dC_L/d\alpha}\right)\frac{z_a}{c} \tag{5–9}$$

The wing contribution to stability is therefore a function of the c.g position with respect to the wing aerodynamic center. If the c.g. is ahead of the aerodynamic center, x_a will be negative and the first term will be stabilizing, whereas if the c.g. is aft of the aerodynamic center, x_a will be positive and the first term will be destabilizing. To study the effects of the second term, average values substituted for the terms

the effects of the second term, average values substituted for the terms in the brackets give $-\frac{C_L}{10}\frac{z_a}{c}$. If the c.g. position is under the wing aerodynamic center, z_a will be negative and the term will be stabilizing, whereas if the c.g. is over the wing aerodynamic center, the term will be unstable. The second term is usually very small with respect to the first term except at high values of the lift coefficient. At a $C_L = 1$

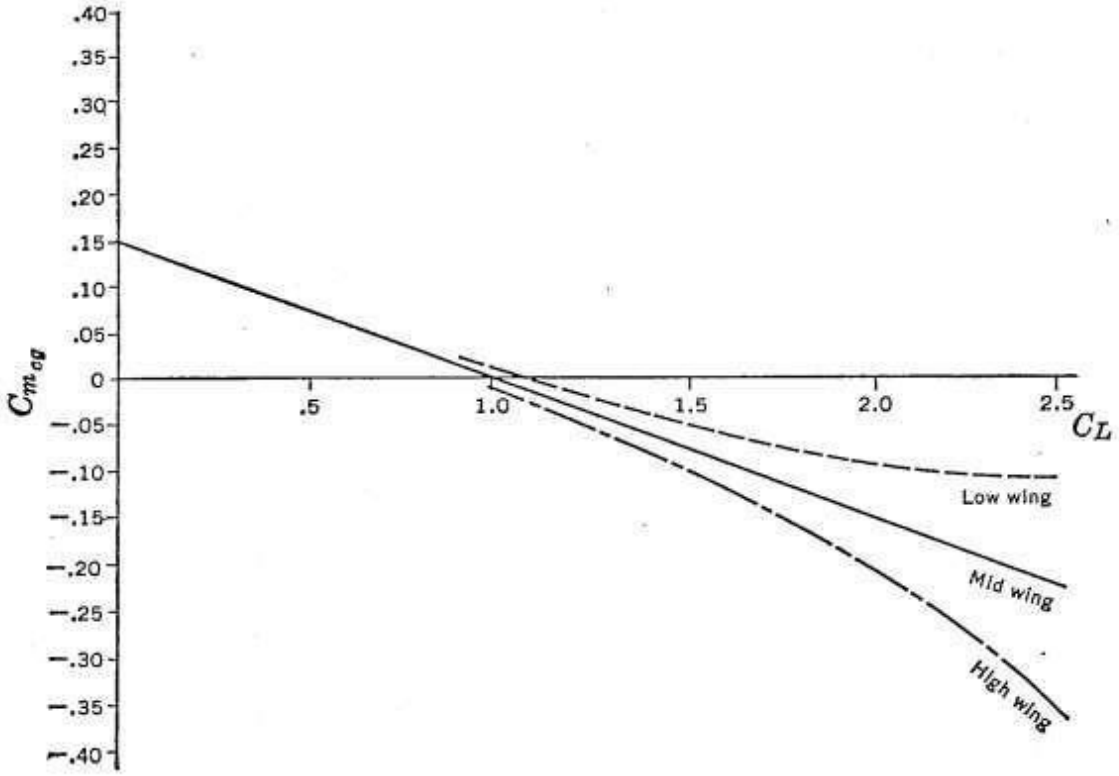


FIGURE 5–4. Effect of vertical location of c.g. on pitching moments.

the second term is only one-tenth as powerful as the first term for equal horizontal and vertical shifts of the center of gravity. The effects of the second term, or the drag term, as it is often referred to, can usually be neglected. It will increase the stability of the high wing airplane at high lift coefficient and destabilize the low wing airplane at high lift coefficient. At a low C_L the term is negligible. These effects are shown in Figure 5–4.

In the rest of this study the drag term will be neglected. This leaves for the wing contribution the very simple relationship

$$\left(\frac{dC_m}{dC_L}\right)_{Wing} = \frac{x_a}{c} \qquad (5\text{–}10)$$

which can be written as

$$\left(\frac{dC_m}{dC_L}\right)_{Wing} = x_{cg} - x_{ac} \qquad (5\text{–}11)$$

where x_{cg} and x_{ac} are the locations of the airplane center of gravity and the wing aerodynamic center in percentage of the wing mean aerodynamic chord.

A study of this expression shows that if the airplane's c.g. is at the wing aerodynamic center, the stability contribution of the wing will be zero, and that the numerical value of the wing contribution is equal to the per cent aerodynamic chord between the c.g. and the aerodynamic center. If the c.g. is 1 per cent aft of the wing aerodynamic center, the slope of the pitching moment curve dC_m/dC_L will be .01.

5–3 Tail Contribution

The tail contribution to the static longitudinal stability of the airplane can be analyzed by studying the last term of equation (5–5). The derivative dC_{N_t}/dC_L is not approximately equal to unity as was assumed for dC_N/dC_L because of the effects of the wing downwash. The evaluation of this derivative can be developed as follows:

$$C_{N_t} = \left(\frac{dC_N}{d\alpha}\right)_t \alpha_t \qquad (5\text{–}12)$$

The angle of attack of the horizontal tail can be seen from Figure 5–2 to be

$$\alpha_t = \alpha_w - \epsilon + i_t - i_w \qquad (5\text{–}13)$$

The angle of attack of the horizontal tail is modified by the downwash from the wing and any incidence between the wing and the horizontal tail.

Equation (5–12) therefore becomes

$$C_{N_t} = \left(\frac{dC_N}{d\alpha}\right)_t (\alpha_w - \epsilon + i_t - i_w) \qquad (5\text{–}14)$$

and the derivative dC_{N_t}/dC_L becomes:

$$\frac{dC_{N_t}}{dC_L} = \left(\frac{dC_N}{d\alpha}\right)_t \left(\frac{d\alpha_w}{dC_L} - \frac{d\epsilon}{dC_L}\right) \qquad (5\text{–}15)$$

which can be reduced to

$$\frac{dC_{N_t}}{dC_L} = \frac{(dC_N/d\alpha)_t}{(dC_N/d\alpha)_w}\left(1 - \frac{d\epsilon}{d\alpha}\right) \qquad (5\text{–}16)$$

The tail contribution to stability becomes:

$$\left(\frac{dC_m}{dC_L}\right)_{Tail} = -\frac{(dC_N/d\alpha)_t}{(dC_N/d\alpha)_w}\frac{S_t}{S_w}\frac{l_t}{c}\eta_t\left(1-\frac{d\epsilon}{d\alpha}\right) \qquad (5\text{–}17)$$

A short-hand notation is introduced in order to simplify the writing of this expression, which appears many times in the course of the developments that follow. This notation is:

$$\left(\frac{dC_N}{d\alpha}\right)_t = a_t$$

$$\left(\frac{dC_N}{d\alpha}\right)_w = a_w$$

$$\frac{S_t}{S}\frac{l_t}{c} = \bar{V} \qquad (5\text{–}18)$$

With this new notation equation (5–17) becomes

$$\left(\frac{dC_m}{dC_L}\right)_{Tail} = -\frac{a_t}{a_w}\bar{V}\eta_t\left(1-\frac{d\epsilon}{d\alpha}\right) \qquad (5\text{–}19)$$

In order to evaluate the tail term of the stability equation, it is necessary to evaluate the slopes of the two lift curves a_t and a_w. The slope of the lift curve (C_N or C_L versus α) is a function of the wing aspect ratio, A, and the infinite aspect ratio or section slope of the lift curve a_0.

$$a_w = \frac{a_0}{1+\dfrac{57.3ra_0}{\pi A}} \qquad (5\text{–}20)$$

Curves of the slope of the lift curve versus aspect ratio are given in Figure 5–5, corrected at low aspect ratio from experimental data. A correction, r, for end plates is given in Figure 5–6 for use in equation (5–20).

The contributions of the horizontal tail surface to the airplane's equilibrium and static longitudinal stability are affected seriously by the downwash from the wing due to the wing vortex system, and therefore, in order to determine these contributions, it is necessary to evaluate the downwash at the horizontal tail for a given airplane design.

A wing's vortex system can be represented as shown in Figure 5–7, consisting of the "bound" vortex located at the wing quarter chord and a vortex sheet streaming from the wing trailing edge, rolling up to form the familiar two trailing vortices.

The wing wake centerline, which is coincident with the vortex sheet, is displaced downward and deformed by the influence of the bound vortex and the powerful trailing vortices. The whole flow field behind the wing is determined by this vortex system, with the major influence

coming from the trailing vortices and the flow of air into the wake. The strength of the vortex system is proportional to the lift coefficient,

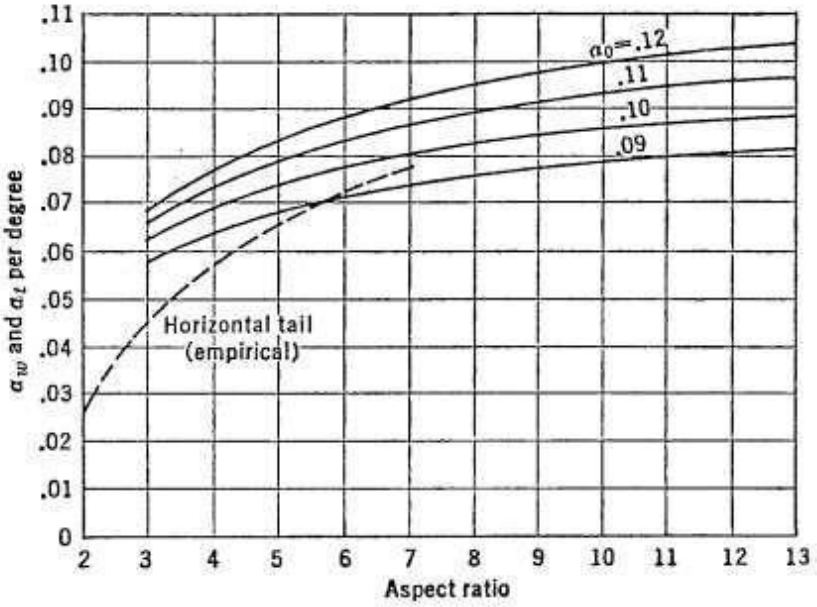


FIGURE 5–5. Slope of lift curve versus aspect ratio.

and therefore the downwash at any particular point will be proportional to the lift coefficient.

$$\epsilon = f(C_L) \tag{5-21}$$

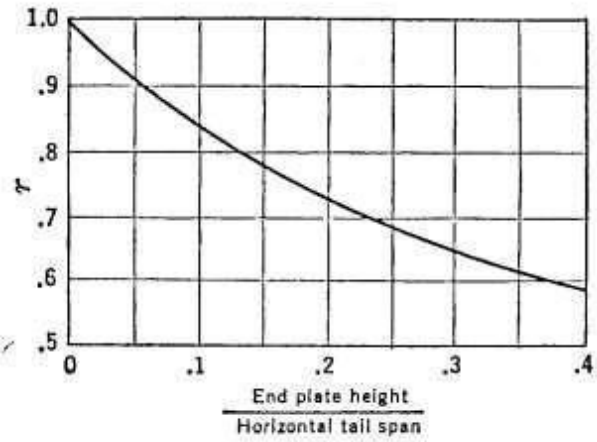


FIGURE 5–6. End plate correction to slope of lift curve.

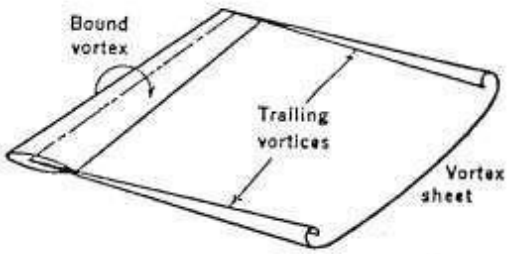


FIGURE 5–7. The trailing vortex sheet.

from the wing the upwash is measured. A sketch of the wing upwash and downwash distribution is shown in Figure 5–8.

The theoretical downwash at the wing aerodynamic center for an elliptical lift distribution is given as

$$\epsilon_{ac}^{\circ} = \frac{57.3 C_L}{\pi A} \tag{5–22}$$

Directly behind the wing quarter chord the downwash builds up very rapidly because of the strong influence of the bound vortex; for positions farther aft of the wing trailing edge the effect of the bound vortex

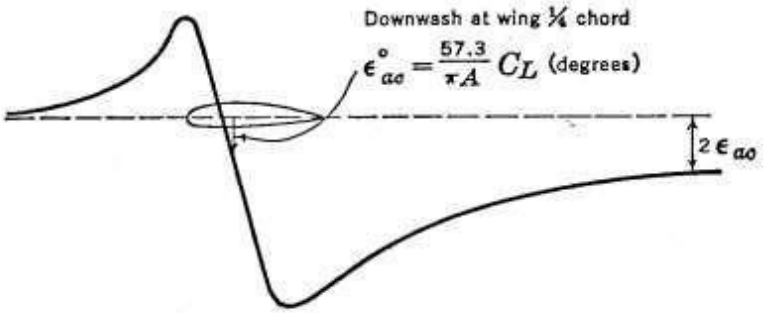


FIGURE 5–8. Downwash distribution in front of and behind a finite wing.

rapidly diminishes and the downwash becomes almost entirely a function of the two trailing vortices. At infinity the downwash theoretically becomes equal to $2\epsilon_{ac}$. Actually, however, the trailing vortices are dissipated quite rapidly because of the effects of viscosity, and the downwash will eventually disappear completely. For first estimates of the downwash at the tail, it is usual to assume the theoretical value of the downwash at infinity:

$$\epsilon^{\circ} = \frac{114.6 C_L}{\pi A} \tag{5–23}$$

The rate of change of downwash, $d\epsilon/d\alpha$, required in the evaluation of the tail term to stability can be obtained, also to a first approximation, by differentiating (5–23) with respect to wing angle of attack.

$$\frac{d\epsilon}{d\alpha} = \frac{114.6}{\pi A} a_w \tag{5–24}$$

The downwash at the tail can be calculated with good accuracy by referring to NACA TR 648, in which design charts are given for predicting downwash angles behind plain and flapped wings. This report is recommended to the reader for a precise evaluation for downwash angles or rates of change of downwash.

Neutral Point (Stick-fixed)

$$\frac{dC_m}{dC_L} = \frac{x_a}{c} + \left(\frac{dC_m}{dC_L}\right)_{\substack{Fus \\ Nac}} - \frac{a_t}{a_w}\bar{V}\eta_t\left(1 - \frac{d\epsilon}{d\alpha}\right)$$

and the equilibrium equation:

$$C_{m_{cg}} = C_L \frac{x_a}{c} + C_{m_{ac}} + C_{m_{\substack{Fus \\ Nac}}} - \frac{a_t}{a_w} C_L \bar{V} \eta_t \left(1 - \frac{d\epsilon}{d\alpha}\right) - a_t(\alpha_0 - i_w + i_t)\bar{V}\eta_t \quad (5\text{–}33)$$

The contributions of the various airplane components to its static longitudinal stability (dC_m/dC_L) are shown in Figure 5–17.

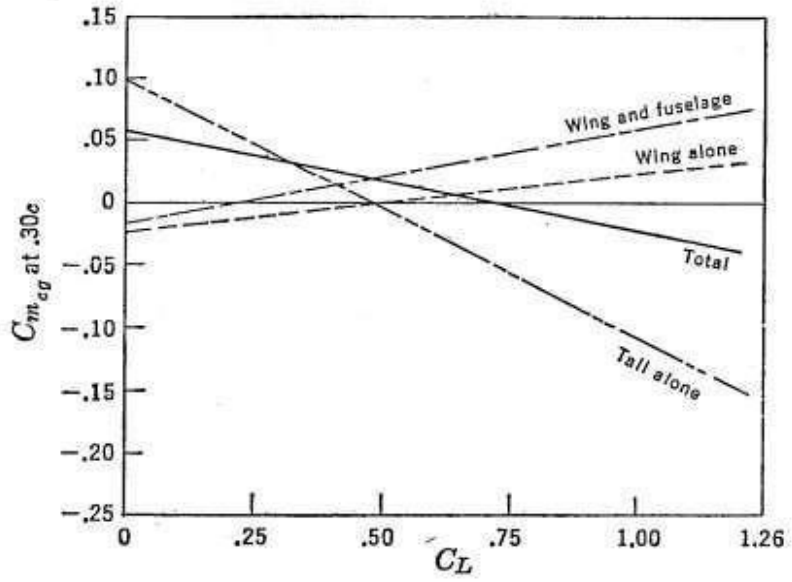


FIGURE 5–17. Typical longitudinal stability breakdown.

For any given airplane, the stability equation (5–32) is fixed except for movement of the center of gravity, which affects the wing term through the variation of x_a. A shift of center of gravity has very small influence on the tail term, a negligible influence on the fuselage or nacelle term, and a very strong influence on the wing term. The wing contribution can be written as in equation (5–11):

$$\left(\frac{dC_m}{dC_L}\right)_{Wing} = x_{cg} - x_{ac} \quad (5\text{–}34)$$

This indicates that for every per cent of the m.a.c. that the c.g. is moved aft, $(dC_m/dC_L)_{Wing}$ will increase positively 1 per cent. Center of gravity movement, therefore, has a powerful influence on the airplane's static longitudinal stability, and as will be shown in the following pages, is probably the most important variable in airplane design for stability. The effect of c.g. shift on the pitching moment coefficient of an assumed airplane is shown in Figure 5–18. It will be noted that all the curves rotate about a fixed C_m at $C_L = 0$.

An examination of Figure 5–18 shows that as the c.g. is moved aft the slope of the pitching moment curve (dC_m/dC_L) for the assumed airplane becomes more positive. When a c.g. position of 30 per cent is reached, the slope becomes zero. At this point the airplane is neutrally stable ($dC_m/dC_L = 0$), and this c.g. position is called the stick-fixed neutral point and is denoted by the symbol (N_0). Equations (5–32) and (5–34) can be used to solve for this neutral point.

$$N_0 = x_{cg(dC_m/dC_L=0)} = x_{ac} - \left(\frac{dC_m}{dC_L}\right)_{\substack{Fus \\ Nac}} + \frac{a_t}{a_w}\bar{V}\eta_t\left(1 - \frac{d\epsilon}{d\alpha}\right) \qquad (5\text{–}35)$$

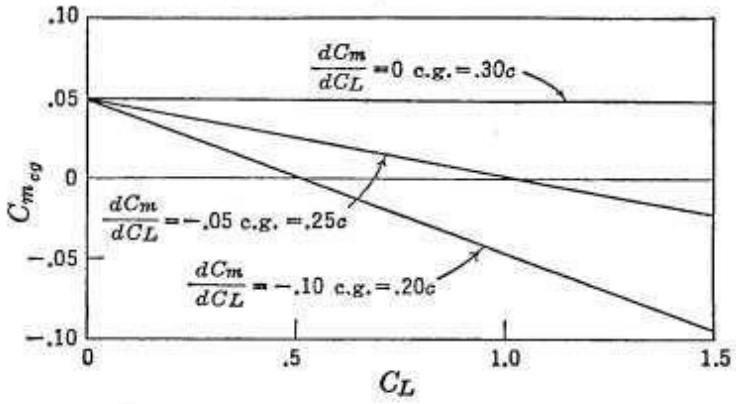


FIGURE 5–18. Typical effect of c.g. shift on pitching moments.

The neutral point gives the most aft location at which the c.g. can be placed before making the airplane unstable. It therefore places one limit on the airplane's permissible c.g. travel, for the airplane should never be balanced aft of this point if a stable airplane is desired.

The neutral point is also very convenient to obtain, for once the neutral point is known, the stability at any other c.g. position may be obtained with good accuracy from the following relation:

$$\frac{dC_m}{dC_L} = x_{cg} - N_0 \qquad (5\text{–}36)$$

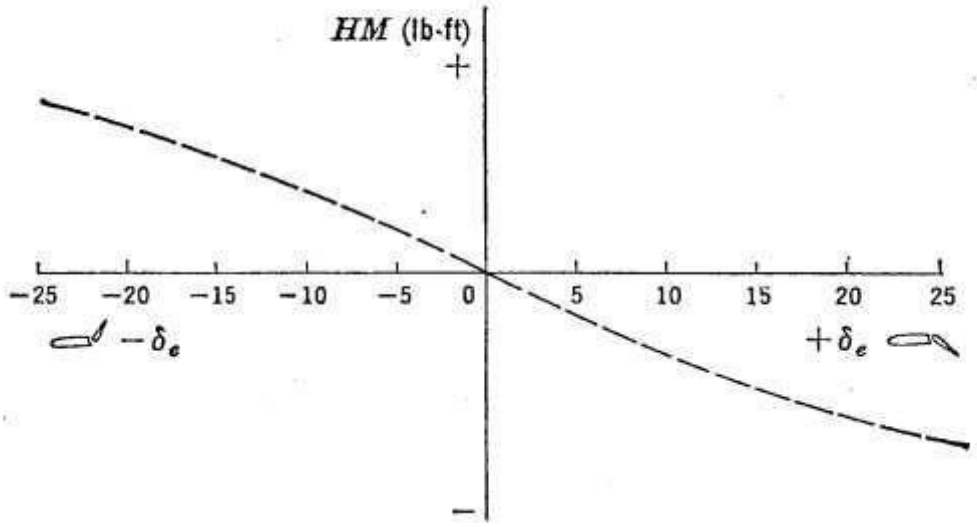
In other words, the slope of the pitching moment coefficient versus C_L is numerically equal to the difference between the c.g. location and the neutral point, both expressed in percentage of the mean aerodynamic chord.

The Hinge Moment Parameters

The control surface, in almost all cases, is a plain flap making up the rear portion of an airfoil section. The pressure distribution over this section and, therefore, over the flap is governed by two major variables. The first of these is the angle of attack of the airfoil, and the second is the deflection of the flap chord with respect to the airfoil chord. Changes in angle of attack and flap deflection alter the pressure distribution over the entire airfoil and consequently change the magnitude of the flap hinge moment.

A typical pressure distribution over a horizontal tail at zero angle of attack, but with varying elevator angles, is shown in Figure 6–1.

The suction created on the upper surface as the elevator is deflected downward can be represented as the reaction, R, acting at the centroid of the pressure area, creating the hinge moment, HM, about the hinge line. For up elevator deflections, the picture is the same, but the suction will then be on the bottom surface. The variation of the hinge moment with elevator deflection for zero angle of attack is shown in



Hinge moment versus control deflection.

Figure 6–2, showing that for the plain flap of normal section with the hinge at the nose radius, the slope of the hinge moment versus control surface deflection will be negative, under the assumption that a positive hinge moment tends to rotate the control surface trailing edge down.

The pressure distribution picture shown in Figure 6–1 is quite simple, but unfortunately it is complicated by the effect of angle of attack. As any airfoil section is given an increment in angle of attack, the pressure distribution over it changes rapidly with large suction pressures appearing close to the leading edge of the section. These distributions affect the pressure distribution over the whole section and therefore affect the control surface hinge moments. A typical variation of the pressure distribution with angle of attack is shown in Figure 6–3.

The suction over the upper surface, which increases with angle of attack, again creates a hinge moment which tends to make the elevator "float" up as the angle of attack is increased. The variation of hinge moment with angle of attack for a plain flap of normal section with the hinge line at the nose radius is shown in Figure 6–4. It should be

noted that the slope of control surface hinge moment versus angle of attack is negative for this type of control.

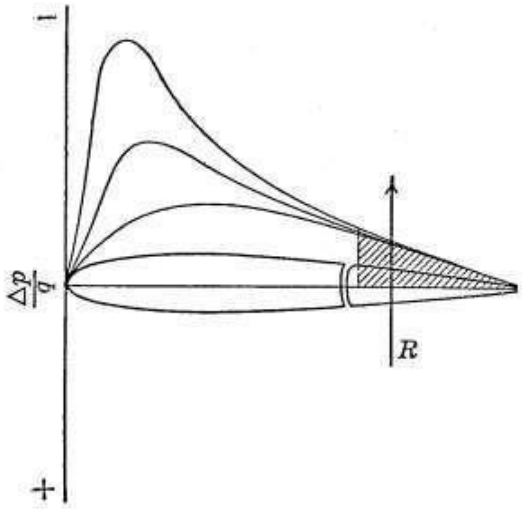


FIGURE 6–3. Pressure distribution due to angle of attack.

The magnitude of the hinge moment for any combination of angle of attack and control surface deflection is developed from the complicated

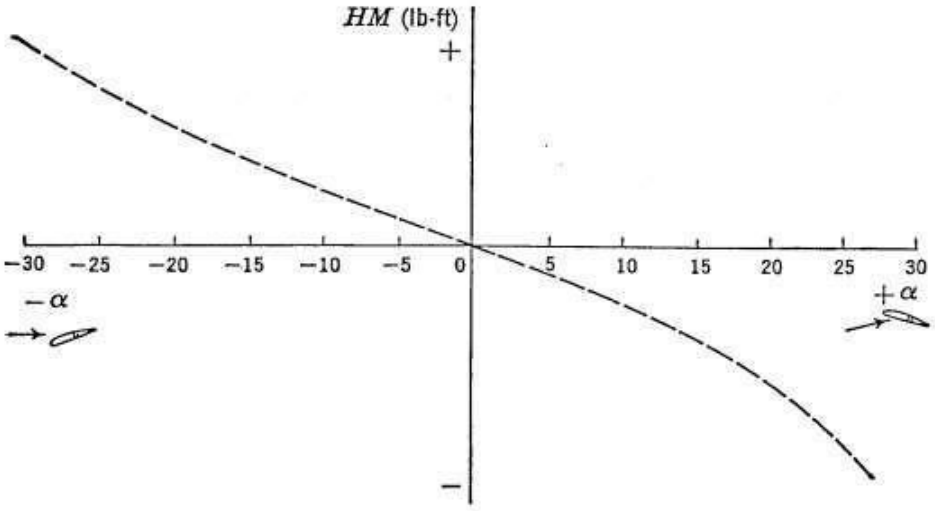


FIGURE 6–4. Hinge moment versus angle of attack.

pressure distribution created by both the angle of attack and control surface deflection operating together. It has been assumed, and the assumption is well justified, that the total hinge moment, HM, can be thought of as the addition of the two effects taken separately. In other words the hinge moment due to a control surface deflection of 5° at a 5° angle of attack is the sum of the hinge moment due to a 5° deflection at zero angle of attack plus a 5° angle of attack at zero deflection. Mathematically, this can be thought of as a total differential being made up of partial derivatives.

$$dHM = \left(\frac{\partial HM}{\partial \alpha}\right)_{\delta} d\alpha + \left(\frac{\partial HM}{\partial \delta}\right)_{\alpha} d\delta \qquad (6\text{–}1)$$

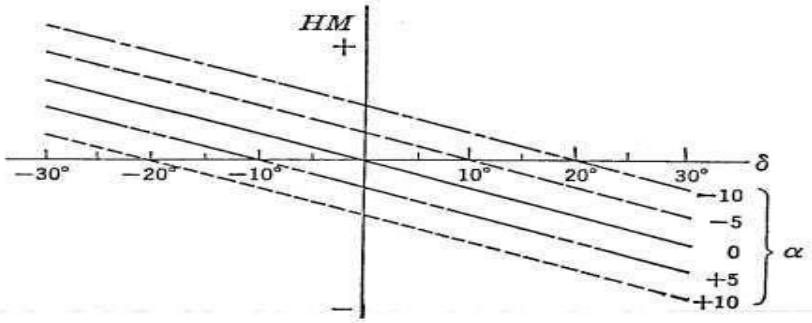


FIGURE 6–5. Typical hinge moment curves.

Under the assumption that the variation of hinge moment with angle of attack and deflection is linear, the absolute value of the hinge moment, angle of attack, and deflection can replace the differentials.

$$HM = \left(\frac{\partial HM}{\partial \alpha}\right)_{\delta} \alpha + \left(\frac{\partial HM}{\partial \delta}\right)_{\alpha} \delta \qquad (6\text{–}2)$$

The variation of the hinge moment with both control surface deflection and angle of attack represented by equation (6–2) is shown as the family of curves in Figure 6–5.

It has been found very convenient to non-dimensionalize the hinge moment, as was done with the lift, drag, and pitching moment. The hinge moment coefficient, C_h, is obtained by dividing the hinge moment in pound-feet by the dynamic pressure, q, in pounds per square foot, the area of the control surface back of the hinge line, S_c, in square feet, and the root mean square chord of the control surface aft of the hinge line, $\bar{c}_c$, in feet.

$$C_h = \frac{HM}{qS_c\bar{c}_c} \qquad (6\text{–}3)$$

The hinge moment coefficient can be expressed in terms of partial derivatives as was done in equation (6–2).

$$C_h = \left(\frac{\partial C_h}{\partial \alpha}\right)_\delta \alpha + \left(\frac{\partial C_h}{\partial \delta}\right)_\alpha \delta \tag{6–4}$$

The partial derivatives $(\partial C_h/\partial \alpha)_\delta$ and $(\partial C_h/\partial \delta)_\alpha$ are usually written in the short-hand form, C_{h_α} and C_{h_δ}, and will be written in this form throughout the remainder of this book. In this notation equation (6–4) is written as follows:

$$C_h = C_{h_\alpha} \alpha + C_{h_\delta} \delta \tag{6–5}$$

One further addition is necessary to make this equation perfectly general, and that is to include a term to allow for a residual hinge moment at zero angle of attack and deflection for cambered surfaces. The coefficient of this residual hinge moment is given as C_{h_0}.

$$C_h = C_{h_0} + C_{h_\alpha} \alpha + C_{h_\delta} \delta \tag{6–6}$$

As most control surfaces use symmetric airfoil sections, the term C_{h_0} will not be included unless a specific case requiring this term is under investigation.

6–3 Control Surface Floating Characteristics and Aerodynamic Balance

One of the important characteristics of any control surface is its so-called floating characteristics. If the control surface is mounted on frictionless bearings and the whole surface given an angle of attack, the hinge moment developed for the unbalanced flap of Figures 6–1 and 6–3 will tend to make the control float up. This tendency is a function of the parameter C_{h_α}. However, as soon as the control surface starts to float up, a new pressure distribution due to the parameter C_{h_δ} is added which tends to prevent the elevator from floating up. An equilibrium floating angle is reached when the "floating tendency," C_{h_α}, is just opposed by the "restoring tendency," C_{h_δ}. The floating angle can be solved mathematically from equation (6–5) by equating C_h to zero and solving for the control surface angle, δ.

$$\delta_{\text{Float}} = -\frac{C_{h_\alpha}}{C_{h_\delta}}\alpha \tag{6–7}$$

The control surface floating angle, therefore, varies with angle of attack, and if C_{h_α} and C_{h_δ} both have negative signs, the elevator will float up more and more with increasing angle of attack. The rate that the elevator floats with angle of attack can be obtained readily by

differentiating equation (6–7) with respect to α, obtaining:

$$\frac{d\delta_{\text{float}}}{d\alpha} = -\frac{C_{h\alpha}}{C_{h\delta}} \tag{6–8}$$

The floating characteristics of the control surface as well as the magnitude of the hinge moment coefficient depend on the magnitudes of the parameters $C_{h\alpha}$ and $C_{h\delta}$. On most control surfaces it is im-

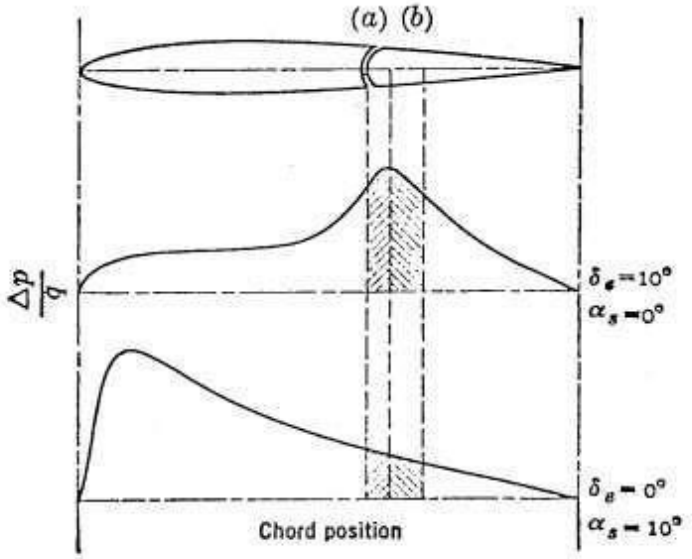


FIGURE 6–6. Set-back hinge line.

portant to reduce the floating tendency to as near zero as practicable. This means C_{h_α} should be made to have a zero slope. At the same time C_{h_δ} must be made to have a low slope if the pilot is to be able to move the control at all, especially for large, very fast airplanes. Methods for controlling the parameters C_{h_α} and C_{h_δ} are therefore of prime interest and are referred to as methods of aerodynamic balancing control surface.

One of the most frequently used methods of aerodynamic balancing control surfaces is the set-back hinge. This is a very powerful method, and its mechanics can be demonstrated by restudying Figures 6–1 and 6–3. The pressure distribution over the surface can be redrawn as in Figure 6–6.

With the hinge line at the nose radius, the summation of moments due to the pressure distribution will create a resultant moment tending to raise the control surface in each case. However, as the hinge line is moved aft, the summation of moments about the hinge line will yield a total moment somewhat smaller, and finally, if the hinge line is moved far enough aft, the sign of the resultant moment can be reversed. When the hinge line is moved aft far enough so that the resultant moment is reversed, the control is said to be overbalanced. The hinge moment parameters C_{h_α} and C_{h_δ} are therefore very sensitive to hinge line position, becoming more positive as the hinge line is moved aft.

The Stick-free Neutral Point

From the discussion of control surface hinge moments, the total hinge moment coefficient of the elevator may be given as:

$$C_{h_e} = C_{h_0} + C_{h_\alpha}\alpha_s + C_{h_{\delta_e}}\delta_e + C_{h_{\delta_t}}\delta_t \qquad (6\text{–}19)$$

from which the floating angle of the elevator can be obtained by equating C_{h_e} to zero and solving for δ_e. For the case of $C_{h_0} = 0$ and $\delta_t = 0$

$$\delta_{e_{\text{Float}}} = -\frac{C_{h_\alpha}}{C_{h_\delta}}\alpha_s \qquad (6\text{–}20)$$

where α_s is the angle of attack of the stabilizer.

In Chapter 5 the elevator angle required for equilibrium at each lift coefficient was obtained, and the derivative $d\delta_e/dC_L$ given as the criterion for stick-fixed longitudinal stability. Now, if the elevator floating angle at each lift coefficient just happened to be the same as

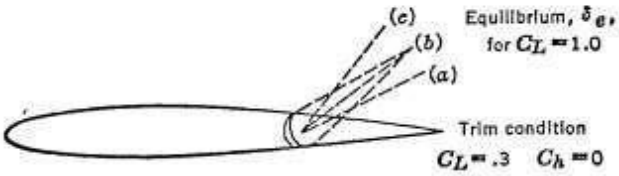


FIGURE 6–14. Typical floating conditions of the elevator.

the elevator angle required for equilibrium, the pilot would not have to apply any force to hold any other air speed than the original trim speed, and the airplane would be termed neutrally stable stick-free. If for an increase in lift coefficient from trim the elevator floats up, but at a smaller angle than required of the elevator for equilibrium, the pilot will have to supply a pull force at the stick to hold the equilibrium elevator angle, and the airplane would be considered stick-free stable. Finally, if the floating angle exceeded the equilibrium angle required for an increase in C_L from trim, the pilot would have to supply a push force at the stick which would be termed stick-free instability. The three conditions are shown schematically in Figure 6–14.

To evaluate the static longitudinal stability stick-free, it is necessary to determine the value of the derivative $(dC_m/dC_L)_{\text{Free}}$. This is obviously the value of $(dC_m/dC_L)_{\text{Fix}}$ plus the destabilizing effect of freeing the elevators.

$$\left(\frac{dC_m}{dC_L}\right)_{\text{Free}} = \left(\frac{dC_m}{dC_L}\right)_{\text{Fix}} + \left(\frac{dC_m}{dC_L}\right)_{\substack{\text{Free} \\ \text{elevator}}} \tag{6–21}$$

and can be restated as follows:

$$C_{m_t} = -C_{L_t}\bar{V}\eta_t \tag{6-22}$$

and

$$C_{m_\delta} = -\frac{dC_{L_t}}{d\delta_e}\bar{V}\eta_t \tag{6-23}$$

The stability contribution of the free elevator is therefore

$$\left(\frac{dC_m}{dC_L}\right)_{\substack{\text{Free}\\ \text{elevator}}} = \left(\frac{d\delta_e}{dC_L}\right)_{C_{he}=0} \times C_{m_\delta} \tag{6-24}$$

The value of the floating angle derivative $(d\delta_e/dC_L)_{C_{he}=0}$ comes from equation (6–20).

$$(\delta_e)_{C_{he}=0} = -\frac{C_{h_\alpha}}{C_{h_\delta}}\alpha_s \tag{6-25}$$

$$\left(\frac{d\delta_e}{dC_L}\right)_{C_{he}=0} = -\frac{C_{h_\alpha}}{C_{h_\delta}}\frac{d\alpha_s}{dC_L} \tag{6-26}$$

The angle of attack of the stabilizer, α_s, was given in Chapter 5 as follows:

$$\alpha_s = \alpha_w - \epsilon - i_w + i_t \tag{6–27}$$

therefore

$$\frac{d\alpha_s}{dC_L} = \frac{d\alpha}{dC_L}\left(1 - \frac{d\epsilon}{d\alpha}\right) = \frac{(1 - d\epsilon/d\alpha)}{a_w} \tag{6–28}$$

also

$$\frac{dC_{L_t}}{d\delta_e} = \left(\frac{dC_L}{d\alpha}\right)_t \tau = a_t \tau \tag{6–29}$$

Substituting (6–23), (6–26), (6–28), and (6–29) into (6–24),

$$\left(\frac{dC_m}{dC_L}\right)_{\substack{\text{Free}\\ \text{elevator}}} = \frac{C_{h_\alpha}}{C_{h_\delta}} \frac{a_t}{a_w} \bar{V}\eta_t \tau \left(1 - \frac{d\epsilon}{d\alpha}\right) \tag{6–30}$$

finally

$$\left(\frac{dC_m}{dC_L}\right)_{\text{Free}} = \left(\frac{dC_m}{dC_L}\right)_{\text{Fix}} + \frac{C_{h_\alpha}}{C_{h_\delta}} \frac{a_t}{a_w} \bar{V}\eta_t \tau \left(1 - \frac{d\epsilon}{d\alpha}\right) \tag{6–31}$$

The stability criterion $(dC_m/dC_L)_{\text{Fix}}$ was developed in Chapter 5 as follows for the props-off case:

$$\left(\frac{dC_m}{dC_L}\right)_{\text{Fix}} = \frac{x_a}{c} + \left(\frac{dC_m}{dC_L}\right)_{\substack{Fus\\ Nac}} - \frac{a_t}{a_w} \bar{V}\eta_t \left(1 - \frac{d\epsilon}{d\alpha}\right) \tag{6–32}$$

Substituting (6–32) into (6–31) and rearranging,

$$\left(\frac{dC_m}{dC_L}\right)_{\text{Free}} = \frac{x_a}{c} + \left(\frac{dC_m}{dC_L}\right)_{\substack{Fus \\ Nac}} - \frac{a_t}{a_w}\bar{V}\eta_t\left(1 - \frac{d\epsilon}{d\alpha}\right)\left(1 - \frac{C_{h_\alpha}}{C_{h_\delta}}\tau\right) \quad (6\text{–}33)$$

This is the equation for static longitudinal stability, stick-free, props off. The stick-free stability, props windmilling, can be written as follows:

$$\left(\frac{dC_m}{dC_L}\right)_{\text{Free}} = \frac{x_a}{c} + \left(\frac{dC_m}{dC_L}\right)_{\substack{Fus \\ Nac}} - \frac{a_t}{a_w}\bar{V}\eta_t\left(1 - \frac{d\epsilon}{d\alpha}\right)\left(1 - \frac{C_{h_\alpha}}{C_{h_\delta}}\tau\right)$$

$$+ \frac{N(dC_N/d\alpha)_{pT=0}\,(d\beta/d\alpha)l_pS_p}{S_wca_w} + \frac{a_t}{a_w}\frac{\bar{V}\eta_t}{.07}\left(\frac{d\beta}{d\alpha}\right)\left(\frac{dC_N}{d\alpha}\right)_{pT=0} \quad (6\text{–}34)$$

The effect of freeing the elevator enters the tail term as the multiplying factor $\left(1 - \tau\frac{C_{h_\alpha}}{C_{h_\delta}}\right)$. For an airplane equipped with an elevator having no change in hinge moment with angle of attack ($C_{h_\alpha} = 0$), this term becomes unity, and the stick-fixed and stick-free stabilities are equal. However, if the elevator has a large floating tendency (the ratio $C_{h_\alpha}/C_{h_\delta}$ large and positive), the stability contribution of the horizontal tail can be reduced materially. For instance, for a ratio of $C_{h_\alpha}/C_{h_\delta} = 2$ and a normal value of τ equal to .5, the floating of the elevator can obviate the whole tail contribution to stability. From this brief analysis the importance of careful elevator balance design to insure proper hinge moment characteristics and thereby good stick-free stability can be readily appreciated.

The stick-free neutral point, N_0', can be obtained directly from equation (6–33) for the propeller-off condition.

$$N_0' = x_{ac} - \left(\frac{dC_m}{dC_L}\right)_{\substack{Fus \\ Nac}} + \frac{a_t}{a_w}\bar{V}\eta_t\left(1 - \frac{d\epsilon}{d\alpha}\right)\left(1 - \frac{C_{h_\alpha}}{C_{h_\delta}}\tau\right) \quad (6\text{–}35)$$

The difference between the stick-fixed and stick-free neutral points is as follows:

$$N_0 - N_0' = \frac{a_t\bar{V}\eta_t C_{h_\alpha}}{a_w C_{h_\delta}}\tau\left(1 - \frac{d\epsilon}{d\alpha}\right) = -\frac{C_{m_\delta}}{a_w}\frac{C_{h_\alpha}}{C_{h_\delta}}\left(1 - \frac{d\epsilon}{d\alpha}\right) \quad (6\text{–}36)$$

UNIT–II

Aircraft Equations of Motion

Aircraft Axis Systems

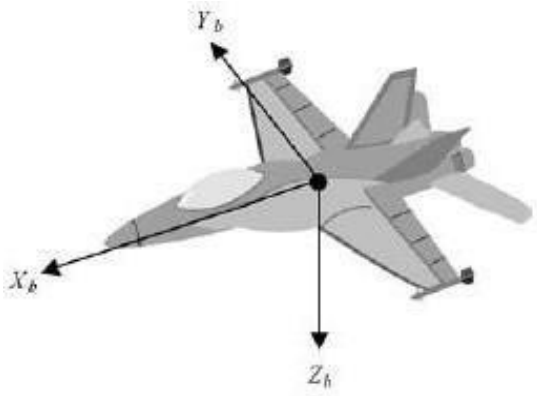
In this chapter we will concern ourselves with three axis systems. These include the body axis system fixed to the aircraft, the Earth axis system, which we will assume to be an inertial axis system fixed to the Earth, and the stability axis system, which is defined with respect to the relative wind. Each of these systems is useful in that they provide a convenient system for defining a particular vector, such as, the aerodynamic forces, the weight vector, or the thrust vector.

Body Axis System

The body axis system is fixed to the aircraft with its origin at the aircraft's center of gravity. The x axis is defined out the nose of the aircraft along some reference line. The reference line may be chosen to be the chord line of the aircraft or may be along the floor of the aircraft, as is often the case in large transports. The y axis is defined out the right wing of the aircraft, and the z axis is defined as down through the bottom of the aircraft in accordance with the right-hand rule, as shown in Fig. 4.1. The pilot sits in the body axis system, making it a very useful reference frame. Additionally, it is relatively easy to determine the moments and products of inertia in the body axis system because it is fixed to the aircraft.

Earth Axis System

The Earth axis system is fixed to the Earth with its z axis pointing to the center of the Earth. The x axis and y axis are orthogonal and lie in the local horizontal plane with the origin at the aircraft center of gravity. Often, the x

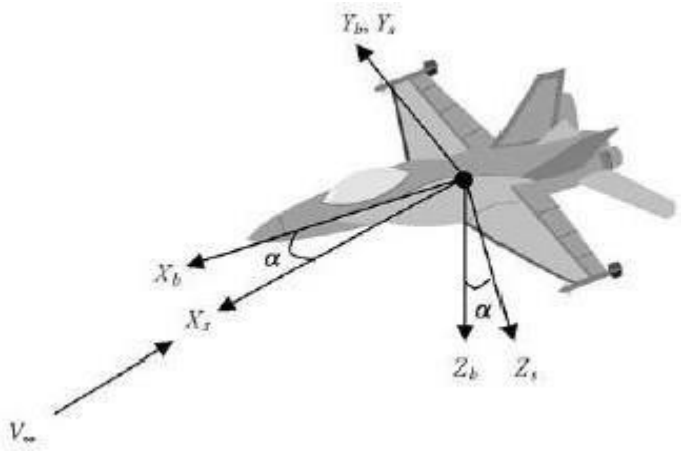


Body axis system.

axis is defined as North and the y axis defined as East. The Earth axis system is assumed to be an inertial axis system for aircraft problems. This is important because Newton's 2nd law is valid only in an inertial system. While this assumption is not totally accurate, it works well for aircraft problems where the aircraft rotation rates are large compared to the rotation rate of the Earth.

Stability Axis System

The stability axis system is rotated relative to the body axis system through the angle of attack. This means that the stability x axis points in the direction of the projection of the relative wind onto the xz plane of the aircraft. The origin of the stability axis system is also at the aircraft center of gravity. The y axis is out the right wing and coincident with the y axis of the body axis system. The z axis is orthogonal and points downward in accordance with the right-hand rule. This is illustrated in Fig. 4.2. The stability axis system is particularly useful in defining the aerodynamic forces of lift and drag.



Stability axis system.

Coordinate Transformations

As stated previously, it is convenient to express certain vectors in a particular coordinate system. For example, the weight vector of the aircraft is conveniently represented in the Earth axis system where there is only a component in the positive z direction because the vector acts toward the center of the Earth, that is,

$$\bar{F}_{\text{Weight}_{\text{Earth}}} = \bar{F}_{\text{Gravity}_{\text{Earth}}} = \bar{F}_{G_E} = \begin{bmatrix} 0 \\ 0 \\ W \end{bmatrix}_{\text{Earth}} = \begin{bmatrix} 0 \\ 0 \\ mg \end{bmatrix}_{\text{Earth}} \quad (1)$$

The aerodynamic forces are conveniently displayed in the stability axis, where drag acts in the negative x direction and lift acts in the negative z axis, that is,

$$\bar{F}_{\text{Aero}} = \bar{F}_A = \begin{bmatrix} -D \\ F_{A_y} \\ -L \end{bmatrix}_{\text{Stability}} \quad (2)$$

Likewise, the thrust vector can easily be expressed in the body axis as

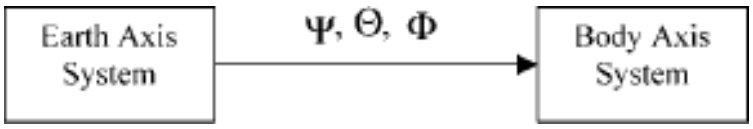
$$\bar{F}_{\text{Thrust}} = \bar{F}_T = \begin{bmatrix} T\cos(\phi_T) \\ 0 \\ -T\sin(\phi_T) \end{bmatrix}_{\text{Body}} \quad (3)$$

where ϕ_T is the angle between the x-body axis and the thrust vector, T. While these equations are conveniently displayed in a particular axis system, they must be all transformed into the same axis system before they can be summed in the equations of motion. As a result, it is very important to understand how to transform a vector from one axis system to another.

Earth Axis to Body Axis Transformation

Transforming a vector from the Earth axis system to the body axis system requires three consecutive rotations about the z axis, y axis, and x axis, respectively. In flight mechanics, the Euler angles are used to rotate the "vehicle carried" Earth axis system into coincidence with the body axis system. The Euler angles are expressed as yaw (Ψ), pitch (Θ), and roll (Φ). Euler angles are very useful in describing the orientation of the aircraft with respect to inertial space. The proper order or rotation is illustrated in Fig. 4.3.

The yaw angle, Ψ, is defined as the angle between the projection of the x-body axis onto the horizontal plane and the x axis of the Earth axis system. With the Earth x axis defined as North, the yaw angle is the same as the vehicle heading angle. The pitch angle, Θ, is the angle measured in a vertical plane between the x-body axis and the horizontal plane. The roll angle, Φ, is the angle measured in the yz plane of the vehicle body axis system, between the y-body axis and the horizontal plane. This is the same as the bank angle



Earth to body transformation.

for a given ψ and θ, and is a measure of the rotation about the x axis to put the aircraft in the desired position from a wing's horizontal condition. The accepted limits on the Euler angles are

$$0 \leq \Psi \leq 360 \text{ deg}$$
$$-90 \text{ deg} \leq \Theta \leq 90 \text{ deg}$$
$$-180 \text{ deg} \leq \Phi \leq 180 \text{ deg}$$

The importance of the sequence of the Euler angle rotations cannot be overemphasized. If the sequence is performed in a different order than ψ, θ, and ϕ, the final result will be incorrect. The following illustrates the transformation of a vector, $\bar{F}_E$, in the Earth axis system into the body axis system, where

$$\bar{F}_E = X_E \hat{i}_E + Y_E \hat{j}_E + Z_E \hat{k}_E \qquad (4)$$

and $\hat{i}_E$, $\hat{j}_E$, and $\hat{k}_E$ are the unit vectors in the Earth axis system. Therefore,

$$\bar{F}_E = \begin{bmatrix} X_E \\ Y_E \\ Z_E \end{bmatrix}_{\text{Earth}} \qquad (5)$$

If we rotate through the yaw angle, ψ, about the z-Earth axis, $\hat{k}_E$, we end up in some intermediate axis system $\hat{i}'$, $\hat{j}'$, and $\hat{k}'$. (See Fig. 4.4.)

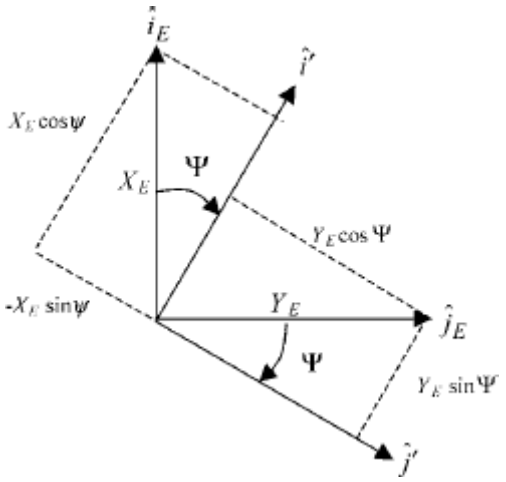
The vector in the intermediate axis system ($\hat{i}'$, $\hat{j}'$, and $\hat{k}'$) is:

$$\bar{F}' = X'\hat{i}' + Y'\hat{j}' + Z'\hat{k}' \qquad (6)$$

where

$$X' = X_E \cos\Psi + Y_E \sin\Psi$$
$$Y' = -X_E \sin\Psi + Y_E \cos\Psi$$
$$Z' = Z_E$$

7



Rotation through Ψ.

so

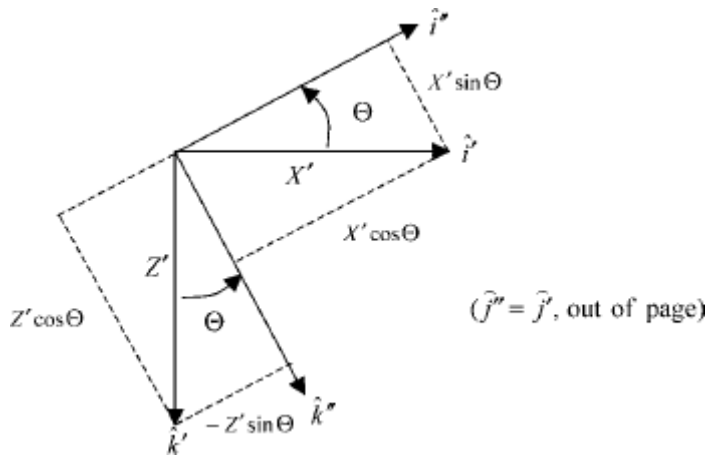
$$\begin{bmatrix} X' \\ Y' \\ Z' \end{bmatrix} = \underbrace{\begin{bmatrix} \cos\Psi & \sin\Psi & 0 \\ -\sin\Psi & \cos\Psi & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{R_3(\Psi)} \begin{bmatrix} X_E \\ Y_E \\ Z_E \end{bmatrix}$$

8

$$\bar{F}' = R_3(\Psi)\bar{F}_E$$

9

We will now rotate this intermediate vector, $\bar{F}'$, through some pitch angle, Θ, to some other intermediate axis system, $\hat{i}''$, $\hat{j}''$, and $\hat{k}''$ as shown in Fig. 4.5.



Rotation through Θ.

The vector in the intermediate axis is

$$\bar{F}'' = X''i'' + Y''j'' + Z''k'' \qquad \text{(10)}$$

where

$$\begin{aligned} X'' &= X'\cos\Theta - Z'\sin\Theta \\ Y'' &= Y' \\ Z'' &= X'\sin\Theta + Z'\cos\Theta \end{aligned} \qquad \text{(11)}$$

$$\begin{bmatrix} X'' \\ Y'' \\ Z'' \end{bmatrix} = \underbrace{\begin{bmatrix} \cos\Theta & 0 & -\sin\Theta \\ 0 & 1 & 0 \\ \sin\Theta & 0 & \cos\Theta \end{bmatrix}}_{R_2(\Theta)} \begin{bmatrix} X' \\ Y' \\ Z' \end{bmatrix} \qquad \text{(12)}$$

so,

$$\bar{F}'' = R_2(\Theta)\bar{F}' = R_2(\Theta)R_3(\Psi)\bar{F}_e \qquad \text{(13)}$$

Finally we will rotate the vector, $\bar{F}''$, through some roll angle, Φ, into the body axis system, $\hat{i}, \hat{j}$, and $\hat{k}$ as shown in Fig. 4.6.

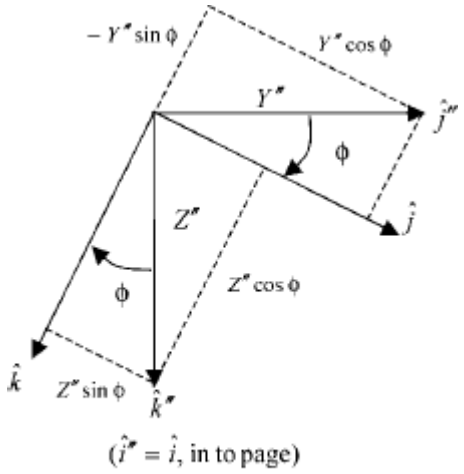


Fig. 4.6 Rotation through Φ.

The vector in the body axis system is

$$\bar{F}_B = X_B\hat{i} + Y_B\hat{j} + Z_B\hat{k} \qquad \rightarrow (14)$$

where

$$\begin{aligned} X_B &= X'' && (\hat{i}\text{ direction}) \\ Y_B &= Y''\cos\Phi + Z''\sin\Phi && (\hat{j}\text{ direction}) \\ Z_B &= -Y''\sin\Phi + Z''\cos\Phi && (\hat{k}\text{ direction}) \end{aligned} \qquad \rightarrow (15)$$

$$\begin{bmatrix} X_B \\ Y_B \\ Z_B \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\Phi & \sin\Phi \\ 0 & -\sin\Phi & \cos\Phi \end{bmatrix}}_{R_1(\Phi)} \begin{bmatrix} X'' \\ Y'' \\ Z'' \end{bmatrix} \qquad \rightarrow (16)$$

so,

$$\bar{F}_B = R_1(\Phi)\bar{F}'' = R_1(\Phi)R_2(\Theta)R_3(\Psi)\bar{F}_E$$ 17

Therefore, any vector in the Earth axis system can be transformed into the body axis system using the following transformation.

$$\bar{F}_B = R_1(\Phi)R_2(\Theta)R_3(\Psi)\bar{F}_E$$ 18

This transformation is very useful in transforming the weight vector of an aircraft expressed in the Earth axis into the body axis system. As shown earlier in Eq. (4.1), the aircraft's weight vector in the Earth axis system is

$$\bar{F}_{\text{Gravity}_{\text{Earth}}} = \bar{F}_{G_E} = \begin{bmatrix} 0 \\ 0 \\ W \end{bmatrix}_{\text{Earth}} = \begin{bmatrix} 0 \\ 0 \\ mg \end{bmatrix}_{\text{Earth}}$$ 19

$$\bar{F}_{\text{Gravity}_B} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\Phi & \sin\Phi \\ 0 & -\sin\Phi & \cos\Phi \end{bmatrix} \begin{bmatrix} \cos\Theta & 0 & -\sin\Theta \\ 0 & 1 & 0 \\ \sin\Theta & 0 & \cos\Theta \end{bmatrix} \begin{bmatrix} \cos\Psi & \sin\Psi & 0 \\ -\sin\Psi & \cos\Psi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ mg \end{bmatrix}_E$$ 20

This yields the following weight vector in the body axis system:

$$\bar{F}_{\text{Gravity}_B} = \begin{bmatrix} -mg\sin\Theta \\ mg\sin\Phi\cos\Theta \\ mg\cos\Phi\cos\Theta \end{bmatrix}_B$$ 21

In going from the body axis system to the Earth axis system, we go through $-\Phi$, then $-\Theta$, and then $-\Psi$. So the transformation is

$$\bar{F}_E = R_3(-\Psi)R_2(-\Theta)R_1(-\Phi)\bar{F}_B$$ 22

For these orthonormal transformation matrices

$$R_1(-\Phi) = R_1^T(\Phi)$$
$$R_2(-\Theta) = R_2^T(\Theta)$$
$$R_3(-\Psi) = R_3^T(\Psi)$$ 23

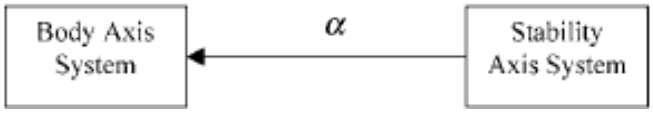
where the superscript T indicates the transposition of the matrix.
Therefore,

$$\bar{F}_E = R_3^T(\Psi)R_2^T(\Theta)R_1^T(\Phi)\bar{F}_B$$ 24

This is convenient for transforming the acceleration or velocity vector in the body axis system into a vector in the Earth axis system, as might be measured by a radar site tracking the aircraft.

Stability Axis to Body Axis Transformation

It is also important to transform a vector in the stability axis system into the body axis system. This is useful in transforming the aerodynamic forces from their convenient axis system, Eq. (4.7), into the body axis system. This is accomplished by rotating the stability axis system through a positive angle of attack, as shown in Fig. 4.7.



Stability to body axis transformation.

The transformation from stability to body is simply a rotation about the y axis through the angle α, or an $R_2(\alpha)$ transformation, namely,

$$\bar{F}_{\text{Aero}_{\text{Body}}} = R_2(\alpha)\bar{F}_{\text{Aero}_{\text{Stability}}} \longrightarrow \text{25}$$

$$\begin{bmatrix} F_{A_x} \\ F_{A_y} \\ F_{A_z} \end{bmatrix}_{\text{Body}} = \underbrace{\begin{bmatrix} \cos\alpha & 0 & -\sin\alpha \\ 0 & 1 & 0 \\ \sin\alpha & 0 & \cos\alpha \end{bmatrix}}_{R_2(\alpha)} \begin{bmatrix} -D \\ F_{A_y} \\ -L \end{bmatrix}_{\text{Stability}} \longrightarrow \text{26}$$

Therefore,

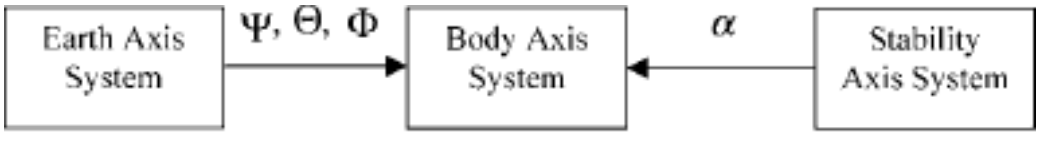
$$\begin{aligned} F_{A_{X_B}} &= -D\cos\alpha + L\sin\alpha \\ F_{A_{y_B}} &= F_{A_{y_S}} \\ F_{A_{z_B}} &= -D\sin\alpha - L\cos\alpha \end{aligned} \longrightarrow \text{27}$$

Aircraft Force Equations

This section develops the three aircraft force equations. The force equations consist of aircraft response (in terms of accelerations) on the left-hand side of the equations, and the applied forces on the right-hand side of the equations. Newton's 2nd law states that the time rate of change of linear momentum is equal to the summation of the applied forces acting on the aircraft's center of gravity:

$$\left[\frac{\mathrm{d}(m\bar{V})}{\mathrm{d}t}\right]_{\text{Inertial}} = \bar{F} \longrightarrow \text{1}$$

It is extremely important to understand that Newton's 2nd law is only valid in an inertial reference frame. An inertial reference frame is an axis system that is



fixed in space with no relative motion. For the purposes of analyzing aircraft, it will be assumed that the Earth axis system is an inertial reference frame, even though it does have rotation. It is assumed that the rotation rate of the Earth is small compared to the rotation rates of the aircraft.

Aircraft Response

In developing the response side of the aircraft force equations, several additional assumptions will be made. First, it is assumed that the aircraft is a rigid body. This assumes that the different parts of the aircraft are not moving with respect to each other. The mass of the aircraft is also assumed to be constant, which is reasonable over a relatively short duration of time. This assumption allows Newton's 2nd law to be rewritten as

$$m\left[\frac{\mathrm{d}(\bar{V})}{\mathrm{d}t}\right]_{\text{Inertial}} = m\bar{a}_{\text{Inertial}} = \bar{F} \qquad (2)$$

While Newton's 2nd law is only valid with respect to an inertial reference frame, the equations can be expressed in the vehicle body axis system. If the equations are expressed in the body axis system, the fact that the system is rotating with respect to an inertial reference frame must be taken into account. This is accomplished using

$$(\bar{a}_{\text{Inertial}})_{Body} = \dot{\bar{V}}_{\text{Body}} + \bar{\omega}_{\text{Body}} \times \bar{V}_{\text{Body}} \qquad (3)$$

The velocity vector in the body axis system, $\bar{V}_{\text{Body}}$, is defined as

$$\bar{V}_{\text{Body}} = U\hat{i} + V\hat{j} + W\hat{k} \qquad (4)$$

where U, V, and W are the velocities in the x, y, and z body axes, respectively. The aircraft angular rate in the body axis system, $\bar{\omega}_{\text{Body}}$, is defined as

$$\bar{\omega}_{\text{Body}} = P\hat{\boldsymbol{i}} + Q\hat{\boldsymbol{j}} + R\hat{\boldsymbol{k}}$$ → 5

where P, Q, and R are the roll, pitch, and yaw rates, respectively, expressed in the body axis. Therefore,

$$(\bar{a}_{\text{Inertial}})_{Body} = \begin{bmatrix} \dot{U} \\ \dot{V} \\ \dot{W} \end{bmatrix}_{\text{Body}} + \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ P & Q & R \\ U & V & W \end{vmatrix}_{\text{Body}}$$ → 6

This results in

$$(\bar{a}_{\text{Inertial}})_{Body} = \begin{bmatrix} \dot{U} + QW - RV \\ \dot{V} + RU - PW \\ \dot{W} + PV - QU \end{bmatrix}_{\text{Body}}$$ → 7

Multiplying the inertial acceleration in the body axis system by the mass of the aircraft yields the three force equations

$$m\begin{bmatrix} \dot{U} + QW - RV \\ \dot{V} + RU - PW \\ \dot{W} + PV - QU \end{bmatrix}_{\text{Body}} = \begin{bmatrix} F_x \\ F_y \\ F_z \end{bmatrix}_{\text{Body}} = \bar{F}_{\text{Body}}$$ → 8

Therefore,

$$\begin{aligned} m(\dot{U} + QW - RV) &= F_x \\ m(\dot{V} + RU - PW) &= F_y \\ m(\dot{W} + PV - QU) &= F_z \end{aligned}$$ → 9

Applied Forces

The previous section developed the left-hand side, or response side, of the force equations. The right-hand side of each equation consists of the applied forces that act on the aircraft. They consist of the gravity forces, the aerodynamic forces, and the thrust forces.

$$m(\dot{U} + QW - RV) = F_{G_x} + F_{A_x} + F_{T_x}$$
$$m(\dot{V} + RU - PW) = F_{G_y} + F_{A_y} + F_{T_y}$$
$$m(\dot{W} + PV - QU) = F_{G_z} + F_{A_z} + F_{T_z}$$

10

$$m(\dot{U} + QW - RV) = -mg\sin\Theta + (-D\cos\mathrm{A} + L\sin\mathrm{A}) + T\cos\Phi_T$$
$$m(\dot{V} + RU - PW) = mg\sin\Phi\cos\Theta + F_{A_y} + F_{T_y}$$
$$m(\dot{W} + PV - QU) = mg\cos\Phi\cos\Theta + (-D\sin\mathrm{A} - L\cos\mathrm{A}) - T\sin\Phi_T$$

11

Moment Equations

The three moment equations are determined by applying Newton's 2nd law in a manner similar to the three force equations. Newton's 2nd law states that

the time rate of change in the angular momentum of the aircraft is equal to the applied moments acting on the aircraft, namely,

$$\left[\frac{\mathrm{d}\bar{H}}{\mathrm{d}t}\right]_{\text{Inertial}} = \bar{M}$$

12

$\bar{H}$ is the angular momentum of the aircraft and is defined as

$$\bar{H} = \bar{r} \times (m\bar{V})$$

13

Response Side of Moment Equations

A six-step procedure will be used to methodically build up the response side of the three moment equations. This provides both a mathematical and physical insight into the equations.

4.4.1.1 Step 1. The first step is to examine a small elemental mass, dm, of the aircraft that is located at some distance from the aircraft's center of gravity. It will be assumed that the elemental mass is rotating about the aircraft center of gravity with a positive roll rate, pitch rate, and yaw rate (P, Q, and R, respectively). The distance from the center of gravity to the small mass is defined as

$$\bar{r}_{dm} = x\hat{i} + y\hat{j} + z\hat{k} \qquad (14)$$

where x, y, and z are the distances in the x, y, and z axes of the body axis system, respectively. This is shown in Fig. 4.9.

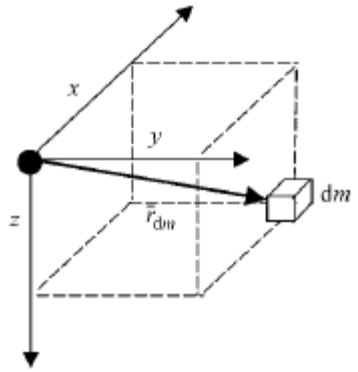


Fig. 4.9 Differential mass in body axis system.

4.4.1.2 Step 2. Next an expression is developed for the velocity of the small mass, dm, solely because of its rotation about the center of gravity. The velocity for the movement of the center of gravity of the aircraft was taken into account in the development of the three force equations. The velocity, $\bar{V}_{dm}$, of the mass relative to the center of gravity is determined using the expression

$$\bar{V}_{dm} = \left[\frac{d\bar{r}_{dm}}{dt}\right]_{Body} + \bar{\omega}_{Body} \times \bar{r}_{dm} \qquad (15)$$

Because the aircraft was previously assumed to be a rigid body, $\bar{r}_{dm}$ is constant, so

$$\left[\frac{d\bar{r}_{dm}}{dt}\right]_{Body} = 0 \qquad (16)$$

and therefore,

$$\bar{V}_{dm} = \bar{\omega}_{Body} \times \bar{r}_{dm} \qquad (17)$$

Mathematically this yields

$$\bar{V}_{dm} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ P & Q & R \\ x & y & z \end{vmatrix} \qquad (18)$$

$$\bar{V}_{dm} = (Qz - Ry)\hat{i} + (Rx - Pz)\hat{j} + (Py - Qx)\hat{k} \qquad (19)$$

4.4.1.3 Step 3. Next an expression is developed for the linear momentum of dm solely because of its rotation about the center of gravity. The linear momentum is found simply by multiplying the mass times the velocity, namely,

$$\text{linear momentum} = dm\bar{V}$$
$$= dm[(Qz - Ry)\hat{i} + (Rx - Pz)\hat{j} + (Py - Qx)\hat{k}] \qquad (20)$$

4.4.1.4 Step 4. An expression for the angular momentum of the differential mass, dm, is developed using

$$d\bar{H}_{dm} = \bar{r}_{dm} \times (dm\bar{V}_{dm}) \qquad (21)$$

Therefore,

$$d\bar{H}_{dm} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ x & y & z \\ dm(Qz - Ry) & dm(Rx - Pz) & dm(Py - Qx) \end{vmatrix} \qquad (22)$$

After carrying out the cross product and regrouping the terms, the three components of the angular momentum are:

$$dH_x = P(y^2 + z^2)dm - Qxy\ dm - Rxz\ dm$$
$$dH_y = Q(x^2 + z^2)dm - Ryz\ dm - Pxy\ dm$$
$$dH_z = R(x^2 + y^2)dm - Pxz\ dm - Qyz\ dm$$

23

4.4.1.5 Step 5. The next step is to integrate the expressions for the angular momentum of dm over the entire aircraft. Because P, Q, and R are not functions of the mass, they can be taken outside of the integration. Therefore, the three components for the angular momentum of the entire aircraft are

$$H_x = \int dH_x = P\int(y^2 + z^2)dm - Q\int xy\ dm - R\int xz\ dm$$
$$H_y = \int dH_y = Q\int(x^2 + z^2)dm - R\int yz\ dm - P\int xy\ dm$$
$$H_z = \int dH_z = R\int(x^2 + y^2)dm - P\int xz\ dm - Q\int yz\ dm$$

24

The **moments of inertia** are defined as

$$I_{xx} = \int(y^2 + z^2)dm$$
$$I_{yy} = \int(x^2 + z^2)dm$$
$$I_{zz} = \int(x^2 + y^2)dm$$

25

The moments of inertia are indications of the resistance to rotation about that axis (that is, I_{xx} indicates the resistance to rotation about the x axis of the aircraft). The **products of inertia** are

$$I_{xy} = \int xy\ dm$$
$$I_{xz} = \int xz\ dm$$
$$I_{yz} = \int yz\ dm$$

26

$$H_x = PI_{xx} - QI_{xy} - RI_{xz}$$
$$H_y = QI_{yy} - RI_{yz} - PI_{xy}$$
$$H_z = RI_{zz} - PI_{xz} - QI_{yz}$$

27

where

28

$$\bar{H} = H_x\hat{i} + H_y\hat{j} + H_z\hat{k}$$

This can also be easily found by applying an expression for angular momentum usually developed in basic physics courses, which is

$$\bar{H} = \bar{I}\bar{\omega}$$

29

where $\bar{I}$ is the aircraft's inertia tensor and $\bar{\omega}$ is the aircraft's angular rate. The inertia tensor for an aircraft is

$$\bar{I} = \begin{bmatrix} I_{xx} & -I_{xy} & -I_{xz} \\ -I_{xy} & I_{yy} & -I_{yz} \\ -I_{xz} & -I_{yz} & I_{zz} \end{bmatrix}_{\text{Body}}$$

30

Therefore,

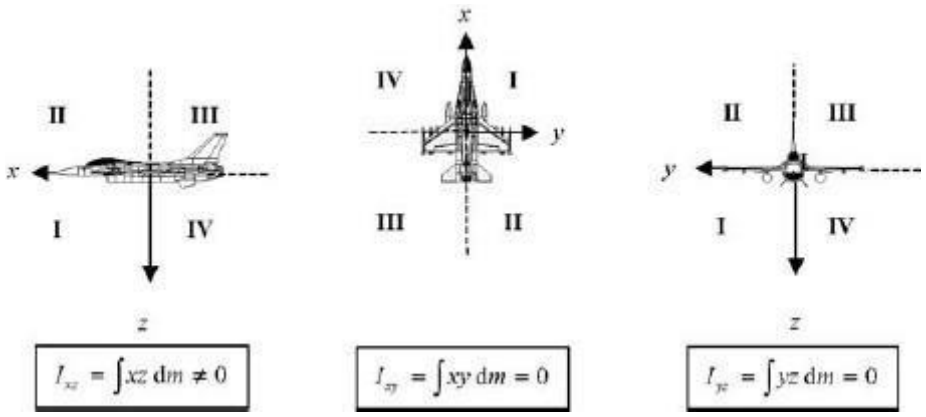
$$H_B = \begin{bmatrix} I_{xx} & -I_{xy} & -I_{xz} \\ -I_{xy} & I_{yy} & -I_{yz} \\ -I_{xz} & -I_{yz} & I_{zz} \end{bmatrix} \begin{bmatrix} P \\ Q \\ R \end{bmatrix}$$

31

so,

$$H_x = PI_{xx} - QI_{xy} - RI_{xz}$$
$$H_y = QI_{yy} - RI_{yz} - PI_{xy}$$
$$H_z = RI_{zz} - PI_{xz} - QI_{yz}$$

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Aircraft products of inertia.

The angular momentum components for the aircraft become

$$H_x = PI_{xx} - RI_{xz}$$
$$H_y = QI_{yy}$$
$$H_z = RI_{zz} - PI_{xz}$$

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or

$$\bar{H} = (PI_{xx} - RI_{xz})\hat{i} + (QI_{yy})\hat{j} + (RI_{zz} - PI_{xz})\hat{k}$$

4.4.1.6 Step 6. After the angular momentum vector of the aircraft has been determined, the final step is to take the time rate of change of the angular momentum vector with respect to inertial space but represented in the aircraft body axis system. The same relationship used in developing the acceleration with respect to an inertial reference frame from the force equations can be used, namely,

$$\left[\frac{d\bar{H}}{dt}\right]_{\text{Inertial}} = \left[\frac{d\bar{H}}{dt}\right]_{\text{Body}} + \omega_{\text{Body}} \times \bar{H}_{\text{Body}}$$

$$\left[\frac{d\bar{H}}{dt}\right]_{\text{Body}} = \begin{bmatrix} \dot{P}I_{xx} - \dot{R}I_{xz} + P\dot{I}_{xx} - R\dot{I}_{xz} \\ \dot{Q}I_{yy} + Q\dot{I}_{yy} \\ \dot{R}I_{zz} - \dot{P}I_{xz} + R\dot{I}_{zz} - P\dot{I}_{xz} \end{bmatrix}_{\text{Body}}$$

Assuming that the mass distribution of the aircraft is constant, such as neglecting fuel slosh, the moments and products of inertia do not change with time, that is, $\dot{I}_{xx}$, $\dot{I}_{yy}$, $\dot{I}_{zz}$, and $\dot{I}_{xz}$ are all zero. Therefore,

$$\left[\frac{d\bar{H}}{dt}\right]_{\text{Body}} = \begin{bmatrix} \dot{P}I_{xx} - \dot{R}I_{xz} \\ \dot{Q}I_{yy} \\ \dot{R}I_{zz} - \dot{P}I_{xz} \end{bmatrix}_{\text{Body}}$$

Finally,

$$\omega \times \bar{H}_{\text{Body}} = \begin{vmatrix} i & j & k \\ P & Q & R \\ (PI_{xx} - RI_{xz}) & (QI_{yy}) & (RI_{zz} - PI_{xz}) \end{vmatrix}_{\text{Body}}$$

$$= \begin{bmatrix} Q(RI_{zz} - PI_{xz}) - RQI_{yy} \\ R(PI_{xx} - RI_{xz}) - P(RI_{zz} - PI_{xz}) \\ PQI_{yy} - Q(PI_{xx} - RI_{xz}) \end{bmatrix}_{\text{Body}}$$

Grouping terms yields

$$\left[\frac{d\bar{H}}{dt}\right]_{\text{Inertial}_{\text{Body}}} = \begin{bmatrix} \dot{P}I_{xx} + QR(I_{zz} - I_{yy}) - (\dot{R} + PQ)I_{xz} \\ \dot{Q}I_{yy} - PR(I_{zz} - I_{xx}) + (P^2 - R^2)I_{xz} \\ \dot{R}I_{zz} + PQ(I_{yy} - I_{xx}) + (QR - \dot{P})I_{xz} \end{bmatrix}_{\text{Body}}$$

$$\dot{P}I_{xx} + QR(I_{zz} - I_{yy}) - (\dot{R} + PQ)I_{xz} = L$$

$$\dot{Q}I_{yy} - PR(I_{zz} - I_{xx}) + (P^2 - R^2)I_{xz} = M$$

$$\underbrace{\dot{R}I_{zz}}_{\substack{\text{angular} \\ \text{acceleration} \\ \text{terms}}} + \underbrace{PQ(I_{yy} - I_{xx})}_{\substack{\text{gyro} \\ \text{precession} \\ \text{terms}}} + \underbrace{(QR - \dot{P})I_{xz}}_{\text{coupling terms}} = N$$

L, M, and N are the rolling moment, pitching moment, and yawing moment, respectively. Unfortunately, the letter L is used to also represent lift. This can be confusing and the reader is advised to carefully check the context of its use in any aeronautical engineering text. Recall that the assumptions made in developing the equations of motion were: the mass of the aircraft is constant, the aircraft is a rigid airframe, the Earth axis system is an inertial reference frame, the mass distribution of the aircraft is constant, and the aircraft has an xz plane of symmetry. It is, therefore, extremely important to realize that these equations are valid for flight conditions where these assumptions are reasonable.

$$\dot{P}I_{xx} = L$$

$$-I_{yy}QR = L$$

$$-R\underbrace{(I_{yy}Q)}_{\text{angular momentum in pitch}} = L$$

$$R\underbrace{(I_{xx}P)}_{\text{angular momentum in roll}} = M$$

$$P^2 I_{xz} = -I_{yy}\dot{Q}$$

We will assume the aircraft is doing a high-speed fly-by and has a negative I_{xz}. If the pilot does a snap roll to either direction, this equation predicts that a positive angular acceleration ($\dot{Q}$) will result. This will cause the aircraft to pitch up and could lead to serious effects if the pilot does not anticipate the pitch up. It can, of course, be counteracted with a nose down stick input. This particular case is referred to as roll coupling, which resulted in the crash of several aircraft in the 1940s and 1950s, before a full understanding of coupling.

Applied Moments

The applied moments consist of the aerodynamic rolling, pitching, and yawing moments, L_A, M_A, and N_A, respectively, and the rolling, pitching, and yawing moments because of thrust, L_T, M_T, and N_T, respectively. There are no moments because of gravity because the weight vector acts through the center of gravity and the moment arms are zero. Also, any moments because of rotating masses (such as jet engines) on or within the aircraft have been neglected. Therefore,

$$\begin{aligned}
&\dot{P}I_{xx} + QR(I_{zz} - I_{yy}) - (\dot{R} + PQ)I_{xz} = L_A + L_T \\
&\dot{Q}I_{yy} - PR(I_{zz} - I_{xx}) + (P^2 - R^2)I_{xz} = M_A + M_T \\
&\dot{R}I_{zz} + PQ(I_{yy} - I_{xx}) + (QR - \dot{P})I_{xz} = N_A + N_T
\end{aligned}$$

→ a

The makeup of each of these moments will be discussed in detail in subsequent chapters.

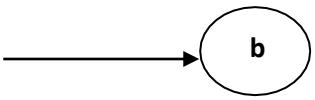
Longitudinal and Lateral-Directional Equations of Motion

The six aircraft equations of motion (EOM) can be decoupled into two sets of three equations. These are the three longitudinal EOM and the three lateral-directional EOM. This is convenient in that it requires only three equations to be solved simultaneously for many flight conditions. For example, an aircraft in wings-level flight with no sideslip and a pitching motion can be analyzed using only the longitudinal EOM because the aircraft does not have any lateral-directional motion.

Longitudinal Equations of Motion

The three longitudinal EOM consist of the x force, z force, and y moment equations, namely,

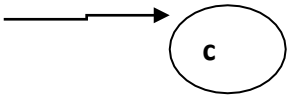
$$m(\dot{U} + QW - RV) = -mg\sin\Theta + (-D\cos\alpha + L\sin\alpha) + T\cos\Phi_T$$
$$\dot{Q}I_{yy} - PR(I_{zz} - I_{xx}) + (P^2 - R^2)I_{xz} = M_A + M_T$$
$$m(\dot{W} + PV - QU) = mg\cos\Phi\cos\Theta + (-D\sin\alpha - L\cos\alpha) - T\sin\Phi_T$$



Lateral-Directional Equations of Motion

The lateral-directional EOM consist of the y force, x moment, and z moment equations, namely,

$$\dot{P}I_{xx} + QR(I_{zz} - I_{yy}) - (\dot{R} + PQ)I_{xz} = L_A + L_t$$
$$m(\dot{V} + RU - PW) = mg\sin\Phi\cos\Theta + F_{A_y} + F_{T_y}$$
$$\dot{R}I_{zz} + PQ(I_{yy} - I_{xx}) + (QR - \dot{P})I_{xz} = N_A + N_T$$



For any lateral-directional motion the xz plane would move out of some xz plane fixed in space. Translation in the y direction, roll about the x axis, and yaw about the z axis would all cause the xz plane of the aircraft to move out of that arbitrarily fixed xz plane in space.

Small Perturbation Approach

Linearization of the aircraft equations of motion begins with consideration of **perturbed flight**. Perturbed flight is defined relative to a steady-state (trimmed) flight condition using a combination of steady-state and perturbed variables for aircraft motion parameters and for forces and moments. Simply stated, each motion variable, Euler angle, force, and moment in the equations of motion (EOM) are redefined as the summation of a steady-state value (designated with the subscript "1") and a perturbed value (designated with lower case symbols) as summarized in Eq. (6.1).

$$
\begin{aligned}
U &\equiv U_1 + u & V &\equiv V_1 + v & W &\equiv W_1 + w \\
P &\equiv P_1 + p & Q &\equiv Q_1 + q & R &\equiv R_1 + r \\
\Psi &\equiv \Psi_1 + \psi & \Theta &\equiv \Theta_1 + \theta & \Phi &\equiv \Phi_1 + \phi \\
F_A &\equiv F_{A_1} + f_A & F_T &\equiv F_{T_1} + f_T & & \\
L_A &\equiv L_{A_1} + l_A & M_A &\equiv M_{A_1} + m_A & N_A &\equiv N_{A_1} + n_A \\
L_T &\equiv L_{T_1} + l_T & M_T &\equiv M_{T_1} + m_T & N_T &\equiv N_{T_1} + n_T
\end{aligned}
$$

Longitudinal Linearized EOMs

$$
\begin{aligned}
&m[\dot{U}_1 + \dot{u} + (Q_1 + q)(W_1 + w) - (R_1 + r)(V_1 + v)] \\
&\quad = -mg\sin(\Theta_1 + \theta) + F_{A_{x_1}} + f_{A_x} + F_{T_{x_1}} + f_{T_x} \\
&I_{yy}(\dot{Q}_1 + \dot{q}) - (P_1 + p)(R_1 + r)(I_{zz} - I_{xx}) + [(P_1 + p)^2 - (R_1 + r)^2]I_{xz} \\
&\quad = M_{A_1} + m_A + M_{T_1} + m_T \\
&m[\dot{W}_1 + \dot{w} + (P_1 + p)(V_1 + v) - (Q_1 + q)(U_1 + u)] \\
&\quad = mg\cos(\Phi_1 + \phi)\cos(\Theta_1 + \theta) + F_{A_{z_1}} + f_{A_z} + F_{T_{z_1}} + f_{T_z}
\end{aligned}
$$

Applying Steps 2–4 of Sec. 6.1, these reduce to

$$
\begin{aligned}
&m(\dot{u} - V_1 r - R_1 v + W_1 q + Q_1 w) = -mg\theta\cos\Theta_1 + f_{A_x} + f_{T_x} \\
&I_{yy}\dot{q} + (I_{xx} - I_{zz})(P_1 r + R_1 p) + I_{xz}(2P_1 p - 2R_1 r) = m_A + m_T \\
&m(\dot{w} - U_1 q - Q_1 u + V_1 p + P_1 v) = -mg\theta\cos\Phi_1\sin\Theta_1 \\
&\qquad\qquad - mg\phi\sin\Phi_1\cos\Theta_1 + f_{A_z} + f_{T_z}
\end{aligned}
$$

Lateral-Directional Linearized EOMs

$$
\begin{aligned}
&(\dot{P}_1 + \dot{p})I_{xx} + (Q_1 + q)(R_1 + r)(I_{zz} - I_{yy}) - [\dot{R}_1 + \dot{r} + (P_1 + p)(Q_1 + q)]I_{xz} \\
&\quad = L_A + l_A + L_T + l_T \\
&m[\dot{V}_1 + v + (R_1 + r)(U_1 + u) - (P_1 + p)(W_1 + w)] \\
&\quad = mg\sin(\Phi_1 + \phi)\cos(\Theta_1 + \theta) + F_{A_{y_1}} + f_{A_y} + F_{T_{y_1}} + f_{T_y} \\
&(\dot{R}_1 + r)I_{zz} + (P_1 + p)(Q_1 + q)(I_{yy} - I_{xx}) + [(Q_1 + q)(R_1 + r) - (\dot{P}_1 + \dot{p})]I_{xz} \\
&\quad = N_{A_1} + n_A + N_{T_1} + n_T
\end{aligned}
$$

Applying Steps 2–4 of Sec. 6.1, these reduce to

$$
\begin{aligned}
&I_{xx}\dot{p} - I_{xz}\dot{r} - I_{xz}(P_1 q + Q_1 p) + (I_{zz} - I_{yy})(R_1 q + Q_1 r) = l_A + l_T \\
&m(\dot{v} + U_1 r + R_1 u - W_1 p - P_1 w) = -mg\theta\sin\Phi_1\sin\Theta_1 \\
&\qquad + mg\phi\cos\Phi_1\cos\Theta_1 + f_{A_y} + f_{T_y} \\
&I_{zz}\dot{r} - I_{xz}\dot{p} + (I_{yy} - I_{xx})(P_1 q + Q_1 p) + I_{xz}(Q_1 r + R_1 q) = n_A + n_T
\end{aligned}
$$

UNIT-III

Lateral, Directional & Dynamic Stability-Response to Control

Static stability of motion about x-axis–dihedral effect

The lateral stability analysis deals with the motion about x-axis. In this context the following three points may be noted.

(a) The rotation about x-axis leads to the bank angle

(b) A disturbance would change the bank angle from Φ to $(\Phi+\Delta\Phi)$.

(c) For static stability about x-axis, an airplane should develop a rolling moment to bring the airplane to the original bank angle.

To examine the lateral static stability, consider an airplane in a steady, level flight. In this flight the x-axis coincides with the velocity vector (**V**) and $\Phi= 0$. Let the airplane be given a bank angle Φ gently so that the rate of roll is negligible. It is noticed that even in the banked position the aerodynamic field remains symmetric about the plane of symmetry. Hence, no restoring rolling moment is produced. Thus, the airplane is neutrally stable about x-axis. The restoring moment is brought about in the following manner.

(1) when an airplane acquires a bank angle a component of the weight,

$W \sin\Phi$, acts in the y-direction and the airplane begins to sideslip. Consider a roll to right, i.e. right wing down. Due to $W \sin\Phi$, the airplane begins to sideslip to right or experiences a relative wind from right to left. This produces a positive β.

(2) If the airplane is rolled to right with an angular velocity 'p' then, as pointed out in subsection (on adverse yaw), the airplane develops yaw to left. Which results again in a positive β.

(3) When an airplane has a sideslip it produces both rolling moment and yawing moment. If the rolling moment so produced, tends to restore the airplane to the original attitude of $\Phi = 0$, then it can be considered as a stabilizing effect.

Rolling moment due to sideslip is called dihedral effect.

Rolling moment and its convention

Rolling moment is denoted as L′ to distinguish it from lift which is denoted by L. Rolling moment coefficient is denoted as C'_l i.e.

$$C'_l = \frac{L'}{\frac{1}{2}\rho V^2 Sb}$$

Convention for rolling moment

A rolling moment which causes roll to right or right wing down, is taken as positive. It may be recalled that a moment is positive in clockwise direction when looking along the positive direction of the axis.

Criterion for stabilizing dihedral effect

As mentioned earlier, a bank to right produces positive β. This β should produceanegativeL′tobringtheairplanebacktozeroroll.Hence,forstabilizing effect, $C'_{l\beta}$ should be negative.

The contribution to $C'_{l\beta}$can be expressed as:

$$C'_{l\beta}=(C'_{l\beta})_w+(C'_{l\beta})_{f,n,p}+(C'_{l\beta})_{vt}$$

Contribution of wing to $C'_{l\beta}$

The contributions of wing to $C'_{l\beta}$ Are due to the dihedral angle (Γ) and the sweep (Λ).

Contribution of wing dihedral angle to $C'_{l\beta}$

A wing is said to have a dihedral, when the tips of the wing are at a higher level than the root of the wing .The contribution to $C'_{l\beta}$ due to dihedral angle (Γ) can be calculated with the following steps.

(a) Assume that the airplane rolls to right.

(b) It develops positive β.

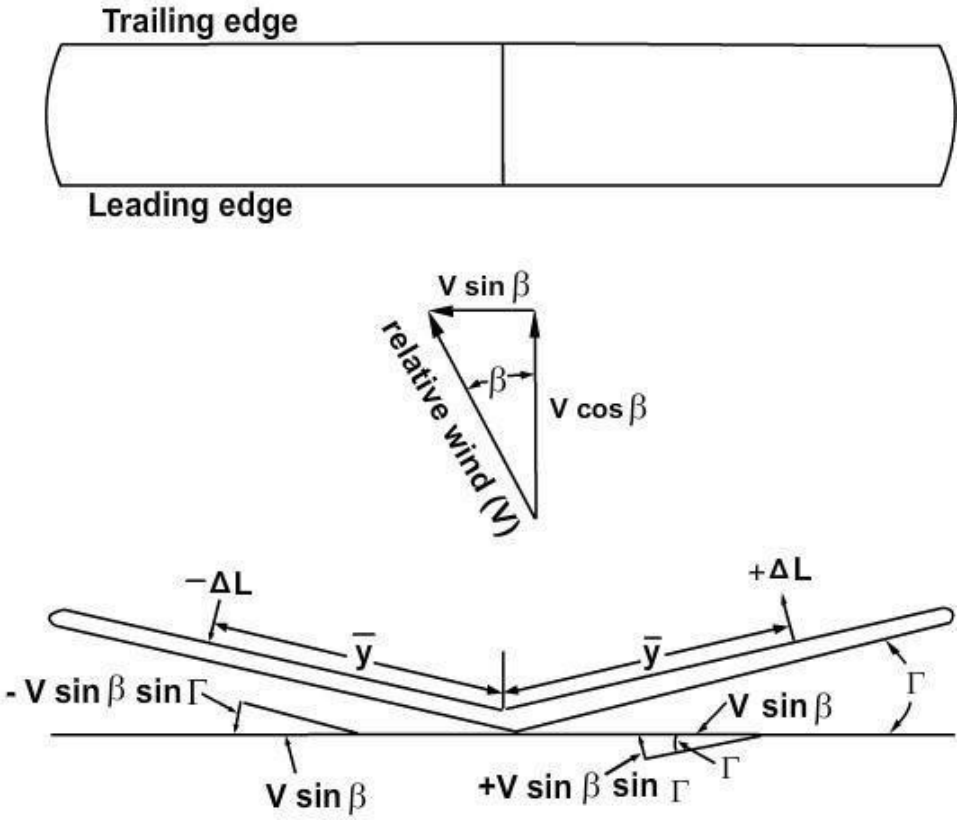
(c) V sinβ is the sideward component of the relative velocity(side wind).

(d) The component of the side wind (V sinβ)perpendicular to the wing is V sin βsin Γ.But,it is up ward on the right wing and downward on the left wing (Fig.6.1).

(e) Δα, the magnitude of the change in the angle of attack on the two wing halves, is:

$$\Delta\alpha = \frac{V\sin\beta\sin\Gamma}{V} \quad \Gamma\beta = \Gamma\frac{v}{V}$$

v= Vsinβ is the sideward velocity.



Contribution of dihedral angle to C'_l

(f) However, $\Delta\alpha$ on the right wing = $\beta\Gamma$ and $\Delta\alpha$ on the leftwing=-$\beta\Gamma$.Hence, the lifts on the two wing halves are unequal and a rolling moment is produced. Rolling moment due to $\Delta\alpha$ on the right wing is:

$$L'_{wr} = -\frac{1}{2}\rho V^2 \frac{dC_L}{d\alpha} \Delta\alpha \int_0^{b/2} c\,y\,dy \qquad \text{Note: } \Delta C_L = \Delta\alpha (dC_L/d\alpha)$$

Rolling moment due to $\Delta\alpha$ on the left wing is:

$$L'_{wl} = -\frac{1}{2}\rho V^2 \frac{dC_L}{d\alpha} \Delta\alpha \int_0^{b/2} c\,y\,dy \qquad \text{Note: } \Delta\alpha \text{ is negative on left wing.}$$

Finally,

$$(L')_{w\Gamma} = -2\frac{1}{2}\rho V^2 \frac{dC_L}{d\alpha} \Delta\alpha \int_0^{b/2} c\,y\,dy$$

Substituting $\Delta\alpha = \beta\Gamma$ and $\bar{y} = \frac{2}{S}\int_0^{b/2} c\,y\,dy$, gives:

$$(L')_{w\Gamma} = -2\left(\frac{1}{2}\rho V^2 \frac{dC_L}{d\alpha}\right)\Gamma\beta\left(\frac{S}{2}\right)\bar{y}$$

Or

$$(C'_{lw})_\Gamma = (L'_w)_\Gamma / (-\rho V S b) = -\Gamma\beta\, 2\left(\frac{dC_L}{d\alpha}\right)\bar{y}$$

Hence, $$(C')_{l\beta\Gamma} = -\Gamma\left(\frac{dC_L}{d\alpha}\right)\bar{y}$$

i) It may be noted that the contribution of dihedral to $C'_{l\beta}$ is negative. Since, $C'_{l\beta}$ should be negative for static stability, the contribution of dihedral to $C'_{l\beta}$ is called a stabilizing contribution.

ii) For a wing witht a taperratio λ,Eq gives:

$$(C'_{l\beta\Gamma}) = -0.25\Gamma\left(\frac{dC_L}{d\alpha}\right)\left[\frac{2(1+2\lambda)}{3(1+\lambda)}\right] \text{ perradian}$$

iii) Reference1.12chapter3andRef.2.2,section5.1.2giverefinedestimatesof $(C'_{l\beta\Gamma})$.

6.5.2 Contribution of wing to sweep to $C'_{l\beta}$

While discussing the contribution of wing sweep to $C_{n\beta}$,it was pointed out that for a wing with sideslip, the normal components of free stream velocity (V) are different on the two wing halves .This gives rise to different drags on the two wing halves and contributes to yawing moment. Similarly, the difference in normal velocity components would result in the lift on the two wing halves being different in this case. This would give rise to a rolling moment. The contribution of sweep to $C'_{l\beta}$ is obtained by the following steps.

The contributions of the right wing and left wing to the rolling moment are:

$$(L')_{wr\Lambda} = -C_L \frac{S}{2}\left(\frac{1}{2}\rho V^2\right)\bar{y}\cos^2(\Lambda-\beta)$$

$$(L')_{wl\Lambda} = C_L \frac{S}{2}\left(\frac{1}{2}\rho V^2\right)\bar{y}\cos^2(\Lambda+\beta)$$

Hence, $$(L')_{w\Lambda} = -C_L \frac{S}{2}\left(\frac{1}{2}\rho V^2\right)\bar{y}\{\cos^2(\Lambda-\beta)-\cos^2(\Lambda+\beta)\}$$

$$= -C_L \frac{S1}{22} \rho V_\infty^2 \bar{y}\{4\cos\Lambda\cos\beta\sin\Lambda\sin\beta\}$$

where, $\bar{y}$ is the location of the resultant lift on the wing half. Since, β is small,

$$(L'_{w\Lambda}) = -C_L \frac{1}{2} \rho V^2 \bar{S} y \beta \sin 2\Lambda$$

$$(C_{l\beta w\Lambda}) = -C_L \frac{y}{b} \beta \sin 2\bar{\Lambda}$$

$$(C_{l\beta w\Lambda}) = -C_L \frac{y}{\bar{b}} \sin 2\Lambda$$

i) The contribution due to sweep back is negative and hence stabilizing. Note that it is proportional to C_L.

ii) Reference 1.12 gives an improved estimate of $(C_{l\beta w})_\Lambda$. The value of $(C_{l\beta w})_\Lambda$ Given by this reference, increases monotonically with □.

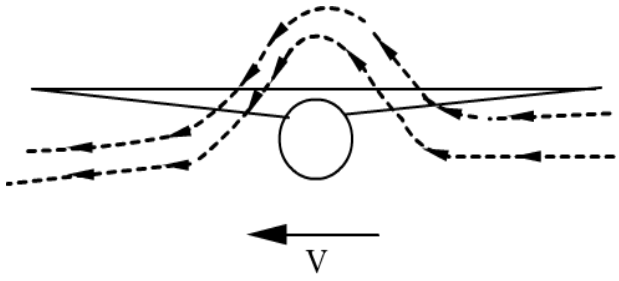
Contribution of fuselage to C'$_{l\beta}$

The contribution of fuselage to $C'_{l\beta}$ arises due to the interference effect. Consider an airplane having positive β or the sideward velocity component (v = V sinβ) from right to left. Figure 6.2 shows the displacement of stream lines for high wing and low wing configurations. It is observed that for the high wing case the change in angle of attack, Δα, is positive on the right wing and negative on the left wing. This would result in a negative rolling moment which is a stabilizing contribution. For a low wing configuration the effect would be opposite and a destabilizing contribution. Give elaborate methods of estimating effect of wing fuselage interference. gives the following approximate estimates for high wing, mid-wing and low wing cases.

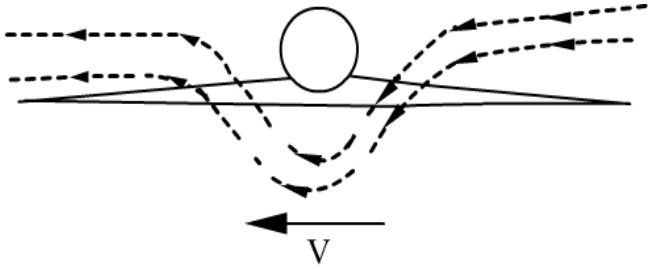
$\Delta C'_{l\beta} = -0.0006 \text{deg}^{-1}$ for high wing,

$= 0.00$ for mid-wing and

$= +0.0006 \text{deg}^{-1}$ for low wing configuration.



High wing configuration

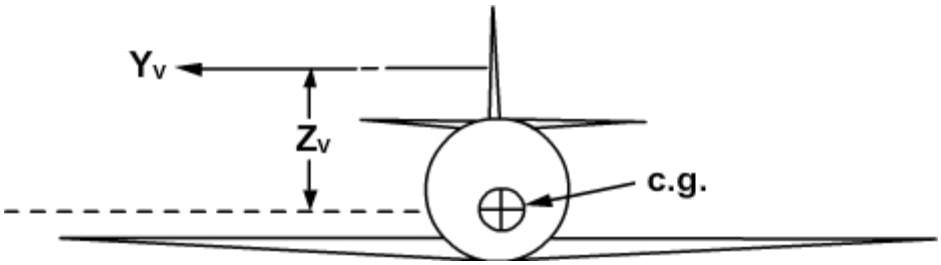


Low wing configuration

Effect of wing location on $C'_{l\beta}$

Contribution of vertical tail to $C'_{l\beta}$

It is shown that a vertical tail with a positive β causes a negative side force. This side force generally acts above the c.g and would contribute a negative rolling moment (Fig.6.3).



Contributionofverticaltail to $C'_{l\beta}$ The contribution of vertical tail can be expressed as:

$$L' = -Y_v Z_v$$

$$= -\frac{1}{2}\rho V^2 \eta_v S_v \left(\frac{dC_L}{d\alpha}\right)_v \beta Z_v$$

Hence, $(C')_{l\beta v} = -\eta_v \frac{S_v Z_v}{Sb} C_{L\alpha v}$

i) When the airplane has an angle of attack (α) the height Z_v will depend on the tail length l_v and the angle of attack α.
ii) Generally $(C_{l\beta})_v$ is small.

Contribution due to propeller and flaps to $C'_{l\beta}$

Figure shows a propeller in positive sideslip. As pointed out in section the slipstream is rendered asymmetric due to sideslip. A larger portion of the left wing is influenced by the slipstream as compared to that on the right wing. Hence, the left wing will experience a higher dynamic pressure and consequently will produce more lift as compared to the right wing. This causes a positive rolling moment. Hence, $C'_{l\beta}$ is positive, i.e. a destabilizing contribution. This influence worsens when the flaps are deflected. However, the contributions due to propeller and flap to $C'_{l\beta}$ are small.

Selection of dihedral angle

As noted earlier, the levels of longitudinal and directional static stability ($C_{m\alpha}$ and $C_{n\beta}$) can be adjusted by changing the areas of the horizontal tail (S_t) and the vertical tail (S_v) respectively. The level of $C'_{l\beta}$, can be adjusted by choosing an appropriate dihedral angle. To arrive at the dihedral angle needed for an airplane, the contributions due to wing sweep, fuselage, power plant and vertical tail are first calculated. Then, the difference between the sum of these contributions and the desirable level of $C'_{l\beta}$ is provided by choosing an appropriate dihedral angle.

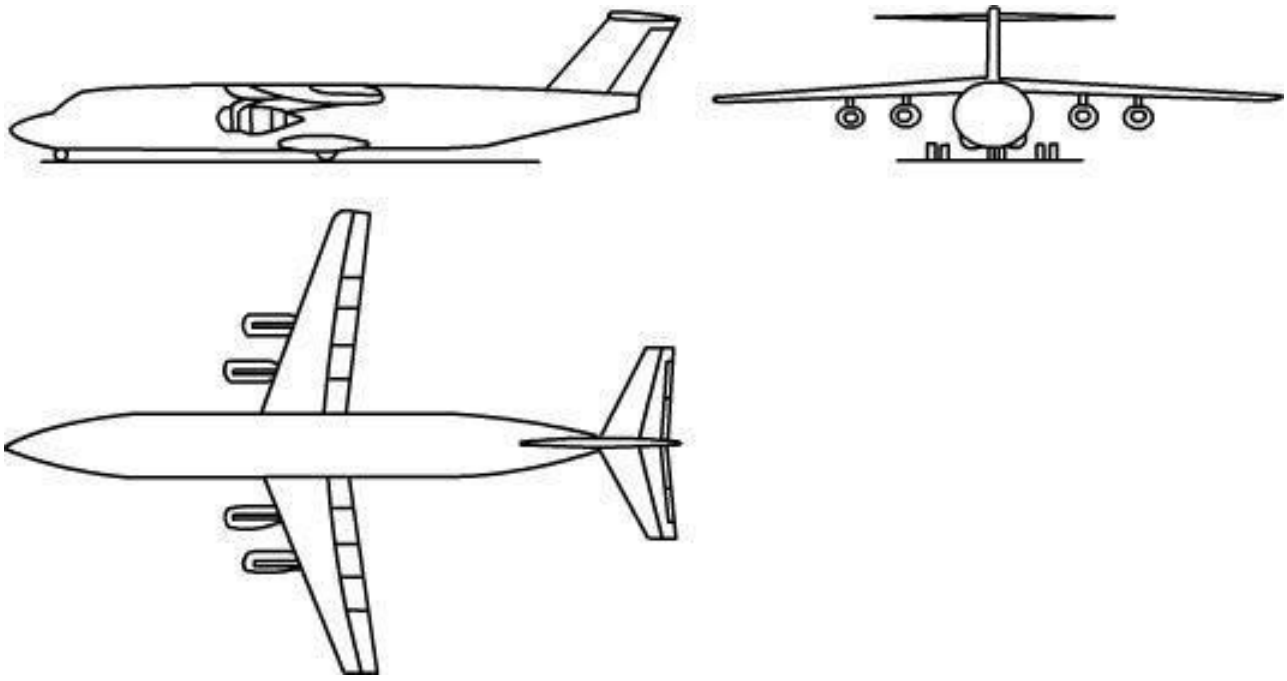
$(C'_{l\beta})_{desirable} = -(C_{n\beta}/2)$

However, the data on seven airplanes given in appendix‘ B’of reference indicates that Eq may be approximately valid for military airplanes. For other airplanes, $C'_{l\beta}$could be equal to or higher than $C_{n\beta}$. Actual values of $C'_{l\beta}$ and

$C_{n\beta}$ are arrived at after carrying out the lateral dynamic stability analysis

6.9.1 Wing with anhedral

As mentioned earlier, $C'_{l\beta}$ should not be too high. When an airplane has highly swept wings and in addition has high wing configuration then the contributions due to these factors may be large. Sometimes, such airplanes have negative dihedral which is called an hedral.



An airplane with anehedral (Note:swept and high wing configuration)

Directional static stability and control

In this case, the motion of the airplane takes place in the plane of symmetry i.e. along x- and z- axes and about y- axis.

This chapter and the next one, deal with the motions along y-axis and about x- and z-axes. These motions lie outside the plane of symmetry. The translatory motion along y-axis is sideslip and rotations about x- and z-axes are the rolling and yawing respectively. The directional stability and control, deal with the equilibrium and its maintainability about the z-axis. The lateral stability and control, deal with the equilibrium and its maintainability about the x-axis. However, the lateral and directional motions cannot be separated completely because a change in one of them leads to change in the other.

For example, when an airplane has a rate of roll, the unequal changes in the drag of the two wing halves create a yawing moment. Besides the rolling and yawing motions, the sideslip also creates forces and moments affecting lateral and directional motions. The six effects caused by rolling, yawing and sideslip are listed below.

i. Rolling moment due to rate of roll. It is called damping in roll.
ii. Yawing moment due to rate of yaw. It is called damping in yaw.
iii. Rolling moment due to rate of yaw. It is called cross effect.
iv. Yawing moment due to rate of roll. It is called adverse yaw.
v. Rolling moment due to side slip. It is called dihedral effect.
vi. Yawing moment due to side slip. It is called weathercock effect.

The directional static stability and control are considered in this chapter.

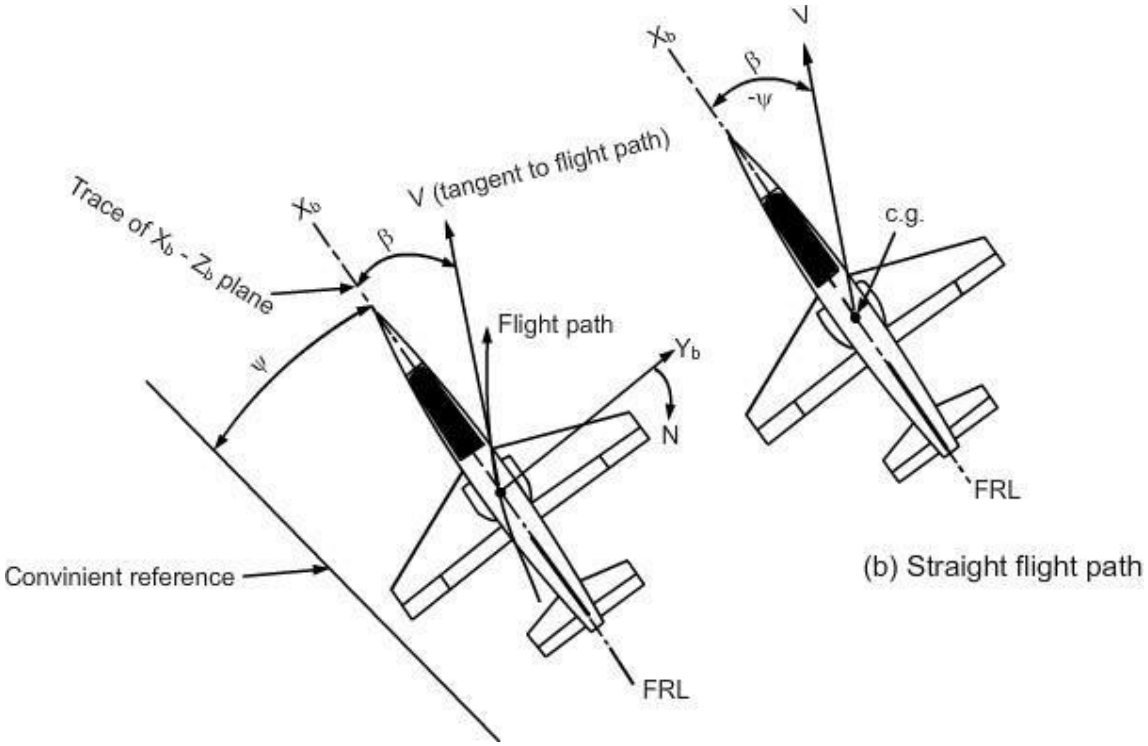
Criteria for equilibrium and static stability about z-axis

In an equilibrium flight, the airplane flies in the plane of symmetry with sideslip and yawing moment both being zero. Before discussing the criteria for equilibrium and static stability about z- axis, it is useful to recapitulate a few relevant concepts.

Sideslip and yaw

Sideslip is the angle between the plane of symmetry of the airplane and the direction of motion. It is taken as positive in the clockwise sense. It is denoted by 'β'. It may be recalled that the tangent to the flight path is the direction of motion. It may be further pointed out that a positive β is due to a positive sideslip velocity which is the component of airplane velocity along the y-axis.

Angle of yaw is the angular displacement of the airplane centerline, about a vertical axis, from a convenient horizontal reference line. It is measured from the arbitrarily chosen reference direction and taken as positive in the clockwise direction. It is denoted by 'Ψ'.



Slideslipandyaw

i) The sideslip angle and the yaw angle are not equal. For example in a 360º turn, the airplane yaws through 360º, but there may not be any sideslip if the airplane axis is aligned with the tangent to the flight path at all points during the turn.

ii) If the flight path is a straight line and the arbitrary axis chosen to measure the yaw is taken as the direction of flight, then yaw and sideslip angles are equal in magnitude, but opposite in sign.

iii) In wind tunnel tests, the models of airplane are tested by rotating the airplane center line with respect to the air stream and the angle between the plane of symmetry of the airplane and the air stream is called the angle of yaw. The results are reported as variations of yawing moment with□.

iv) In flight test work however,sideslip angleβ is generally used.

In the subsequent analysis, the case of straight flight path is considered.
Even if the flight path is curved, the analysis can be carried out by taking into considerationa small segment of the path; however, the damping due to angular velocity will need to be taken into account.
the subsequent analysis is carried out in terms of β.

5.2.2

Yawing movement and its convention

The moment about z-axis i.e. yawing moment is denoted by N.

Consideringcontributionsfrommajorcomponentsof theairplane,Ncanbe written as:

$$N=(N)_w+(N)_f+(N)_n+(N)_p+(N)_{vt} \quad (1)$$

Where,the suffixesw, f,n,pandv$_t$ indicatecontributionsfrom wing,fuselage, nacelle, power and vertical tail.

Equation5.1isexpressedinnon-dimensionalformas:

$$C_n=\frac{N}{\frac{1}{2}\rho V^2 Sb}=(C_n)_w+(C_n)_{f,n,p}+(C_n)_{vt} \quad (2)$$

i) Convention

The yawing moment (N) is taken positive when the right wing goes back (see Figs.5.1a and 1.8). Recalling the convention for pitching moment, it is observed that in stability analysis a moment is taken positive in a clockwise direction when looking along the axis under consideration.

ii) In Eq.(5.2), the wing span (b) is used as the reference length for non-dimensionalization of yawing moment, where as the mean aerodynamic chord($\bar{c}$) isusedasthereferencelengthfornon-dimensionalizationofpitchingmoment.

To explain this difference in choice of reference length, it may be recalled that a moment is the product of a force and the distance perpendicular to the axis. Hence, reference length for non-dimensionalization of a moment should be a representative length perpendicular to the axis under consideration.

Criterion for equilibrium about z-axis

The criteria for equilibrium about z-axis is that the yawing moment should be zero i.e. for equilibrium, $C_n = 0$. 3

Criterion for directional static stability

Figures a and b show the conventions for positive yawing moment and sideslip (β). Consider that in equilibrium flight, the airplane is flying with $\beta = 0$.

Now, let a disturbance cause the airplane to develop positive sideslip of $\Delta\beta$. It is observed that to bring the airplane back to equilibrium position i.e. $\beta = 0$, a positive yawing moment (ΔN) should be produced by the airplane. Similarly, a disturbance causing a negative $\Delta\beta$ should result in $-\Delta N$ i.e.for static directional stability, $dC_n / d\beta$ or $C_{n\beta}$ should be positive. Hence,

$C_{n\beta}$>0forstaticdirectionalstability

=0forneutraldirectionalstability and 4

<0fordirectional in stability.

Differentiating Eq.(5.2)yields:

$C_{n\beta}=(C_{n\beta})_w +(C_{n\beta})_{f,n,p}+(C_{n\beta})_{vt}$ 5

It may be recalled that $C_{m\alpha}$should be negative for longitudinal static stability whereas $C_{n\beta}$ should be positive for directional static stability. This difference in sign is due to conventions used for α and β. CompareFigs.1.14 and 1.15b. However, it may also be pointed out that when α is positive, the 'w' componentofflightvelocityisalongpositivez-directionandwhenβispositive, the sideslip velocity 'v' is along positive y-axis (see Fig.1.15).

The contributions of major components to $c_n$and $c_{n\beta}$ are discussed in the next four sections.

Contribution of wing to $C_{n\beta}$

For straight (or un swept) wings, there is no significant contribution of wing to $C_{n\beta}$. However, for swept wings, there is a small contribution. Explains this contribution based on certain simplifying assumptions. This approach is explained below.

The explanation is based on the argument that for a swept wing, the component of the free stream velocity normal to the quarter chord line mainly decides the aerodynamic forces. Consider an airplane with wings which have sweep Λ. When this wing is subjected to sideslip β, the components of the free stream velocity normal to the quarter chord line on the two wing halves will be unequal i.e. V cos (□-□) on the right wing and V cos (□+□) on the left wing. Consequently, even ifthe two wing halves are at the same angle of attack, they would experience unequal effective dynamic pressures and their drags will be different. This will cause a yawing moment.The contribution of a swept wing to $C_{n\beta}$ can be derived in the following manner.

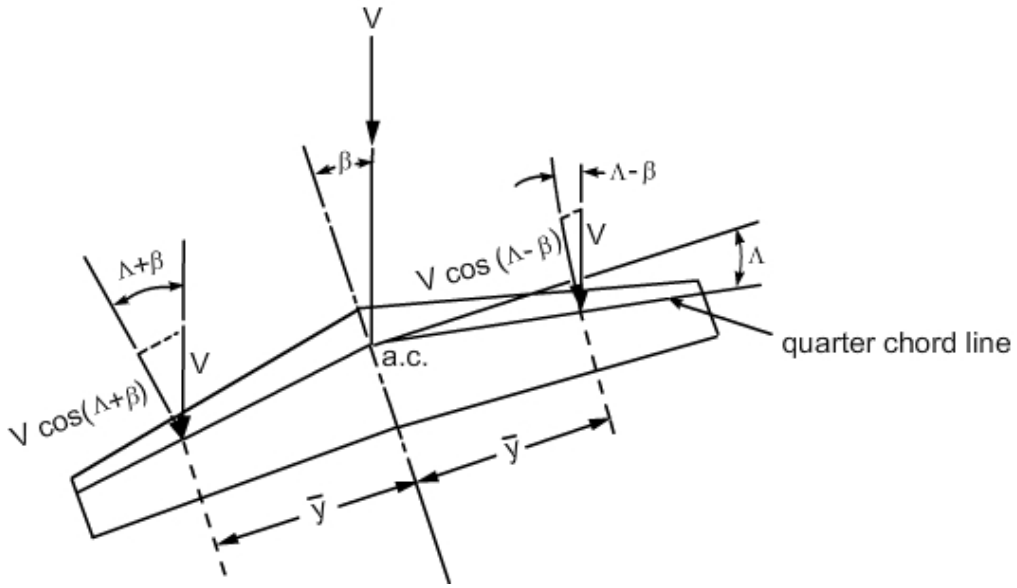


Fig.Sweptwingwithsideslip

In general, the chord of the wing and the span wise lift distribution varies withthespanwisecoordinate(y).However,forthesakeofexplanationletybe $\bar{y}$ thespanwiselocationoftheresultantdragontherightwing.Similarlylet–y be the location of $\bar{y}$ the resultant drag on the left wing. Then, the yawing moments due to the right and the left wing halves $(N_w)_r$ and $(N_w)_l$are:

$$(N)_{wr} = \frac{1}{2}\rho V^2 \frac{C}{2} S \bar{y} \cos^2(\Lambda - \beta) \qquad 6$$

$$(N)_{wl} = -\frac{1}{2}\rho V^2 C_D S_y \cos^2(\Lambda+\beta) \quad (7)$$

Consequently, total yawing moment due to the wingis:

$$N_w = \frac{1}{2}\rho V C_D^2 \frac{S}{2} y \{\cos(\Lambda-\beta)-\cos(\Lambda+\beta)\}$$

$$= \frac{1}{2}\rho V C_D^2 \frac{S}{2} y \{4\cos\Lambda\sin\Lambda\cos\beta\sin\beta\} \quad (8)$$

Forsmall β,sinβ=βandcosβ=1.Hence,

$$N = C S y \frac{1}{2}\rho V^2 \frac{\beta}{57.3} \sin 2\Lambda; \beta \text{ indegrees.} \quad (9)$$

$$w \quad D$$

$$\text{Hence,} (C)_{nw} = \frac{N_w}{\frac{1}{2}\rho V^2 S b} = C_D y \frac{\beta}{b 57.3} \sin 2\Lambda \quad (10)$$

DifferentiatingEq.(5.10)withβyields:

$$(C)_{n\beta w} = C_D y \frac{1}{b 57.3} \sin 2\Lambda \quad (11)$$

i) As mentioned earlier Eq.(11) is an approximate estimate, based on DATCOM , gives a more accurate formula for $C_{n\beta w}$ due to sweep. The formula shows that $C_{n\beta w}$ depends also on the wing aspect ratio and thedistancebetweenthea.c.andc.g..Furtherthecontributionisproportionalto C_L^2 and would be small during cruise.

ii) A wing with a dihedral also contributes to $C_{n\beta w}$.

iii) It may be noted that the contribution of wing to $C_{n\beta}$ is positive. Since,$C_{n\beta}$ should be positive for directional static stability, appositive contribution to $C_{n\beta}$ is called stabilizing contribution.

Contribution of fuselage to $C_{n\beta}$

In subsection 2.5.1 it is shown that a fuselage at an angle of attack produces a pitching moment and also contributes to $C_{m\alpha}$.Similarly ,a fuselage in sideslip produces a yawing moment and contributes to $C_{n\beta}$of the airplane. However,inanairplane,theflowpastafuselageisinfluencedbytheflowpast

the wing. Hence, instead of an isolated fuselage, the contributions of wing and fuselage to $C_{n\beta}$ are estimated togather. It is denoted by $C_{n\beta wf}$. Based on, the following formula is presented for $C_{n\beta wf}$.

$$C_{n\beta wf} = -k k_n k_{Rl} \frac{S_{fs} l_f}{S_w \overline{b}} \text{ deg}^{-1} \qquad 12$$

where, k_n is the wing body interference factor which depends on the following fuselage parameters.

(a) Length of fuselage (l_f). (b) Projected side area of fuselage (S_{fs}).

(c) Heights (h_1 and h_2) of fuselage at $l_f/4$ and $3l_f/4$. (d) Distance, from nose, of the station where the height of fuselage is maximum (x_m).

Figure 5.3 illustrates the procedure to obtain k_n when $x_m/l_f = 0.586$, $l_f^2/S_{fs} = 10$, $(h_1/h_2)^{1/2} = 1.0$ and $h/w_f = 1.0$.

The quantity k_{Rl} is an empirical factor which depends on the Reynolds number of the fuselage ($R_{lf} = V\rho l_f/\mu$) (see Fig. 5.4).

Appendix 'C' illustrates the procedure to obtain $C_{n\beta wf}$.

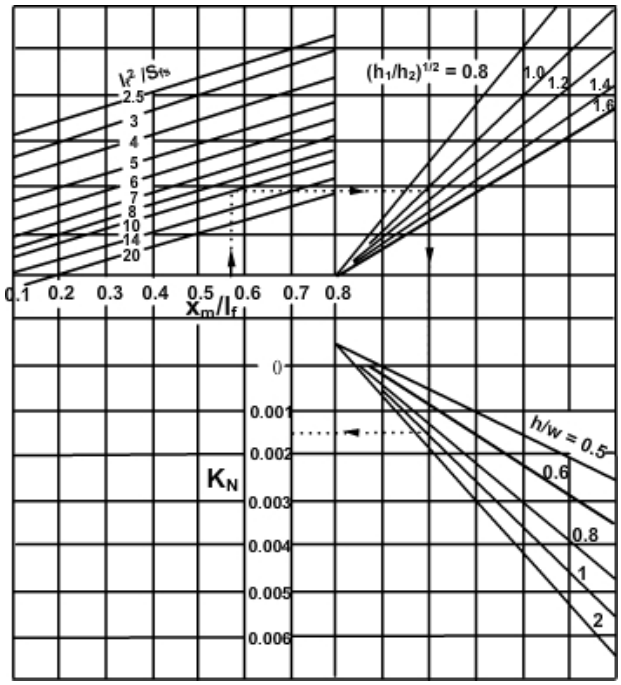


Fig. 5.3 Wing body interference factor

(Adapted from Ref. 2.2, section 5.2.3.1)

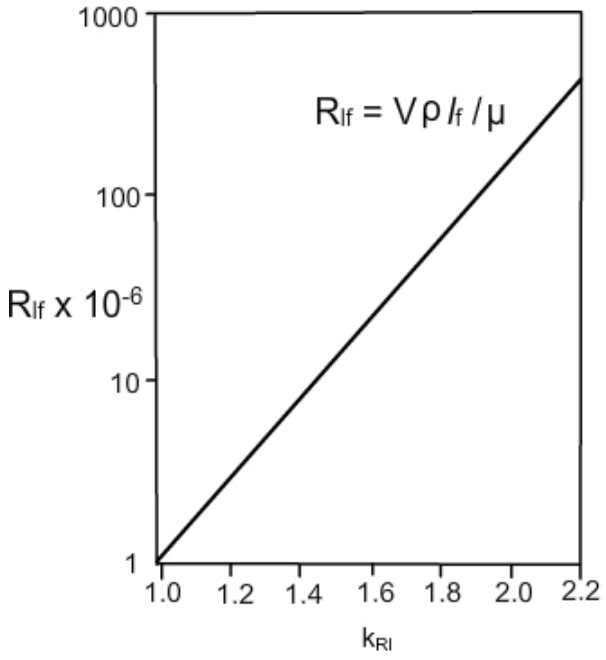


Fig.CorrectionfactorforReynoldsnumber

AircraftLateralDynamics

- Using a procedure similar to the longitudinal case, we can develop theequationsofmotionforthe**lateraldynamics**

$$\dot{x}=Ax+Bu, x=\begin{bmatrix} v \\ p \\ r \\ \varphi \end{bmatrix}, u=\begin{bmatrix} \delta_a \\ \delta_r \end{bmatrix}$$

and $\dot{\psi}=r\sec\theta_0$

$$A=\begin{bmatrix} \frac{Y_v}{m} & \frac{Y_p}{m} & \frac{Y_r}{m}-U_0 & g\cos\theta_0 \\ \left(\frac{L_v}{I^r_{xx}}+I^r_{zx}N_v\right) & \left(\frac{L_p}{I^r_{xx}}+I^r_{zx}N_p\right) & \left(\frac{L_r}{I^r_{xx}}+I^r_{zx}N_r\right) & 0 \\ \left(I^r_{zx}L_v+\frac{N_v}{I^r_{zz}}\right) & \left(I^r_{zx}L_p+\frac{N_p}{I^r_{zz}}\right) & \left(I^r_{zx}L_r+\frac{N_r}{I^r_{zz}}\right) & 0 \\ 0 & 1 & \tan\theta_0 & 0 \end{bmatrix}$$

where

$$I^r_{xx}=(I_{xx}I_{zz}-I^2_{zx})/I_{zz}\quad I^r_{zx}=(I_{xx}I_{zz}-I^2_{zx})/I_{xx}\quad I^r_{zz}=I_{zx}/(I_{xx}I_{zz}-I^2_{zx})$$

and

$$B=\begin{bmatrix} (m)^{-1} & 0 & 0 \\ 0 & (I^r_{xx})^{-1} & I^r_{zx} \\ 0 & I^r_{zx} & (I^r_{zz})^{-1} \\ 0 & 0 & 0 \end{bmatrix}\cdot\begin{bmatrix} Y_{\delta_a} & Y_{\delta_r} \\ L^a_\delta & L^r_\delta \\ N^a_\delta & N^r_\delta \\ 0 & 0 \end{bmatrix}$$

Lateral Stability Derivatives

- A key to understanding the lateral dynamics is **roll•yawcoupling**.

- L_p rolling moment due to roll rate:
  - Roll rate p causes right to move wing down, left wing to move up
    →Vertical velocity distribution over the wing $W=py$
  - Leads to a span wise change in the AOA:$\alpha_r(y)=py/U_0$
  - Creates lift distribution(chord wises trips)

$$\delta L_w(y)=\frac{1}{2}\rho U_0^2 C_{l_\alpha}\alpha_r(y)c_y dy$$

  - Net result is higher lift on right, lower on left
  - Rolling moment:

$$L=\int_{-b/2}^{b/2}\delta L_w(y)\cdot(-y)dy=-\rho U_0\frac{1}{2}\int_{-b/2}^{b/2}C_{l_\alpha}\frac{py^2}{U_0}c_y dy\Rightarrow L_p<0$$

  - **Key point: positive roll rate ⇒negative roll moment.**

- L_r rolling moment due to yaw rate:
  - Positive r has left wing advancing, right wing retreating
    →Horizontal velocity distribution over wing $U=U_0-ry$
  - Creates lift distribution over wing(chord wise strips)

$$L_w(y)\sim\frac{1}{2}\rho U^2 C_l c dy\approx\frac{1}{2}\rho(U_0^2-2U_0 ry)C_l c_y dy$$

  - Net result is higher lift on the left, lower on the right.
  - Rolling Moment:$L=\int_{-b/2}^{b/2}L_w(y)\cdot(-y)dy\approx\rho U_0 r\int_{-b/2}^{b/2}C_l c_y y^2 dy$
  - For large aspect ratio rectangular wing(crude)

$$L_r\approx(\frac{1}{6}\text{ to }\frac{1}{4})C_{L_\alpha}>0$$

  - **Key point:positive yaw rate⇒positive rollmoment.**

- N_p yawing moment due to roll rate:
  - Rolling wing induces a change in span wise AOA, which changes the span wise **lift** and **drag**.
  - Distributed drag change creates a yawing moment. Expect higher drag on right(lower on left)→positive yaw moment
  - There is both a change in the lift (larger on downward wing be• cause of the increase in α) and a rotation(leans forward on down• ward wing because of the largerα).→negative yaw moment
  - In general hard to know which effect is larger. Nelson suggests that for a rectangular wing, crude estimate is that

$$N_p \approx \frac{1}{2}\rho U_0^2 Sb\left(\frac{C_L}{8}\right) < 0$$

- $N_r$yawing moment due to yawrate:
  - Keyindeterminingstabilityproperties–mostlyfromfin.
  - Positiverhasfinmovingtotheleftwhichincreasestheapparent angleofattackby

$$\Delta\alpha_f = \frac{r l_f}{(U_0)_f}$$

- Createsincreaseinliftatthetailfinby

$$\Delta L_f = \frac{1}{2}\rho (U_0)_f^2 S_f C_{L_{\alpha_f}} \Delta\alpha_f$$

  - Createsachangeintheyawmomentof

$$N = -l_f \Delta L_f = -\frac{1}{2}\rho (U_0)_f^2 S_f C_{L_\alpha} r l_f^2$$

  - So$N_r = -\frac{1}{2}\rho (U_0)_f S_f C_{L_{\alpha_f}} l_f^2 < 0$
  - **Keypoint:positiveyawrate⇒negativeyawmoment.**

	L	N
p	<0	?
r	>0	<0

NumericalResults

- The code gives the numerical values for all of the stability derivatives. Cansolvefortheeigenvaluesofthematrix$A$tofindthemodesof thesystem.

$$-0.0331 \pm 0.9470i$$
$$-0.5633$$
$$-0.0073$$

–Stable,butthereisoneveryslowpole.

- There are 3 modes, but they are a **lotmorecomplicated** than the longitudinalcase.

Slowmode	•0.0073	⇒**SpiralMode**
Fastreal	•0.5633	⇒**RollDamping**
Oscillatory	$-0.0331 \pm 0.9470i$	⇒**DutchRoll**

Canlookatnormalizedeigenvectors:

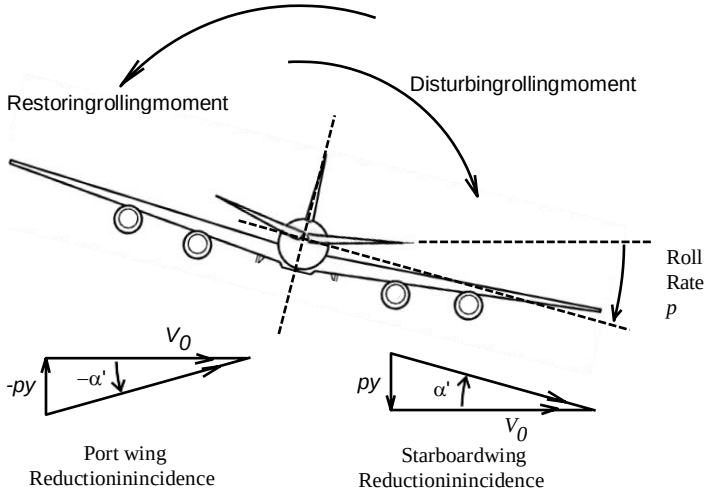
	Spiral	Roll	Dutch Roll	
$\beta = w/U_0$	0.0067	•0.0197	0.3269	•28°
$\hat{p} = p/(2U_0/b)$	•0.0009	•0.0712	0.1198	92°
$\hat{r} = r/(2U_0/b)$	0.0052	0.0040	0.0368	•112°
φ	1.0000	1.0000	1.0000	0°

Notasenlighteningasthelongitudinalcase.

LateralModes

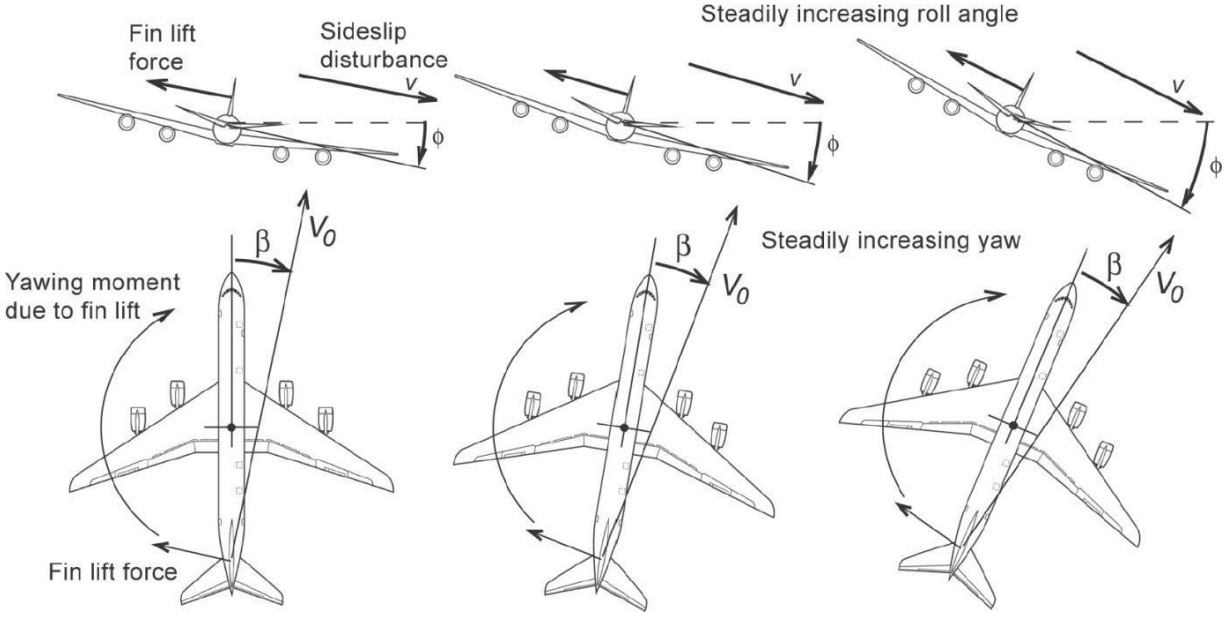
RollDamping•welldamped.

- Astheplanerolls,thewinggoingdownhasanincreasedα (windiseffectively“comingup”moreatthewing)
- Oppositeeffectforotherwing.
- Thereisadifferenceintheliftgeneratedbyboth wings →moreonsidegoingdown
- The differential lift creates a **moment** that tends to **restore** the equilibrium. RecallthatL_p<0
- After a disturbance, the roll rate builds up exponentially until the restoringmomentbalancesthedisturbingmoment,andasteady rollisestablished.



SpiralMode•slow,often unstable.

- From level flight, consider a disturbance that creates a small roll angle$\varphi>0\rightarrow$Thisresultsinasmallside-slip$v$(vehicles*lides downhill*)
- Now the tail fin hits on the oncoming air at an incidence angleβ $\rightarrow$extrataillift$\rightarrow$positiveyawingmoment
- Moment creates positive yaw rate that creates positive roll mo• ment($L_r>0$)thatincreasestherollangleandtendstoincrease theside•slip $\rightarrow$makesthingsworse.
- If unstable and left unchecked, the aircraft would fly a slowly diverging path in roll, yaw, and altitude$\Rightarrow$it would tend to *spiral* in to the ground!!



- Can get a restoring torque from the wing **dihedral**
- Want a small tail to reduce the impact of the spiral mode.

DutchRoll•dampedoscillationinyaw,thatcouplesintoroll.

- Frequencysimilartolongitudinalshortperiodmode,notaswell damped (fin less effective than horizontal tail).
- Consideradisturbancefromstraight•levelflight

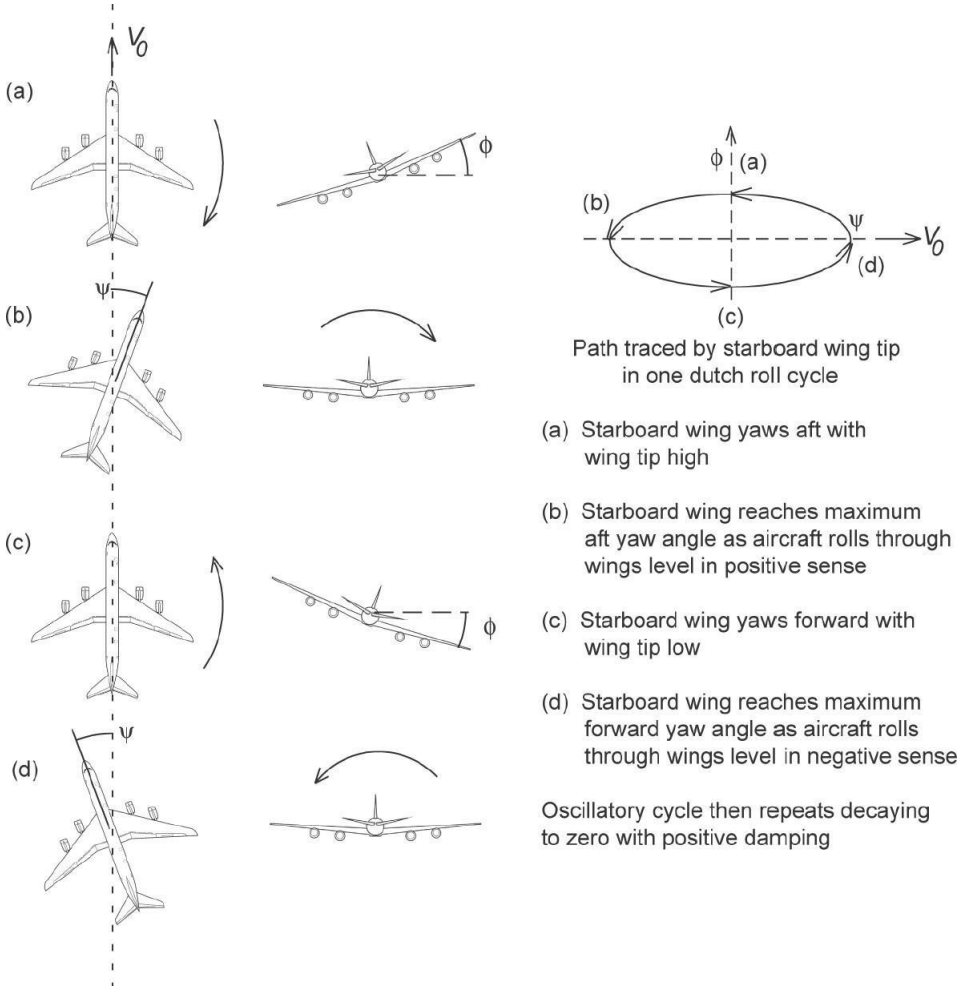
  →Oscillationinyawψ(finprovidesthe*aerodynamicstiffness*)

  →Wingsmovingbackandforthduetoyawmotionresultinoscil- latory differential lift/drag (wing moving forward generates more lift) $L_r > 0$

  →Oscillationinroll φthatlags $\psi$by approximately90°

  ⇒*Forward goingwing is low*

  Oscillatingroll⇒sideslipindirectionoflowwing.



Path traced by starboard wing tip in one dutch roll cycle

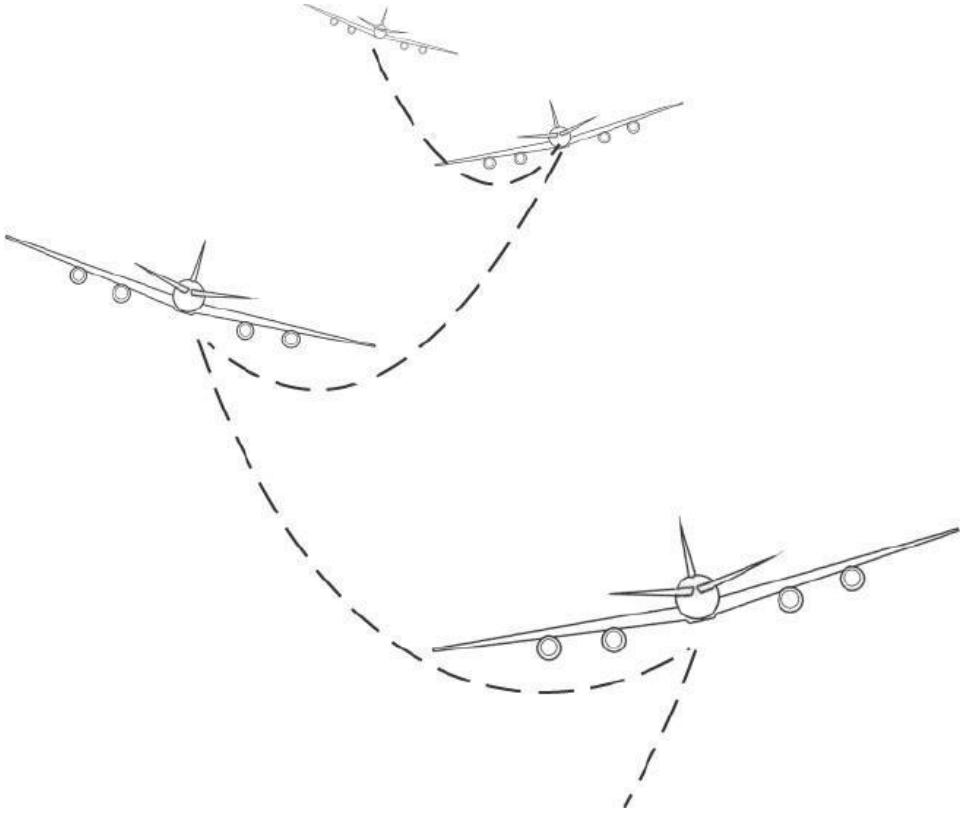
(a) Starboard wing yaws aft with wing tip high

(b) Starboard wing reaches maximum aft yaw angle as aircraft rolls through wings level in positive sense

(c) Starboard wing yaws forward with wing tip low

(d) Starboard wing reaches maximum forward yaw angle as aircraft rolls through wings level in negative sense

Oscillatory cycle then repeats decaying to zero with positive damping



- Doyouknowtheoriginsonthenameofthe mode?
- DamptheDutchrollmodewithalargetailfin.

UNIT – IV
Control System Modeling and Feedback Control

Index

SlNo	Name of the topic
1	Components of control systems
2	Linear & non linear system
3	Time invariant system
4	Modelling of control systems
5	Linearization of non-linear systems
6	Steady state error
7	Control system components, sensors, transducers
8	Block diagram reduction

Componentsofcontrolsystem.Thebasiccomponentsofacontrolsystemare:

(a) Input or Objective of control

(b) Plant or control system components

(c) Outputs or Results.

The basic relationship between these three components is shown infig1.1 below.

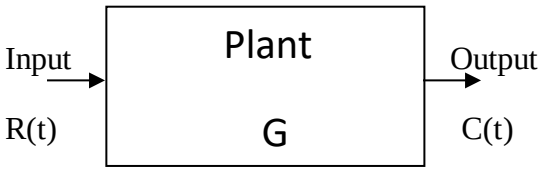


Fig:1.1

In technical terms the objectives can be identified with inputs or actuating signal, u, and the results are called outputs or the controlled variable y. In general, the objective of a control is to control the output in some predetermined manner by the inputs through the elements of control systems.

Plant: a plant may be a piece of equipment, perhaps just a set of machine parts functioning together, the purpose of which is to perform a particular operation.

Process: any operationtobecontrolledisdalledaprocess.Examplesarechemical,economic & biological processes.

TransferFunction.Foranycontrolsystemthereexistsaninputtermedasexcitationorcause (denoted as R) which operates through a transfer operation termed as transfer function (denoted as G) and produces an effect resulting in output or response termed as controlled variable denoted as C). The cause and effect relationship between the output and input is related to each other through a transfer function. This relationship between the output and the input is represented by a diagram known as block diagram. The transfer function is expressedastheratioofLaplacetransformofoutputtoLaplacetransformofinputwithzero initial condition.

G(s)=Laplace transform ofC(t)/Laplace trans form ofR(t)

BlockDiagramRepresentation.Acontrolsystemmayconsistofanumberofcomponents.To show the functionperformedby eachcomponent,incontrolengineering,we commonly use adiagramcalledtheblockdiagram.Ablockdiagramofasystemispictorialrepresentationof the function performed by each component and of the flow of signals. In block diagram all systemvariablesare linked toeachotherthroughfunctionalblocks.The functionalblock isa symbolforthemathematicaloperationontheinputsignaltotheblockthatproducesthe

output. The transfer function of the components is usually entered in the corresponding blocks,whichare connectedbyarrowstoindicate thedirectionoftheflow ofsignals.Fig1.2 below shows the elements of the block diagram.

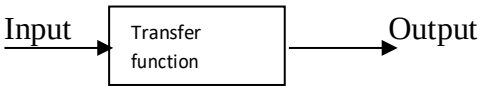


Fig1.2

BlockDiagramofaClosedLoopSystemforasingleinputsingleoutputSystem.Fig1.3below shows the block diagram of a closed loop system. The output C(s)is feedback to the summing point,whereitiscomparedwithreferenceinputR(s).TheoutputoftheblockC(s)isobtained by multiplying the transfer function G(s) by input to the block E(s). Output is measured by a sensor or measuring device whose transfer function is denoted by H(s).

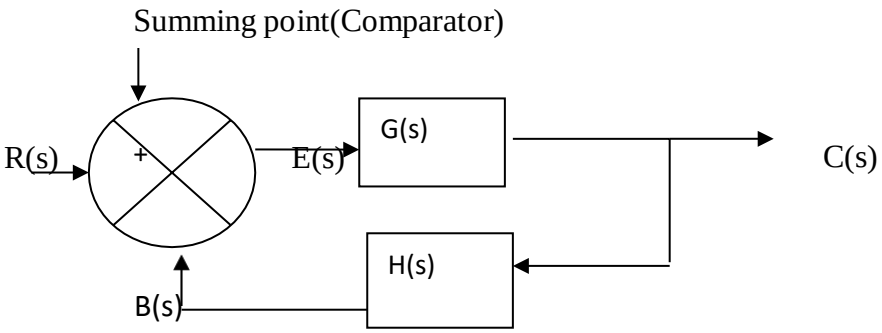


Fig1.3

Open loop control, Closed loop Control, effect of feedback on overall gain, stability, sensitivity and on noise:1

The Concept of Feed back: a system that maintains a prescribed relationship between the output and the reference input by comparing them using the difference as means of control is called feedback control system.

Open Loop Systems: In open loop system, there is no feedback from output to input. Example of open loop control system is conventional washing machine, because amount of machinewashtimeisdeterminedentirelybythejudgment&estimationofthehuman operator. Another example is control of traffic light. The element of an open loop control system can be divided into two parts: the controller and the controlled process as shown below (fig 1.4)

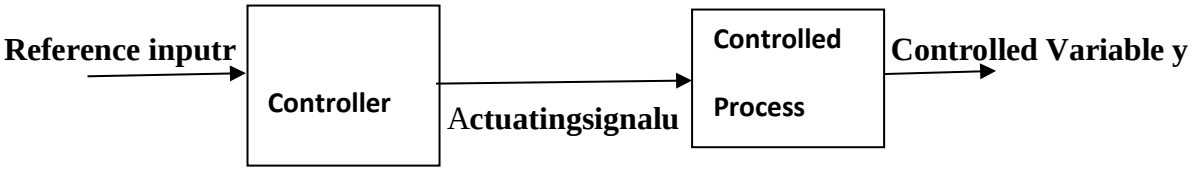


Fig1.4: Elements of an Open-Loop system

Aninputsignalorcommandrisappliedtothe controller,whereoutputactsasthe actuating signalu,theactuatingsignalthencontrolsthecontrolledprocessthatthecontrolledvariable ywillperform accordingtoprescribed standards.Insimple casescontrollercan be amplifier, mechanical linkage, filter or other control element, depending on the nature of the system. In more sophisticated cases, controller can be a computer such as a microprocessor. Open loop systems find application in many non-critical applications because of **simplicity and economy**.

Closed Loop Control System: What is missing in the open loop control system for more accurate & more adaptable control is the link or feedback from the output to input of the system. To obtain more accurate control, the controlled signal y should be fed back & compared with the reference input, and the actuating signal proportional to the difference of input and the output must be sent through the system to correct the error. Such a system is calledclosedloopsystem.Exampleofclosedloopsystemisshowninthefig1.4below which is a room heating system.

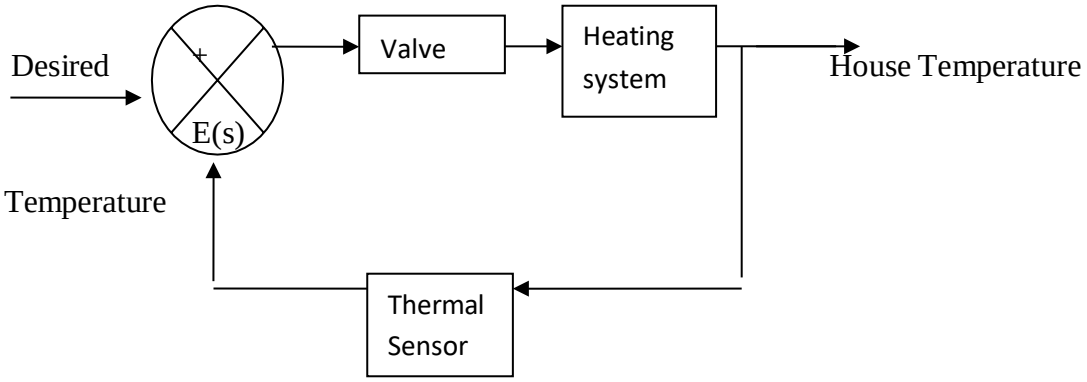


Fig1.4:Home Heating System

Athermostat senses thetemperatureandif it is lowerthanaset valuethefurnaceis turned on. The furnace is turned off when the temperature exceeds the set value.

MajorAdvantagesofOpen-loopcontrolsystemare:

1. Simpleconstructionandeaseofmaintenance.
2. Lessexpensivethancorrespondingclosedloopsystem.
3. Thereisnostabilityproblem.
4. Convenient when output is hard to measure or measuring the output precisely is economically not feasible. For example, in the washing machine, it would be quite expensive to provide a device to measure the quality of the washer's output, cleanliness of the clothes.

Themajordisadvantagesofopenloopsystems areas follows:

1. Disturbanceandchangesincalibrationcauseerrors,andthe output maybe different from what is desired.
2. Tomaintaintherequiredqualityintheoutput,recalibrationisnecessaryfromtimeto time.

Effectoffeedbackonoverallgain,stability,sensitivity&on noise: Inmanycontrol system application, the system designed must yield the performance that is robust i.e. insensitive to external disturbance, noise and parameter variations. Feedback in control system has the inherent ability of reducing the effect of external disturbance and parameter variations.

Feedback is not only for reducing the error betweenthe reference input and the system output,it hasmany othersignificanteffects ontheperformance ofthe controlsystem.It has effects on such system performance characteristic such as stability, bandwidth, overall gain, disturbance and sensitivity.

Letusconsiderasimpleexample offeedbackcontrolsystemshowninfig1.5

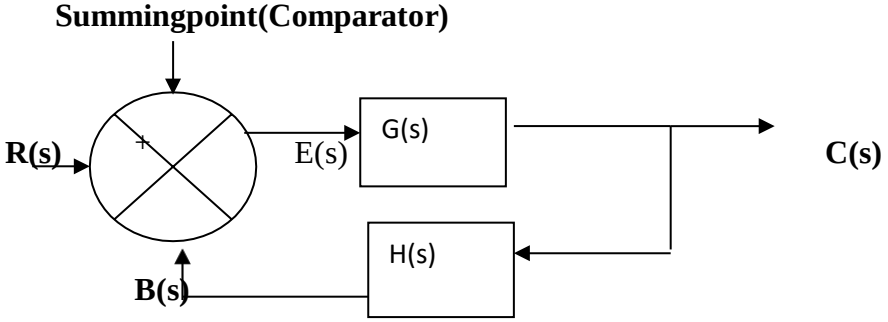


Fig1.5

Weknowthatoverallgainofthesystemis

M(s)=Laplacetransformofoutput/Laplacetransformofinput.

M(s)=C(s)/R(s)=G(s)/(1+G(s)H(s))

(a) **Effect of feedback on overall gain**. The feedback affects the gain G(s) of a non-feedbacksystembyafactorof1+G(s)H(s).ThequantityG(s)H(s)mayincludeaminus sign, so the general effect of a feedback is that itmay increase or decrease the gain G(s). In practical control system, G(s) and H(s) are functions of frequency, so the magnitudeofthe1+G(s)H(s)maybegreaterthanoneinonefrequencyrangebutless than one in other frequency range. So the feedback can increase the system gain in one frequency range but decrease it in other.

(b) **Effectof Feedback on Stability**. Stability is notion that describes whetherthe system will be able to follow the input command, or be useful in general. A system is said to be unstable if it is out of control. If G(s) H(s) = -1 the output is infinite for any finite input.Therefore,wemaysaythatfeedbackcancauseasystemthatisoriginallystable to become unstable. Feedback when used improperly can be harmful. It can be demonstrated that one of the advantages of incorporating feedback is that it can stabilize an unstable system.

(c) **Effect of Feedback on Sensitivity**. All physical elements have properties that change withtheenvironmentandage;wecannotalwaysconsidertheparametersofacontrol system to be completely stationary over the entire operating life of the system. For example, winding resistance of an electric motor changes as the temperature of the motor rises during the operation. In general, a good control system should be very insensitivetoparametervariations but sensitivetotheinput command. Weconsider G(s) to be gain parameter that may vary. The sensitivity of the gain of the overall system, M(s) to variation of G(s) is defined as

SensitivityofM(s)withrespecttoG(s)=

∂M(s)/M(s))/(∂G(s)/G(s))=%changeinM(s)/%changeinG(s).

Sensitivity of M with respect to G = 1/ [G(s) + H(s)].

This relationship shows that if G(s) H(s) is positive constant, the magnitude of sensitivitycan be made arbitrarily small by increasing G(s) H(s), provided the systemremainsstable.Inopen loopsystem sensitivity = 1. We should note that G(s) H(s) is a function of frequency, the magnitude 1 + G(s) H(s) may be less than unity over somefrequency range,sothat feedbackcouldbe harmfultothesensitivityto parameter variations in certain cases.

(d) **Effect of Feedback on External Disturbance or Noise**. The system with noise input n is shown in the figure 1.6

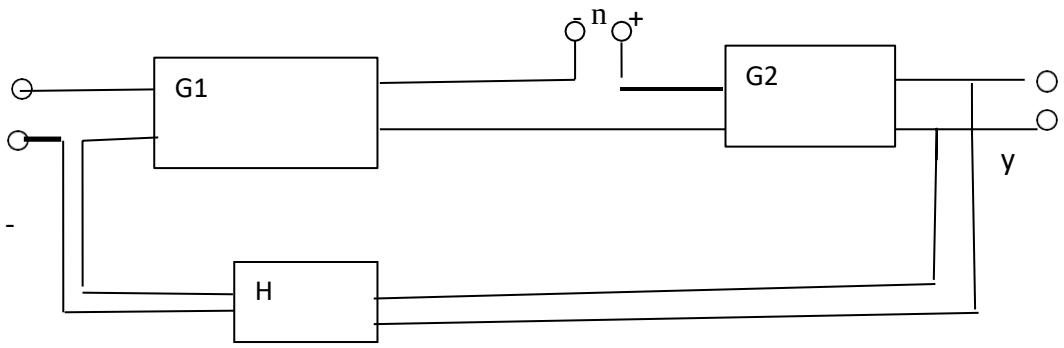


Fig1.6:Feedbackwith externalnoise.

In the absence of feedback

Y= G2 n (1)

Withpresenceoffeedback,thesystem outputduetonoisenactingalone (i.e.r= 0) Y = G2

n/ (1+ G1 G2 H) (2)

Comparing equation (1) and (2) shows that noise component in output is reduced by a factor of 1 + G1G2H. If the latter is greater than unity and system is kept stable.

In summary we can say that feedback if used properly will make the system robust byreducing the effect parameter variations, noise and external disturbance.

LinearandNon-LinearSystem,TimevaryingandTimeinvariantlinearsystem:

Linear and Non- Linear System: A system is called linear if the principle of superposition applies.Theprincipleofsuperpositionstatesthattheresponseproducedbythesimultaneous application of the different forcing functions is thesum of two individual responses. Hence for the linear system, the response to several inputs can be calculated by treating one input at a time and adding the results. It is this principle that allows one to build up complicated solutions to the linear differential equations from simple solutions. In an experimental investigation of a dynamical system, if cause and effect are proportional, thus implying that the principle superposition holds, then the system can be considered linear.

Example:Anamplifiercanbe consideredas linear system ifoutput varies proportionaltoan input. This maybe true ifthe input signalis notverylargeandamplifierdoes not saturateas shown in fig 1.7

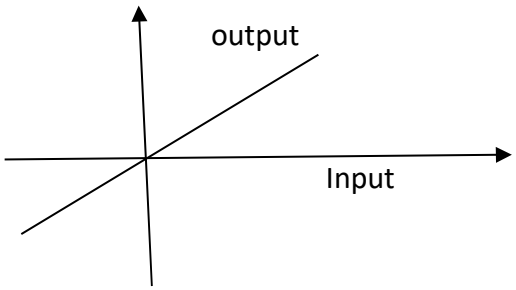


Fig1.7:Linearsystem

A non- linear system is one where principle of superposition cannot be applied. Thus for nonlinearsystemtheresponsetoinputscannotbecalculatedbytreatingoneinputatatime and adding the result. Although many physical systems are often represented by linear equations, inmost cases actualrelationships are not quite linear. Incarefulstudy ofphysical systems reveals that even so called linear systems are readily linear only in the limited operating range. For example, output of an amplifier may saturate for large input signals. There may be a dead space that affects the small signal. Dampers used in physical systems may be linear for low velocity operation but may become non- linear at high velocities, and the damping force may become proportional to the square of the operating velocity. This is shown in fig 1.8.

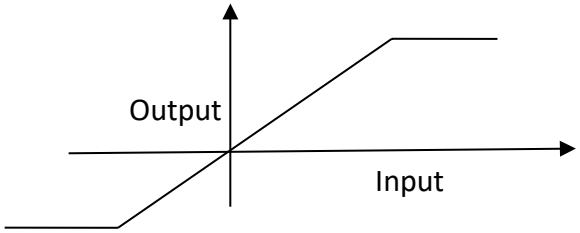


Fig1.8:Non-Linearsystem

Time- Invariant System Linear System: When parameters of a linear control system are stationary withrespect totime during the operationofthe system,the system is called time invariant linearsystem.Forexample,inmass,springdampersystemdiscussedabovesystem parameters are spring constant k, damping force constant b and mass m. In case these parameters remain constant we say it is linear time invariant system. Most physical systems contain elements that drift or vary with temperature. For example winding resistance of the motor will vary when motor is first excited & its temperature is rising. In guided missile system,massofthemissiledecreasesasthefuelonboardisbeingconsumedduringtheflight.

ModelingofDynamicalsystembyDifferentialEquation:

A dynamical system can be modeled using the differential equations. The differential is derivedbyfindingtherelationbetweeninputandoutputusingmathematicalequations

governing thesystem. Thiscanbe demonstrated uinga mechanical system consistingof spring, mass, damper system as shown in fig 1.9.

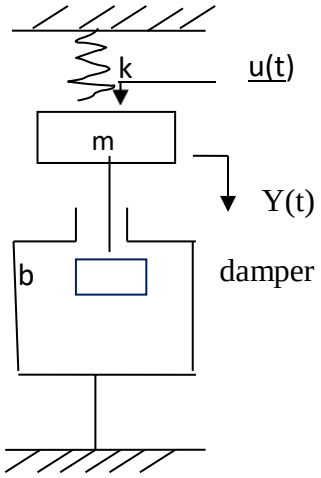


Fig1.9:Mass,spring,damper system.

K is spring constant and b is coefficient of viscous damping. Spring force = k ×x and Viscous forceexertedbydamperisb×dy/dt;where dy/dtis thevelocityofthemassm. Theexternal forceu (t)istheinputtothesystemanddisplacementy(t)ismeasuredfromtheequilibrium position in the absence of the external force. The system is single input and single output system. We can write the system equation after drawing the free body diagram of the mass which is shown in fig: 1.10

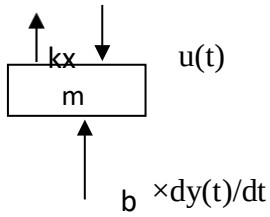


Fig1.10:Freebody diagram

Fromthediagram,the systemequationis(usingNewton'ssecondlawofmotion):

$u(t)-k\,y(t)-b(\,dy/dt) = m\,d^2y/dt^2$

$u(t)=md^2y/dt^2 +ky(t)+bdy/dt$

TakingtheLaplacetransformofbothside u(s) =

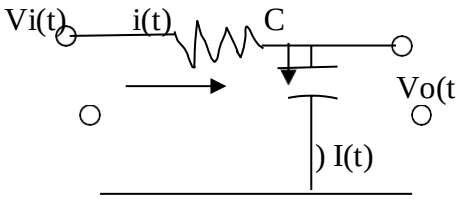
$(\,ms^2 +b\,s +k\,)\,y(s)$

$y(s)/u(s)=G(s)=1/(ms^2+bs+k)$

Order of thesystem: It is order of the differential equation governing the input and output. In this case system is governed by second order differential equation; hence order of the system is two. Order of the system can also be defined as highest power of s in the denominatorofthe transferfunction. Inthis example highest powerofs is two;hence it is a second order system.

SystemParameters:Intheaboveexamplesystemparametersaremassm,springconstantk, and coefficient of viscous force b.

Anotherexampleofmodelingdynamicalsystemusingdifferentialequation: R



IntheaboveRC networkwhichis also called low passfilter,input isapplied voltageVi(t) and Output is Vo(t). Resistance is R and capacitance is C. Let us derive the model of this system using differential equation.

Currentpassingthroughthecircuitisi(t). Vi(t) =

R i(t) + Vo(t)

Chargeonthecapacitorq=C Vo(t)

Currenti(t)is rateofchangeofchargeq.hence dq /dt

= c d Vo(t)/dt = i(t). Hence

Vi(t) =RCdVo(t)/dt+ Vo(t)

This is the first order differential equation; hence it is **first order system**.Solution of this equation will give the output for a given input. In this case **system parameters** are R and C.

TakingtheLaplacetransformofbothsidesandassumingzeroinitialconditionweget Vi(s) =

RCs Vo(s) + Vo(s)

Therefore,transferfunctionG(s)=1/(1+RCs).

nearization of Non- Linear System: To obtain a linear model we assume that variables deviate only slightly from some operating condition. Consider a system whose input is x (t) and output is y (t).The relationship between y(t) and x(t) is given by:

Y=f(x)

If the normal operating condition corresponds to $\bar{x}$,, then above equation may be may be expanded into a Taylor series about this point as follows:

$$y = f(x) = f(\bar{x}) + \frac{df}{dx}(x-\bar{x}) + \frac{1}{!2}\frac{d^2f}{dx^2}(x-\bar{x})^2 + \ldots$$

Where $\frac{df}{dx}$ and d^2f/dx^2 are evaluated at $x=\bar{x}$. If variation $x-\bar{x}$ is $x=\bar{x}$ small enough, we may
Neglect higher order term sinx-$\bar{x}$ then equation can be written as:

$y=\bar{y}+k(x-\bar{x})$; (3)

Where $\bar{y}=f(\bar{x})$ and $k=\frac{df}{dx}$ Evaluate dat $x=\bar{x}$

Then equation(3)can be written as:

$y-\bar{y}=k(x-\bar{x})$ (4)

Which indicates that y-$\bar{y}$ is proportional to x-$\bar{x}$.Equation(4)above gives linear mathematicalmodelforthesystemgivenbyequation(1)neartheoperatingpointx-$\bar{x}$,y-$\bar{y}$

Next consider anon-linear system whose output yisfunctionoftwoinputsx1&x2,so that Y =

f(x1, x2) (5)

Toobtainalinearapproximationtothismodel,wemayexpandequationintoTaylorseries
About point $\bar{x}1,\bar{x}2$ Then equation (5) is:

$$y=f(\bar{x}1,\bar{x}2)+\left[\frac{\partial f}{\partial x1}(x1-\bar{x}1)+\frac{\partial f}{\partial x2}(x2-\bar{x}2)\right]+\frac{1}{!2}\left[\frac{\partial^2 f}{(\partial x1)^2}(x1-\bar{x}1)^2+2\frac{\partial^2}{\partial x1\partial x2}(x1-\bar{x}1)(x2-\bar{x}2)+\frac{\partial^2 f}{(\partial x2)^2}(x2-\bar{x}2)^2\right]+\ldots.$$

Where partial derivatives are evaluated at x = $\bar{x}1$, x2 = $\bar{x}2$ Near the normal operating point, the higher order terms may be neglected. The linear mathematical model of the non linear system in the neighborhood of the normal operating condition is given by

$$Y - K1(x1-\bar{x}1) + K2(x2-\bar{x}2)$$

Where $\bar{y}=f(\bar{x},\bar{y})$

K1= $\frac{\partial f}{\partial x1}$,evaluatedatx1= $\bar{x}1$,x2=$\bar{x}2$

K2= $\frac{\partial f}{\partial x2}$,evaluatedatx1= $\bar{x}1$,x2=$\bar{x}2$

Exampleproblem:Linearizethenonlinearequationz=xy,intheregion5≤x≤7,10≤y≤12. Findtheerrorifthe linearized equationisusedtocalculatethevalueofzwhenx=5;y=10.

Solution:Sincetheregion consideredis,5≤x≤7,10 ≤y≤12,choose$\bar{x}$ 6, $\bar{y}$1 then$\bar{z}$=$\bar{x}$ $\bar{y}$=66.Letusobtainalinearized equationforthenonlinear equationnearapoint$\bar{x}$=6 and $\bar{y}$=11.

Expandingthenon-linearequationintoTaylorseriesaboutthepointx=$\bar{x}$,y=$\bar{y}$and neglecting the high order terms,

z-$\bar{z}$ =K1(x-$\bar{x}$)+K2(y-$\bar{y}$)

WhereK1= $\frac{\partial z}{\partial x}$ evaluatedatx=$\bar{x}$,y =$\bar{y}$K1=11

WhereK2= $\frac{\partial z}{\partial y}$ evaluatedatx=$\bar{x}$,y=$\bar{y}$K2=6

Hencelinearizedequationis

z-66= 11(x-6)+6(y-11) or z=11x+ 6y -66

Whenx=5,y=10,z=11*5+6*10-66 =49.

Theexactvalueisz=xy=50.Theerroristhen50-49=1.Inpercentage,theerroris2%.

SystemType,SteadyStateError,Error Constant:

SystemType: Acontrolsystemtransferfunctioncanbe represented as:

$G(s)=(K(1+T_1s)(1+T_2s)\ldots(1+Tm_1s+Tm_2s^2))/(s^j(1+Tas)(1+Tbs)\ldots(1+Tn1s+Tn2s^2)$

; Where K and all T's are real constants. The system type represents order of the pole of G(s) at s=0. Thus, the closed loop system having the forward path transfer function of above equation is type j, where j = 0, 1, 2… The following example illustrates the system type with reference to the form of G(s):

$G(s)=K(1+0.5s)/ ((s(1+s)(1+2s)(1+s+s^2))$ **Type1**

$G(s)=K(1+2s)/(s^3)$ **Type3**

teady State Error: One of the objectives of control system is that the system output responsefollowsaspecificreferencesignalaccuratelyinthesteadystate.Thedifference betweentheoutputand&referenceinputinthesteadystateisdefinedasthesteady

state error. In the real world, because of friction and other imperfection, & the natural composition of the system, the steady state output response seldom agrees with the reference. Therefore, steady state errors in control system are almost unavoidable. In a design problem, one of the objectives is to keep the steady state error to minimum, or below a certain tolerable value,and the same time the transient response must satisfy a certain set of specifications. The accuracy requirement on control systems depends to a great extent on the control objectives of the system.

Definition of steady state error with respect to system configuration: Let us consider a system as shown in fig 1.11 below. Error of the system may be defined as:

e(t)=referencesignal –y(t)

Where reference signalinthesignalthattheoutputy(t)istotrack.Whenthesystemhas unity feedback (H (s) = 1), the input r(t) is the reference signal, and the error is simply

e(t)=r (t)–y(t)

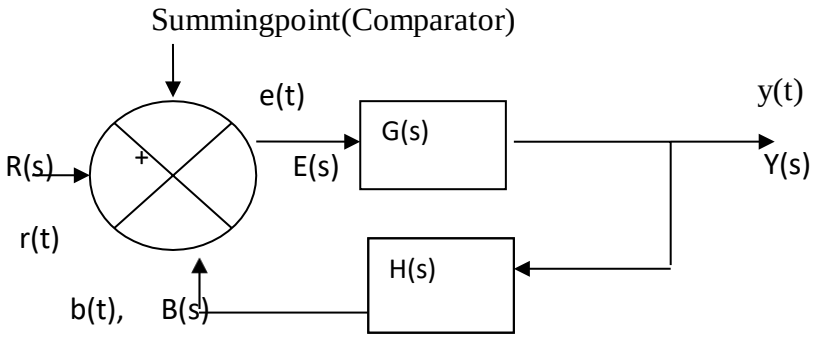


Fig1.11.Feedbackcontrolsystem

The steadystateerrorisdefinedas ess

$$= \lim_{t \to \infty} (\square)$$

TypesofControlSystem:Unityfeedbacksystem.Consideracontrolsystemwithunity feedback. It can be represented by or simplified to the block diagram in fig 1.12 below.

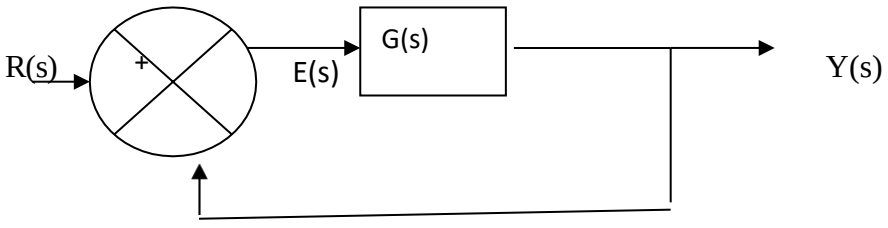


Fig1.12:Unityfeedbacksystem

The steady state error of the system is written as

$$ess=\lim_{s\to\infty} e(s)=\lim_{s\to 0} sE(s)=\lim_{s\to 0}[s(R(s)-B(s))]$$

$$=\lim_{s\to 0}[R(s)-G(s)H(s)R(s)/(1+G(s)H(s)]$$

$$=\lim_{s\to 0}(s)/(1+G(s)) \quad (1)$$

Clearlyessdepends on the characteristics of G(s). More specifically,we can show thatess depends on the number of poles that G(s)hasats=0. Thisnumberisknownasthe **TYPE of the control system**. Orsimply,**system Type**.We can show that the steadystate error ess depends on the type of the control system. Ingeneral G(s) can be represented by:

$$\mathbf{G(s)=(K\ (1+T_1s)(1+T_2s)\ldots(1+Tm_1s+Tm_2s^2))/(s^j(1+Tas)(1+Tbs)\ldots(1+Tn1\ s+Tn2s^2))} \quad (2)$$

Where K and all T's are real constants. The system type represents order of the pole of G(s) at s=0. Thus the closed loop system having the forward path transfer function of above equation is type j, where j = 0, 1, 2… The following example illustrates the system type with reference to the form of G(s):

$G(s)=K(1+0.5s)/((s(1+s)(1+2s)(1+s+s^2))$ **Type1**

$G(s)=K(1+2s)/(s^3)$ **Type3**

SteadystateerrorofasystemwithStepinput:wheninputr(t)tothecontrolsystemisastep function with magnitude R, R(s) = R/s. the steady state error is written from equation (1)

$$ss=\lim_{s\to 0} sE(s)/(1+G(s)=\lim_{s\to 0} s/(1+G(s)=R/(1+\lim_{s\to 0} G(s))$$

For convenience, wede fine $Kp=\lim_{s\to} G(s)$

Hence ess=R/(1+Kp) (3)

Wecanseefromequation(3)thatforesstobezeroKpmustbeinfinite.IfG(s)isasshownin equation(2)thenforKptobeinfinitejmustbeatleastequaltounity,thatis,G(s)musthave at least one pole at s = 0. Therefore, we can summarize the steady state error due to step function input as follows:

IMPORTANT

Type0system:ess=R/(1+ Kp)=constant. Type 1
or higher system: ess = 0.

Kpisknownas**positionerrorconstant**.

SteadyStateerrorwithaRampfunctioninput:Whentheinputisarampfunctionwith magnitude R, r(t) = R t where R is real constant, the Laplace transform of r(t) is R(s)

$R(s)=R/s^2$

Thesteadystate errorisess=$\lim_{\square\to 0}\square/(\square+\square\square(\square))=\lim_{\square\to 0}\square/(\square\square(\square))$

Wedefinetheramperrorconstantas Kv;where Kv

$=\lim_{\square\to 0}(\square)$

Theness=R/Kv

Henceforesstobezero,Kvmustbe infinite.Usingequation(2)we obtain Kv

$=\lim_{\square\to 0}\square(\square)=\lim_{\square\to 0}\square/s^{j-1}$

Thus,forKvtobeinfinite,jmustatleastbeequalto2,orthesystemmustbetype2orhigher. The following conclusions may be stated with regard to steady state error with ramp input.

Type0system:ess=∞
Type1system:ess=R/Kv=constant Type 2
or higher order: ess = 0

SteadystateerrorwithofsystemwithParabolicInput:Wheninputisdescribedbythe standard parabolic form,

$r(t)=Rt^2/2$

TheLaplacetransformofr(t) =r/s^3

Thesteadystateerrorisess= $\frac{\square}{\lim_{\square\to 0(\square)}\square^2}$;defining theparabolicerror constantas Ka

$Ka=\lim_{\square\to 0}\square 2(\square)$;thesteadystateerror becomes

ess = R/Ka

Following the patternset with the step & ramp input,the steady state error due to parabolic input is zero if the system is 3orgreater. The following conclusions are made with regardto steady state error of a system with parabolic input.

Type0system:ess=∞ Type 1 system: ess =∞ Type2system:ess=R/Ka=constant Type 3 or higher order: ess = 0

Example: Consider a closed loop unity feedback system has the following transfer functions. The error constants and steady state errors are calculated for three basic inputs using the error constants;

(a) $G(s)=\frac{(□+3.5)}{(□+1.5)(□+0.5)}$,H(s)=1

For step input:step errorc onstantKp=∞,ess=R/(1+Kp)= 0 Ramp input: Kv = 4.2 K, ess = R/(4.2k)
Parabolic Input:ka=0,ess=R/ka=∞

(b) LetG(s)=5(s+1)/((s^2(s+12)(s+5))

We can calculate the error constants and steady state error for three basic inputs:
Step input: Kp = ∞ ; ess = R/(1+Kp) = 0
Ramp input: Kv = ∞ , ess= R/Kv =
0Parabolicinput:Ka=1/12,ess=R/Ka=12R.

ControlSystemComponents-sensors,transducers,servomotors,actuators,filters-modeling, transfer function:

Sensor: We can define a sensor as a device that converts a physical stimulus or input intoareliableoutput,whichtodaywouldpreferablybeelectronic,butwhichcanalsobe communicatedbyothermeanssuchasvisualandacoustic.Thegenericblockdiagramfora

sensor is shown in fig 1.13 which highlights the role of a sensor as an interface between acontrol system and the physical world.

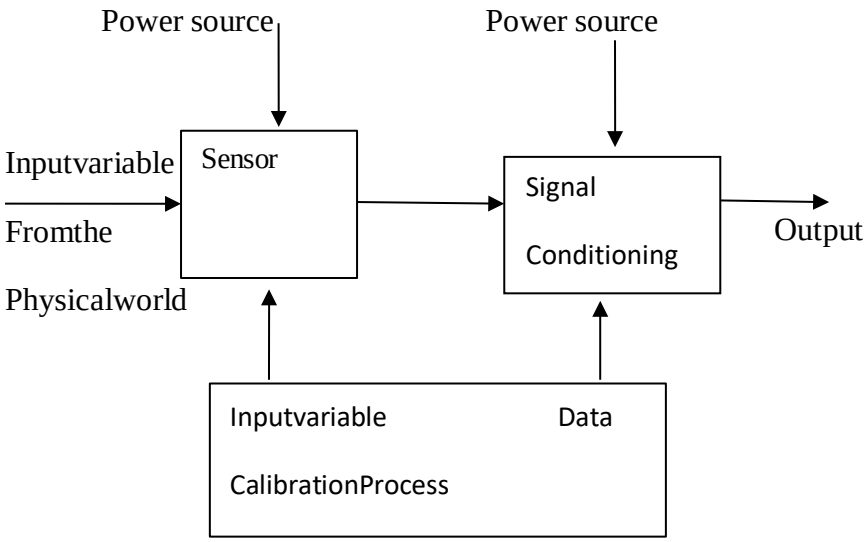


Fig:1.13:SensorBlockDiagram

Sensors used in aircraft control system:

(a) Pitot-static sensor: for sensing Pitot &Static pressure.
(b) Temperature sensor:Thermo-couple,resistance based.
(c) Roll, Pitch,Yaw sensor:Mechanical,LaserGyros.
(d) Acceleration sensors: Inertial navigation based.
(e) Velocity sensors:INS-GP Ssystem based.
(f) Angle of attack sensors
(g) Angle of sideslip sensors.

Application of sensor in aircraft auto-pilot: This is shown in Fig 1.14

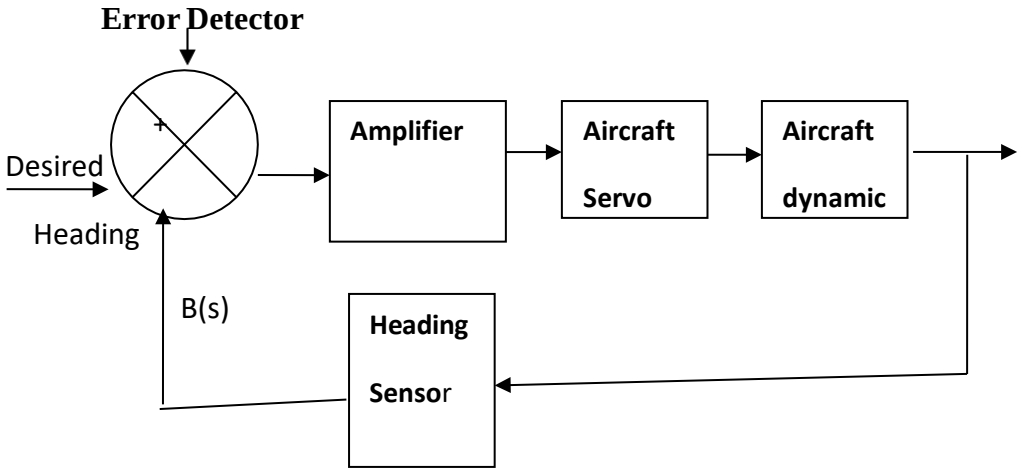


Fig1.14:AnoverviewoftheHeadingholding system

MathematicalModelingandtransferfunctionofsensors:Mathematicalmodelingofsensors can be derived by writing the differential equation governing input and output. Then taking the Laplace transform of bothside of differential equation we canfind the transfer function of the sensor. For example, Gyros are generally very accurate in low frequency measurements,but not sogood in highfrequency regions. So,we can model a gyroas a low pass filter, being

$H_{gyro}(s)=1/(s+\omega_{br})$

The gyrobreak frequency, (above which the performance starts to decrease) is quite high.I n fact,it isusually higher than any of the important frequencies of the aircraft. Therefore,gyro can often be simply modeled as H(s) =1. In other words, it can be assumed that the gyro is sufficiently accurate.

Transducer:Atransducerisadevicethattransformsoneformofenergyintoanother.

Transducers are generally made as small as possible, and the energy being transferred is small. Conversion between input and output is done quantitatively using a calibration process. Transducers use basic physical laws to measure physical parameters using sensing elements that is the part of transducer. The parameters measured in a servo control system are position and motion while parameters measured in process control systems are temperature, flow, level, pressure and others.

Examples of transducer: Potentiometer, LVDT (Linear variable differential transformers) tachometer, encoders.

Mathematical Modeling and transfer function of transducer. A potentiometer is an electromechanical transducer that coverts mechanical energy into electrical energy. The input to the device is in the form of a mechanical displacement. When a voltage is applied acrossfixed terminals,the output voltage,whichismeasuredvariable,isproportionaltothe

input displacement as shown in fig 1.15. E is the applied voltage across fixed terminal. The output voltage is proportional to the shaft position θ(t).

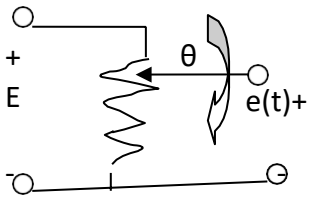


Fig1.15: Potentiometer

Then, e (t) = K θ(t) ; where K is proportionality constant. Hence E(s) = K θ(s); or transfer Function is

E(s)/θ(s)=K

MathematicalmodelandTransferfunctionoftachometer:Tachometeriselectromechanical device that convert mechanical energy into electrical energy. The output voltage is proportionaltothe angularvelocity ofthe input shaft. The dynamics ofa tachometercan be represented by the equation

e(t)= K_t(d□/dt)= Kt ω(t)

Where e(t) is the output voltage(t) is the rotor displacement in radians, ω(t) is the rotor velocity,$K_t$istachometerconstantinV/rad/sec.thevalueof$K_t$isgivenasacatalogparameter in volts per 1000 rpm. Transfer function of the tachometer is obtained by taking the Laplace transform of both sides.

E(s)/□(s)=sK_t

Servomotor:servo motors are widely used in control system as position controller. A DCservomotorisbasicallyatorquetransducerthatconvertselectricalenergyintomechanical energy. The torque developed on the motor is directly proportional to the field flux and armaturecurrent.Therelationshipamongthetorquedeveloped,the flux ϕandcurrent$i_a$is

T_m=K_mϕi_a

Inadditiontothevoltage,thebackemf,whichis proportionaltotheshaftvelocity,tendsto oppose the current flow. The relationship between the back emf and shaft velocity is

$_{Eb}$=K_bϕ$\omega_{m;}$where$\omega_m$istheshaft velocity.

MathematicalmodelingofaDCMotor:Mathematicalmodelisshowninfig1.16

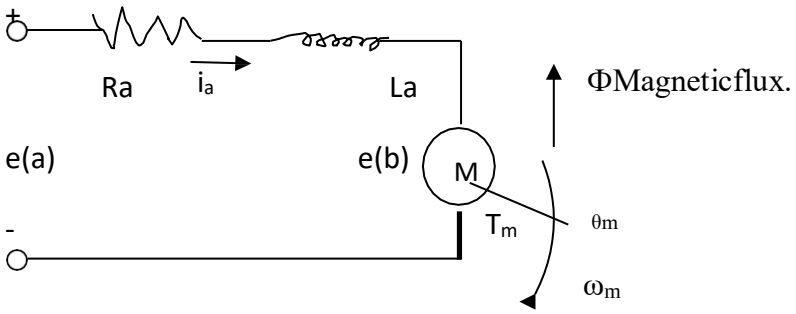


Fig:1.16:MathematicalModelofPermanentMagnetMotor

$T_m(t)=K_i i_a(t)$

$e_a(t)=R_a\, ia(t)+L_a d(i_a)/dt+e_b(t);T_m(t)=Kii_a(t)\ e_b(t)=$

$K_b d\Theta_m/dt = K_b\omega_m(t)$

$J_m d^2\Theta/dt^2=T_m(t)–B_m d\Theta_m/dt$

Where$i_a(t)$=armaturecurrent;L_a=armatureinductance $E_a(t)=$

applied voltage;$E_b(t)$ = back emf

$T_m(t)$=Motor torque;K_i=Torqueconstant

K_b=Backemfconstant;Φ=magnetic flux

$\omega_m(t)$= rotorarmaturevelocity;J_m= rotorinertia $B_m=$

Viscous friction force

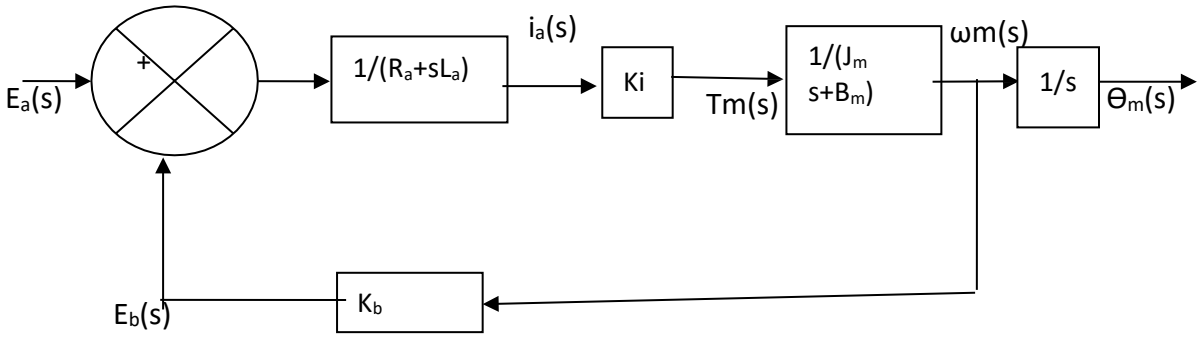


Fig:1.17: transfer Function Block diagram

hetransfer function between the motor displacement &the input voltageis given by (refer Fig 1.17):

□m(s)/E(s)= $\frac{\square\square}{\square\square\square\square^3+(\square\square\square\square+\square\square\square\square)\square^2+(\square\square\square\square+\square\square\square\square)\square}$
a

Actuators Function, Modeling and Transfer function: An example of a controller for anaircraftsystemisahydraulicactuatorusedtomovetothecontrolsurface.Acontrolvalve ontheactuatorispositionedbyeitheramechanicalorelectricalinput,thecontrolvalveports hydraulicfluidunderpressuretotheactuator,andtheactuatorpistonmovesuntilthecontrol valveshutsoffthehydraulicfluid.Ahydraulicactuatorisshownbelowinfig1.18.

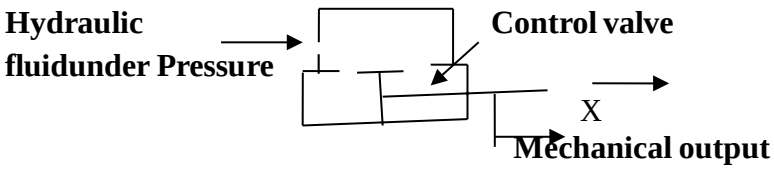


Fig1.18:Hydraulic actuator

Clearly actuator piston cannot move instantaneously because it takes a finite time for the hydraulic fluid to move through the ports from the control valve. In response to a step unit, the resulting motion (x) of hydraulic actuator can be modeled as an exponential.

$x(t)=Z(1-\square^{-\square\square})$

,Where Z isthefinaldisplacementvalueoftheactuator.Generalizedtransferfunctionofthe actuatoris: $X(s)/E(s) = \frac{\square}{\square+\square}$

Where X(s) is the Laplace transform of the output &E(s) is Laplace transform of input. Block diagram of actuator with transfer function is shown in fig 1.19

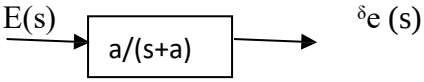
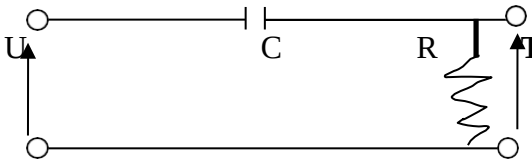


Fig1.19:transferfunctionofaActuator

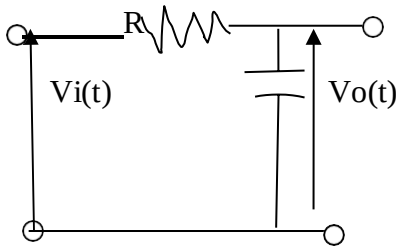
Filters, Purpose, Modeling and Transfer Function: A powerful tool available to thecontrolengineeriscompensationfilters.Compensationfilterscanvariousformsandarevery Affectiveintail oringtheaircraft response. They are of the following types:

(a) **Lead Compensator/High Frequency Filter: Purpose, Modeling and Transfer Function**: A lead compensator is used to quicken the system response by increasing natural frequency and/ or decreasing time constant. A lead compensator also increases the overall stability of the system. A simple lead compensator using simple RC network is shown below:



Transfer function=τs/(τs+1).where,τ=RC

(b) **LagCompensator/HighFrequencyFilter:Purpose,modelingandTransferFunction**.They areused toslowthe systemresponsebydecreasingthenatural frequencyand/or increasing thetimeconstant.Theyalsotendtodecreasethe overallstabilityofthesystem.Asimplelag network is shown in the figure below. They attenuate high frequencies like noise and disturbance.



TransferFunction=1/(τs+1).

Whereτ=RC

Fig:AsimpleLag Network

(c) **LeadLagfilter**.Thecombinedbenefitsofleadcompensatorandlagcompensatormay berealizedusinglead-lagcompensation.Commonuseoflead-lagcompensatoristhe attenuationofaspecificfrequencyrange(sometimescallednotchfilter).Forexample, an aircraft structural resonant frequency can be filtered out with a lead-lag compensatorifa feedback sensoriserroneously affected by that frequency. Transfer function of a lead-lag compensator can be represented by:

$TF=\frac{\square\square(\square+\square)(\square+\square)}{(\square+\square)(\square+\square)}$ wherea>b;a<c;c<d.(s+a)/(s+b)component corresponds to lag filter,s+c/s+dcomponentrepresentstheleadfilter.

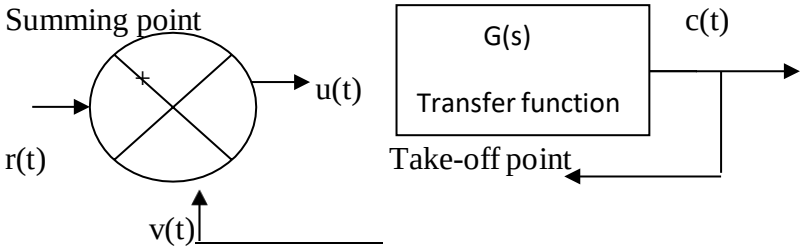
(d) **Washout Filter**. Another type of high pass filter which is used commonly in aircraft SAS is wash out filter. It is simply a case of the lead compensator where the zero is actually a differentiator. It has the transfer function as

$TF=\frac{\square\square\square}{\square+\square}$

Low frequency signals are attenuated, or washed out. Only changes in the input are passed through. This valuable for aircraft feedback control because feeding back a parameter such asrollratewithawashoutfilter,theSASwouldconstantlyopposetherollrateanddecrease the performance. The gain for high frequencies is determined by the corner frequency& the wash out filter gain Kw. Additionally, phase lead is added at higher frequencies.

Composition,reductionofBlockdiagramsofcomplexsystems-rulesandconventions:

There are four basic components of a block diagram. First there are blocks themselves, describing the relationship between input and output quantities through a transfer function. There are summingpointswhereoutputpartsoftwoormoreblocksareaddedalgebraically. Third componentisatake-offpointwhichrepresentstheapplicationofthetotaloutputfrom that point as input to some other block. Finally, diagrams contain arrows, indicating unidirectional flow of signal in these diagrams. This will be clear from the following fig.

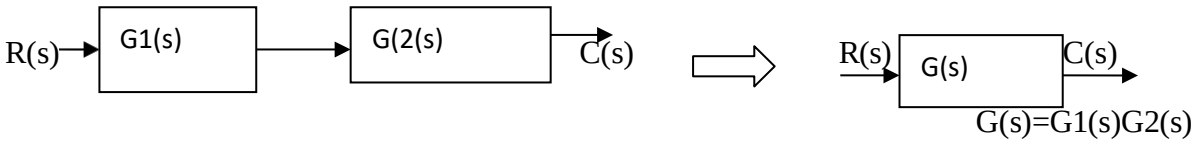


u(t)=r(t)–v(t)

C(s)=G(s)U(s)

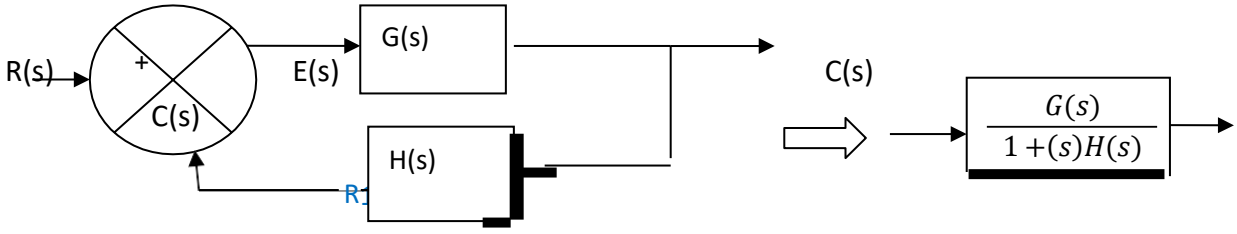
Rules of Block Diagram Algebra:

1. **Combining cascade Blocks**: Blocks connected in cascade can be replaced by a simpleblockwithtransferfunctionequaltotheproductoftherespectivetransfer function. This is shown below:



This is valid only if no loading effect on first block due to second block.

2. **Elimination of a feedback control: LetG**(s)be the transfer function in the forward path, H(s) is transfer function of feedback path. Overall transfer function is shown in the fig below.



C(s)H(s)

Example: Determine the overalltransferfunctionofthesystemshowninFig1.16.1below.

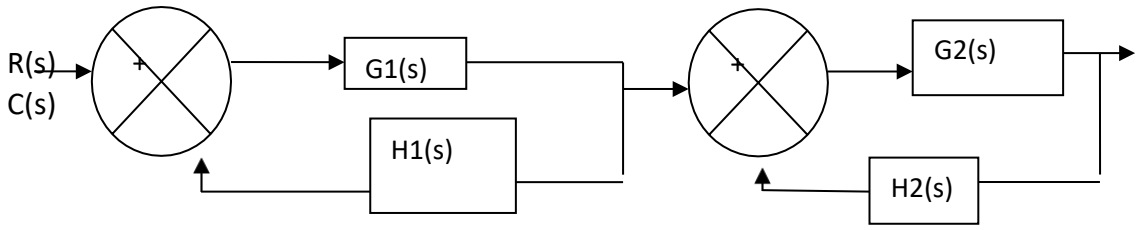
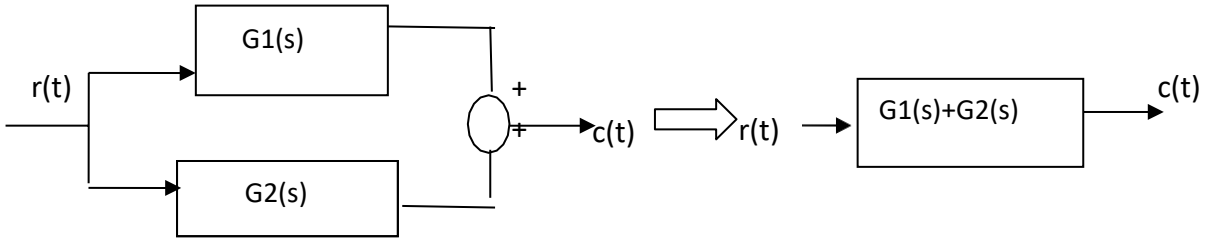


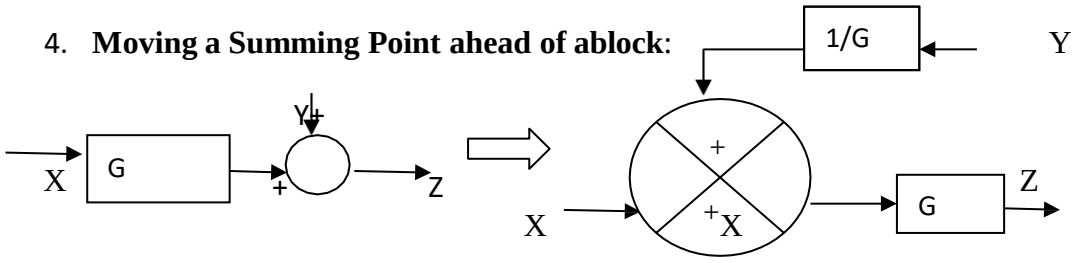
Fig1.16.1

Ans: $\frac{\square 1(\square)\square 2(\square)(1+\square 1(\square)(}{\square 1(\square))(1+\square 2(\square)\square 2(\square))}$

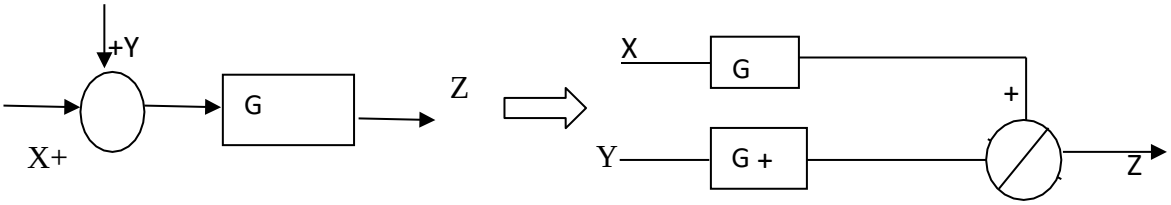
3. **Parallel Block**: Blocks are said to be parallel if they have common input & the overalloutputissumoftheindividualoutputs.Thetransferfunctionoftheparallel combination is simply the sum of the transfer function of the parallel blocks. This is shown below:



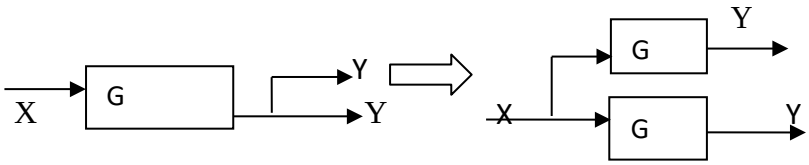
4. **Moving a Summing Point ahead of ablock**:



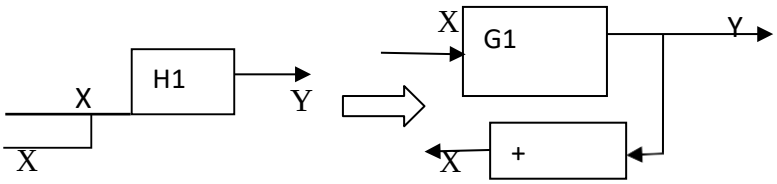
5. **Moving a Summing point behind a Block**:



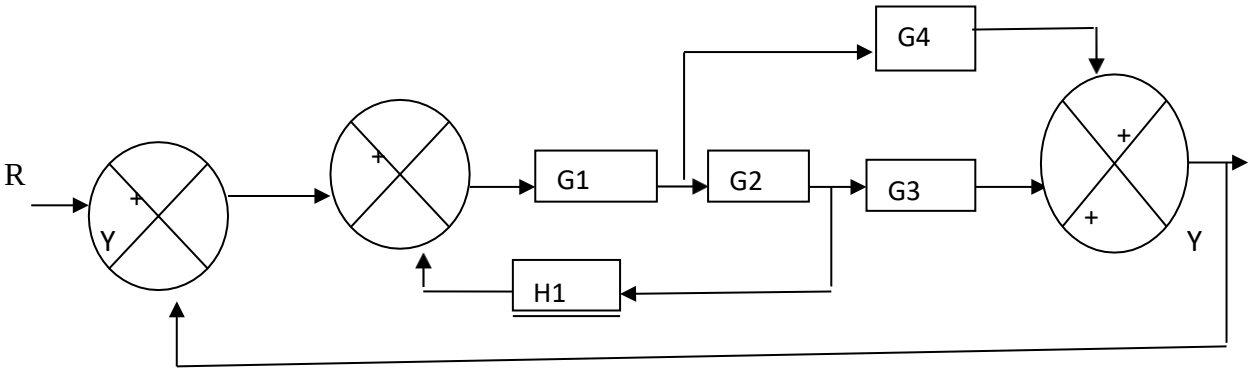
6. **Moving apick-off point ahead of aBlock**:



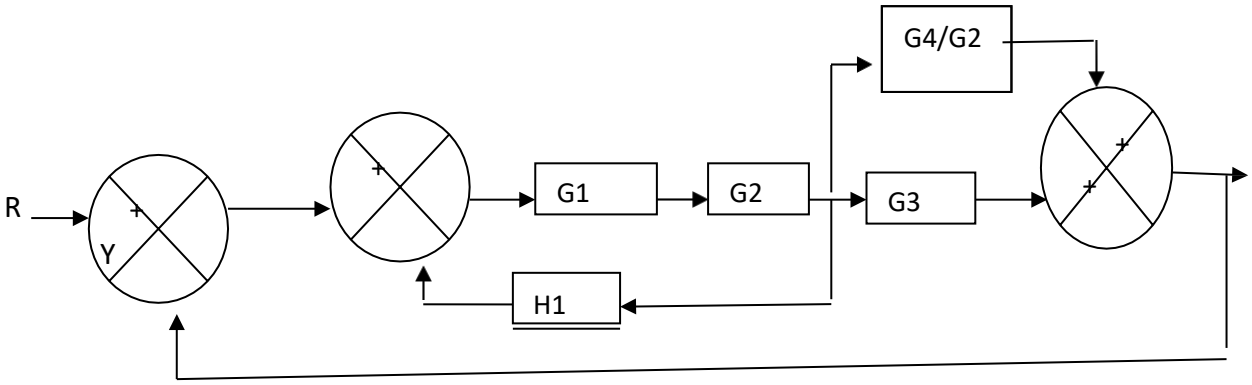
7. **Moving a Pick-off Point behind a Block**:



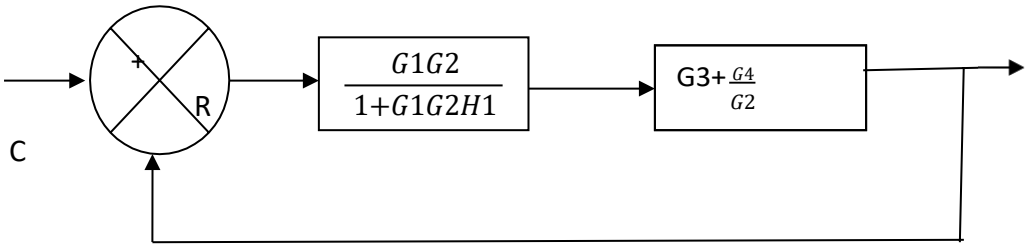
Examples of Block Reduction: FindY/R for the following blocks**:**



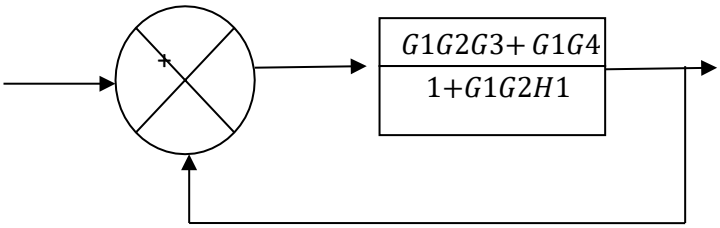
Solution:Aboveblockcanbereducedasfollows:Weshiftpick-offpointafterblockG2.



This can be further reduced to:



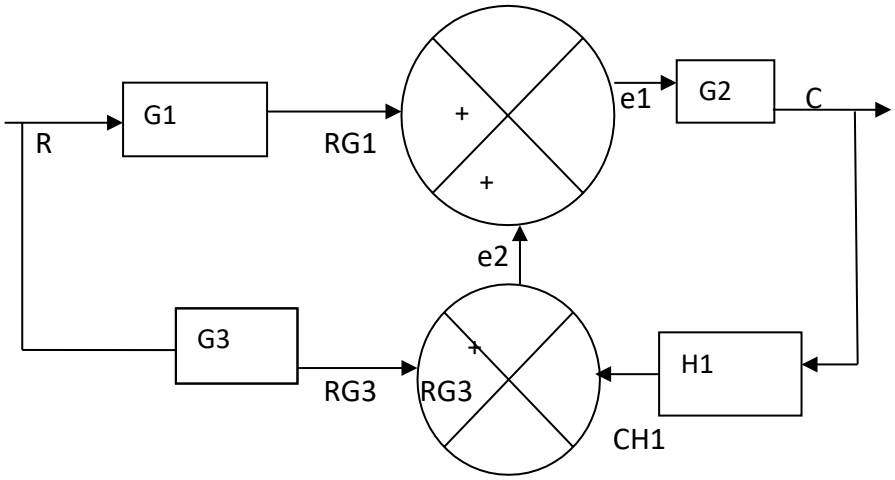
This can be reduced as follows:



Hence final transfer function is:

C(s)/R(s) = $\frac{□1□4+□1□2□31+□1}{□2□1+□1□2□3+□1□4}$ Ans.

Problem2: Reduce the block diagram & find C/R



Solution: e2=RG3– CH1

e1=RG1+e2=RG1+RG3-H1C ;

C=e1G2(substitutingvalueofe1) C

= (RG1+RG3 –H1C) G2

RG1G2+RG3G2 =G2H1C+C;

Hence, C/R= $\frac{□1□2+□3□2}{1+□1□2}$

UNITV
Design of Aircraft Controller, Stability and Control Augmentation & Auto Pilots

pproximation to aircraft transfer functions. The longitudinal and lateral equations of motion are described by a set of linear differential equations. The transfer function gives the relationship between the output and input to a system. The transfer function is defined as the Laplace transform of output to the Laplace transform of input, with all initial conditions set tozero. Following assumptions are made in **approximation to aircraft transfer functions**.

(a) We assume that aircraft motion consists of small deviations from its equilibrium flight conditions.

(b) We assume that the motion of the aircraft can be analyzed by separating the equation into Longitudinal and Lateral motion (later consists of yawing motion and roll motion).

Longitudinal Transfer Function Approximations: The longitudinal motion of an airplane(controlsfixed)disturbedfromitsequilibriumflightconditionischaracterizedbytwo oscillatorymodesofmotion.Fig4.1belowillustratesthesebasicmodes.Weseethatonemode islightlydamped &hasa longperiod.Thismotioniscalledthelong-periodorphugoid mode. It occurs at constant angle of attack. The second basic mode is heavily damped & has a very short period & it is appropriately called the short-period mode.

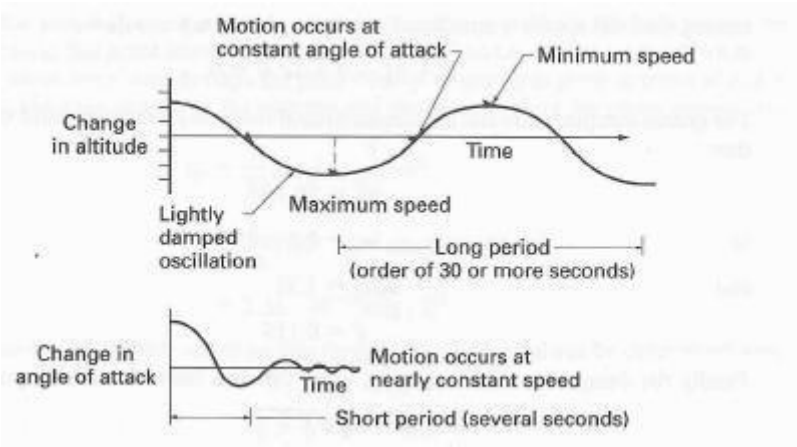


Fig 4.1 Phugoid and Short Period Oscillations

Longitudinaldifferentialequationscanbewrittenas:

$$\left(\frac{d}{dt} - X_u\right)\Delta u - X_w + (g\cos\theta_0)\Delta\theta = X_\delta \Delta\delta + X_{\delta_T}\Delta\delta_T$$

$$-Z_u \Delta u + \left[(1 - Z_{\dot{w}})\frac{d}{dt} - Z_w\right]\Delta w - \left[(u_0 + Z_q)\frac{d}{dt} - g\sin\theta_0\right]\Delta\theta = Z_\delta \Delta\delta + Z_{\delta_T}\Delta\delta_T$$

$$-M_u \Delta u - \left(M_{\dot{w}}\frac{d}{dt} + M_w\right)\Delta w + \left(\frac{d^2}{dt^2} - M_q \frac{d}{dt}\right)\Delta\theta = M_\delta \Delta\delta + M_{\delta_T}\Delta\delta_T$$

Where Δ□ andΔ□□are the aerodynamic and propulsive controls, respectively. Ifwe take the Laplace transform of above equations and divide by control deflection we can find the transfer functionΔ□/Δ ,Δ□/Δ□,Δ//Δ□. These equationscan be solved byCramer's Rule to find the transfer functions.

Transferfunctionscanbeexpressed astwopolynomials

$$\Delta u/\Delta\delta=\frac{A_u s^3+B_u s^2+C_u s+\mathrm{Du}}{\Delta_{longitude}}$$

$$\Delta w/\Delta\delta=\frac{A_w s^3+B_w s^2+\mathrm{Cw}s+\mathrm{Dw}}{\Delta_{longitude}}$$

$$\Delta\theta/\Delta\delta=\frac{A_\theta s^2+B_\theta s+\mathrm{C}_\theta}{\Delta_{longitude}}$$

$$\Delta_{longitude}=As^4+Bs^3+Cs^2+Ds+E$$

State Variable Representation of Equation of Motion: When equations are written as a system of first-order differential equations, they are called state space or state variable equations and expressed mathematically as

□=Ax+Bη;wherexisthestatevectorandηiscontrolvector&theA&Bcontaintheaircraft's dimensional stability derivative. The above differential equations of longitudinal motion can be further simplified as follows.

In practice, the force derivatives Z_q and □ usually are neglected because they contribute very little to the aircraft response. Therefore too simplify our presentation of the equations of motion in the state-space form we will neglect both these derivatives. Rewriting the equations in the state-space form

$$\begin{bmatrix}\Delta u\\ \Delta w\\ \Delta q\\ \Delta\theta\end{bmatrix}=\begin{bmatrix}X_u & X_w & 0 & -g\\ Z_u & Z_w & u_0 & 0\\ M_u+Z_u M_{\dot{w}} & M_w+Z_w M_{\dot{w}} & M_q+M_{\dot{w}}u_0 & 0\\ 0 & 0 & 1 & 0\end{bmatrix}\begin{bmatrix}\Delta u\\ \Delta w\\ \Delta q\\ \Delta\theta\end{bmatrix}+\begin{bmatrix}X_\delta & X_{\delta_T}\\ Z_\delta & Z_{\delta_T}\\ M_\delta+Z_\delta M_{\dot{w}} & X_{\delta_T}+M_w Z_{\delta_T}\\ 0 & 0\end{bmatrix}\begin{bmatrix}\Delta\delta\\ \Delta\delta_T\end{bmatrix}$$

Wherethestatevectorxand controlvectorηaregivenby

$$x=\begin{bmatrix}\Delta\square\\ \Delta w\\ \Delta q\\ \Delta\theta\end{bmatrix},\eta=\begin{bmatrix}\Delta\delta\\ \Delta\delta_T\end{bmatrix};\text{andthematricesA andBaregivenby}$$

$$A==\begin{bmatrix}X_u & X_w & 0 & -g\\ Z_u & Z_w & u_0 & 0\\ M_u+Z_u M_{\dot{w}} & M_w+Z_w M_{\dot{w}} & M_q+M_{\dot{w}}u_0 & 0\\ 0 & 0 & 1 & 0\end{bmatrix}$$

$$B=\begin{bmatrix} X_\delta & X_{\delta_T} \\ Z_\delta & Z_{\delta_T} \\ M_\delta+Z_\delta M_{\dot{w}} & X_{\delta_T}+M_{\dot{w}} Z_{\delta_T} \\ 0 & 0 \end{bmatrix}$$

P lModeApproximation:Inthismodethereisnochangeinangleofattack.

$\Delta\alpha=\frac{\Delta w}{u_0}$ $\Delta\alpha=0\rightarrow\Delta w=0$;;Makingtheseassumptions,thehomogeneouslongitudinalstate equationsreducetothefollowing:

$$\begin{bmatrix} \Delta\dot{u} \\ \Delta\dot{\theta} \end{bmatrix}=\begin{bmatrix} X_u & -g \\ \frac{-Z_u}{u_0} & 0 \end{bmatrix}\begin{bmatrix} \Delta u \\ \Delta\theta \end{bmatrix}$$

Theeigenvaluesofthelongperiodapproximationareobtainedbysolvingtheequation

$|\lambda I-A|=0$

$$\begin{vmatrix} \lambda-X_u & g \\ \frac{Z_u}{u_0} & \lambda \end{vmatrix}=0$$

Expandingthedeterminantyields

$\lambda^2-X_u\lambda-\frac{Z_u g}{u_0}=0$;or

$\lambda_p=\left[X_u\pm\sqrt{X_u^2+4\frac{Z_u g}{u_0}}\right]/2.0$

Thefrequencyanddampingratiocanbeexpressed as

$\omega_{n_p}=\sqrt{\frac{-Z_u g}{u_0}}$

$\xi_p=\frac{-X_u}{2\omega_{n_p}}$;Ifweneglectcompressibilityeffects,thefrequencyanddampingratiosforthelong-periodmotioncanbe approximatedbythe followingequation:

$\omega_{n_p}=\sqrt{2}\frac{g}{u_0}$

$\xi_p=\frac{1}{\sqrt{2}}\frac{1}{L/D}$

Notice that the frequency of oscillation and the damping ratio are inversely proportional to the forward speed and the lift-to-drag ratio, respectively. We see from this approximation thatthephugoiddampingisdegradedastheaerodynamicefficiency(L/D)isincreased.When pilots are flying an airplane under visualflight rules the phugoid damping andfrequency can varyoverawiderangeandtheywillstillfindtheairplaneacceptabletofly.Ontheotherhand, iftheyareflyingtheairplaneunderinstrumentflightruleslowphugoiddampingwillbecome veryobjectionable.Toimprovethedampingofthephugoidmotion,thedesignerwouldhave

toreduce the lift-to-drag ratio ofthe airplane. Because this would degrade the performance oftheairplane,thedesignerwouldfindsuchchoiceunacceptableandwouldlookforanother alternative, such as an automatic stabilization system to provide the proper damping characteristics.

rt-Period Approximation: An approximation to the short period mode of motion canbeobtainedbyassuming Δu =0anddroppingtheX-forceequation.Thelongitudinalstate-space equations reduce to the following:

$$\begin{bmatrix}\Delta \dot{w}\\ \Delta \dot{q}\end{bmatrix}=\begin{bmatrix} Z_w & u_0 \\ M_w+M_{\dot{w}}Z_w & M_q+M_{\dot{w}}u_0 \end{bmatrix}\begin{bmatrix}\Delta w\\ \Delta q\end{bmatrix}$$

Thisequationcanbewrittenintermsoftheangleof attackbyusingtherelationship

$$\Delta\alpha=\frac{\Delta w}{u_0}$$

In addition, one can replace the derivative due to w and □with derivative due to α andy using the following equations. The definition of the derivative □$_a$is

$$M_\alpha=\frac{1}{I_y}\frac{\partial M}{\partial \alpha}=\frac{1}{I_y}\frac{\partial M}{\partial w}\frac{\Delta w}{(u_0)}=\frac{u_0}{I_y}\frac{\partial M}{\partial w}=u_0 M_w$$

Inasimilar way we can show that

$$Z_\alpha=u_0 Z_w \text{ and } M_{\dot{\alpha}}=u_0 M_{\dot{w}}$$

Using these expressions, the state equations for the short-period approximate on can be written as

$$\begin{bmatrix}\Delta \dot{\alpha}\\ \Delta \dot{q}\end{bmatrix}=\begin{bmatrix} \frac{Z_\alpha}{u_0} & 1 \\ M_\alpha+M_{\dot{\alpha}}\frac{Z_\alpha}{u_0} & M_q+M_{\dot{\alpha}} \end{bmatrix}\begin{bmatrix}\Delta \alpha\\ \Delta q\end{bmatrix}$$

Theeigenvaluesofthestateequationcanagainbedeterminedbysolvingtheequation

$$|\lambda I-A|=0$$

Which yields

$$\begin{vmatrix} \lambda-\frac{Z_\alpha}{u_0} & -1 \\ -M_\alpha-M_{\dot{\alpha}}\frac{Z_\alpha}{u_0} & \lambda-(M_q+M_{\dot{\alpha}}) \end{vmatrix}=0$$

The characteristic equation for this determinant is

$$\lambda^2-(M_q+M_{\dot{a}}+\frac{Z_a}{u_0})+M_q\frac{Z_a}{u_0}-M_a=0$$

Theapproximateshort-periodrootscanbe obtainedeasilyfromthecharacteristicequation,

$$\lambda_{sp}=(M_q+M_{\dot{a}}+\frac{Z_a}{u_0})/2\pm[(M_q+M_{\dot{a}}+\frac{Z_a}{u_0})^2-4(M_q\frac{Z_a}{u_0}-M_a)]^{1/2}/2$$

Orintermsofthedampingandfrequency

$$\omega_{n_{sp}}=[M_q\frac{Z_a}{u_0}-M_a)]^{1/2}$$

$$\xi_{sp}=-[M_q+M_{\dot{\alpha}}+\frac{Z_\alpha}{u_0}]/(2(\omega_{n_{sp}})$$

Lateral Approximation of aircraft transfer function.The characteristic equation of aircraft lateral motion is characterized by the following equation.

$A\lambda^4+B\lambda^3+C\lambda^2+D\lambda+E=0$

Where A, B,C,D &E arethe functionsofstabilityderivative, massand inertia characteristic of the airplane.

Ingeneralwefindthattherootsofthecharacteristicequationtobecomposedoftworealroots andfairofcomplexroots.Therootswillbesuchthattheairplaneresponsecanbecharacterized by the following motions.

(a) Aslowlyconvergentordivergentmotion, calledthespiralmode.
(b) Ahighlyconvergentmotion,calledtherollingmode.
(c) Alightlydumpedoscillating motionhavinga lowfrequency,calledtheDutchroll.

Spiralmode isshowninfig4.2. Rollmode infig4.3andDutchrollmotioninFig4.4.

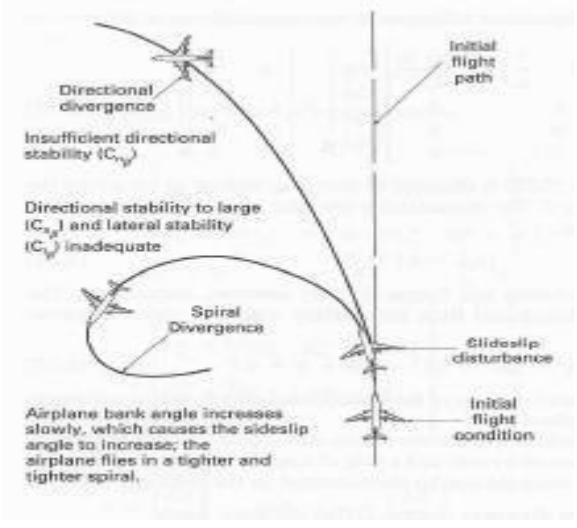


Fig4.2:SpiralMode

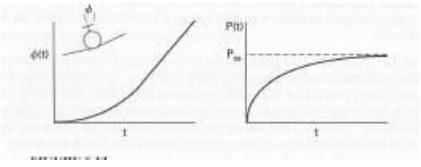


Fig4.3RollMode.

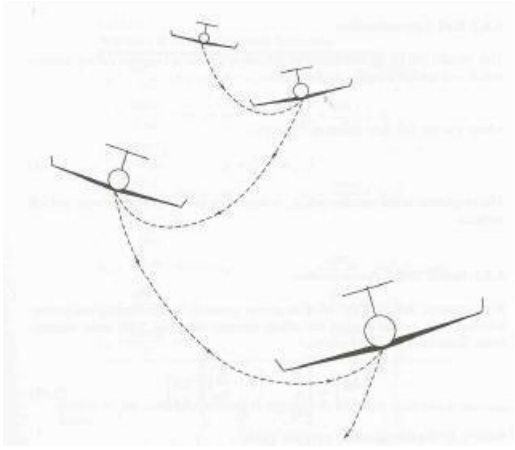


Fig4.4:DutchRollMotion.

(a) **SpiralApproximation**.Thecharacteristicrootofthespiralmodeis

$\lambda_{spiral} =$ $\frac{\square\square\square-\square\square\square\square}{\square\square}$

The stability derivative (dihedral effect) &(yaw rate damping), are usually negative quantities. On the other hand, (directional stability) &(Roll moment due to yaw rate) are generally positive quantities. Hence condition for stable spiral mode is

$L_\beta N_r > L_r N_\beta$

Increasing the dihedral effect $\square\square$and/or the yaw damping can be used to make the spiral mode stable.

(b) **RollApproximation**: $\lambda_{roll} = L_p = -1/\tau$

ThemagnitudeofrolldampingLpcan bedeterminedbythewing&tail surfaces.

(c) **Dutch Roll approximations**: If we consider that Dutch roll consists of side slipping & yawing motions, we get

$$\begin{bmatrix}\Delta\beta \\ \Delta r\end{bmatrix} = \begin{bmatrix}\frac{Y_\beta}{u_0} & -(1-\frac{Y_r}{u_0}) \\ N_\beta & N_r\end{bmatrix}\begin{bmatrix}\Delta\beta \\ \Delta r\end{bmatrix}$$

Solvingforthecharacteristicequationyields

$$\lambda^2 - \left(\frac{Y_\beta + u_0 N_r}{u_0}\right) + \frac{Y_\beta N_r - N_\beta Y_r + u_0 N_\beta}{u_0}$$

Fromthisexpressionwecan determinetheundampednaturalfrequencyandthedamping ratio as follows:

$$\omega_{nDR} = \sqrt{\frac{Y_\beta N_r - N_\beta Y_r + u_0 N_\beta}{u_0}}$$

$\xi_{DR} = \frac{-1}{2\omega_{nDR}} \left(\frac{y_\beta + u_0 N_r}{u_0}\right)$

Response ofaircraft to pilot's controlinputs.

Response of aircraft to Pilot's control input: Response of an aircraft to control input or atmosphere can be done by considering step input and sinusoidal input. The step and sinusoidal input functions are important for two reasons. First, the input to many physical systems takes the form of either a step change or sinusoidal signal. Second, an arbitrary function can be represented by a series of step changes or a periodic function can be decomposed by means of Fourier analysis into a series of sinusoidal waves. If we know the response of a linear system to either a step or sinusoidal input, then we can construct the system's response to an arbitrary input by the principle of superposition.

Of particular importance to the study of aircraft response to control or atmospheric inputs is the steady-state response to a sinusoidal input. If the input to a control system is sinusoidal, then after the transients have died out the response of the system also will be sinusoid of the same frequency. Theresponseofthesystemis completelydescribed bytheratio oftheoutput toinputamplitudeandthephasedifferenceoverthefrequencyrangefromzerotoinfinity.The magnitudeandphaserelationship betweenthe input andoutput signalsiscalledthe frequency response.Thefrequencyresponsecanbeobtainedreadilyfromthesystemtransferfunctionby replacingtheLaplacevariablesbyjω.Thefrequencyresponseinformationisusuallypresented ingraphicalformusingeitherrectangular,polar, log-logorsemi-logplotsasdiscussedinunit-II.Considerthetransferfunction,givenby

$G(s) = \frac{(1+T_a s)(1+T_b s)\ldots}{(1+Ts)(1+Ts)\ldots\left(1+\frac{2\zeta}{\omega_n}s+\frac{s^2}{\omega_n^2}\right)}$

Replacingthesbyjωandrewritingthetransferfunctioninpolarform yields

$M(\omega) = |G(j\omega)| = \frac{|k|\times|1+T_a j\omega|\times|1+T_a j\omega|\times|1+T_b j\omega|\ldots}{|(j\omega)^m|\times|1+Tj\omega|\ldots|1-\left(\frac{\omega}{\omega_n}\right)+2\zeta\frac{\omega}{\omega_n}|\ldots} \times \exp[j(\varphi_a+\varphi_b\ldots-\varphi_1\varphi_2\ldots)]$

Now,ifwetakethelogarithmof thisequation,we obtain

$\text{Log M}(\omega) = \log|G(j\omega)| = \log k + \log|1+T_a j\omega| + \log|1+T_b j\omega| \ldots - m\log|j\omega| - \log|1+T_1 j\omega| -$

$\log|1+Tj\omega| - \log\left|1-\left(\frac{\omega}{\omega_n}\right)^2+2\zeta\frac{\omega}{\omega_n}\right| - \ldots$ **(1)**

Andphase of$G(j\omega)$; $\angle G(j\omega) = \tan^{-1}\omega T_a + \tan^{-1}\omega T_b + \ldots - m(90°) - \tan^{-1}\omega T_1 - \ldots \tan^{-1}\left(\frac{2\zeta\omega_n}{\omega_n^2-\omega^2}\right)$ **(2)**

Inpractice, the log magnitude isoftenexpressed indecibels(dB).The magnitude indecibels is found by multiplying each term in equation (1) by 20:

MagnitudeindB=20 $\log_{10}|(\square\square|$

Thefrequencyresponseinformationofatransferfunctionisrepresentedbytwographs,oneof the magnitude and other of the phase angle, both versus the frequency on a logarithm scale. TheplotsarereferredasBodediagramsafterH.W.Bodewhomadesignificantcontributionto frequency response analysis.

Let us see, how these plots can be used to analyze the response of aircraft to control inputs. Let us consider the longitudinalpitchangle to elevatortransfer functionthat can be shownas indicated below, where the coefficient $A_\theta$and B $_\theta$, and so forth are functions of the aircraft stability derivatives. The longitudinalpitch angle to elevator transfer function is as follows:

$$\frac{\theta(s)}{\delta_e(s)} = \frac{A_\theta s^2 + B_\theta s + C_\theta}{As^4 + Bs^3 + Cs^2 + Ds + E}$$

Thiscanbe writteninthefactored form:

$$\frac{(s)}{\delta_e(s)} \qquad \frac{(T_{\theta 1}s+1)(T_{\theta 2}s+1)}{(\frac{s^2}{\omega_{nsp}^2} + \frac{2\xi_{sp}}{\omega_{nsp}}s+1)(\frac{s^2}{\omega_{np}^2} + \frac{2\xi_{sp}}{\omega_{np}}s+1)}$$

Themagnitudeandphaseangleforthecontrol transferfunctionisobtainedby replacings by $\square\square$ as follows:

$$\left|\frac{(j\omega)}{\delta_e(j\omega)}\right| = \frac{|k_{\theta\delta}||T_{\theta 1}j\omega+1| \qquad |T_{\theta 2}j\omega+1|}{\left|\frac{(j\omega)^2}{\omega_{nsp}^2} + \frac{2\xi_{sp}}{\omega_{nsp}}j\omega+1\right| \left|\frac{(j\omega)^2}{\omega_{np}^2} + \frac{2\xi_{sp}}{\omega_{np}}j\omega+1\right|}$$

$$\text{Phaseangle}\left[\frac{(j\omega)}{(j\omega)}\right. = \tan^{-1}\omega T_{\theta 1} + \tan^{-1}\omega T_{\theta 2} - \tan^{-1}\left(\frac{2\omega\xi_{sp}\omega_{nsp}}{\omega_{nsp}^2 - ^2}\right) - \tan^{-1}\left(\frac{2\omega\xi_p\omega_{np}}{\omega_{np}^2 - \omega^2}\right)$$

The frequency response for pitch attitude to control deflection for a typical business jet aircraft is shown in fig4.5. The amplitude ratios at both the phugoid and short-period frequenciesareofcomparablemagnitude.Atverylargefrequencies,theamplituderatioisvery small, which indicates that the elevator has negligible effect on the pitch attitude in this frequencyrange. The frequencyresponse for the change in forward speed and angle ofattack isshowninFig4.6and4.7respectively.Forthespeedelevatortransferfunctiontheamplitude ratio is large at phugoid frequencyand verysmallatthe short-period frequency. It isbecause short-period motion occurs at essentially constant speed Fig 4.7 shows the amplitude ratio of theangleofattacktoelevatordeflection;hereweseethattheangleofattack(AOA)isconstant atlowfrequencies.ItisbecauseinPhugoidmodeAOAremainsconstant.Thephaseplotwill showthatthere isa largephase lag intheresponse ofthespeedchangetoelevatorinputs.The phase lags for α/□is much smaller, which means that the AOA will respond faster than the change in forward speed to an elevator input.

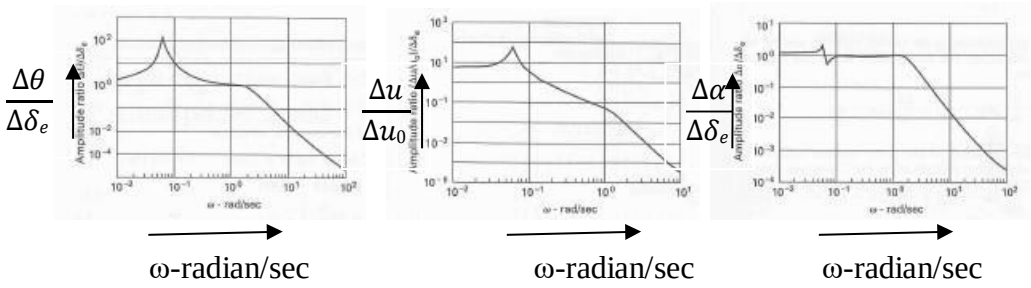


Fig4.5Magnitudeplot($\frac{\Delta\theta}{\Delta\delta_e}$Vsω)Fig4.6Magnitudeplot($\frac{\Delta u}{\Delta u_0}$sω)Fig4.7Plotof$\frac{\Delta\alpha}{\Delta\delta_e}$Vs ω

4.3Thecontroltaskofthepilot: Thecontroltaskofthepilotistoflytheaircraftsafelyinthe assigned mission of the aircraft. For a passenger aircraft mission profile will consist of take- off, cruise andlanding at the designatedairport. Similarly,a military aircraft being a weapon delivery platform should be able to strike the designated target accurately. To accomplish thesemissionspilotshouldbeabletocontrolandflythe aircraftaccuratelyandmaintainthe designatedroutewithout fatigue.The aircraft shouldbe controllableevenwhenit isdisturbed fromitsequilibriumpositioneitherbypilot'sactionorbyatmosphericturbulence.Anairplane must have sufficient stability such that the pilot does not become fatigued by constantly havingtocontroltheairplaneowingtoexternaldisturbance.Althoughairplaneswithlittleor no inherent aerodynamic stability can be flown, they are unsafe to fly unless they are provided artificial stability by stability augmentation system. Two conditions are necessary foranairplanetoflyitsmissionsuccessfully.Theairplanemustbeabletoachieveequilibrium flight and it must have the capability to maneuver for a wide range of flight velocity and altitude.Thestabilityandcontrolcharacteristicofanairplanearereferredtoasthevehicle's handling or flying qualities. Airplane with poor handling qualities will be difficult to fly and could be dangerous. An airplane will be considered of poor design if it is difficult to handle regardless of how outstanding the airplane's performance might be.

Precisiontaskssuchaslandingapproach,tracking,andformationflyinginmilitaryaircraftcan only beaccomplishedsuccessfully iftheaircraft's dynamicstability characteristics arewithin theacceptablelimits.Alsopilot'sshouldhavesufficientcontrolauthority(usuallyreferredto ascontrolpower)totrimandmaneuvertheairplanethroughouttheflightenvelop.Forceper g should be uniform throughout the flight envelop.

Flyingqualitiesofaircraft-relationtoairframetransferfunction.

1 Flying Qualities of an Aircraft: The flying qualities of an airplane are related to the stability and control characteristics and can be defined as those stability and control characteristics important in forming the pilot's impression of the aircraft. The pilot forms a subjectiveopinionabouttheeaseordifficultyofcontrollingtheairplaneinsteadyand

maneuvering flight. In addition to the longitudinal dynamics, the pilot's impression of the airplane is influencedbythefeeloftheairplane, whichisprovidedbythestickforceandstick force gradients. The Department ofDefense and FederalAviationAdministrationhas a list of specifications dealing with airplane handling qualities. These requirements are used by the procuringagenciestodeterminewhetheranairplaneisacceptableforcertification.Thepurpose oftheserequirements isto ensurethattheairplane has flying qualitiesthat place no limitation in the vehicle's flight safety nor restrict the ability of the airplane to perform its intended mission. Military standard MIL-F_875C gives the requirements for military aircraft.

As one might guess, the flying qualities expected by the pilot depend on the type of aircraft and the flight phase. Aircraft are classified according to size and maneuverability. Following are classifications, categories and levels of flying qualities defined as per **MIL-F_875C requirements**.

(a) Classificationofairplanes:Airplanecanbeplacedinoneofthefollowingclasses:

ClassI:Small,lightairplanes

ClassII:Mediumweight, low-to-mediummaneuverabilityairplane

ClassIII:Large,heavy,low-to-mediummaneuverabilityairplanes.

ClassIV:Highmaneuverabilityairplanes

(b) FlightPhaseCategory: Flight Phasesdescriptionsofmostmilitaryairplanemissionare:

Category A: Those non- terminal Flight Phases that require rapid maneuvering, precision tracking,orpreciseflight-pathcontrol.Examplesareair-to-aircombat,groundattack,in-flight refueling, and close formation flying.

CategoryB:Thosenon- terminalFlight phasesthat arenormallyaccomplishedusinggradual maneuvers and without precision tracking, although accurate flight path control may be required. Examples are climb, cruise, and descent.

Category C: terminal Flight Phases normally accomplished using gradual maneuvers and usually require flight-path control. Examples are take-off, approach, go-around, and landing.

(c) Leveloffflyingqualities:TheLevelsare:

Level1: Flyingqualitiesclearlyadequateforthemission FlightPhase

Level 2: Flying qualities adequateto accomplishthe missionFlight Phase, but some increase in pilot work load or degradation in mission effectiveness, or both, exists.

Level3:Flyingqualities suchthat theairplanecanbecontrolledsafely, but pilot workload is excessive or mission effectiveness is inadequate, or both.

ıgitudinal flying qualities- relation to airframe transfer function: Extensive research has been done to relate the flying qualities of airplane with stability and control characteristic of an aircraft. The fig 4.8 shows the relationship between the level of flying qualities and the damping ratio and un-

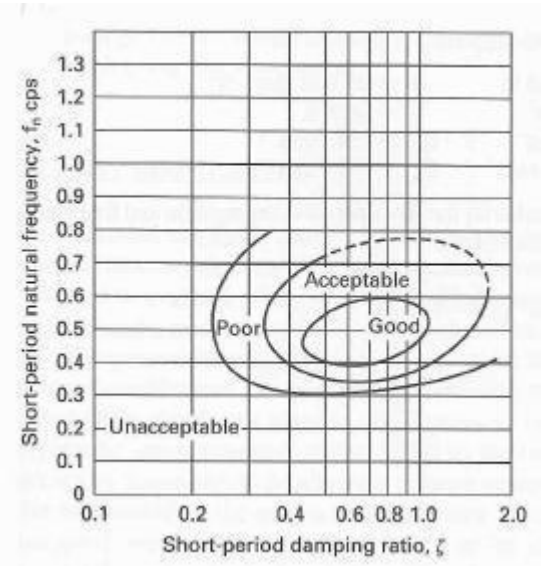


Fig4.8Shortperiodflyingqualities

dampednaturalfrequencyofshortperiodmode.

(a) Phugoidstability.Thelongperiodoscillationswhichoccurwhentheairplaneisdisturbed from a stabilized airspeed following a disturbance shall meet the following requirements:

Level1: ξ_{ph}atleast0.04

Level2: ξ_{ph}atleast0

Level3: T_2atleast55 seconds (WhereT_2istimeto double amplitude)

Short period damping ratio limits: The equivalent short-period damping ratio, shall be within the limits of table 4.4.2

Table4.4.2:Short PeriodDampingRatioLimits

Level	CategoryA&C FlightPhase		CategoryB FlightPhase	
	Minimum	Maximum	Minimum	Maximum
1	0.35	1.30	0.30	2.0
2	0.25	2.00	0.20	2.0
3	0.15	-	0.15	-

Lateralflyingqualities- relationtoairframetransferfunction:

(a) DutchRoll:The frequencyω_{nd}anddampingratio $\zeta_d$ofthe lateral-directionaloscillations following a yaw disturbance input shall exceed the minimum value given in table 4.4.3

(b) Rollmode:Theroll- modetimeconstant,τ_R,shallbenogreaterthantheappropriatevalue in table 4.4.4.

Table4.4.3:MinimumDutchrollfrequency anddamping

Level	FlightPhase Category	Class	Minimum ζ_d	Minimum $\omega_{nd}\zeta_d$rad/s	Minimum ω_{nd}rad/s
1	A[Combat& Ground Attack]	IV	0.4	-	1.0
	A	I,IV II,III	0.19 0.19	0.35 0.35	1.0 0.4
	B	All	0.08	0.15	0.4
	C	I,IV II-Landing, III	0.08 0.08	0.15 0.10	1.0 0.4
2	All	All	0.02	0.05	0.4
3	All	All	0	0	0.4

Table4.4.4:Maximum rolltimeconstant,seconds

Flight Phase Category	Class	Level		
		1	2	3
A	I,IV	1.0	1.4	
	II,III	1.4	3.0	
B	All	1.4	3.0	10
C	I,IV	1.0	1.4	
	II-Ldg,III	1.4	3.0	

(c) SpiralStability:Thecombinedeffectofspiralstability,flight-control-systemcharacteristic androllingmomentchangewithspeedshallbesuchthatfollowingadisturbanceinbankofup to 20 degrees, the time for the bank angle to double shall be greater than the value in table 4.4.5.

Table4.4.5:Spiralstability-minimumtimetodouble amplitude

Flight Phase Category	Level1	Level 2	Level 3
A&C	12sec	8sec	4sec
B	20sec	8sec	4sec

lot's opinion rating: Flying qualities ofan airplane is assessed bytest pilot's comment obtained fromsimulations and test flying of the aircraft. A structured rating scale for aircraft handling qualities was developed byNASA inthe late 1960s called the Cooper-Harper rating scale. This rating applies to specific pilot-in-loop tasks such as air-to-air tracking, formation flying,andapproach.Itdoesnotapplytoopen-loopaircraftcharacteristicssuchasyaw

responseto agust.Table4.5presentstheCooper-Harperratingscale.Aircraft controllability, pilot compensation(workload), andtaskperformancearekeyfactors inthepilot'sevaluation. ACooper-Harperratingof"one"ishighestorbestandaratingof"ten"istheworst,indicating the aircraft cannot be controlled during a portion of the task and that improvement is mandatory. Rating of one through three generally correspond to Level 1 flying qualities, a rating of four through six corresponds to level 2 flying qualities, and a rating of seven trough nine corresponds to Level 3.

Table4.5:Cooper-HarperScale

Pilot rating	**Aircraft Characteristic**	**DemandofPilot**	**Overall Assessment**
1	Excellent, highlydesirable	Pilotcompensationnotafactorfordesired performance	Goodflying Qualities
2	Good,negligible deficiencies	Pilotcompensationnotafactorfordesired performance	
3	Fair, some mildly unpleasant deficiencies	Minimalpilotcompensationrequiredfor desired performance	Flyingqualities warrant improvement
4	Minor but annoying deficiencies	Desiredperformancerequires moderatepilot compensation	
5	Moderately objectionable deficiencies	Adequate performance requires considerable pilot compensation	
6	Very objectionablebut tolerable deficiencies	Adequateperformancerequiresextensive pilot compensation	
7	Major deficiencies	Adequateperformancenotattainablewith maximum tolerable pilot compensation; Controllabilitynotinquestion	Flying quality deficiencies require improvement
8	Major deficiencies	Considerablepilotcompensationisrequired forcontrol	
9	Major deficiencies	Intensepilotcompensationisrequiredto retaincontrol	
10	Major deficiencies	Control will belostduringsomeportion of required operation	Improvement Mandatory

ability Augmentation System- displacement and rate feedback: Stability Augmentation Systems (SAS) were generally the first feedback control systems intended toimprovedynamicstabilitycharacteristic.Theywerealsoreferredtoasdampers,stabilizersand stability augmenters. These systems generally fed back an aircraft motion parameter, such aspitch rate, to provide a control deflection that opposed the motion and increased dampingcharacteristics. TheSAS hasto be integrated withprimaryflight controlsystemoftheaircraft

consisting of the stick, pushrod, cables, and bell cranks leading to the control surface or the hydraulic actuator that activated the control surface. Fig 4.9 presents a simplified SAS. SAS sensors and computers are normally dual redundant to improve the reliability.

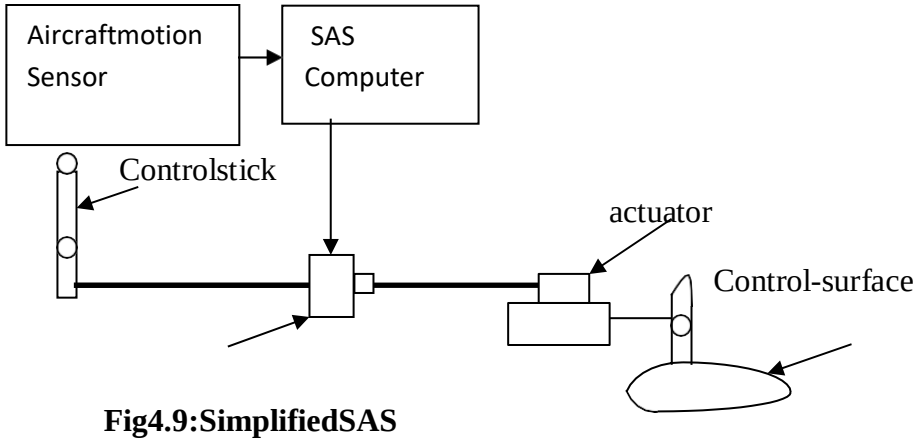


Fig4.9:SimplifiedSAS

Aclosedloopsystemillustratingfunctionsperformedwithinaflightcontrolcomputerisshown in fig 4.10.

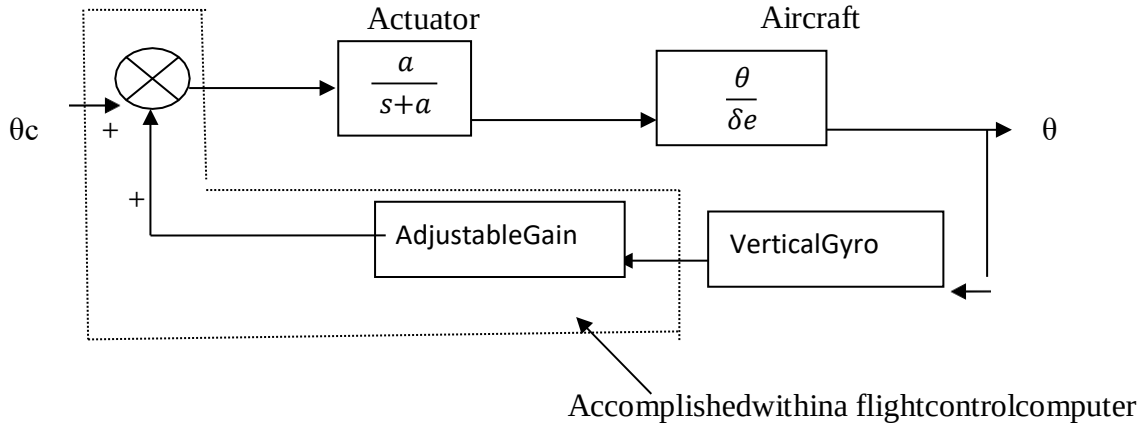


Fig4.10Closedloopsystemillustratingthefunctionsperformedwithinaflight control computer

he command signal and verticalgyro signalare input to the computer in the formof voltage or digital signals. Computer software multiplies the vertical gyro signal by the value of the adjustable gain (which is fixed for a final configuration), and then performs the comparator subtraction.Finally,thecomputeroutputstheerrorsignal(E)to anelectromechanicalactuator inthe formofavoltage. Theelectro-mechanicalactuatorconvertsthevoltageto amechanical displacement, which is input into the control valve of the aircraft hydraulic actuator. Many aircraft integrate the electromechanical actuator with the hydraulic actuator as one unit.

Displacement (Position) feedback as a tool in SAS. A generalized transfer function (TF) of a second order system can be written as:

X(s)/Y(s)= $\frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2}$;X(s)isoutput;Y(s)isinput.

The TF may represent short-period mode of an aircraft with natural frequency and damping ratio representing the dynamic characteristic of the basic airframe. Fig 4.11 represents a simple closed loop position feedback system. The term "position" refers to the fact that output variable (x) is feedback as itself (not as derivative of x).

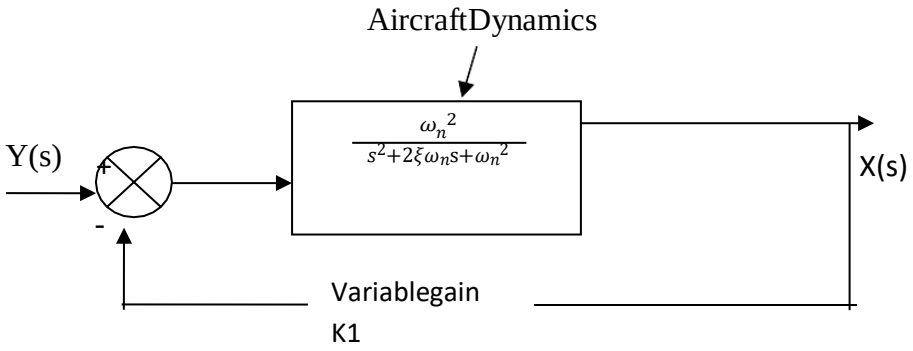


Figure4.11: Positionfeedbacksystem.

Closedlooptransferfunctionofthepositionfeedbacksystemshowninfig4.11 is:

X(s)/Y(s) = $\frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2(1+K_1)}$

Theclosedloopcharacteristicequationofthissystemis

$s^2 + 2\xi\omega_n s+\omega_n^2(1 + K_1) = 0$

Fromthisequationwecanseethat

$\omega_{nposition}=\omega_n\sqrt{(1+K_1)}$ (1)
$_{feedback}$

Likewise,closed-loopdampingratiohasbecome

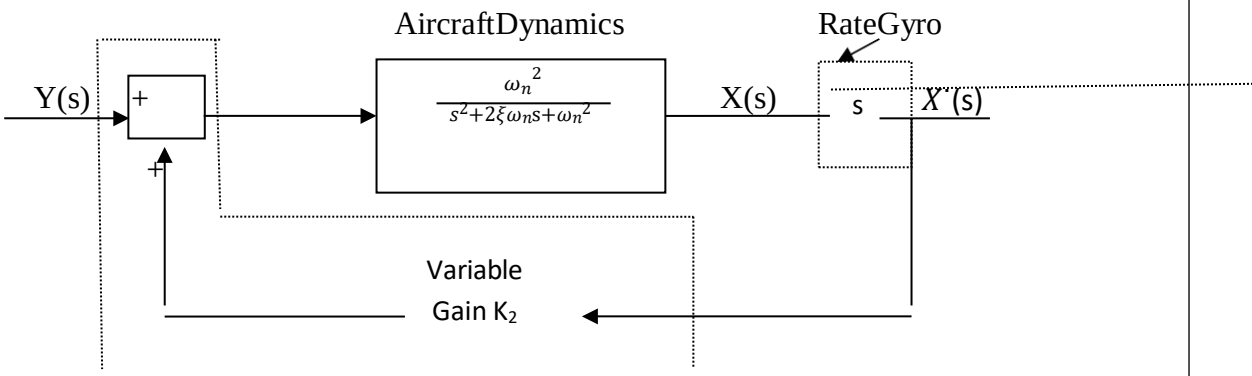
$\xi_{positionfeedback} = \frac{\xi}{\sqrt{(1+K_1)}}$ (2)

Theimportantpointisthattheclosedlooppositionfeedbacksystemprovidestheopportunity to change both the basic airframe natural frequency and damping ratio by adjusting the variable gain K_1. As can be seen from equation (1), position feedback allows the designer to increase the natural frequency of the closed loop system as $K_1$is increased positively from zero. It is unfortunate that the closed-loop damping ratio (equation (2)) decreases as $K_1$is increased.Closed-looptimeconstant(1/($\omega_{nposition}$*$\xi_{positionfeedback}$)remainsconstant.
$_{feedback}$

Thus,a**positionfeedbacksystemprovidestheadvantage**ofautomaticcontrolofamotion

variable (as in pitch hold example) but will be accompanied by an **increase in natural frequency**, a **decrease in damping ratio**, and **no change in time constant**.

Rate feedback System: Fig 4.12 presents a simple closed-loop rate feedback system. Rate refers to the fact that the derivative of the output variable (x) is feedback. Not that (s) is generated by simply multiplying x(s) by the Laplace transform operator s.



Accomplished within Flight Control Computer

Fig4.12: Rate feedback SAS

Closed-loop transfer function is

$$X(s)/Y(s) = \frac{\omega_n^2}{s^2+(2\xi\omega_n+K_2\omega_n^2)s+\omega^2}$$

The closed-loop characteristic equation for the system is

$$s^2+(2\xi\omega_n+K_2\omega_n^2)s+\omega^2=0$$

We can see from the above characteristic equation that the natural frequency of the open-loop & closed-loop system remains constant and is not affected by the value of K2. The closed loop damping ratio becomes

$$\xi_{ratefeedback} = \frac{2\xi+K_2 m_n}{2}$$

Rate feedback allows the designer to increase the damping ratio as K2 is increased positively from zero. This provides a powerful design tool to tailor the handling qualities of an aircraft and meet dynamic stability damping ratio requirements. A rate feedback system typically involves adding a rate gyro to the aircraft to provide measurement and feedback signal shown in fig 4.12. The figure illustrates where the rate gyro fits into the system. A rate gyro is a sensor that outputs a voltage proportional to an angular rate. Most highly augmented aircraft have pitch rate (Q), roll rate (P), and yaw rate (R) gyros to tailor dynamic stability and response characteristics for all three rotational degrees of freedom.

Acceleration feedback: Fig 4.13 below shows a simple closed-loop acceleration feedback system. Acceleration refers to the fact that the second derivative of the output variable (x) is feedback.

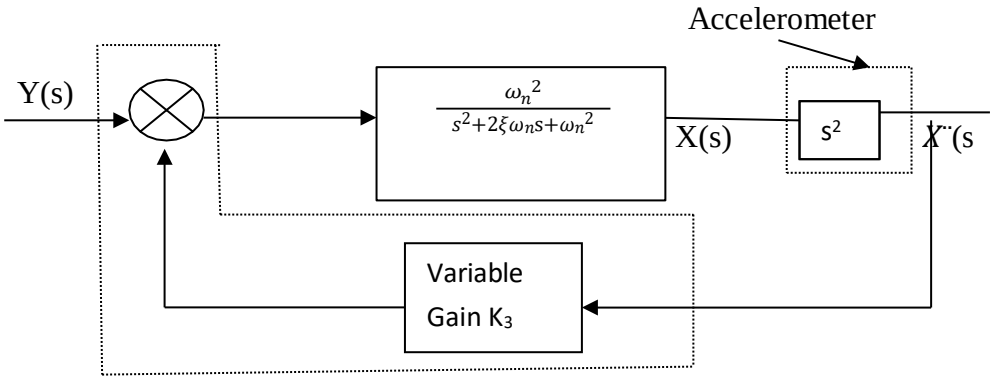


Fig4.13:Acceleration feedback

Closed-looptransferfunctionis

$$X(s)/Y(s) = \frac{\square^2}{(1+\square_3\square^2)\square^2+2\square\square\square\square+\square^2}$$

Theclosedloopcharacteristicequationforthesystemis

$$\square^2+\frac{2\square\square\square}{1+\square_3\square^2_n}s+\frac{\square\square^2}{\square_3\square^2_n}=0$$

Theclosedloopfrequencybecomes

$$\omega_{nacceleration\ feedback} = \frac{\omega_n}{\sqrt{1+K_3\omega_n^2}}$$

Closedloopdampingratio $\xi_{acceleration\ feedback} = \frac{\xi}{\sqrt{1+K_3\omega_n^2}}$

These equations indicate that natural frequency & damping ratio are decreased as K3 is increased positively from zero. With position, rate & acceleration feedback, we have the abilitytoincreaseordecreasethenaturalfrequency&dampingratioofanopenloopsystem. Handling qualities of an aircraft can be tailored with these tools & the roots of the characteristicequationcanbepositionedinthecomplexplanetomeetstatedrequirements. In some cases, a combination of position, rate, and/or acceleration feedback is needed to achievethedesiredcharacteristic.Amultiloopsystemusingallthreetypesof systemisshown in fig 4.14.

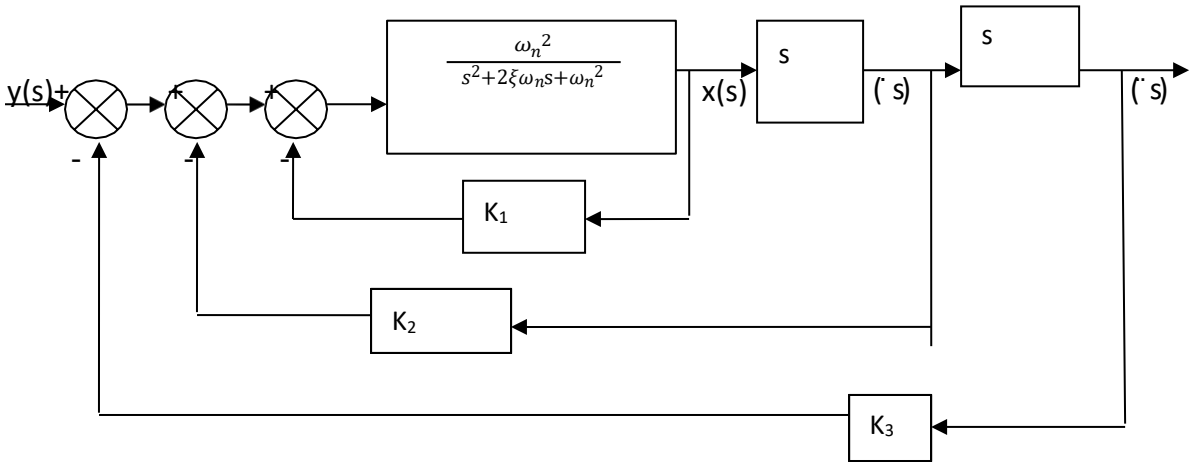


Fig4.14: Amultiloopsystemusingtreefeedbackloops.

Control Augmentation system: Control augmentation system (CAS) added a pilot commandinputintotheflightcontrolcomputer.Aforcesensoronthecontrolstickwasusually used to provide this command input. With CAS, a pilot stick input is provided to FCS in two ways- through the mechanical system and through the CAS electrical path. The CAS design eliminated the SAS problem ofpilot inputs being opposed bythe feedback. Fig 4.15 presents a simplified CAS.

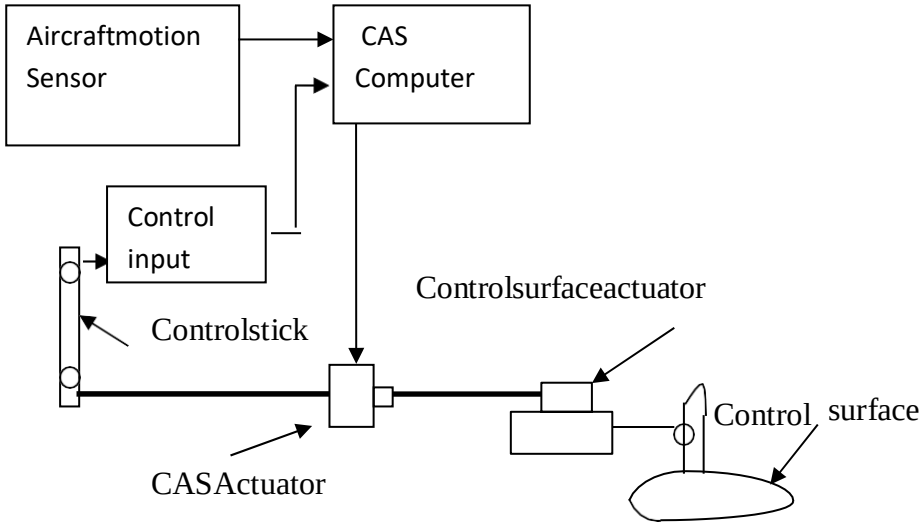


Fig4.15:SimplifiedCAS

With CAS, aircraft dynamic response is typically well-damped, and control response is scheduled with the control system gains to maintain desirable characteristic throughout the flight envelop. A block diagram of a typical CAS is presented in fig 4.16.

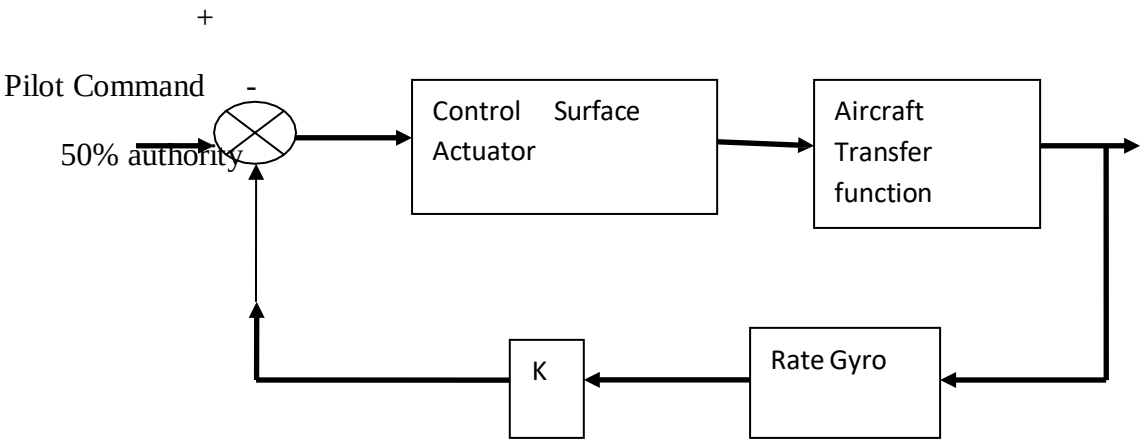


Fig4.16:SimplifieddiagramofCAS

CASprovidesdramaticimprovement inaircrafthandlingqualities.Bothdynamicstabilityand controlresponsecharacteristicscould betailoredandoptimized forthemissionoftheaircraft. Incaseofhigh-performancemilitaryaircraft,wherethepilotmayhavetomaneuvertheaircraft to its performance limits and perform tasks such as precision tracking of targets, specialized CAS are needed.FCS can provide the pilot with selectable "task tailored control laws". For example,althoughtheroleofafighteraircrafthaschangedtoincludelaunchingmissilesfrom long range, the importance of the classical dogfight is still recognized. A dogfight places a premium on high maneuverability and "agility" (ability to maneuver quickly) in the aircraft and control system that allows the pilot to take advantage of this maneuverability. In this situationa suitable controlled variable for pitchaxis is the normalaccelerationofthe aircraft. This is the component of acceleration in the negative direction of body-fixed z-axis. It is directlyrelevanttoperformingamaximum-rateturnandmustbecontrolleduptothestructural limitsofthe airframe, orthepilot'sphysical limits. Therefore, for adogfight, a"g-command" control system is an appropriate mode of operation of the FCS. Another common mode of operation for a pitch-axis CAS system is a pitch rate command system. When a mission requires precise tracking of a target, by means of a sighting device, it has been found that a deadbeat response to pitch-rate commands is well suited to the task. Control of pitch rate is alsothepreferredsystemfor approachandlanding.ApitchrateCASisshowninthefig4.17.

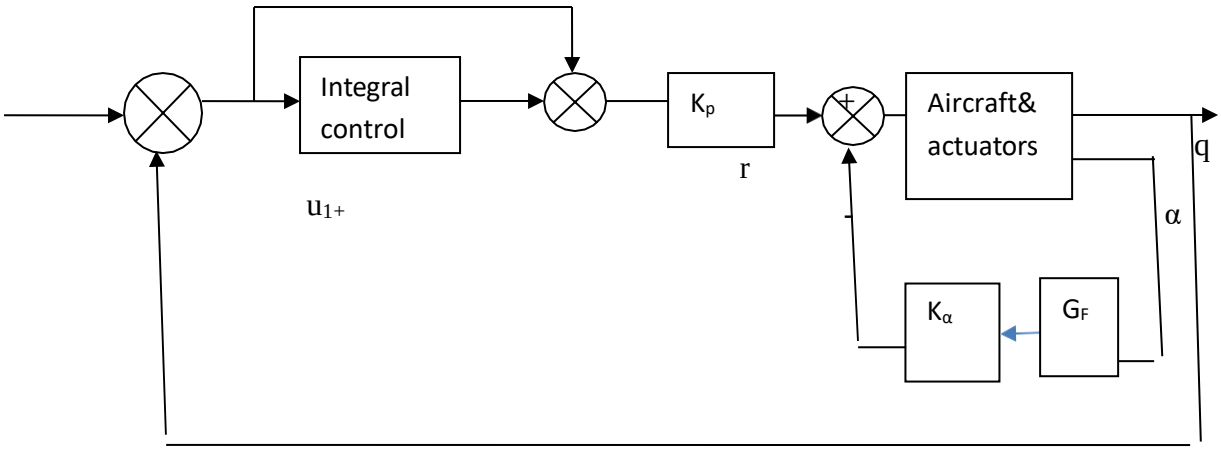


Fig4.17:Pitch-ratecontrol-augmentation

Anormal-accelerationcontrolaugmentationsystemisshowninfig4.18.

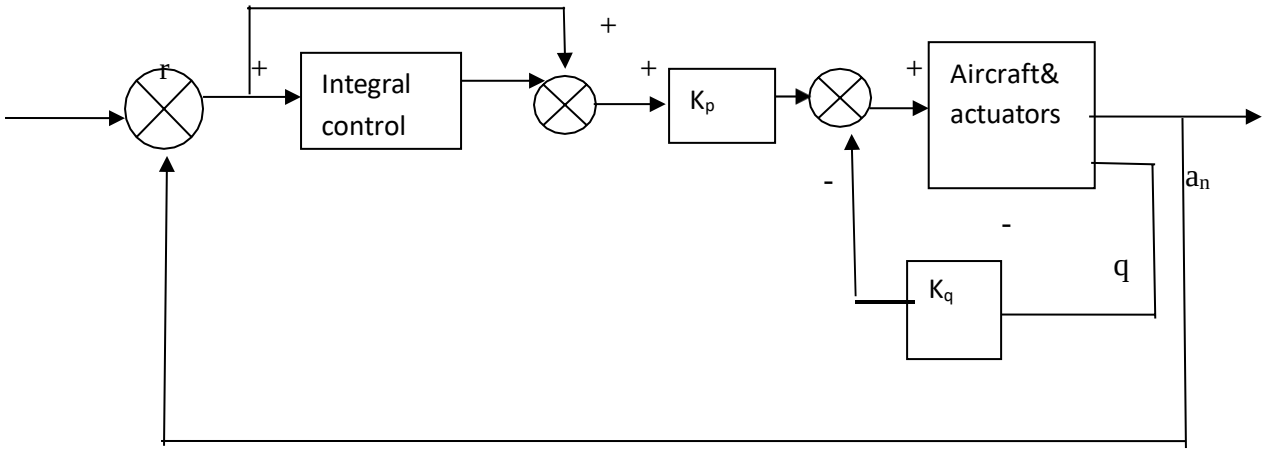


Fig4.18:NormalAcceleration CAS

Full authority fly-by-wire control:Full authority fly-by-wire (FBW) system has no mechanical link from the control stick to actuator system. Basically, FBW systems are CAS systemwithoutmechanicalcontrolsystemandprovidetheCASfullauthority. Theinput from control stick, pedal and from motion sensors are converted into electrical signals and sent to FBW computer. Software inside the FBW computer contains the control law which will command the control surfaces to move. However, to improve the reliability, triple and quad redundancyinsystemcomponentsalongwithself-test softwareisused. Aircraft suchasF-16, Mirage-2000andTejashaveFBWFCS.ThefullauthorityprovidedbyFBWallowssignificant tailoringofstabilityandcontrolcharacteristics.ThisabilityhasledtoFBWsystemswith

several feedback parameters and weighting of feedback gains based on flight condition andother parameter. Fig 4.19 presents a simplified FBW system.

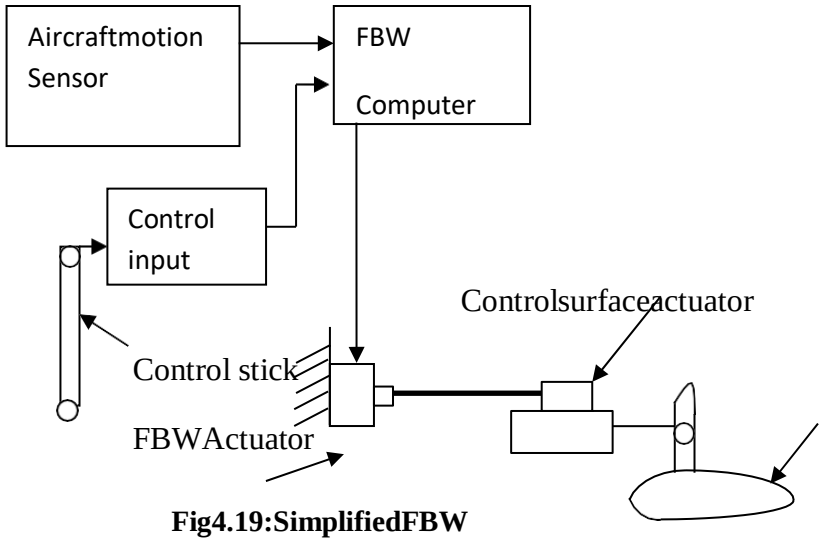


Fig4.19:SimplifiedFBW

BlockdiagramofF-16longitudinalFBWsystemisshownin fig 4.20.

Pilotcommand AircraftTransferFunctions

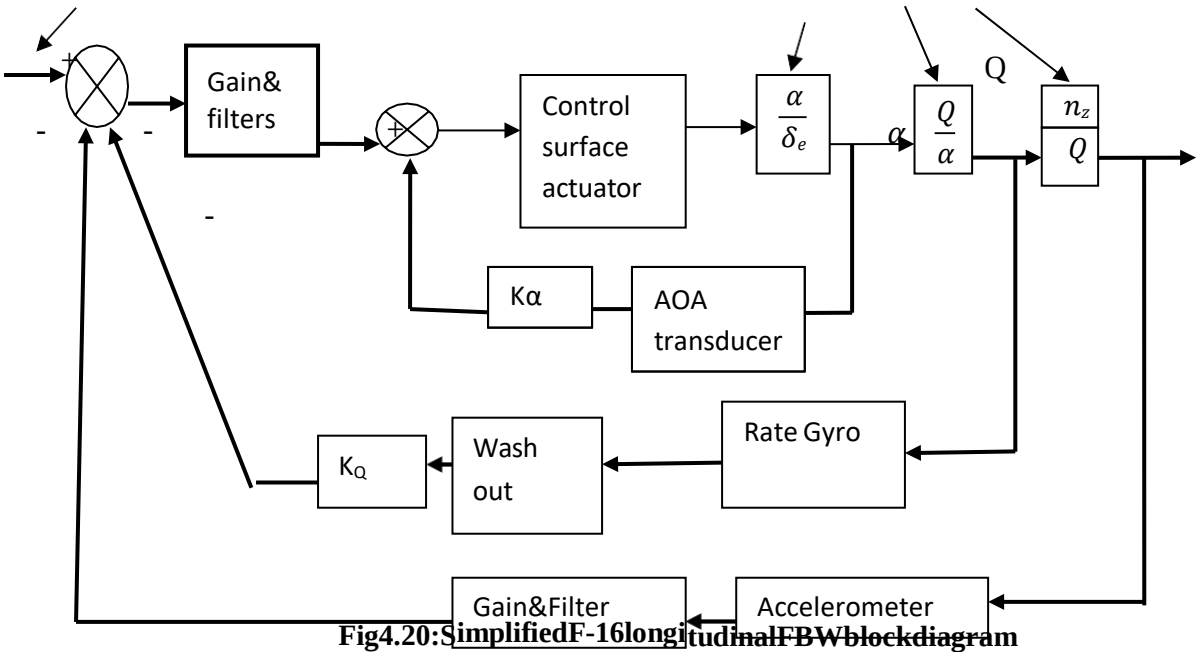


Fig4.20:SimplifiedF-16longitudinalFBWblockdiagram

AdvantagesofFBWControl:

(a) Increasedperformance:FBWenablesasmallertailplane,finrudderto beused, thereby reducing both aircraft weight and drag, active control of the airplane and rudder making up for the reduction in natural stability. For a civil airliner, reducing the stability margin and compensating for the reduction with a FBW system thus results in lighteraircraft withbetterperformance and betteroperating economicsand flexibility than a conventional design, for example, the ability to carry additional freight. For a military aircraft, such as an air superiority fighter, the FBW system enablesaircraft configurationswithnegativestabilitytobeused.Thisgivesmorelift, asthetrimlift ispositive,sothatalighter,moreagile fightercan beproduced-agility defined as the ability to change the direction of the aircraft's velocity vector. An increase in instantaneous turn rate of 35% is claimed for some of the new agile fighters.

(b) Reduced weight. Electrically signaled controls are lighter than mechanically signaled controls. FBW eliminates the bulk and mechanical complexity of mechanically signaled controls with their disadvantages of friction, back lash (mechanicallostmotion),structureflexureproblem,periodicriggingandadjustments.

(c) FBW control stick: FBW flight control enables a small, compact pilot's control stick to be used allowing more flexibility in cockpit layout. The displays are un-obscured.

(d) Automaticstabilization.

(e) CarefreeManeuvering.TheFBWcomputercontinuouslymonitorstheaircraft's stateto assess how close it isto its maneuver boundaries. It automatically limit'sthe pilot's command inputs to ensure that the aircraft does not enter an unacceptable attitude or approach too near its limiting incidence angle (approaching the stall) or carryoutmaneuverwhichwouldexceedthestructurelimitsoftheaircraft.Anumber ofaircraft are lost each year due to flying too close to their maneuver limits and the very high workload in the event of a subsequent emergency. The FBW system can thus make a significant contribution to flight safety.

(f) Ability to integrate additional controls. These controls need to be integrated automatically to avoid an excessive pilot-work-too many things to do at once:

(i) Leading and trailing edge flaps for maneuvering and not just for take-off and landing

(ii) Variablewingsweep

(iii) Thrustvectoring

(g) Ease of integration of the autopilot. The electrical interface and the maneuver command controlofthe FBW systemgreatly ease the autopilot integration task. The autopilot providessteeringcommandsaspitchrateorrollratecommandstotheFBW system.Therelativelyhighbandwidthmaneuvercommand'innerloop'FBWsystem ensures that response to the outer loop autopilot commands is fast and welldamped, ensuring goodcontrolofthe aircraft flight pathinthe autopilot modes. Ademanding autopilotmodeperformanceisrequiredforapplicationssuchasautomaticlanding,or,

automatic terrain following at 100-200 ft above the ground at over 600 knots where the excursions from the demanded flight path must be kept small.

(h) Aerodynamicsversus'Stealth': Theconceptofreducingtheradarcross-section of an aircraft so that it is virtually undetectable has been given the name 'stealth' in the USA. Radar reflection returns are minimized by faceted surfaces which reflect radar energy away from the direction of the source, engine intake design and the extensive use of radar energy absorbing materials in the structure. Stealth considerations and requirements can conflict with aerodynamics requirements and FBW flight control is essential to give acceptable, safe handling across the flight envelop.Thefeedbackmustbeadjustedaccordingtoflightcondition.Theadjustment process is called gain scheduling because, in its simplest form, it involves only changing the amount of feedback as a function of scheduling variable. These scheduling variables will normally be measured dynamic pressure and/or Mach number. The signals from rate gyros, accelerometers, air data computer, and other sources are processed by the flight-control computer (FCC).

4.10Need forautomatic Control: Fig 4.21 showsthe altitude-Machenvelope ofa modernhigh-performanceaircraft;theboundariesofthisenveloparedeterminedbya number of factors. The low-speed limit is set by the maximum lift that can be generated (the alpha limit in the figure), and the high-speed limit follows a constant dynamic pressure contour (because of structural limits, including temperature). At high altitudes the speed becomes limited bythe maximumengine thrust (which falls offwithaltitude).Thealtitudelimit imposedontheenvelopiswherethecombination ofairframeandenginecharacteristicscannolongerproducea minimumrateofclimb (this is the "service ceiling"). The basic aerodynamic coefficients (stability derivatives) vary with Mach number. Because of the large changes in aircraft dynamics, a dynamic mode that is stable and adequately damped in one flight condition may become unstable, or at least inadequately damped, in another flight condition. Alightlydampedoscillatorymodemaycauseagreatdealofdiscomfortto passengers or make it difficult for the pilot to controlthe trajectoryprecisely. These problems are overcome by using feedback control to modify the aircraft dynamics. The aircraft motion variables are sensed and used to generate signals that can be fed into the aircraft control-surface actuators, thus modifying the dynamic electrical output of the FCC is used to drive electro hydraulic valves, and these superimpose additional motion on the hydro mechanical control system.

OnemayaskastowhyuseanFCCinsteadofpilot?Therareseveralreasonsforthis. First ofall, a computer hasa much higher reaction velocitythana pilot. Also, it isn't subject to concentration losses and fatigue. Finally, a computer can more accurately know the stateofthe aircraft is in. (Computer can handle huge amount ofdata better andalso don'tneedtoreadasmallindicatortoknow,forexample,thevelocityorthe height oftheaircraft.)

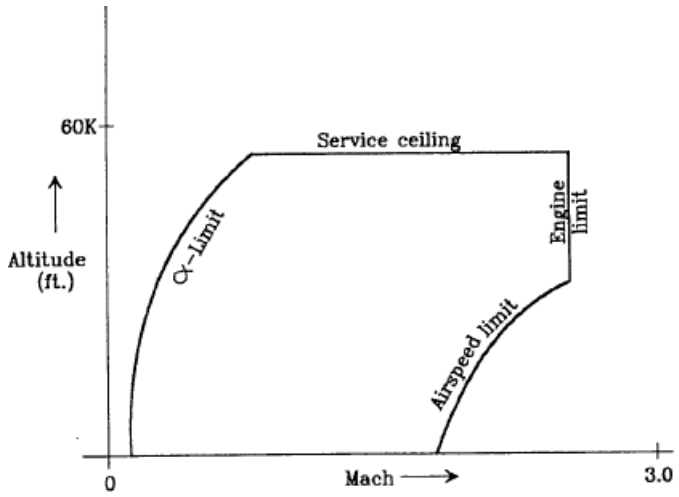


Fig4.21Aircraft altitude-Machenvelope

Fig 4.22 shows how a fully powered aircraft control system might be implemented with mechanical, hydraulic, and electrical components.

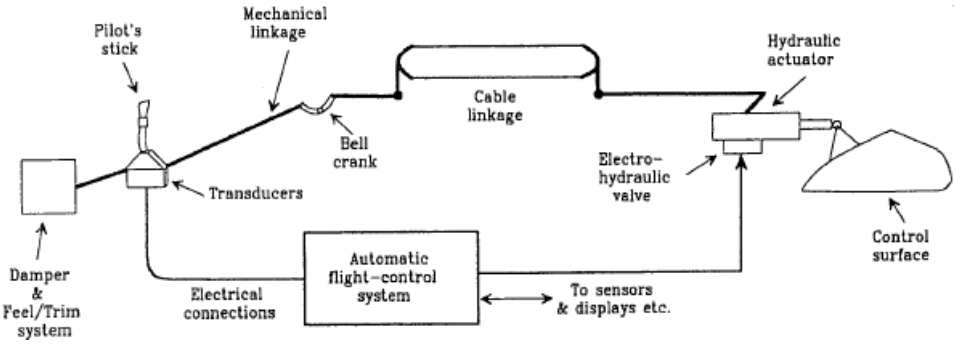


Fig4.22:Anelectro-mechanicalcontrolsystem

Autopilots-purpose, functioning-inputs-hold-command-track.

Autopilots-PurposeandFunctioning,inputs-hold,command,track: Basic purpose of an auto-pilot is to reduce the pilot work load (pilot- relief auto-pilot). The auto-pilots are capable of maintaining (holding) constant attitude (pitch, roll, and heading), velocity, and altitude.Theycanalso becoupledtoinstrument landingsystemduring landing inbadweather conditions.Inautomaticterrainfollowingmodetheycanbeusedtoflyinahillyterrainwithout much work load onthe pilot. Theycan also be used as SAS. Auto-pilots are used fortracking a command instead of holding a reference value. In such cases reference command may be pitch-rate or normal acceleration. Maneuvering auto-pilots can be used in high performance fighter aircraft to give desired normal acceleration, turn rate and pitch-rate during various modesofcombat(exampledogfight,air-to-groundtargettracking).Inholdautopilotaconstant outputismaintainedlikeinheadingholdmodepresentheadingismaintainedoncetheheading hold mode of the auto-pilot is engaged. In command input, auto-pilot is commanded (e.g. a new given heading or bank angle) to new state (bank angel, altitude, heading).

DisplacementAutopilots-Pitch,yaw,bank,altitudeandvelocityhold-purpose, relevant simplified aircraft transfer functions, feedback signals:

Displacement autopilot-pitch, yaw autopilot:One of the earliest auto-pilots to be usedforaircraftcontrolistheso-calleddisplacementauto-pilot.Adisplacementtypeautopilot can be used to controlthe angular orientation ofthe airplane. Conceptually, the displacement autopilot works in the following manner. In a pitch attitude displacement autopilot, the pitch angle is sensed bya verticalgyro andcomparedwiththedesiredpitchangleto createanerror angle. The difference or error in pitch attitude is used to produce proportional displacements of the elevator so that the error signal is reduced. Figure 4.23 is a block diagram of either a pitch or roll angle displacement autopilot. The heading angle of the airplane also can be controlled inasimilarscheme.Theheadingangle issensedbyadirectionalgyro andtheerror signal is used to displace the rudder to reduce the error signal. A displacement heading auto pilot is shown in fig 4.24.

BankAttitudeautopilot.Therollattitudeofanairplanecanbecontrolled byasimple bank angle autopilot as illustrated in fig 4.25. Conceptually the roll angle of the airplane can be maintained at whatever angle one desires. IN practice we would typically design the autopilottomaintainawingslevelattitudeorφ=0.Theautopilotiscomposedofacomparator, aileron actuator, aircraft equation of motion (i.e. transfer function), and an attitude gyro to measure the airplane's roll angle.

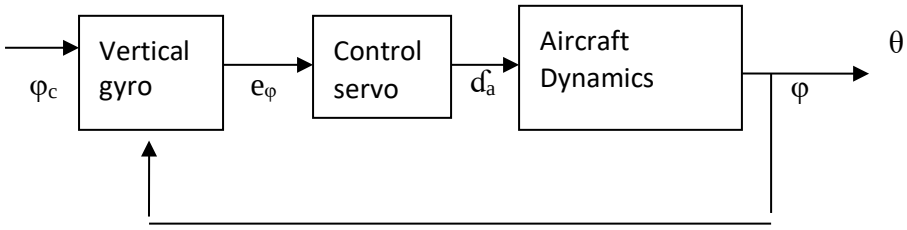


Fig4.23:Arollorpitchdisplacementautopilot

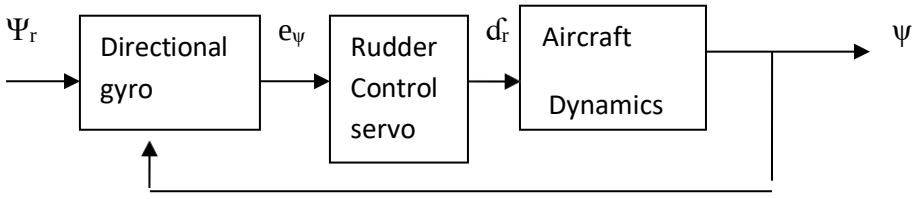


Fig4.24: Aheadingdisplacement autopilot

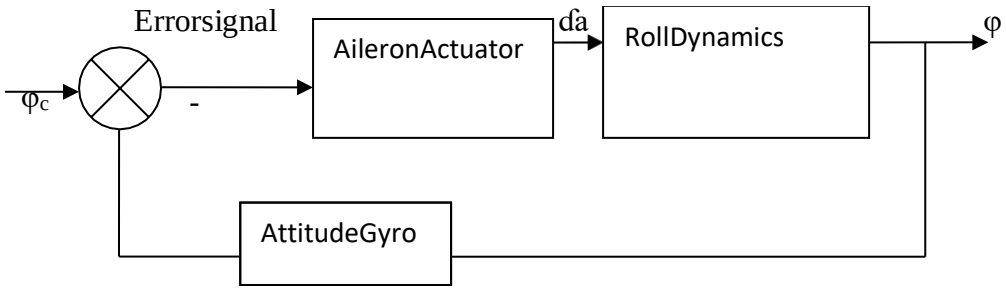


Fig4.25:Simplerollattitudecontrolsystem

Altitude hold autopilot: The altitude ofan airplane can be maintained by an altitudeholdautopilot.Asimplealtitudeholdautopilotisshowninfig4.26.Basically, the autopilot is constructedto minimize the deviationbetweenthe actualaltitude and thedesiredaltitude.To analyze howsuchanautopilot would functionweexaminean idealized case. First, we assume that the airplane's speed will be controlled by a separate control system, second, we neglect any lateral dynamics. With these restrictions we are assuming that the only motion possible is in vertical plane. The transferfunctionsnecessarytoperformtheanalysisareelevatorservoandaircraft dynamics.Theelevatortransferfunctioncanberepresentedasafirstorderlagas□□ □ =□□ □+10.Theaircraftdynamicscanberepresentedbyshortperiodapproximations.

Next,weneedtofindthetransferfunction $\Delta h/\Delta\square\square$.Thiscanbeshownas

$$\frac{\Delta h(\square)}{\Delta\square_{\square}(\square)} = \frac{\square_0}{\square}\left[\frac{\Delta\square(\square)}{\Delta\square_{\square}(\square)} - \frac{\Delta\square(\square)}{\Delta\square_{\square}(\square)}\right]$$

Thetransferfunction$\frac{\Delta(\square)}{\Delta\square_{\square}(\square)}$ canbeobtainedfrom$\Delta(\square)/\Delta\square\square(\square)$inthefollowingways

$\Delta\square=\Delta\square$;hence$\Delta\square(s) = s\Delta\square(\square)$

Hence$\frac{\Delta(\square)}{\Delta(\square)} = \frac{1}{\square}(\Delta(\square)/\Delta\square\square(\square)) = \frac{\square_{\square}\square+\square_{\square}}{\square(\square\square^2+\square\square+\square)}$

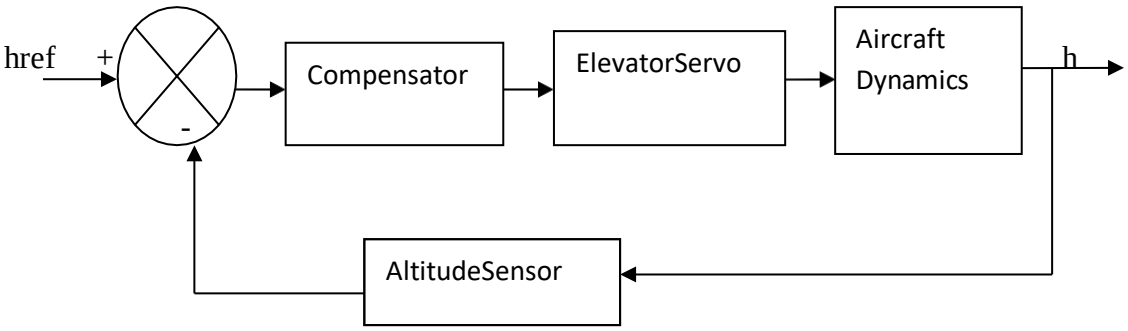


Fig4.26: Altitudeholdautopilot

Velocity Hold Autopilot:The forward speed of an airplane can be controlled by changing the thrust produced by the propulsion system. The function of the speed control systemis to maintainsome desired flight speed. This is accomplished bychanging the engine throttlesettingtoincreaseordecreasetheenginethrust.Figure4.27issimplifiedconceptfora speedcontrolsystem.Thecomponentsthatmakeupthesystemincludeacompensator,engine throttle, aircraft dynamics, and feedback path consisting of the velocity and acceleration feedback.

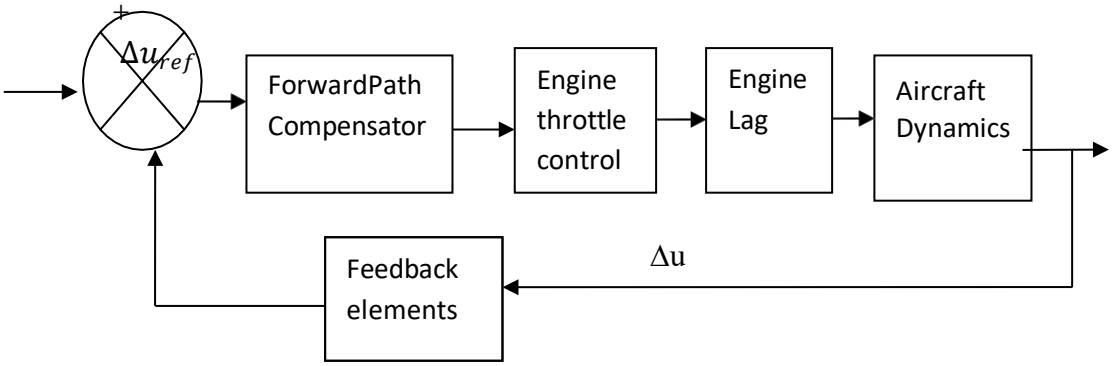


Figure4.27:Ablockdiagramfora speed controlsystem

4.13 Autopilot design by displacement & rate feedback-iterative methods, design by displacement feedback and series PID compensator-Zeigler & Nichols method:

Designofautopilotbydisplacement&ratefeedbackusingiterativemethods:Design ofanautopilotbydisplacement&ratefeedbackisexplainedwithanexampleofapitch attitude hold auto pilot of a transport aircraft. Basic block diagram of a pitch hold auto pilot isshowninfig4.33.Forthisdesignreference the reference pitchangle iscompared withthe actualpithanglemeasuredbythepitchgyrotoproduceanerrorsignaltoactivatethecontrol surface actuator to deflect the control surface. Movement of the control surface causes the aircrafttoachieveanewpitchorientation,whichisfeedbacktoclosetheloop. Todesignthe control system for this auto pilot we need the transfer function of each component. The transfer function of the elevator servocan be represented as a first order system

$\theta e/v=k_a/(\tau s+1)$; where δe,v, k_a and $\tau$are the elevator deflection angle, input voltage,elevator servo gain and servo motor time constant. The time constant can be assumed to be 0.1s. we can represent the aircraft dynamics by short-period approximation. The short period TF for the a typical jet transport for example can be written as:

$$\frac{\Delta\square}{\Delta\square\,\square} = -2.0(s+0.3)/(s(s^2+0.65s+2.15))$$

Thefig 4.28isthe block diagram representationofthe autopilot.The problemnow is oneofdeterminingamplifiergain$k_a$sothatthecontrolsystemwillhavethedesired

performance. Selection of $k_a$can be determined using a root locus plot of transfer function. Fig 4.29 is the root locus plot for the typicaljet aircraft pitch controlautopilot. As the gain is increased from zero, the system damping decreases rapidly and the system becomes unstable. Even for low values k_a, the system damping would be too low for satisfactory dynamic performance. The reason for poor performance is that the airplane has very little natural damping. To improve the design we could increase the damping of the short period mode by adding an inner loop feedback loop. Fig 4.30 is a block diagram representing of displacementautopilotwithpitchratefeedbackforimprveddamping.Intheinnerloopthe pitchrateismeasuredbyarategyroandfedbacktobeaddedwitherrorsignalgeneratedby the difference in pitch attitude. Fig 4.31 shows the block diagram for the business jet where pitchrateisincorporatedintothedesign.Forthisproblemwehavetwoparameterstoselect, namelythegains$k_a$andk_{rg}.Theroot locusmethodcanbe usedtopickbothparameters.The procedure is essentially a trial-and-error method. First,the root locus diagram is determined for the inner loop; a gyro gain is selected, and then the outer root locus plot is constructed. Severaliterations may be requireduntil the desiredoverallsystem performance is achieved.

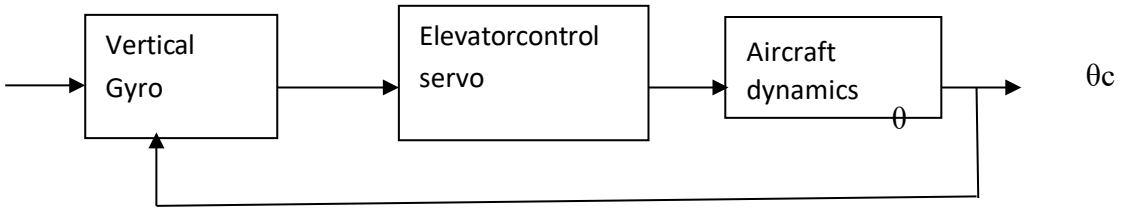


Fig4.28Apitchdisplacementautopilot

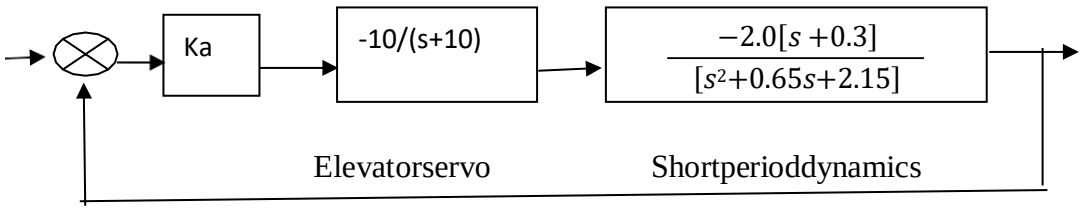


Fig4.29Apitchdisplacementautopilot

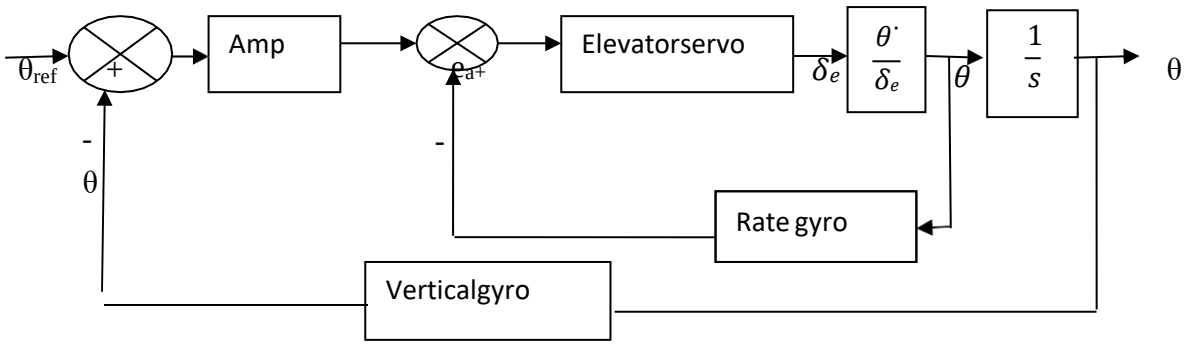


Fig4.30:Apitchattitudeautopilotemployingpitchratefeedback

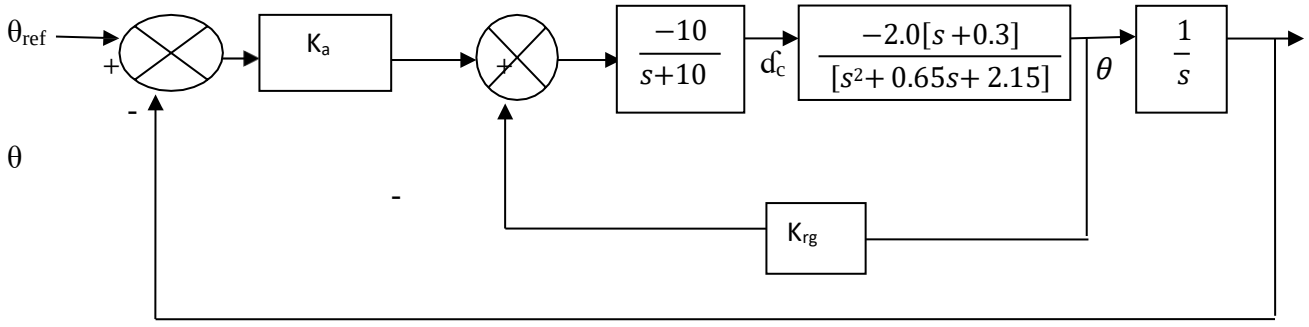


Fig4.31:Apitchattitudeautopilotemployingpitchrate feedback

PID controller or Ziegler-Nichols tuning rules: The simplest feedback controller is oneforwhichthecontrolleroutputisproportionaltotheerrorsignal.Suchacontrolleriscalled aproportionaltotheerrorsignal.Suchacontrolleriscalledaproportionalcontrol.Obviously. the controller's main advantage is its simplicity. It has the disadvantage that there may be a steady state error. The steady-state error can be eliminated by using an integral controller e(s)=k $\int_0^t e(t)dt$ore (s)=k$^{(s)}$wherekistheintegralgain.Theadvantageoftheintegral

o i o i$_s$ i

controller is that the output is proportional to the accumulated error. The disadvantage of the integralcontrolleristhatwemakethesystemlessstablebyaddingthepoleattheorigin.Recall thattheadditionofapoletotheforward-pathtransferfunctionistobendtherootlocustoward

therighthalfs-plane.Itisalsopossibletousea derivativecontrollerdefinedasisthatthe controller:

$e_o(t) = k_d \frac{de}{dt}$ or $e_o(s) = k_d s(s)$

The advantage of the derivative controller is that the controller will provide large corrections before the error becomes large. The major disadvantage of the derivative controller is that it will not produce a control output if the error is constant. Another difficulty of the derivative controllerisitssusceptibilitytonoise.Thederivativecontrollerinitspresentformwouldhave difficultywithnoiseproblem. Thiscanbeavoidedbyusingaderivativecontrolleroftheform

$e_o(s) = k_d \frac{s}{rs+1} e(s)$

The term $1/(rs + 1)$ attenuates the high- frequency components in the error signal, that is, noise,thusavoidingthenoiseproblem.Eachofthecontroller-providingproportional,integral, and derivative control-has its advantages and disadvantages. The disadvantages of each controller canbe eliminated bycombining allthree controllers into asingle PID controller,or proportional, integral, and derivative, controller.

Theselectionofthegains for thePIDcontroller canbedetermined by a methoddeveloped by ZieglerandNichols,whostudiedtheperformanceofPIDcontrollersbyexaminingtheintegral of the absolute error (IAE):

$IAE = \int_0^\infty |e(t)| dt$

Fromtheiranalysistheyobservedthatwhentheerrorindexwasaminimumthecontrolsystem respondedtoastepinputasshowninfig4.32.Notethatsecondovershootisonequarterofthe magnitude of the maximum overshoot. Based on their analysis they derived a set of rules for selectingthePIDgains.Thegainsk_p,k_i,and$k_d$aredeterminedintermsoftwoparameters,k_{pu}, calledtheultimategain,andT_u,theperiodofoscillationthatoccursattheultimategain.Table 4.9 gives the values for the gains for proportional (P), proportional-integral (PI), and the proportional-integral-derivative (PID) controllers.

To apply this technique the root locus plot for the control system with the integral and derivative gains set to 0 must become marginally stable. That is, as proportional gain is increasedthelocusmustintersecttheimaginaryaxis.Theproportionalgain,k_p,forwhichthis occurs is called the ultimate gain, k_{pu}. The purely imaginary roots, $\lambda = \pm \omega$, determine the value of T_u.

$Tu = \frac{2\pi}{\omega}$

One additionalrestriction must be met: Allother roots of the system must have negative real parts;that is, theymust be inthe left-hand portionofthecomplexs-plane. Iftheserestrictions are satisfied the P, PI, or PID gains easily can be determined.

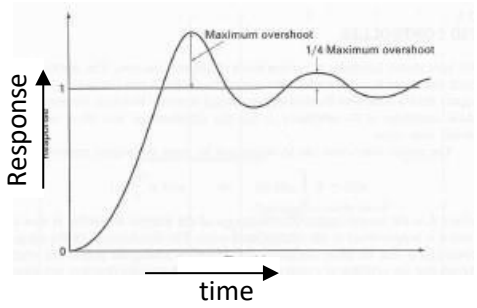


Fig4.32:Thequarter decay

Table4.9GainsforP,PI,andPIDControllers

Typeofcontroller	$\mathbf{k_p}$	$\mathbf{k_i}$	$\mathbf{k_d}$
P(proportionalcontroller)	$k_p=0.5k_{pu}$		
PI (proportional-integral controller)	$k_p=0.45k_{pu}$	$k_i= 0.45\ k_{pu}/(0.83\ T_u)$	
PID(Proportional-integral-derivative controller)	$k_p=0.6k_{pu}$	$k_i=0.6k_{pu}/(0.5T_u)$	$k_d=0.6k_{pu}(0.125T_u)$

ExampleProblem:DesignaPIDcontrollerforthecontroller forthecontrolsystemshownin fig 4.33.

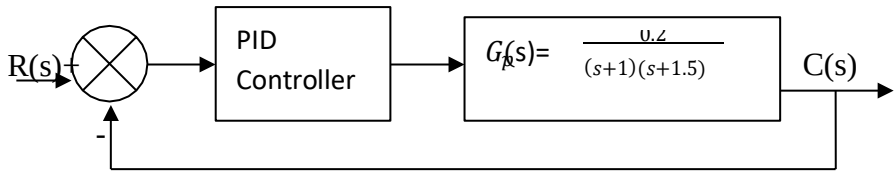


Fig4.33 PIDcontroller

Solution: The gains ofthe PID controller can be estimated using the Ziegler-Nichols method provided the root locus for the plant becomes marginally stable for some value of the proportionalgain □□whentheintegralandderivativecontrolgainshavebeensetto0.Theroot locusplotforG(s)=$\frac{0.2k_p}{(s+1)(s+1.5)}$

isshowninfig4.34.Therootlocusplotmeetstherequirementsfortheziegler-Nicholsmethod. Two branches ofthe root locus cross the imaginaryaxis and all other roots lie in the left half plane. The ultimate gain $k_u$is found by finding the gain when the root locus intersects the imaginaryaxis.The locusintersectsthe imaginaryaxisat s= ±1.25j.Thegaincrossoverpoint can be determined from the magnitude criteria:

$\frac{|0.2k_p|}{|s||s+1|s+1.5||}=1$

Substitutings=1.25jintothemagnitudecriteriayields k_{pu}=

19.8

Theperiodofun-damped oscillationTuisobtained as follows:

$T_u = \frac{2\pi}{\omega} = \frac{2\pi}{1.25} = 5.03$

Knowing$k_{pu}$and$T_u$theproportional,integral,andderivativegainsk_p,k_i,and$k_d$canbe evaluated:

$k_p = 0.6k_{pu} = (0.6)(19.8) = 11.88$

$k_i = 0.6k_{pu}/(0.5T_u) = (0.6)(19.8)/[(0.5)(5.03)] = 4.73$

$k_d = 0.6\ k_{pu}(0.125T_u) = (0.6)(19.8)(0.125)(5.03) = 7.47$

Theresponseofthecontrolsystemtoastep inputisgivenin fig 4.35.

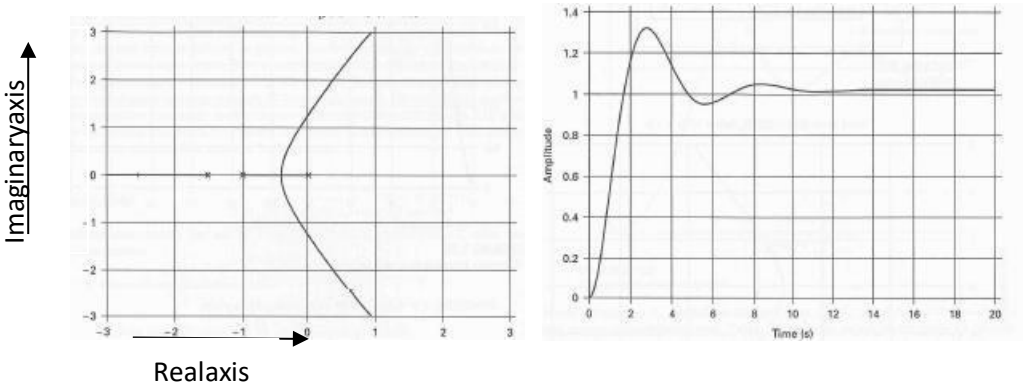


Fig4.34 Rootlocusplot forG(s)= $\frac{\square.\square\square_{\square}}{\square(\square+\square)(\square+\square.\square)}$