LECTURE NOTES

On

NETWORK ANALYSIS & SYNTHESIS (R22A0261)

II B. Tech I – SEM (ECE)

Prepared by:

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MALLA REDDY COLLEGE ENGINEERING & TECHNOLOGY

(Autonomous Institution – UGC, Govt. of India)

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MALLA REDDY COLLEGE OF ENGINEERING AND TECHNOLOGY

II Year B.Tech. ECE- I Sem **L/T/P/C**

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(R22A0261) NETWORK ANALYSIS & SYNTHESIS

COURSEOBJECTIVES:

1. To solve the two port network parameters.
2. To recognize the behavior of R, L, C with DC excitation.
3. Concept of Series , parallel resonance and current locus diagrams
4. To know the pole zero location for driving point and transfer functions
5. To describe Foster and Cauer forms and the properties of immittance functions.

UNIT–I: TWO PORT NETWORKS:

Impedance Parameters, Admittance Parameters, Hybrid Parameters, Transmission (ABCD) Parameters, Conversion of one parameter to another parameter, Conditions for Reciprocity and symmetry, Interconnection of two port networks in series, parallel and cascaded configurations, Illustrative problems.

UNIT–II: D.C.TRANSIENT ANALYSIS (FIRST & SECOND ORDER CIRCUITS):

Introduction to transient response and steady state response, Transient response of series –RL,RC, RLC Circuits for D.C excitation with Initial Conditions, Solutions using Differential Equations approach and Laplace Transform approach ,Illustrative problems.

UNIT-III: LOCUS DIAGRAMS & RESONANCE:

Locus diagrams: Locus diagrams of Series RL, RC circuits with variation of various parameters, parallel RL, RC circuits with variation of various parameters.

Resonance: Resonance-Series and Parallel circuits, Concept of Bandwidth and Quality factor.

UNIT–IV: NETWORK FUNCTIONS: Review of Network functions for one port and two port networks: – pole zero location for driving point and transfer functions-Impulse response of Network functions from pole-zero plots.

UNIT-V: SYNTHESIS OF ONE PORT NETWORKS

Synthesis of reactive one-ports by Foster's and Cauer methods (forms I and II) -Synthesis of LC, RC and RL driving-point functions.

Text Books:

1. K. S. Suresh Kumar, ―Electric Circuit Analysis‖, Pearson Publications, 2013.
2. 2. Ravish R. Singh, "Network Analysis and Synthesis", McGraw-Hill Education, 2013

References:

1. Franklin Kuo, ―Network Analysis and Synthesis‖, 2nd Ed.,Wiley India.

2. Van Valkenburg M.E., ―Introduction to Modern Network Synthesis,‖ Wiley Eastern, 1960 (reprint 1986).

3. Van Valkenburg M.E, ―Network Analysis,‖ Prentice Hall India, 2014.

4. Charles A. Desoer and Ernest S. Kuh, ―Basic Circuit Theory,‖ Tata McGraw Hill Edition.

5. Chakrabarti, A., "Circuit Theory Analysis and Synthesis", Dhanpat Rai& Co., Seventh - Revised edition, 2018

6. S. K. Bhattacharya, ―Network Analysis and Synthesis,‖ Pearson Education India.

COURSE OUTCOMES:

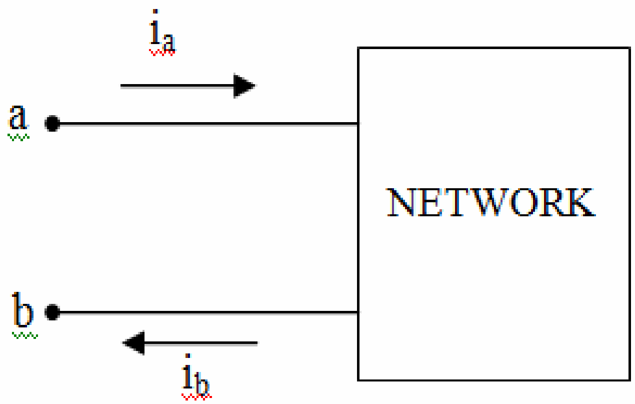
- Able to solve two port network parameters
- Able to analyze the transient and steady state analysis of RLC Circuits.
- Accomplish the computation of Quality factor, band width and current locus diagram for a given electrical circuit.
- Identify the properties and characteristics of network functions.
- Synthesize passive one-port networks using standard Foster and Cauer forms.

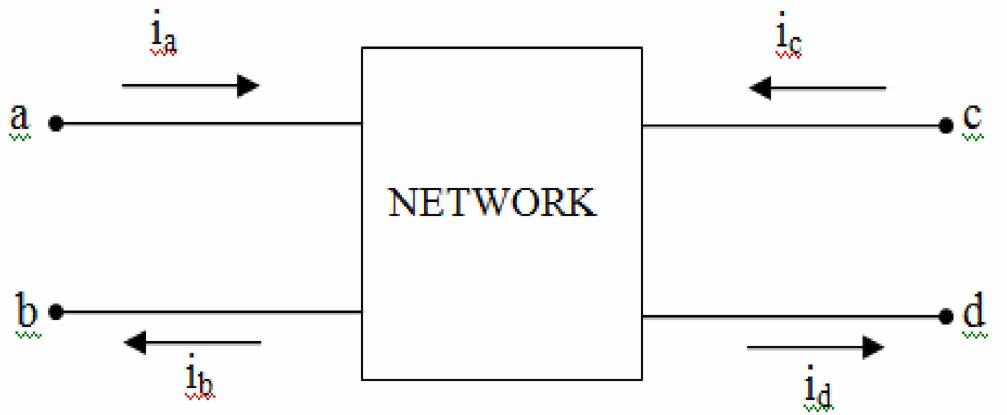
UNIT-1

TWO PORT NETWORKS

A pair of terminals at which a signal may enter (or) leave a network is called a port, and a network having only one such pair of terminals is called a one-port network (or) simply a one-port. Now connections may be made to any other nodes interval to the one-port, and it is there fore evident that 'i_a' must be equal to 'i_b' in the one port network shown in figure. When more than on pair of terminals is present, the network is known as a multi-port network.

In this case $i_a = i_b$ and $i_c = i_d$.





A two-port network is a special case of multi-port network. Each port consists of two terminals, one for entry and other for exit. From the definition of a pot, the current at entry is equal to that at the exit terminal of a port.

Examples:- Transforms, Power Transmission lines, Bridge circuits, Filters etc., Let us consider a network having six terminals to which external connections can be made …….

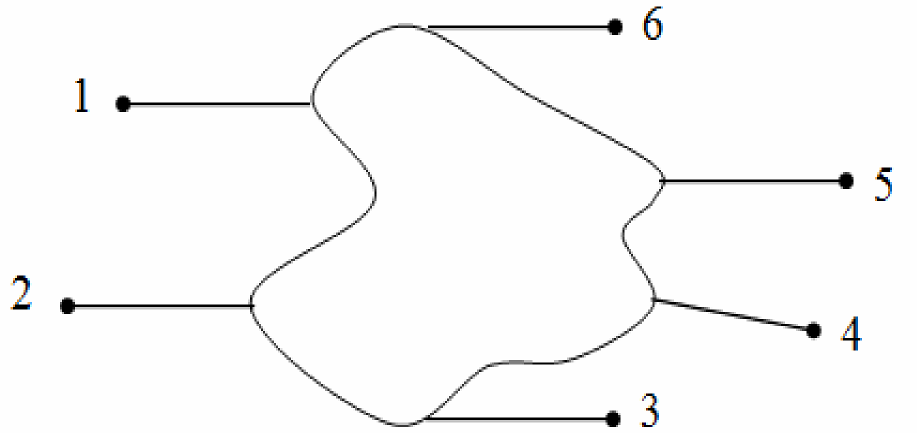


Fig. Six – Terminal network

The special methods of analysis which have been developed for two-port networks, the current and voltage relationships at the terminals of the networks and existing at the specific nature of the currents & voltage within the networks.

RELATIONSHIP OF TWO-PORT VARIABLES

In the two-port network as shown in figure.

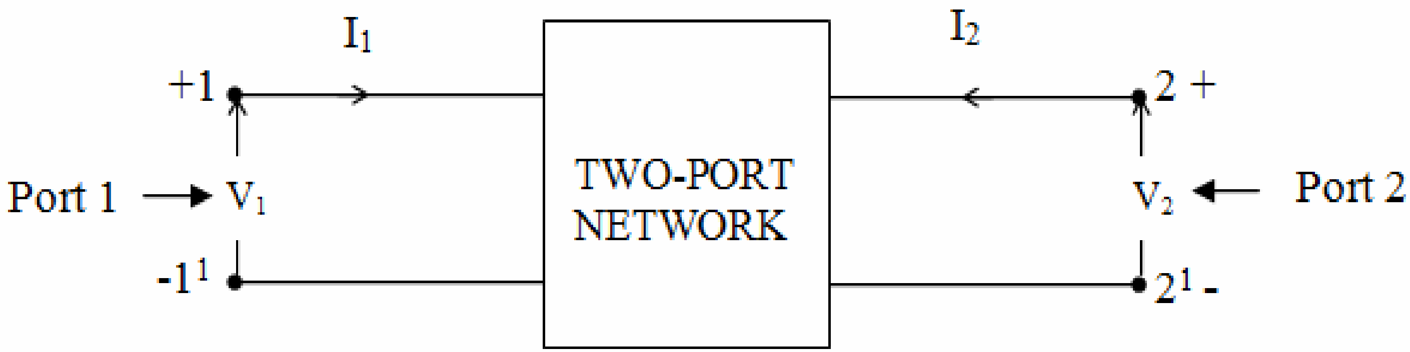


Fig. Two-port network

Here four variables i.e., two voltages and two currents. There are other voltages and currents that are present inside the box. The box enclosing the network has the function of indicating that other voltages and currents are not available for measurement (or) are not important. We assume that the variables are transform quantities and use 'V_1' and 'I_1' as variables at the input i.e., port 1, and V_2 and I_2 as the variables at output port 2. Now only two of the four variables are independent, and the specification of any two of them determines the remaining two. For example, if V_1 and V_2 are specified, the I_1 and I_2 are determined. The dependence of two of the four variables on the other two is described in a number of ways, depending on which of the variables are choosen to be the independent variables.

NAME	FUNCTION		EQUATION
	EXPRESS	INTERMS OF	
OPEN CIRCUIT IMPEDANCE [Z-PARAMETERS]	V_1, V_2	I_1, I_2	$V_1 = Z_{11} I_1 + Z_{12} I_2$ $V_2 = Z_{21} I_1 + Z_{22} I_2$
SHORT CIRCUIT ADMITTANCE [Y-PARAMETERS]	I_1, I_2	V_1, V_2	$I_1 = Y_{11} V_1 + Y_{12} V_2$ $I_2 = Y_{21} V_1 + Y_{22} V_2$
TRANSMISSION [ABCD-PARAMETERS]	V_1, I_1	V_2, I_2	$V_1 = AV_2 - BI_2$ $I_1 = CV_2 - DI_2$
INVERSE TRANSMISSION [$A^1B^1C^1D^1$-PARAMETERS]	V_2, I_2	V_1, I_1	$V_2 = A^1 V_1 - B^1 I_1$ $I_2 = C^1 V_1 - D^1 I_1$
HYBRID [h-PARAMETERS]	V_1, I_2	I_1, V_2	$V_1 = h_{11} I_1 + h_{12} V_2$ $I_1 = h_{21} I_1 + h_{22} V_2$
INVERSE HYBRID [g-PARAMETERS]	I_1, V_2	V_1, I_2	$I_1 = g_{11} V_1 + g_{12} I_2$ $V_2 = g_{21} V_1 + g_{22} I_2$

OPEN CIRCUIT IMPEDANCE – PARAMETERS (OR) Z-PARAMETERS

$(V_1, V_2) = f(I_1, I_2)$ $\quad V_1 = Z_{11} I_1 + Z_{12} I_2 \longrightarrow (1)$

$V_2 = Z_{21} I_1 + Z_{22} I_2 \longrightarrow (2)$

Putting in matrix form,

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

Determination of Z-parameters:

Fig (a)

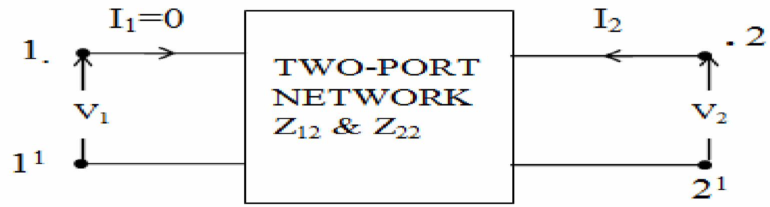


Fig (b)

Mathematically,

$Z_{11} = \left.\dfrac{V_1}{I_1}\right| I_2 = 0 \longrightarrow$ Input driving point impedance with out put port open circuited.

$Z_{12} = \left.\dfrac{V_1}{I_2}\right| I_1 = 0 \longrightarrow$ Reverse transfer function impedance with input port open circuited.

$Z_{21} = \left.\dfrac{V_2}{I_1}\right| I_2 = 0 \longrightarrow$ Forwarded transfer function impedance with put port open circuited.

$Z_{22} = \left.\dfrac{V_2}{I_2}\right| I_1 = 0 \longrightarrow$ Output driving point impedance with input port open circuited.

The Z-parameters equivalent circuit corresponding to equations (1) & (2) is shown in figure (c). The voltage sources $V_1 = Z_{12} I_2$ & $V_2 = Z_{21} I_1$ are called current controlled voltage sources [CCVS] as their voltages are dependent on current I_1 and I_2 respectively.

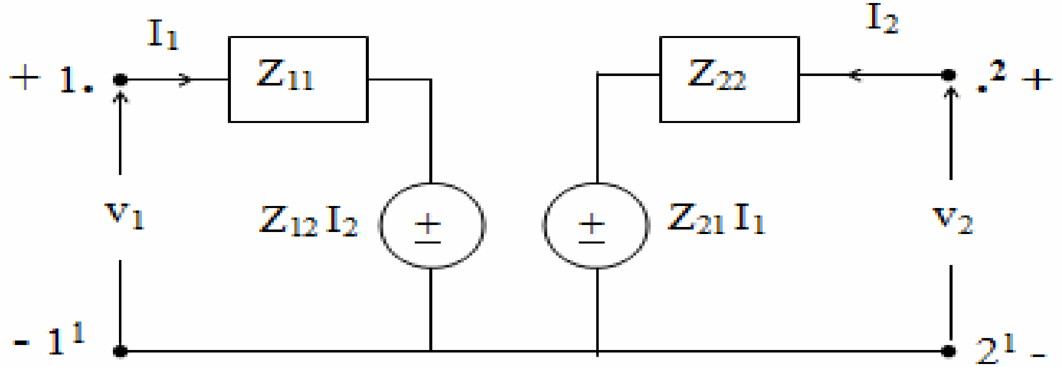


Fig (c) : Two Generator Equivalent circuit using Z-parameters

Writing equation as

$$V_1 = (Z_{11} - Z_{12})\, I_1 + Z_{12}\, (I_1 - I_2) \longrightarrow (3)$$

$$V_2 = (Z_{21} - Z_{12})\, I_1 + (Z_{22} - Z_{12})\, I_2 + Z_{12}\, (I_1 + I_2) \longrightarrow (4)$$

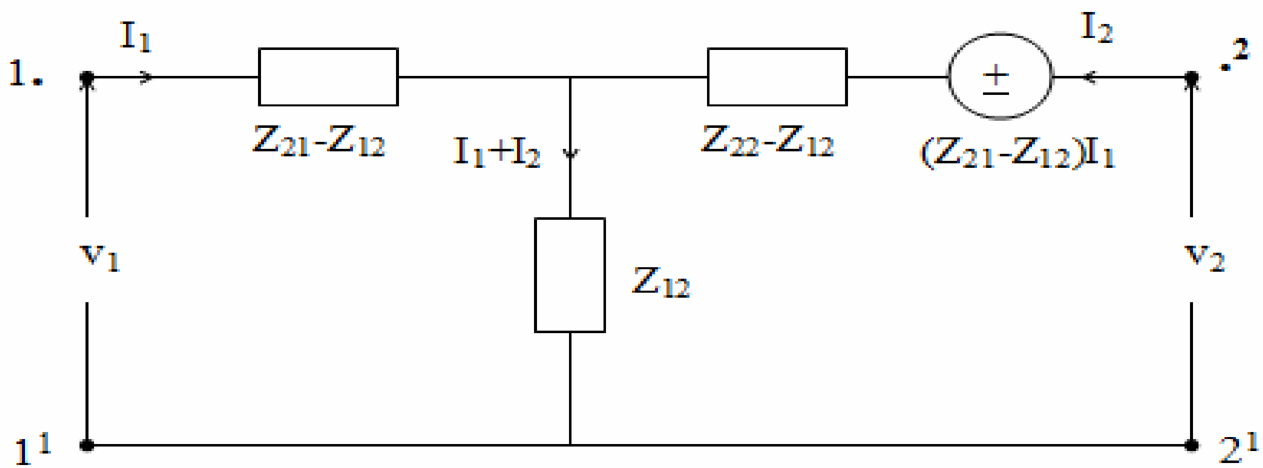


Fig. (d) : One Generator equivalent circuit

SHORT – CIRCUIT ADMITTANCE – PARAMETERS (OR) Y-PARAMETERS

$(I_1, I_2) = f(V_1, V_2)$ $\quad I_1 = Y_{11} V_1 + Y_{12} V_2 \longrightarrow (1)$

$I_2 = Y_{21} V_1 + Y_{22} V_2 \longrightarrow (2)$

Putting in matrix form,

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

Determination of Y-parameters:

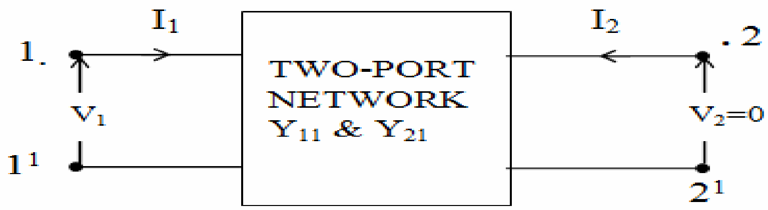


Fig (a)

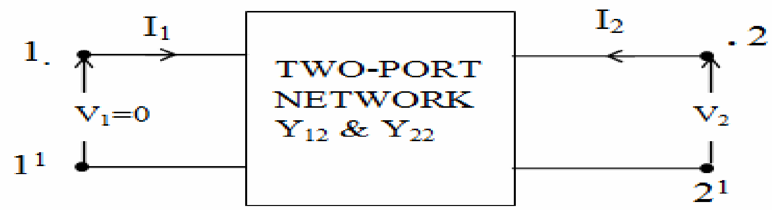


Fig (b)

Mathematically,

$Y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2 = 0} \longrightarrow$ Input driving point admittance with out put port short-circuited.

$Y_{12} = \left.\frac{I_1}{V_2}\right|_{V_1 = 0} \longrightarrow$ Reverse transfer admittance with input port short-circuited.

$Y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2 = 0} \longrightarrow$ Forwarded transfer admittance with out put port short-circuited.

$Y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1 = 0} \longrightarrow$ Output driving point admittance with input port short-circuited.

NOTE: [Short circuit admittance matrix] = [Open circuit impedance matrix] $^{-1}$

And $Y_{ij} \neq \frac{1}{Z_{ij}}$ i.e., $Y_{11} \neq \frac{1}{Z_{11}}$ & $Y_{12} \neq \frac{1}{Z_{12}}$ *etc.*, The Y-parameters equivalent circuit corresponding to equations (1) & (2) is shown in figure (c). The current sources $I_1 = Y_{12} V_2$ & $I_2 = Y_{21} V_1$ are called voltage controlled current sources [VCCS] as the values of their currents are dependent on voltages V_1 and V_2 respectively.

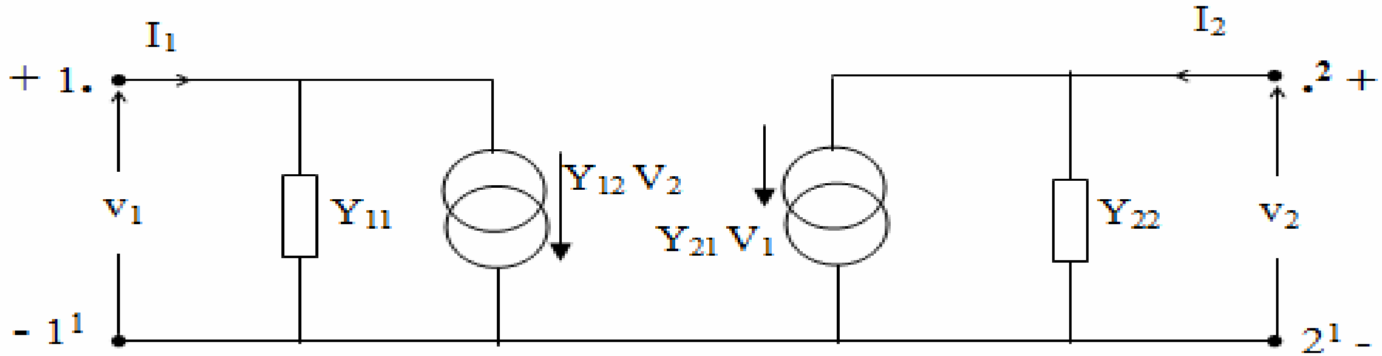


Fig (c) : Two Generator Equivalent circuit using Y-parameters

$$I_1 = (Y_{11} + Y_{12}) V_1 - Y_{12} (V_1 - V_2) \longrightarrow (3)$$

$$I_2 = (Y_{21} - Y_{12}) V_1 + (Y_{22} + Y_{12}) V_2 - Y_{12} (V_2 - V_1) \longrightarrow (4)$$

From which the following equivalent circuit is obtain as shown in figure (d)

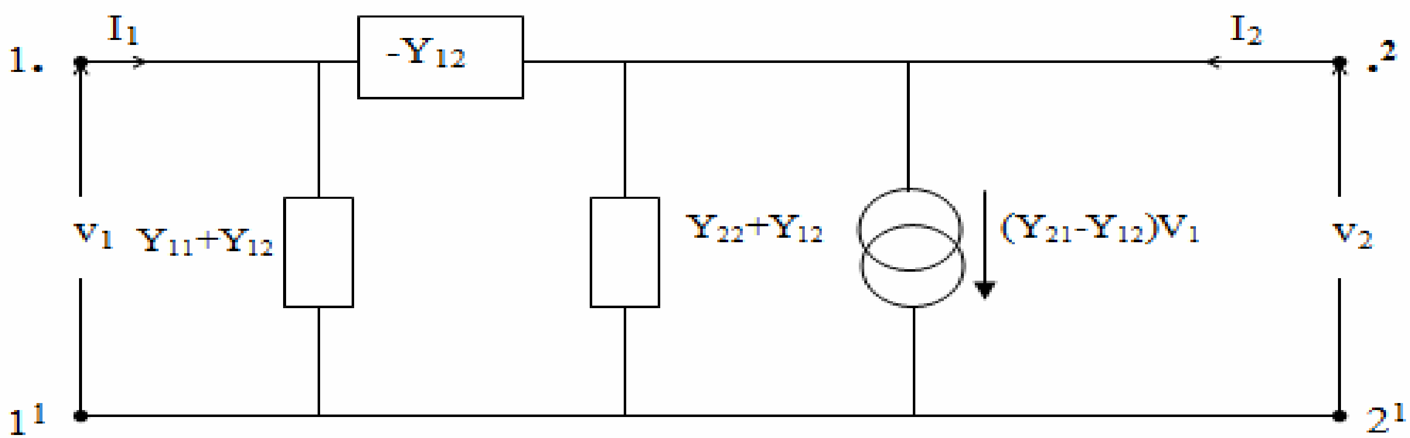


Fig. (d) : One Generator equivalent circuit

HYBRID – PARAMETERS (OR) h -PARAMETERS

The hybrid parameters are wide usage in electronic circuits, especially in constructing models for transistors. In this case, voltage of the input port and the current of the output port are expressed in terms of the current of the input port and the voltage of the output port. The parameters are dimensionally mixed and due to this reason, these parameters are called as "Hybrid Parameters".

$(V_1, I_2) = f(I_1, V_2)$ $\qquad V_1 = h_{11} I_1 + h_{12} V_2 \longrightarrow (1)$

$$I_2 = h_{21} I_1 + h_{22} V_2 \longrightarrow (2)$$

Putting in matrix form,

$$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$$

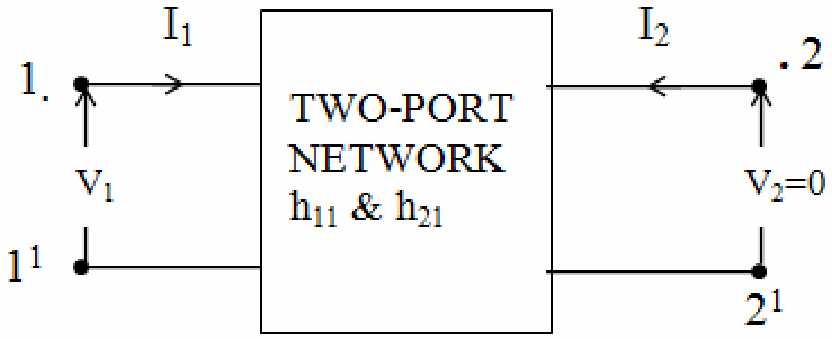


Fig (a)

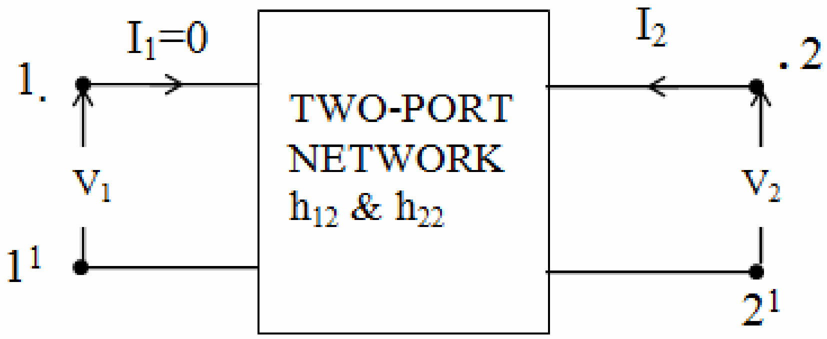


Fig (b)

Mathematically,

$$h_{11} = \left.\frac{V_1}{I_1}\right|_{V_2 = 0} \longrightarrow \text{Input impedance with the out put port short-circuited.}$$

$$h_{12} = \left.\frac{V_1}{V_2}\right|_{I_1 = 0} \longrightarrow \text{Reverse voltage with the input port open circuited.}$$

$$h_{21} = \left.\frac{I_2}{I_1}\right|_{V_2 = 0} \longrightarrow \text{Forward current gain with the out put port short-circuited.}$$

$$h_{22} = \left.\frac{I_2}{V_2}\right|_{I_1 = 0} \longrightarrow \text{Output admittance with the input port open circuited.}$$

The h-parameters equivalent circuit corresponding to equations (1) & (2) is shown in figure (c). Where $V_1 = h_{12} V_2$ & $I_2 = h_{21} I_1$ are voltage controlled voltage sources [VCVS] and currents controlled current source [CCCS] respectively.

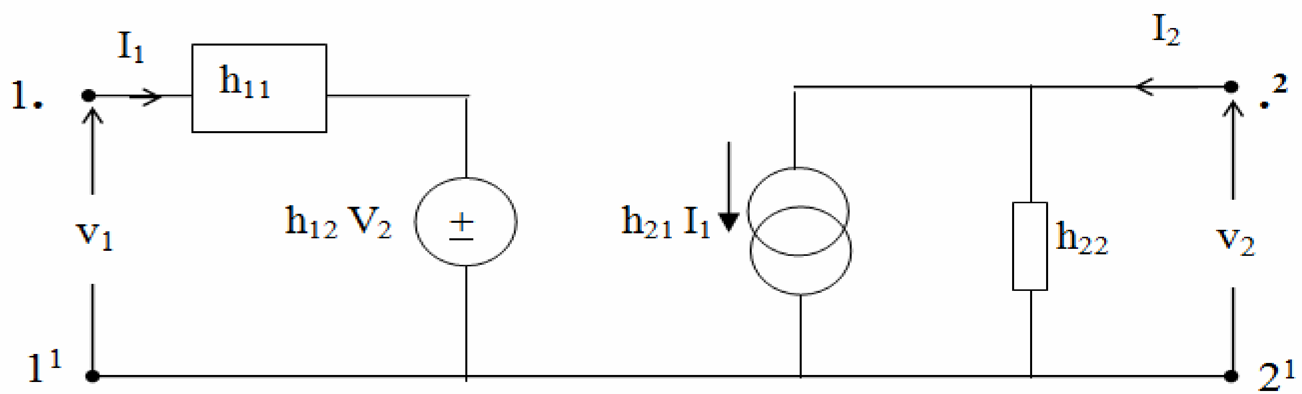


Fig (c) : h-parameters equivalent network

INVERSE - HYBRID – PARAMETERS (OR) g -PARAMETERS

The hybrid parameters and inverse hybrid parameters are dual of each other like Z and Y-parameters.

$$[g] = [h]^{-1}$$

$(I_1, V_2) = f(V_1, I_2)$ $\qquad I_1 = g_{11} V_1 + g_{12} I_2 \longrightarrow (1)$

$$V_2 = g_{21} V_1 + g_{22} I_2 \longrightarrow (2)$$

Determination of g-parameters:

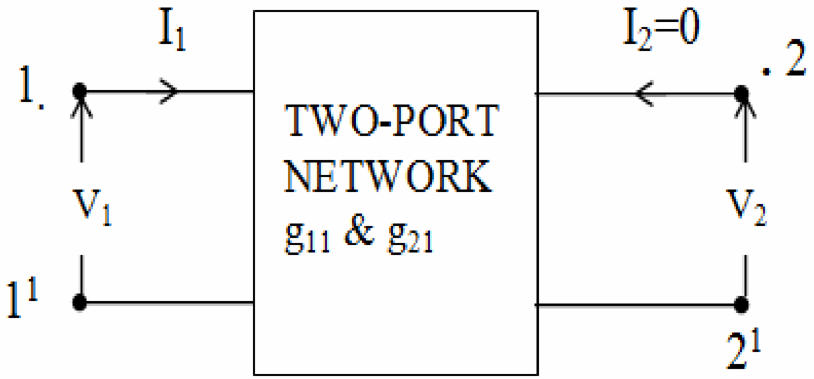


Fig (a)

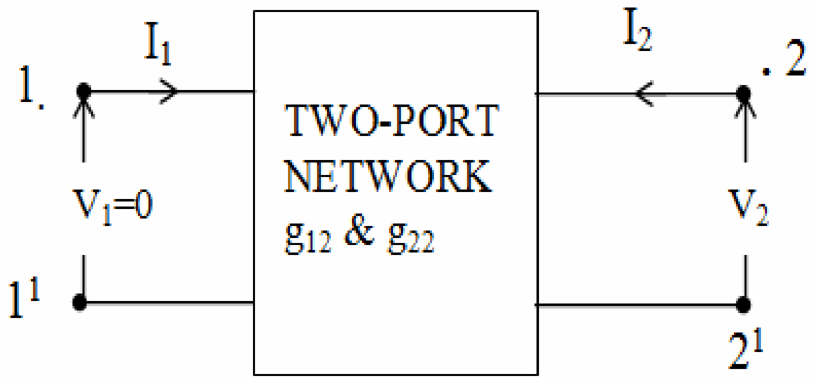


Fig (b)

Mathematically,

$$g_{11} = \left.\frac{I_1}{V_1}\right|_{I_2 = 0} \longrightarrow$$ Input admittance with out put port open circuited.

$$g_{12} = \left.\frac{I_1}{I_2}\right|_{V_1 = 0} \longrightarrow$$ Reverse current gain with input port short-circuited.

$$g_{21} = \left.\frac{V_2}{V_1}\right|_{I_2 = 0} \longrightarrow$$ Forward voltage with out put port open circuited.

$$g_{22} = \left.\frac{V_2}{I_2}\right|_{V_1 = 0} \longrightarrow$$ Output impedance with input port short-circuited.

The g-parameters equivalent circuit corresponding to equations (1) & (2) is shown in figure (c). Where $I_1 = g_{12} I_2$ & $V_2 = g_{21} V_1$ are current controlled current sources [CCCS] and Voltage controlled voltage source [VCVS] respectively.

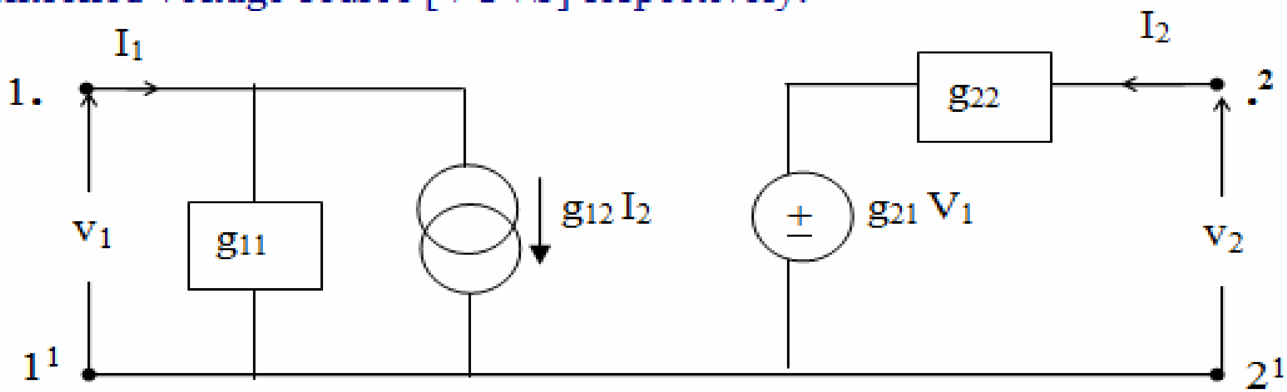


Fig (c) : Equivalent circuit of g-parameters

TRANSMISSION [T] (OR) CHAIN (OR) ABCD-PARAMETERS (OR) GENERAL CIRCUIT PARAMETERS:

The transmission 'T' (or) Chain (or) ABCD – Parameters can be expressed as

$(V_1, I_1) = f(V_2, I_2)$ $\quad V_1 = AV_2 - B I_2 \longrightarrow$ (1)

$I_1 = CV_2 - D I_2 \longrightarrow$ (2)

Putting in matrix form,

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

Determination of ABCD (or) T-parameters:

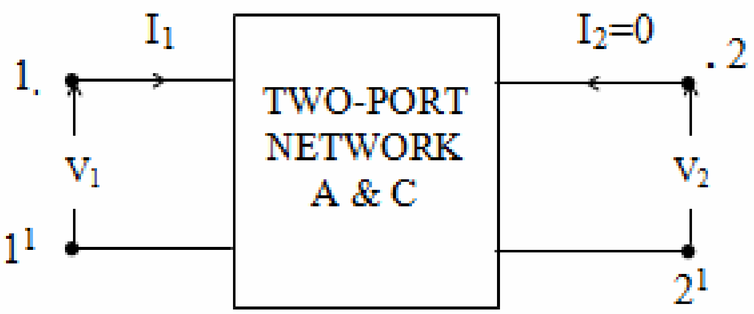


Fig (a)

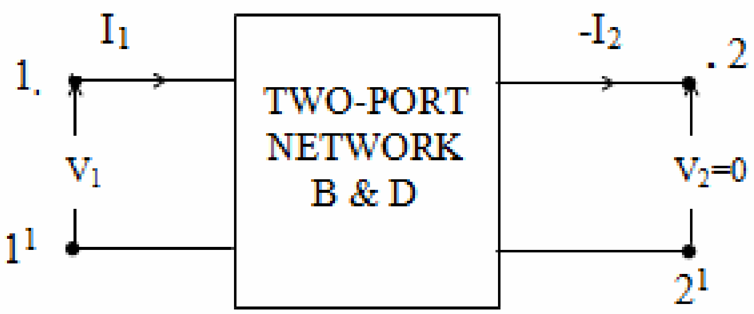


Fig (b)

In order to determine the T-Parameters, open and short the circuit output port [receiving end] and applied some voltage 'V_1' to input port [sending end] as shown in figure (a) & (b) to obtain A,C and B,D respectively.

Mathematically,

$$A = \left.\frac{V_1}{V_2}\right|_{I_2 = 0} \longrightarrow \text{Reverse voltage ratio with the receiving end open circuited.}$$

$$B = \left.\frac{V_1}{-I_2}\right|_{V_2 = 0} \longrightarrow \text{Reverse transfer impedance with the receiving end short-circuited.}$$

$$C = \left.\frac{I_1}{V_2}\right|_{I_2 = 0} \longrightarrow \text{Reverse transfer admittance with the receiving end open circuited.}$$

$$D = \left.\frac{I_1}{-I_2}\right|_{V_2 = 0} \longrightarrow \text{Reverse current ratio with the receiving end short-circuited.}$$

NOTE: For passive network the all the four T-parameters are positive, since I_2 is itself negative (or) $-I_2$ is positive.

INVERSE TRANSMISSION (OR) INVERSE ABCD (OR) T^1 - PARAMETERS:

T^1- parameters can be expressed as output variables in terms of input port variables i.e.,

$(V_2, I_2) = f(V_1, I_1)$ $\quad V_2 = A^1V_1 - B^1 I_1 \longrightarrow (1)$

$I_2 = C^1V_1 - D^1 I_1 \longrightarrow (2)$

Putting in matrix form,

$$\begin{bmatrix} V_2 \\ I_2 \end{bmatrix} = \begin{bmatrix} A^1 & B^1 \\ C^1 & D^1 \end{bmatrix} \begin{bmatrix} V_1 \\ I_1 \end{bmatrix}$$

The equivalent circuit of a two port network is also not possible in terms of T^1 – Parameters.

Determination of T^1-parameters:

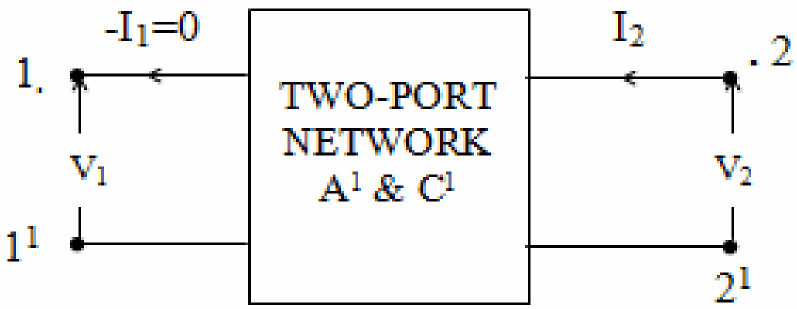


Fig (a)

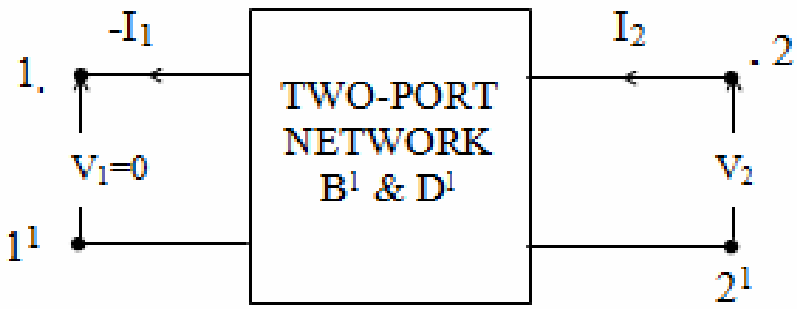


Fig (b)

Mathematically,

$$A^1 = \left.\frac{V_2}{V_1}\right|_{I_1 = 0} \longrightarrow$$ Forwarded voltage ratio with sending end open circuited.

$$B^1 = \left.\frac{V_2}{-I_1}\right|_{V_1 = 0} \longrightarrow$$ Forward transfer impedance with sending end short-circuited.

$$C^1 = \left.\frac{I_2}{V_1}\right|_{I_1 = 0} \longrightarrow$$ Forward transfer admittance with sending end open circuited.

$$D^1 = \left.\frac{I_2}{-I_1}\right|_{V_1 = 0} \longrightarrow$$ Forwarded current ratio with sending end short-circuited.

NOTE: - For passive network in the case also all the four T^1-parameters are positive, as I_1 is itself negative (or) $-I_1$ is positive.

INTER RELATION SHIPS BETWEEN PARAMETERS SETS

If we want to express 'α'-parameters in terms of 'β'-parameters, we have to write 'β'-parameters equations & then the algebraic manipulation, rewrite the equations as needed for 'α'-parameters.

Z-PARAMETERS IN TERMS OF OTHER PARAMETERS

Z-parameters in terms of Y-parameters

We know that Y-parameters as

$$I_1 = Y_{11} V_1 + Y_{12} V_2 \longrightarrow (1)$$

$$I_2 = Y_{21} V_1 + Y_{22} V_2 \longrightarrow (2)$$

Putting in matrix form,

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix}^{-1} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$\therefore [Z] = [Y]^{-1}$$

i.e., $\begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix}^{-1} = \frac{1}{\Delta Y} \begin{bmatrix} Y_{22} & -Y_{12} \\ -Y_{21} & Y_{11} \end{bmatrix}$

Where $\Delta Y = Y_{11} Y_{22} - Y_{12} Y_{21}$

$$\therefore Z_{11} = \frac{Y_{22}}{\Delta Y}; \quad Z_{12} = \frac{-Y_{12}}{\Delta Y}; \quad Z_{21} = \frac{-Y_{21}}{\Delta Y}; \quad Z_{22} = \frac{Y_{11}}{\Delta Y}$$

Z-parameters in terms of T-parameters

We know that T-parameters as

$$V_1 = AV_2 - B I_2 \longrightarrow (1)$$

$$I_1 = CV_2 - D I_2 \longrightarrow (2)$$

From equation (2), $V_2 = \frac{I_1}{C} + \frac{D}{C} I_2$

$\therefore Z_{22} = \frac{D}{C}; \quad Z_{21} = \frac{1}{C}$

Substituting V_2 in equation (1)

$$V_1 = A\left[\frac{I_1}{C} + \frac{D}{C}\right] I_2 - BI_2 = \frac{A}{C} I_1 + \frac{(AD - BC)}{C} I_2$$

Put $\Delta T = AD-BC$

$$V_1 = \frac{A}{C} I_1 + \frac{\Delta T}{C} I_2$$

$\therefore Z_{11} = \frac{A}{C}; \quad Z_{12} = \frac{\Delta T}{C}$

Z-parameters in terms of T^1-parameters

We know that T^1-parameters as

$$V_2 = A^1 V_1 - B^1 I_1 \longrightarrow (1)$$

$$I_2 = C^1 V_1 - D^1 I_1 \longrightarrow (2)$$

From equation (2), $V_1 = \frac{I_2}{C^1} + \frac{D^1}{C^1} I_1 = \frac{D^1}{C^1} I_1 + \frac{I_2}{C^1}$

$\therefore Z_{11} = \frac{D^1}{C^1}; \quad Z_{12} = \frac{1}{C^1}$

Substituting V_2 in equation (1)

$$V_2 = A^1\left[\frac{D^1}{C^1} I_1 + \frac{1}{C^1} I_2\right] - B^1 I_1 = \frac{(A^1 D^1 - B^1 C^1)}{C^1} I_1 + \frac{A^1 I_2}{C^1}$$

Put $\Delta T^1 = A^1 D^1 - B^1 C^1$

$$V_2 = \frac{\Delta T^1}{C^1} I_1 + \frac{A^1 I_2}{C^1}$$

$\therefore Z_{21} = \frac{\Delta T^1}{C^1}; \quad Z_{22} = \frac{A^1}{C^1}$

Z-parameters in terms of h-parameters

We know that h-parameters as

$$V_1 = h_{11} I_1 + h_{12} V_2 \longrightarrow (1)$$

$$I_2 = h_{21} I_1 + h_{22} V_2 \longrightarrow (2)$$

From equation (2) we have

$$V_2 = \frac{I_2}{h_{22}} - \frac{h_{21}}{h_{22}} I_1 = \frac{-h_{21}}{h_{22}} I_1 + \frac{1}{h_{22}} I_2$$

$$\therefore Z_{21} = \frac{-h_{21}}{h_{22}}; \quad Z_{22} = \frac{1}{h_{22}}$$

Substituting V_2 in equation (1)

$$V_1 = h_{11} I_1 + h_{12}\left[\frac{-h_{21}}{h_{22}} I_1 + \frac{I_2}{h_{22}}\right] = \frac{\Delta h}{h_{22}} I_1 + \frac{h_{12}}{h_{22}} I_2$$

Put $\Delta h = h_{11} h_{22} - h_{12} h_{21}$

$$\therefore Z_{11} = \frac{\Delta h}{h_{22}}; \quad Z_{12} = \frac{h_{12}}{h_{22}}$$

Z-parameters in terms of g-parameters

We know that g-parameters as

$$I_1 = g_{11} V_1 + g_{12} I_2 \longrightarrow (1)$$

$$V_2 = g_{21} V_1 + g_{22} I_2 \longrightarrow (2)$$

From equation (1) we have

$$V_1 = \frac{I_1}{g_{11}} - \frac{g_{12}}{g_{11}} I_2$$

$$\therefore Z_{11} = \frac{1}{g_{11}}; \quad Z_{12} = \frac{-g_{12}}{g_{11}}$$

Substituting V_1 in equation (2)

$$V_2 = g_{21}\left[\frac{I_1}{g_{11}} - \frac{g_{12}}{g_{11}} I_2\right] + g_{22}\ I_2 = \frac{g_{21}}{g_{11}} I_1 + \frac{\Delta g}{g_{11}} I_2$$

Put $\Delta g = g_{11} g_{22} - g_{12} g_{21}$

$$V_2 = \frac{g_{21}}{g_{11}} I_1 + \frac{\Delta g}{g_{11}} I_2$$

$$\therefore\ Z_{21} = \frac{g_{21}}{g_{11}}; \ Z_{22} = \frac{\Delta g}{g_{11}}$$

Y-PARAMETERS IN TERMS OF OTHER PARAMETERS

Y-parameters in terms of Z-parameters

We know that Z-parameters as

$$V_1 = Z_{11} I_1 + Z_{12} I_2 \longrightarrow (1)$$

$$V_2 = Z_{21} I_1 + Z_{22} I_2 \longrightarrow (2)$$

Putting in matrix, we have

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix}^{-1} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

$$\therefore [Y] = [Z]^{-1}$$

i.e., $\begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix}^{-1} = \dfrac{1}{\Delta Z} \begin{bmatrix} Z_{22} & -Z_{12} \\ -Z_{21} & Z_{11} \end{bmatrix}$

Where $\Delta Z = Z_{11} Z_{22} - Z_{12} Z_{21}$

$$\therefore Y_{11} = \frac{Z_{22}}{\Delta Z}; \quad Y_{12} = \frac{-Z_{12}}{\Delta Z}; \quad Y_{21} = \frac{-Z_{21}}{\Delta Z}; \quad Y_{22} = \frac{Z_{11}}{\Delta Z}$$

Y-parameters in terms of T-parameters

We know that T-parameters as

$$V_1 = AV_2 - BI_2 \longrightarrow (1)$$

$$I_1 = CV_2 - DI_2 \longrightarrow (2)$$

From equation (1), we have

$$I_2 = \frac{AV_2}{B} - \frac{V_1}{B} = \frac{-1}{B}V_1 + \frac{A}{B}V_2$$

$$\therefore \quad Y_{21} = \frac{-1}{B}; \; Y_{22} = \frac{A}{B}$$

Substituting I_2 in equation (2)

$$I_1 = CV_2 - D\left[\frac{-1}{B}V_1 + \frac{A}{B}V_2\right] = \frac{D}{B}V_1 + \frac{(-(AD-BC))}{B}V_2$$

Put $\Delta T = AD-BC$

$$I_1 = \frac{D}{B}V_1 - \frac{\Delta T}{B}V_2$$

$$\therefore Y_{11} = \frac{D}{B}; \quad Y_{12} = \frac{-\Delta T}{B}$$

Y-parameters in terms of T^1-parameters

We know that the T^1-parameters as

$$V_2 = A^1V_1 - B^1I_1 \longrightarrow (1)$$

$$I_2 = C^1V_1 - D^1I_1 \longrightarrow (2)$$

From equation (1), we have

$$I_1 = \frac{A^1}{B^1}V_1 - \frac{1}{B^1}V_2$$

$$\therefore Y_{11} = \frac{A^1}{B^1}; \quad Y_{12} = \frac{-1}{B^1}$$

Substituting I_1 in equation (2)

$$I_2 = C^1V_1 - D^1\left[\frac{A^1}{B^1}V_1 - \frac{V_2}{B^1}\right] = \frac{-(A^1D^1 - B^1C^1)}{B^1}V_1 + \frac{D^1}{B^1}V_2$$

Put $\Delta T^1 = A^1D^1 - B^1C^1$

$$I_2 = \frac{-\Delta T^1}{B^1}V_1 + \frac{D^1}{B^1}V_2$$

$$\therefore Y_{21} = \frac{-\Delta T^1}{B^1}; \quad Y_{22} = \frac{D^1}{B^1}$$

Y-parameters in terms of h-parameters

We know that h-parameters as

$$V_1 = h_{11} I_1 + h_{12} V_2 \longrightarrow (1)$$

$$I_2 = h_{21} I_1 + h_{22} V_2 \longrightarrow (2)$$

From equation (1), we have

$$I_1 = \frac{V_1}{h_{11}} - \frac{h_{12}}{h_{11}} V_2$$

$$\therefore Y_{11} = \frac{1}{h_{11}}; \quad Y_{12} = \frac{-h_{12}}{h_{11}}$$

Substituting I_1 in equation (2)

$$I_2 = h_{21}\left[\frac{V_1}{h_{11}} - \frac{h_{12}}{h_{11}} V_2\right] + h_{22} V_2 = \frac{h_{21}}{h_{11}} V_1 + \frac{(h_{11}h_{22} - h_{12}h_{21})}{h_{11}} V_2$$

Put $\Delta h = h_{11} h_{22} - h_{12} h_{21}$

$$I_2 = \frac{h_{21}}{h_{11}} V_1 + \frac{\Delta h}{h_{11}} V_2$$

$$\therefore Y_{21} = \frac{h_{21}}{h_{11}}; \quad Y_{22} = \frac{\Delta h}{h_{11}}$$

Y-parameters in terms of g-parameters

We know that g-parameters as

$$I_1 = g_{11} V_1 + g_{12} I_2 \longrightarrow (1)$$

$$V_2 = g_{21} V_1 + g_{22} I_2 \longrightarrow (2)$$

From equation (2) we have

$$I_2 = \frac{-g_{21}}{g_{22}} V_1 + \frac{V_2}{g_{22}}$$

$$\therefore \quad Y_{21} = \frac{-g_{21}}{g_{22}}; \quad Y_{22} = \frac{1}{g_{22}}$$

Substituting I_2 in equation (1)

$$I_1 = g_{11} V_1 + g_{12}\left[\frac{-g_{21}}{g_{22}} V_1 + \frac{V_2}{g_{22}}\right] = \frac{(g_{11} g_{22} - g_{12} g_{21})}{g_{22}} V_1 + \frac{g_{12}}{g_{22}} V_2$$

Put $\Delta g = g_{11} g_{12} - g_{12} g_{21}$

$$I_1 = \frac{\Delta g}{g_{22}} V_1 + \frac{g_{12}}{g_{22}} V_2$$

$$\therefore \quad Y_{11} = \frac{\Delta g}{g_{22}}; \quad Y_{12} = \frac{g_{12}}{g_{22}}$$

T-PARAMETERS IN TERMS OF OTHER PARAMETERS

T-parameters in terms of Z-parameters

We know that Z-parameters as

$$V_1 = Z_{11} I_1 + Z_{12} I_2 \longrightarrow (1)$$

$$V_2 = Z_{21} I_1 + Z_{22} I_2 \longrightarrow (2)$$

From Equation (2), we have

$$I_1 = \frac{V_2}{Z_{21}} - \frac{Z_{22}}{Z_{21}} I_2$$

$$\therefore C = \frac{1}{Z_{21}}; \quad D = \frac{Z_{22}}{Z_{21}}$$

Substituting I_1 in equation (1)

$$V_1 = Z_{11}\left[\frac{V_2}{Z_{21}} - \frac{Z_{22}}{Z_{21}} I_2\right] + Z_{12} I_2$$

$$= \frac{Z_{11}}{Z_{21}} V_2 - \frac{\Delta Z}{Z_{21}} I_2$$

Where $\Delta Z = Z_{11} Z_{22} - Z_{12} Z_{21}$

$$\therefore A = \frac{Z_{11}}{Z_{21}}; \quad B = \frac{\Delta Z}{Z_{21}}$$

T-parameters in terms of Y-parameters

We know that Y-parameters as

$$I_1 = Y_{11} V_1 + Y_{12} V_2 \longrightarrow (1)$$

$$I_2 = Y_{21} V_1 + Y_{22} V_2 \longrightarrow (2)$$

From Equation (2), we have

$$V_1 = \frac{-Y_{22}}{Y_{21}} V_2 + \frac{1}{Y_{21}} I_2$$

$$\therefore \ A = \frac{-Y_{22}}{Y_{21}}; \ B = \frac{-1}{Y_{21}}$$

Substituting V_1 in equation (1)

$$I_1 = Y_{11}\left[\frac{-Y_{22}}{Y_{21}} V_2 + \frac{1}{Y_{21}} I_2\right] + Y_{12} V_2$$

$$= \frac{-\Delta Y}{Y_{21}} V_2 + \frac{Y_{11}}{Y_{21}} I_2$$

Where $\Delta Y = Y_{11} Y_{22} - Y_{12} Y_{21}$

$$\therefore C = \frac{-\Delta Y}{Y_{21}}; \quad D = \frac{-Y_{11}}{Y_{21}}$$

T-parameters in terms of T^1-parameters

We know that the T^1-parameters as

$$V_2 = A^1 V_1 - B^1 I_1 \longrightarrow (1)$$

$$I_2 = C^1 V_1 - D^1 I_1 \longrightarrow (2)$$

Putting in matrix form,

$$\begin{bmatrix} V_2 \\ I_2 \end{bmatrix} = \begin{bmatrix} A^1 & B^1 \\ C^1 & D^1 \end{bmatrix} \begin{bmatrix} V_1 \\ -I_1 \end{bmatrix}$$

Rewriting the above equation,

$$\begin{bmatrix} V_2 \\ -I_2 \end{bmatrix} = \begin{bmatrix} A^1 & -B^1 \\ -C^1 & D^1 \end{bmatrix} \begin{bmatrix} V_1 \\ I_1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A^1 & -B^1 \\ -C^1 & D^1 \end{bmatrix}^{-1} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

$$\therefore [T] \neq [T^1]^{-1}$$

i.e. $\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} A^1 & -B^1 \\ -C^1 & D^1 \end{bmatrix}^{-1} = \frac{1}{\Delta T^1} \begin{bmatrix} D^1 & B^1 \\ C^1 & A^1 \end{bmatrix}$

Where $\Delta T^1 = A^1 D^1 - B^1 C^1$

$$\therefore A = \frac{D^1}{\Delta T^1}; \quad B = \frac{B^1}{\Delta T^1}; \quad C = \frac{C^1}{\Delta T^1}; \quad D = \frac{A^1}{\Delta T^1}$$

NOTE: $\therefore [T] \neq [T^1]^{-1}$

T-parameters in terms of h-parameters

We know that h-parameters as

$$V_1 = h_{11} I_1 + h_{12} V_2 \longrightarrow (1)$$

$$I_2 = h_{21} I_1 + h_{22} V_2 \longrightarrow (2)$$

From equation (2), we have

$$I_1 = \frac{-h_{22}}{h_{21}} V_2 + \frac{1}{h_{21}} I_2$$

$$\therefore \; C = \frac{-h_{22}}{h_{21}}; \; D = \frac{-1}{h_{21}}$$

Substituting I_1 in equation (1)

$$V_1 = h_{11}\left[\frac{-h_{22}}{h_{21}} V_2 + \frac{1}{h_{21}} I_2\right] + h_{12} V_2$$

$$= \frac{-\Delta h}{h_{21}} V_2 + \frac{h_{11}}{h_{21}} I_2$$

Put $\Delta h = h_{11} h_{22} - h_{12} h_{21}$

$$\therefore \; A = \frac{-\Delta h}{h_{21}}; \quad B = \frac{-h_{11}}{h_{21}}$$

T-parameters in terms of g-parameters

We know that g-parameters as

$$I_1 = g_{11} V_1 + g_{12} I_2 \longrightarrow (1)$$

$$V_2 = g_{21} V_1 + g_{22} I_2 \longrightarrow (2)$$

From equation (2) we have

$$V_1 = \frac{V_2}{g_{21}} - \frac{g_{22}}{g_{21}} I_2$$

$$\therefore \; A = \frac{1}{g_{21}}; \; B = \frac{g_{22}}{g_{21}}$$

Substituting V_1 in equation (1)

$$I_1 = g_{11}\left[\frac{V_2}{g_{21}} - \frac{g_{22}}{g_{21}} I_2\right] + g_{12} I_2$$

$$= \frac{g_{11}}{g_{21}} V_2 - \frac{\Delta g}{g_{21}} I_2$$

Put $\Delta g = g_{11} g_{22} - g_{12} g_{21}$

$$\therefore \; C = \frac{g_{11}}{g_{21}}; \; D = \frac{\Delta g}{g_{21}}$$

T^1-PARAMETERS IN TERMS OF OTHER PARAMETERS

T^1-parameters in terms of Z-parameters

We known, that the Z-parameters as

$$V_1 = Z_{11} I_1 + Z_{12} I_2 \longrightarrow \quad (1)$$

$$V_2 = Z_{21} I_1 + Z_{22} I_2 \longrightarrow \quad (2)$$

From Equation (1), we have

$$I_2 = \frac{V_1}{Z_{12}} - \frac{Z_{11}}{Z_{12}} I_2$$

$$\therefore C^1 = \frac{1}{Z_{12}}; \quad D^1 = \frac{Z_{11}}{Z_{12}}$$

Substituting I_2 in equation (2)

$$V_2 = Z_{21} I_1 + Z_{22} \left[\frac{V_1}{Z_{12}} - \frac{Z_{11}}{Z_{12}} I_1 \right]$$

$$= \frac{Z_{22}}{Z_{12}} V_1 - \frac{\Delta Z}{Z_{12}} I_1$$

Where $\Delta Z = Z_{11} Z_{22} - Z_{21} Z_{12}$

$$\therefore A^1 = \frac{Z_{22}}{Z_{12}}; \quad B^1 = \frac{\Delta Z}{Z_{12}}$$

T^1-parameters in terms of Y-parameters

We known, the Y-parameters as

$$I_1 = Y_{11} V_1 + Y_{12} V_2 \longrightarrow (1)$$

$$I_2 = Y_{21} V_1 + Y_{22} V_2 \longrightarrow (2)$$

From Equation (1), we have

$$V_2 = \frac{I_1}{Y_{12}} - \frac{Y_{11}}{Y_{12}} V_1 = \frac{-Y_{11}}{Y_{12}} V_1 + \frac{I_1}{Y_{12}}$$

$$\therefore A^1 = \frac{-Y_{11}}{Y_{12}}; \quad B^1 = \frac{-1}{Y_{12}}$$

Putting V_2 in equation (2)

$$I_2 = Y_{21} V_1 + Y_{22}\left[\frac{-Y_{11}}{Y_{12}} V_1 + \frac{I_1}{Y_{12}}\right]$$

$$= \frac{-\Delta Y}{Y_{12}} V_1 + \frac{Y_{22}}{Y_{12}} I_1$$

Where $\Delta Y = Y_{11} Y_{22} - Y_{12} Y_{21}$

$$\therefore C^1 = \frac{-\Delta Y}{Y_{12}}; \quad D^1 = \frac{-Y_{22}}{Y_{12}}$$

T^1-parameters in terms of T-parameters

We know that the T-parameters as

$$V_1 = AV_2 - B I_2 \longrightarrow (1)$$

$$I_1 = CV_2 - D I_2 \longrightarrow (2)$$

Putting in matrix form,

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

Rewriting the above equation,

$$\begin{bmatrix} V_1 \\ -I_1 \end{bmatrix} = \begin{bmatrix} A & -B \\ -C & D \end{bmatrix} \begin{bmatrix} V_2 \\ I_2 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} V_2 \\ I_2 \end{bmatrix} = \begin{bmatrix} A & -B \\ -C & D \end{bmatrix}^{-1} \begin{bmatrix} V_1 \\ -I_1 \end{bmatrix}$$

$$\therefore [T^1] \neq [T]^{-1}$$

i.e. $$\begin{bmatrix} A^1 & B^1 \\ C^1 & D^1 \end{bmatrix} = \begin{bmatrix} A & -B \\ -C & D \end{bmatrix}^{-1} = \frac{1}{\Delta T}\begin{bmatrix} D & B \\ C & A \end{bmatrix}$$

Where $\Delta T = AD-BC$

$$\therefore A^1 = \frac{D}{\Delta T}; \quad B^1 = \frac{B}{\Delta T}; \quad C^1 = \frac{C}{\Delta T}; \quad D^1 = \frac{A}{\Delta T}$$

NOTE: $\therefore [T^1] \neq [T]^{-1}$

T^1-parameters in terms of h-parameters

We know that h-parameters as

$$V_1 = h_{11} I_1 + h_{12} V_2 \longrightarrow (1)$$

$$I_2 = h_{21} I_1 + h_{22} V_2 \longrightarrow (2)$$

From equation (1), we have

$$V_2 = \frac{V_1}{h_{12}} - \frac{h_{11}}{h_{12}} I_1$$

$$\therefore \quad A^1 = \frac{1}{h_{12}}; \quad B^1 = \frac{h_{11}}{h_{12}}$$

Putting V_2 in equation (2)

$$I_2 = h_{21} I_1 + h_{22}\left[\frac{V_1}{h_{12}} - \frac{h_{11}}{h_{12}} I_1\right]$$

$$= \frac{h_{22}}{h_{12}} V_1 - \frac{\Delta h}{h_{12}} I_1$$

Put $\Delta h = h_{11} h_{22} - h_{12} h_{21}$

$$\therefore \quad C^1 = \frac{h_{22}}{h_{12}}; \quad D^1 = \frac{\Delta h}{h_{12}}$$

T^1-parameters in terms of g-parameters

We know that g-parameters as

$$I_1 = g_{11} V_1 + g_{12} I_2 \longrightarrow (1)$$

$$V_2 = g_{21} V_1 + g_{22} I_2 \longrightarrow (2)$$

From equation (1), we have

$$I_2 = \frac{I_1}{g_{12}} - \frac{g_{11}}{g_{12}} V_1 = \frac{-g_{11}}{g_{12}} V_1 + \frac{1}{g_{12}} I_1$$

$$\therefore \quad C^1 = \frac{-g_{11}}{g_{12}}; \quad D^1 = \frac{-1}{g_{12}}$$

Putting I_2 in equation (2)

$$V_2 = g_{21} V_1 + g_{22}\left[\frac{-g_{11}}{g_{12}} V_1 + \frac{1}{g_{12}} I_1\right]$$

$$= \frac{-\Delta g}{g_{12}} V_2 + \frac{g_{22}}{g_{12}} I_1$$

Put $\Delta g = g_{11} g_{22} - g_{12} g_{21}$

$$\therefore A^1 = \frac{-\Delta g}{g_{12}}; \quad B^1 = \frac{-g_{22}}{g_{12}}$$

h-PARAMETERS IN TERMS OF OTHER PARAMETERS

h-parameters in terms of Z-parameters

We know that Z-parameters as

$$V_1 = Z_{11} I_1 + Z_{12} I_2 \longrightarrow (1)$$

$$V_2 = Z_{21} I_1 + Z_{22} I_2 \longrightarrow (2)$$

From Equation (2), we have

$$I_2 = \frac{V_2}{Z_{22}} - \frac{Z_{21}}{Z_{22}} I_1 = -\frac{Z_{21}}{Z_{22}} I_1 + \frac{V_2}{Z_{22}}$$

$$\therefore h_{21} = \frac{-Z_{21}}{Z_{22}}; \quad h_{22} = \frac{1}{Z_{22}}$$

Put I_2 in equation (1)

$$V_1 = Z_{11} I_1 + Z_{12}\left[\frac{-Z_{21}}{Z_{22}} I_1 + \frac{V_2}{Z_{22}}\right]$$

$$= \frac{\Delta Z}{Z_{22}} I_1 + \frac{Z_{12}}{Z_{22}} V_2$$

$$\because \Delta Z = Z_{11} Z_{22} - Z_{12} Z_{21}$$

$$\therefore \quad h_{11} = \frac{\Delta Z}{Z_{22}}; \quad h_{12} = \frac{Z_{12}}{Z_{22}}$$

h-parameters in terms of Y-parameters

We know, Y-parameters as

$$I_1 = Y_{11} V_1 + Y_{12} V_2 \longrightarrow (1)$$

$$I_2 = Y_{21} V_1 + Y_{22} V_2 \longrightarrow (2)$$

From Equation (1), we have

$$V_1 = \frac{I_1}{Y_{11}} - \frac{Y_{12}}{Y_{11}} V_2$$

$$\therefore \quad h_{11} = \frac{-1}{Y_{11}}; \quad h_{12} = \frac{-Y_{12}}{Y_{11}}$$

Put V_1 in equation (2)

$$I_2 = Y_{21}\left[\frac{I_1}{Y_{11}} - \frac{Y_{12}}{Y_{11}} V_2\right] + Y_{22} V_2$$

$$= \frac{Y_{21}}{Y_{11}} I_1 + \frac{\Delta Y}{Y_{11}} V_2$$

$$\because \Delta Y = Y_{11} Y_{22} - Y_{12} Y_{21}$$

$$\therefore \quad h_{21} = \frac{Y_{21}}{Y_{11}}; \quad h_{22} = \frac{\Delta Y}{Y_{11}}$$

h-parameters in terms of T-parameters

We know that the T-parameters as

$$V_1 = AV_2 - B I_2 \longrightarrow (1)$$

$$I_1 = CV_2 - D I_2 \longrightarrow (2)$$

From equation (2), we have

$$I_2 = \frac{CV_2}{D} - \frac{I_1}{D} = -\frac{I_1}{D} + \frac{C}{D} V_2$$

$$\therefore h_{21} = -\frac{1}{D}; \quad h_{22} = \frac{C}{D}$$

Put I_2 in equation (1)

$$V_1 = AV_2 - B\left[\frac{-I_1}{D} + \frac{C}{D} V_2\right] = \frac{B}{D} I_1 + \frac{\Delta T}{D} V_2$$

$$\because \Delta T = AD - BC$$

$$\therefore h_{11} = \frac{B}{D}; \quad h_{12} = \frac{\Delta T}{D}$$

h-parameters in terms of T^1-parameters

We know that the T^1-parameters as

$$V_2 = A^1 V_1 - B^1 I_1 \longrightarrow (1)$$

$$I_2 = C^1 V_1 - D^1 I_1 \longrightarrow (2)$$

From equation (1), we have

$$V_1 = \frac{V_2}{A^1} + \frac{B^1}{A^1} I_1 = \frac{B^1}{A^1} I_1 + \frac{1}{A^1} V_2$$

$$\therefore\ h_{11} = \frac{B^1}{A^1}; \quad h_{12} = \frac{1}{A^1}$$

Put V_1 in equation (2)

$$I_2 = C^1\left[\frac{B^1}{A^1} I_1 + \frac{1}{A^1} V_2\right] - D^1 I_1 = \frac{-\Delta T}{A^1} I_1 + \frac{C^1}{A^1} V_2$$

$$\because \Delta T^1 = A^1 D^1 - B^1 C^1$$

$$\therefore\ h_{21} = \frac{-\Delta T^1}{A^1}; \quad h_{22} = \frac{C^1}{A^1}$$

h-parameters in terms of g-parameters

We know that the g-parameters as

$$I_1 = g_{11} V_1 + g_{12} I_2 \longrightarrow (1)$$

$$V_2 = g_{21} V_1 + g_{22} I_2 \longrightarrow (2)$$

Putting in matrix, we have

$$\begin{bmatrix} I_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ I_2 \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix}^{-1} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$$

$$\therefore [h] = [g]^{-1}$$

i.e., $$\begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} = \begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix}^{-1} = \frac{1}{\Delta g}\begin{bmatrix} g_{22} & -g_{12} \\ -g_{21} & g_{11} \end{bmatrix}$$

$$\because \Delta g = g_{11} g_{22} - g_{12} g_{21}$$

$$\therefore h_{11} = \frac{g_{22}}{\Delta g}; \quad h_{12} = \frac{-g_{12}}{\Delta g}; \quad h_{21} = \frac{-g_{21}}{\Delta g}; \quad h_{22} = \frac{g_{11}}{\Delta g}$$

g-PARAMETERS IN TERMS OF OTHER PARAMETERS

g-parameters in terms of Z-parameters

We know, Z-parameters as

$$V_1 = Z_{11} I_1 + Z_{12} I_2 \longrightarrow (1)$$

$$V_2 = Z_{21} I_1 + Z_{22} I_2 \longrightarrow (2)$$

From Equation (1), we have

$$I_1 = \frac{V_1}{Z_{11}} - \frac{Z_{12}}{Z_{11}} I_2$$

$$\therefore g_{11} = \frac{1}{Z_{11}}; \quad g_{12} = \frac{-Z_{12}}{Z_{11}}$$

Substituting I_1 in equation (2)

$$V_2 = Z_{21}\left[\frac{V_1}{Z_{11}} - \frac{Z_{12}}{Z_{11}} I_2\right] + Z_{22} I_2$$

$$= \frac{Z_{21}}{Z_{11}} V_1 + \frac{\Delta Z}{Z_{11}} I_2$$

$$\because \Delta Z = Z_{11} Z_{22} - Z_{12} Z_{21}$$

$$\therefore \quad g_{21} = \frac{Z_{21}}{Z_{11}}; \quad g_{22} = \frac{\Delta Z}{Z_{11}}$$

g-parameters in terms of Y-parameters

We know, Y-parameters as

$$I_1 = Y_{11} V_1 + Y_{12} V_2 \longrightarrow (1)$$

$$I_2 = Y_{21} V_1 + Y_{22} V_2 \longrightarrow (2)$$

From Equation (2), we have

$$V_2 = \frac{I_2}{Y_{22}} - \frac{Y_{21}}{Y_{22}} V_1 = -\frac{Y_{21}}{Y_{22}} V_1 + \frac{1}{Y_{22}} I_2$$

$$\therefore \; g_{21} = \frac{-Y_{21}}{Y_{22}}; \quad g_{22} = \frac{1}{Y_{22}}$$

Put V_2 in equation (1)

$$I_1 = Y_{11} V_1 + Y_{12}\left[-\frac{Y_{21}}{Y_{22}} V_1 + \frac{1}{Y_{22}} I_2\right]$$

$$= \frac{\Delta Y}{Y_{22}} V_1 + \frac{Y_{12}}{Y_{22}} I_2$$

$\because \; \Delta Y = Y_{11} Y_{22} - Y_{12} Y_{21}$

$$\therefore \; g_{11} = \frac{\Delta Y}{Y_{22}}; \quad g_{12} = \frac{Y_{12}}{Y_{22}}$$

g-parameters in terms of T-parameters

We know that the T-parameters as

$$V_1 = AV_2 - B I_2 \longrightarrow (1)$$

$$I_1 = CV_2 - D I_2 \longrightarrow (2)$$

From equation (1), we have

$$V_2 = \frac{V_1}{A} + \frac{B}{A} I_2$$

$$\therefore \; g_{21} = \frac{1}{A}; \quad g_{22} = \frac{B}{A}$$

Put V_2 in equation (2)

$$I_1 = C\left[\frac{V_1}{A} + \frac{B}{A} I_2\right] - DI_2 = \frac{C}{A} V_1 + \frac{\Delta T}{A} I_2$$

$\because \Delta T = AD - BC$

$$\therefore \; g_{11} = \frac{C}{A}; \quad g_{12} = \frac{-\Delta T}{A}$$

g-parameters in terms of T^1-parameters

We know that the T^1-parameters as

$$V_2 = A^1 V_1 - B^1 I_1 \longrightarrow (1)$$

$$I_2 = C^1 V_1 - D^1 I_1 \longrightarrow (2)$$

From equation (2), we have

$$I_1 = \frac{C^1}{D^1} V_1 - \frac{I_2}{D^1}$$

$$\therefore \; g_{11} = \frac{C^1}{D^1}; \quad g_{12} = \frac{-1}{D^1}$$

Put I_1 in equation (1)

$$V_2 = A^1 V_1 - B^1 \left[\frac{C^1}{D^1} V_1 + \frac{I_2}{D^1} \right] = \frac{\Delta T}{D^1} V_1 + \frac{B^1}{D^1} I_2$$

$$\because \Delta T^1 = A^1 D^1 - B^1 C^1$$

$$\therefore \; g_{21} = \frac{\Delta T^1}{D^1}; \quad g_{22} = \frac{B^1}{D^1}$$

g-parameters in terms of h-parameters

We know that the h-parameters as

$$V_1 = h_{11} I_1 + h_{12} V_2 \longrightarrow (1)$$

$$I_2 = h_{21} I_1 + h_{22} V_2 \longrightarrow (2)$$

Putting in matrix, we have

$$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} I_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix}^{-1} \begin{bmatrix} V_1 \\ I_2 \end{bmatrix}$$

$$\therefore [g] = [h]^{-1}$$

$$\text{i.e.,} \begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix}^{-1} = \frac{1}{\Delta h} \begin{bmatrix} h_{22} & -h_{12} \\ -h_{21} & h_{11} \end{bmatrix}$$

$$\because \Delta h = h_{11} h_{22} - h_{12} h_{21}$$

$$\therefore g_{11} = \frac{h_{22}}{\Delta h}; \quad g_{12} = \frac{-h_{12}}{\Delta h}; \quad g_{21} = \frac{-h_{21}}{\Delta h}; \quad g_{22} = \frac{h_{11}}{\Delta h}$$

Parameters	[Z]	[Y]	[T]	[T^1]	[h]	[g]
[Z]	$\begin{matrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{matrix}$	$\begin{matrix} \frac{Y_{22}}{\Delta Y} & \frac{-Y_{12}}{\Delta Y} \\ \frac{-Y_{21}}{\Delta Y} & \frac{Y_{11}}{\Delta Y} \end{matrix}$	$\begin{matrix} \frac{A}{C} & \frac{\Delta T}{C} \\ \frac{1}{C} & \frac{D}{C} \end{matrix}$	$\begin{matrix} \frac{D^1}{C^1} & \frac{1}{C^1} \\ \frac{\Delta T^1}{C^1} & \frac{A^1}{C^1} \end{matrix}$	$\begin{matrix} \frac{\Delta h}{h_{22}} & \frac{h_{12}}{h_{22}} \\ \frac{-h_{21}}{h_{22}} & \frac{1}{h_{22}} \end{matrix}$	$\begin{matrix} \frac{1}{g_{11}} & \frac{-g_{12}}{g_{11}} \\ \frac{g_{21}}{g_{11}} & \frac{\Delta g}{g_{11}} \end{matrix}$
[Y]	$\begin{matrix} \frac{Z_{22}}{\Delta Z} & \frac{-Z_{12}}{\Delta Z} \\ \frac{-Z_{21}}{\Delta Z} & \frac{Z_{11}}{\Delta Z} \end{matrix}$	$\begin{matrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{matrix}$	$\begin{matrix} \frac{D}{B} & \frac{-\Delta T}{B} \\ \frac{-1}{B} & \frac{A}{B} \end{matrix}$	$\begin{matrix} \frac{A^1}{B^1} & \frac{-1}{B^1} \\ \frac{-\Delta T^1}{B^1} & \frac{A^1}{B^1} \end{matrix}$	$\begin{matrix} \frac{1}{h_{11}} & \frac{-h_{12}}{h_{11}} \\ \frac{h_{21}}{h_{11}} & \frac{\Delta h}{h_{11}} \end{matrix}$	$\begin{matrix} \frac{\Delta g}{g_{22}} & \frac{g_{12}}{g_{22}} \\ \frac{-g_{21}}{g_{22}} & \frac{1}{g_{22}} \end{matrix}$
[T]	$\begin{matrix} \frac{Z_{11}}{Z_{21}} & \frac{\Delta Z}{Z_{21}} \\ \frac{1}{Z_{21}} & \frac{Z_{22}}{Z_{21}} \end{matrix}$	$\begin{matrix} \frac{-Y_{22}}{Y_{21}} & \frac{-1}{Y_{21}} \\ \frac{-\Delta Y}{Y_{21}} & \frac{-Y_{11}}{Y_{21}} \end{matrix}$	$\begin{matrix} A & C \\ C & D \end{matrix}$	$\begin{matrix} \frac{D^1}{\Delta T^1} & \frac{B^1}{\Delta T^1} \\ \frac{C^1}{\Delta T^1} & \frac{A^1}{\Delta T^1} \end{matrix}$	$\begin{matrix} \frac{-\Delta h}{h_{21}} & \frac{-h_{11}}{h_{21}} \\ \frac{-h_{22}}{h_{21}} & \frac{-1}{h_{21}} \end{matrix}$	$\begin{matrix} \frac{1}{g_{21}} & \frac{g_{22}}{g_{21}} \\ \frac{g_{11}}{g_{21}} & \frac{\Delta g}{g_{21}} \end{matrix}$
[T^1]	$\begin{matrix} \frac{Z_{22}}{Z_{12}} & \frac{\Delta Z}{Z_{12}} \\ \frac{1}{Z_{12}} & \frac{Z_{11}}{Z_{12}} \end{matrix}$	$\begin{matrix} \frac{-Y_{11}}{Y_{12}} & \frac{-1}{Y_{12}} \\ \frac{-\Delta Y}{Y_{12}} & \frac{-Y_{22}}{Y_{12}} \end{matrix}$	$\begin{matrix} \frac{D}{\Delta T} & \frac{B}{\Delta T} \\ \frac{C}{\Delta T} & \frac{A}{\Delta T} \end{matrix}$	$\begin{matrix} A^1 & C^1 \\ C^1 & D^1 \end{matrix}$	$\begin{matrix} \frac{1}{h_{12}} & \frac{h_{11}}{h_{12}} \\ \frac{h_{22}}{h_{12}} & \frac{\Delta h}{h_{12}} \end{matrix}$	$\begin{matrix} \frac{-\Delta g}{g_{12}} & \frac{-g_{22}}{g_{12}} \\ \frac{-g_{11}}{g_{12}} & \frac{-1}{g_{12}} \end{matrix}$
[h]	$\begin{matrix} \frac{\Delta Z}{Z_{22}} & \frac{Z_{12}}{Z_{22}} \\ \frac{-Z_{21}}{Z_{22}} & \frac{1}{Z_{22}} \end{matrix}$	$\begin{matrix} \frac{1}{Y_{11}} & \frac{-Y_{12}}{Y_{11}} \\ \frac{Y_{21}}{Y_{11}} & \frac{\Delta Y}{Y_{11}} \end{matrix}$	$\begin{matrix} \frac{B}{D} & \frac{\Delta T}{D} \\ \frac{-1}{D} & \frac{C}{D} \end{matrix}$	$\begin{matrix} \frac{B^1}{A^1} & \frac{1}{A^1} \\ \frac{-\Delta T^1}{A^1} & \frac{C^1}{A^1} \end{matrix}$	$\begin{matrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{matrix}$	$\begin{matrix} \frac{g_{22}}{\Delta g} & \frac{-g_{12}}{\Delta g} \\ \frac{-g_{21}}{\Delta g} & \frac{g_{11}}{\Delta g} \end{matrix}$
[g]	$\begin{matrix} \frac{1}{Z_{11}} & \frac{-Z_{12}}{Z_{11}} \\ \frac{Z_{21}}{Z_{11}} & \frac{\Delta Z}{Z_{11}} \end{matrix}$	$\begin{matrix} \frac{\Delta Y}{Y_{22}} & \frac{Y_{12}}{Y_{22}} \\ \frac{-Y_{21}}{Y_{22}} & \frac{1}{Y_{22}} \end{matrix}$	$\begin{matrix} \frac{C}{A} & \frac{-\Delta T}{A} \\ \frac{1}{A} & \frac{B}{A} \end{matrix}$	$\begin{matrix} \frac{C^1}{D^1} & \frac{-1}{D^1} \\ \frac{\Delta T^1}{D^1} & \frac{B^1}{D^1} \end{matrix}$	$\begin{matrix} \frac{h_{22}}{\Delta h} & \frac{-h_{12}}{\Delta h} \\ \frac{-h_{21}}{\Delta h} & \frac{h_{11}}{\Delta h} \end{matrix}$	$\begin{matrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{matrix}$

UNIT-2

TRANSIENT RESPONSE

INTRODUCTION: - As we are aware that the terminal characteristics of capacitors and inductors are governed by differential relationships. The connection of these elements with the resistors and energy sources will result in integro-differential (or) simply differential equations with constant coefficients. The solutions of these equations in time – domain gives the "**TRANSIENT RESPONSE**" of the system of equations. The time – domain response of the circuit for different test signals is almost important to synthesize (or) design electronic circuits.

Whenever a circuit is switched from one condition to another, either by a change in the applied source or a change in the circuit elements, there is a transition period during which the branch currents and element voltages change from their former values to new values. The period is called the "**TRANSIENT STATE** (or) **NATURAL RESPONSE**". After the transient period has passed, the circuit is said to be in the "**STEADY STATE** (or) **FORCED RESPONSE**". Thus, the total response of the network is the sum of its transient response and steady state response.

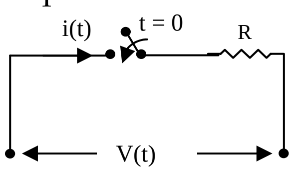
Now, the linear differential equation that describes the circuit will have two parts to its solution, the complementary function corresponding to the transient and the particular solution corresponding to the steady state.

INITIAL VALUES OF NETWORK ELEMENTS

RESISTOR: -

If a circuit is purely resistive, it does not exhibit any transient response. Thus in the circuit, the current instantaneously rises to its steady state value ie.,

$i = \dfrac{V}{R}$, and there is no transient response.



$t = 0^-$ $\rightarrow$ Represents the instant just prior to the closing of the switch at t = 0.

$t = 0^+$ $\rightarrow$ Represents the instant immediately after closing of the switch at t = 0.

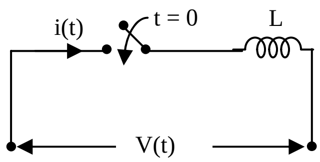
At $t = 0^- \rightarrow i = 0$, but at $t = 0^+ \rightarrow i = \dfrac{V}{R}$

INDCUTOR: -

If a circuit is purely inductor as shown in figure.

If the switch is closed at t = 0, by, applying KVL to the circuit

$V = L\dfrac{di}{dt}$ $\quad$ *for* $t > 0^+$

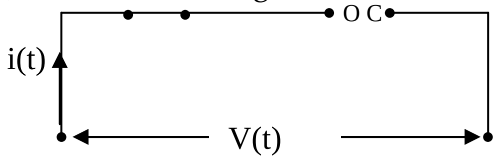


At $t = 0^-$, the current is zero, assuming the circuit to be relaxed [ie., No initial inductor current].

At $t = 0^+$, the current must still be zero, since the current through an inductor cannot become zero instantaneously, even if it is not zero at $t = 0^-$.

$$\therefore i = 0 \quad At\ t = 0^+$$

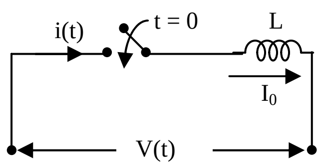
Hence, it is obvious that at $t(0^+)$, the inductor 'L' act as an open circuit. The equivalent circuit at $t(0^+)$ is as shown in figure…



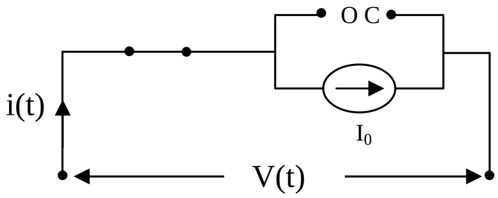
$\Rightarrow$'L' is replaced by an open circuit

However, if at $t(0^-)$ the inductor is already carrying a current due to a previously applied forcing function, it would continue to flow at $t(0^+)$, without change of magnitude.

Let 'I_0' be the initial inductor current as shown in figure…



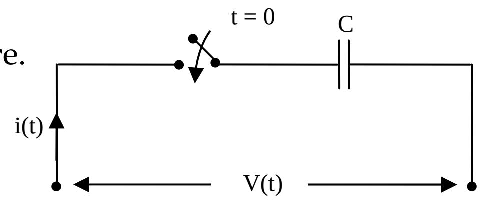
The equivalent circuit at $t(0^+)$ is shown in figure…



'L' is replaced by open and 'I_0' is replaced by equal current source.

CAPACITOR: -

If the circuit contains pure capacitor as shown in figure.
If the switch is closed at t = 0 by, applying
KVL to the circuit



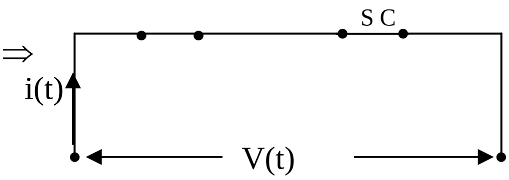
$$V = \frac{1}{C}\int i\,dt \quad for \quad t > 0^+$$

$$i = C\frac{dV}{dt}$$

At $t = 0^-$, the voltage across capacitor is zero, assuming the circuit to be relaxed [ie., No initial capacitor voltage]. Also current, $i = 0$.

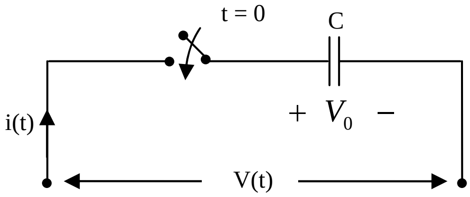
At $t = 0^+$, the voltage across capacitor must be zero, since the voltage across a capacitor cannot become zero instantaneously even if it is not zero at $t = 0^-$. It means that, at $t = 0^+$, the capacitor acts as Short Circuit. The equivalent circuit at $t(0^+)$ is as shown in figure…

Hence 'C' is replaced by short circuit $\Rightarrow$

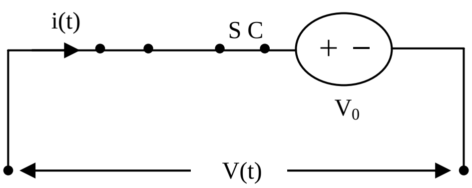


However, at $t = 0^-$, if there is a capacitor voltage due to previously applied forcing function, then at $t = 0^+$ also, it would remain without change in magnitude.

Let 'V_0' be initial capacitor voltage as shown in figure…



The equivalent circuit at $t(0^+)$ is shown in figure…



'C' is replaced by Short Circuit and 'V_0' is replaced by an equal voltage source.

FINAL VALUES OF NETWORK ELEMENTS

We shall next see how we can obtain equivalent circuits under Steady state conditions ie., at t = ∞.

RESISTOR: -

A resistor obviously remains un effected. Hence a resistance R of a given network remains as 'R' only in the equivalent circuit at t = ∞. also.

INDUCTOR: -

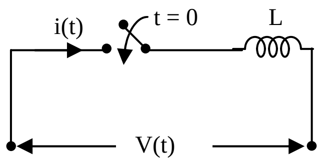
We have induced e.m.f in an inductor across 'L' then

$$V_L = L\frac{di}{dt}$$

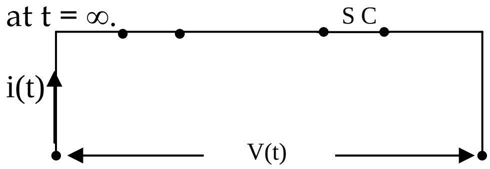
When Steady state has been reached ie., at t = ∞., there is no change of current

$$\text{i.e., } \frac{di}{dt} = 0 \qquad \therefore V_L = 0$$

Since, there is no inductor voltage, it implies that the inductor acts as short – circuit. Hence an inductor acts as open – circuit at $t = 0^+$, but it acts as short – circuit at t = ∞.



Equivalent circuit at t = ∞.



<u>CAPACITOR</u>: -

The current through a capacitor is given as

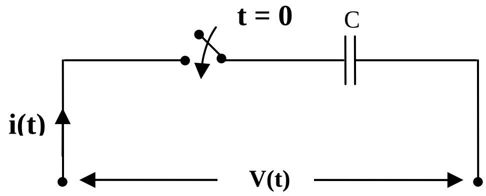
$$i = C\frac{dV_C}{dt} \text{ where } V_C \longrightarrow \text{Capacitor voltage}$$

At Steady state, there is no change of capacitor voltage

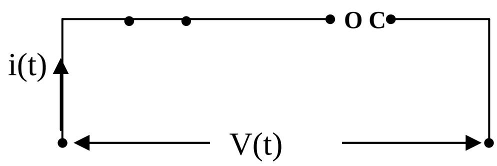
$$\frac{dV_C}{dt} = 0 \qquad \therefore i = 0$$

It implies that the capacitor acts as an open – circuit at t = ∞.

Hence a capacitor acts as short – circuit at $t = 0^+$, but acts as open – circuit at t = ∞.



Equivalent circuit of t = ∞.



<u>SUMMARY:</u>

- ❖ The current through an inductor cannot change instantaneously.
- ❖ Voltage across a capacitor cannot change instantaneously.
- ❖ At t (0^+), an inductor acts as an open – circuit.
- ❖ At t (0^+), a capacitor acts as short – circuit.
- ❖ At t (0^+), with initial inductor current 'I_0' is replaced by an equal current source with the same polarity.
- ❖ At t (0^+), with an initial capacitor voltage 'V_0' is replaced by an equal voltage source with the same polarity.
- ❖ At t (0^+) and at t = ∞, a resistor remains as it is, without any change.

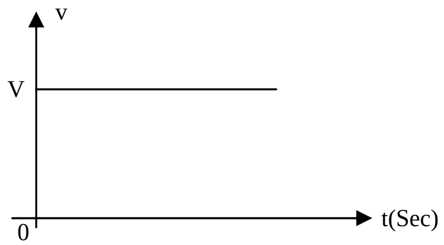
- ❖ An inductor acts as short – circuit, under Steady state conditions, for a forcing function of constant magnitude like step (or) DC voltage.
- ❖ A capacitor acts as open – circuit, under Steady state conditions, for a forcing function of constant magnitude like step (or) DC voltage.

TRANSIENT RESPONSE IN TIME DOMAIN WITH CONSTANT INPUT [DC EXCITATION]

RC CIRCUIT

The constant input as shown in figure….. is called step input (or) constant input. Since it steps from 0 to V volts at a time t=0

Let us assume that the voltage is suddenly applied at t=0 to the RC circuit shown in figure. Let us assume the initial charge on the capacitor is zero.



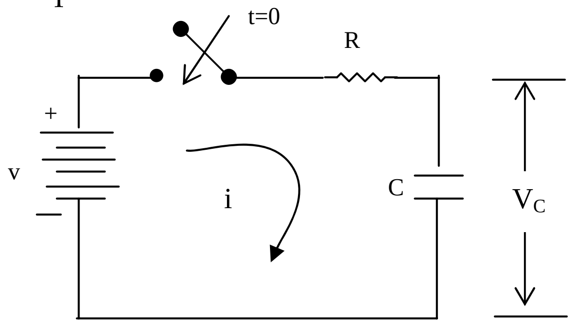


FIG : RC Circuit with Step Input

$\because$ At $t = 0^-$ $\longrightarrow$ $V_C(0^-) = 0$

At $t = 0^+$ $\longrightarrow$ $V_C(0^+) = 0$ [$\because$ Voltage across capacitor cannot change instantaneously]

Initially it act as short circuit

$$\therefore i(O^+) = \frac{V}{R}$$

Let 'i' be the current flowing in the circuit when the switch is closed at t = 0. Using KVL, the equilibrium equation is

$$Ri + \frac{1}{C}\int i\,dt = v \qquad ....(1)$$

Differentiating the equation (1) with respect to time 't'.

$$R\frac{di}{dt} + \frac{1}{C}i = 0 \Rightarrow \frac{di}{dt} + \frac{1}{RC}i = 0 \Rightarrow \left[D + \frac{1}{RC}\right]i = 0 \qquad ....(2)$$

The equation (2) is a first order linear homogeneous equation. Hence the total solution will have only complementary function, and particular integral is zero.

$$i(t) = Ae^{-t/RC} \qquad .... (3)$$

To evaluate the constant 'A' we will use the initial condition i.e.,

$$i(0^+) = \frac{V}{R}$$

$$i(0)\,|_{t=0} = A = \frac{V}{R}$$

$$\therefore i(t) = \frac{V}{R}e^{-t/RC}\ (or)\ i(t) = \frac{V}{R}e^{-t/\tau} \qquad ....(4)$$

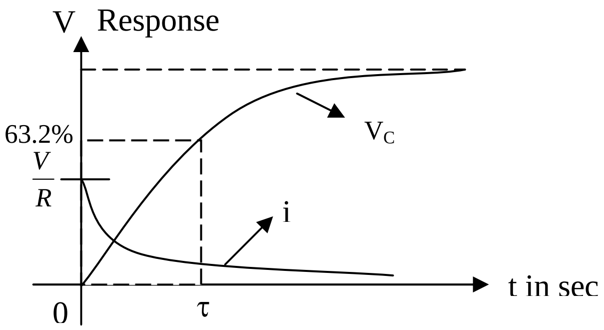
This solution is called Natural response of the circuit and also called as the complementary function. Where τ=RC is called the time constant of an RC circuit.

VOLTAGE ACROSS CAPACITOR:

$$V_C = \frac{1}{C}\int i\,dt = \frac{1}{C}\int_0^t \frac{V}{R}e^{-t/\tau}dt = \frac{V}{RC}\frac{1}{\left[{-1}/{\tau}\right]}\left[e^{-t/\tau}\right]_0^t$$

$$\frac{V}{RC}(-RC)\left[e^{-t/\tau}\right]_0^t = -V\left[e^{-t/\tau}-1\right] = V\left[1-e^{-t/\tau}\right] \quad .....(5)$$

As 't' is varying from 0 to ∞ the time response characteristics of current and voltage across the capacitor from equations (4) and (5) are shown in figure.



The transient solution is a total solution of the circuits i.e., $i(t)=i_{ss}+i_t = \frac{V}{R}e^{-t/\tau}$. Where '$i_{ss}$' is steady state value and 'i_t' is the transient value. The response of the circuit will depend upon 'τ'. If 'R' and 'C' are larger then the circuit takes longer time to settle down to the new steady state value.

TIME CONSTANT

The interpretation of the time constant as 'the time interval during which the response of the circuit starting from any point of time during the transient interval, would have reached its final value if it had maintained its rate of change constant at the value it had at that point of time". If the time is equal to one time constant then $V_C = V(1-e^{-1}) = 0.632V$. ***The time constant can be regarded as the time required for the transient response to attain 63.2% of the steady state value starting from zero***. In two, three, four (or) five time constants the time response values would be 0.864, 0.95, 0.982 and 0.993 of its steady state value. For all practical purposes most of the electrical instruments used for measurement of electrical quantities will have a least count of 1% after approximately five time constants have elapsed.

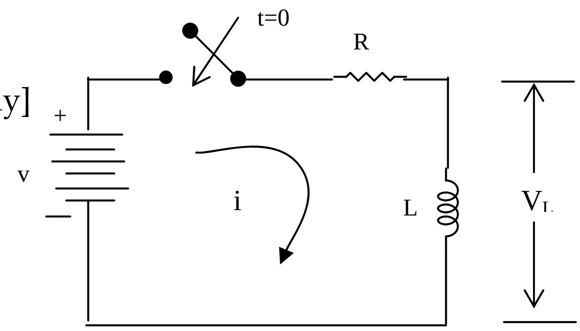
RL CIRCUIT

The RL network is excited by a step input is shown in figure. Let us assume that at the time t=0 the switch is closed and initially the current through the inductor is zero.

At $t = 0^-$ ⟶ $i_L(0^-) = 0$

At $t = 0^+$ ⟶ $i_L(0^+) = 0$

[∵ Current in the inductor cannot change instantaneously]



Using KVL, the equilibrium equation is

$$Ri + L\frac{di}{dt} = v \quad ....(1)$$

$$\frac{di}{dt} + \frac{R}{L}i = \frac{V}{L} \Rightarrow \left[D + \frac{R}{L}\right]i = \frac{V}{L} \quad ....(2)$$

The equation (2) is a first order differential equation and the solution gives the response of the circuit. To get the solution we will obtain the transient part [complementary function] and steady state part [particular integral] separately. The transient part of the solution is obtained by solving the homogenous part of the differential equation by making forcing function to zero.

i.e., $\left[D + \frac{R}{L}\right]i = 0$

The general solution is of the form

$$i_t = Ae^{-\frac{R}{L}t} \quad .....(3)$$

Steady state part of the solution (or) particular integral is obtained from

$$\left[D+\frac{R}{L}\right]i=\frac{V}{L}\Rightarrow i=\frac{V}{L\left[D+\frac{R}{L}\right]}$$

To get the steady state part of the solution, substitute D=0 [for DC excitation].

$$i_{ss}=\frac{V}{L*\frac{R}{L}}=\frac{V}{R} \qquad \ldots.(4)$$

The complete solution is

$$i=i_{ss}+i_t=\frac{V}{R}+Ae^{-\frac{R}{L}t}$$

To evaluate the constant A, we use initial condition i(0 $^+$)=0 At t = 0 $^+$

$$i(0^+)=\frac{V}{R}+A\Rightarrow 0=A+\frac{V}{R}\Rightarrow A=-\frac{V}{R}$$

$\therefore$ The complete solution is

$$i(t)=\frac{V}{R}-\frac{V}{R}e^{-\frac{R}{L}t}=\frac{V}{R}\left[1-e^{-t/\tau}\right] \qquad \ldots.(5)$$

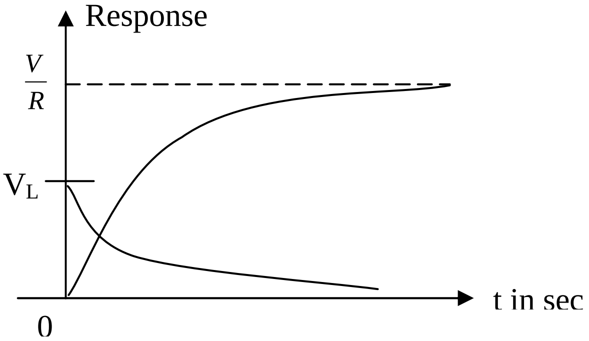
Where 'τ' is the time constant and is equal to $\frac{L}{R}$

VOLTAGE ACROSS THE INDUCTOR:

The voltage across the inductor $V_L=L\frac{di}{dt}$

$$\Rightarrow V_L=L\frac{d\left[\frac{V}{R}\left(1-e^{-t/\tau}\right)\right]}{dt}=\frac{VL}{R}\left(\frac{1}{\tau}\right)e^{-t/\tau}=\frac{VL}{R}*\frac{R}{L}e^{-t/\tau}=Ve^{-t/\tau} \qquad \ldots\ldots.(6)$$

As 't' is varying from 0 to ∞ the time response characteristics of current and voltage across the inductor from equations (5) & (6) are shown in figure……

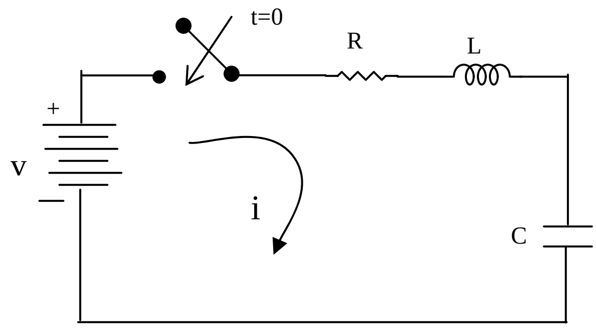


TIME CONSTANT

If the time is equal to one time constant then V_L = V(e^{-1}) = 0.3678 Volts. The time constant can be regarded as the time required for the transient response to attain 36.78% of initial value.

RLC CIRCUIT

The behaviour of an RLC series circuit with constant excitation is presented here. Such RLC circuits are of great importance, since they occur, in many practical situations. In the figure shown above. A battery of voltage 'V' is suddenly applied to the series RLC circuit with no-initial current in the inductor and initial charge on the capacitor



At t = 0 $^-$ ⟶ $i_L(0^-)=0$; $V_C(0^-)=0$

At t = 0 $^+$ ⟶ $i_L(0^+)=0$; $V_C(0^+)=0$

Applying KVL, the equilibrium equation is

$$Ri + L\frac{di}{dt} + \frac{1}{C}\int i\,dt = V \qquad \text{....(1)}$$

Differentiating with respect to time, 't'

$$R\frac{di}{dt} + L\frac{di^2}{dt^2} + \frac{1}{C}i = 0 \Rightarrow \frac{di^2}{dt^2} + \frac{R}{L}\frac{di}{dt} + \frac{1}{LC}i = 0 \Rightarrow \left[D^2 + \frac{R}{L}D + \frac{1}{LC}\right]i = 0 \qquad \text{....(2)}$$

This is a second order differential equation and it is a homogeneous equation.
The solution of this equation is of the form

$$i = Ae^{m_1 t} + Be^{m_2 t} \qquad \text{.....(3)}$$

Where 'A' and 'B' are constants to be determined from the initial conditions of the network and m_1 and m_2 are the roots of characteristic equations.

$$D^2 + \frac{R}{L}D + \frac{1}{LC} = 0$$

The roots of the characteristic equation are

$$m_1, m_2 = \frac{-\frac{R}{L} \pm \sqrt{\left(\frac{R}{L}\right)^2 - 4\left(\frac{1}{LC}\right)}}{2} = \frac{-R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}$$

The response of the network depends on the nature of the roots m_1 and m_2. Also depend up on the value under radical. Three cases of these roots are explained below.

CASE 1 : When $\sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}$ is positive. In this case $\left(\frac{R}{2L}\right)^2 > \frac{1}{LC}$. Hence the roots are negative real. The response of the circuit is with out oscillations as shown in figure, curve 1. In this case the final value is reached more slowly and is said to be over damped.

CASE 2 : When $\sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}$ is equal to zero. In this case $\left(\frac{R}{2L}\right)^2 = \frac{1}{LC}$. Hence the roots are equal to $-\frac{R}{2L}$. In this case the response rises faster than curve 1 without any oscillations and no over-shoot on the final value. This response is called critically damped and is shown by curve 2 in figure. The time of response is shortest.

CASE 3 : When $\sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}$ is negative i.e., $\left(\frac{R}{2L}\right)^2 < \frac{1}{LC}$ then the roots m_1 and m_2 are complex conjugates with negative real parts. The response of the system is oscillatory with over shoots on the final value. This response is termed as under damped. Such a response is often said to be ringing. The under damped behaviour is shown by curve 3 in figure.

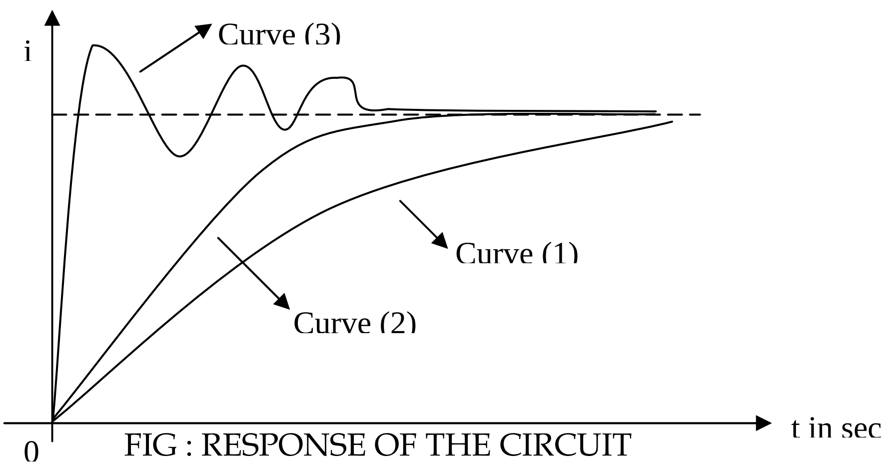


FIG : RESPONSE OF THE CIRCUIT

THE EXPRESSION FOR CIRCUIT CURRENT

The expression for current of an RLC series circuit may be written as

$$\frac{d^2 i}{dt^2} + 2\zeta\omega_n \frac{di}{dt} + \omega_n^2 i = 0$$

Whose roots are $m_1, m_2 = -\zeta\,\omega_n \pm \omega_n \sqrt{\zeta^2 - 1}$ and the solution is

$$i(t) = C_1 e^{\left[-\zeta\,\omega_n + \omega_n\sqrt{\zeta^2 - 1}\right]t} + C_2 e^{\left[-\zeta\,\omega_n - \omega_n\sqrt{\zeta^2 - 1}\right]t}$$

The solution will have different farms depending up on the value of 'ζ'. If 'ζ' is less than one then $R<R_C$ and the response is under damped. If 'ζ' is equal to one then $R=R_C$ and the response is critically damped. If 'ζ' is more than one then $R>R_C$ and the response is over damped.

UNIT-3

LOCUS DIAGRAMS

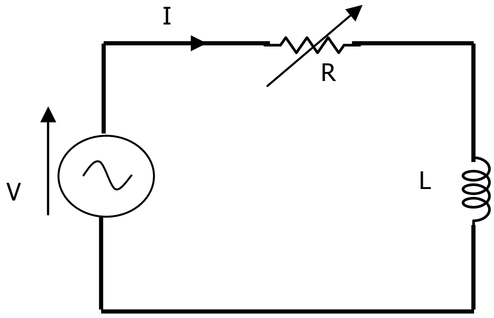
An AC circuit is generally analyzed presuming the input voltage is constant. If the values of R, L, C and f are varied the current in AC circuit gets affected in magnitude and phase. The behavior of the circuit under such conditions can be studied using **LOCUS DIAGRAMS**.

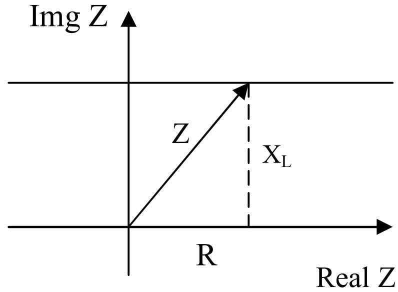
Example: - The performance of induction machine can be determined using the locus diagram. The transmission line performance for different load conditions can be easily evaluated using locus diagrams in power system engineering.

IMPEDANCE LOCUS DIAGRAMS

1. SERIES R – L CIRCUIT WITH CONSTANT 'X_L' AND VARIABLE 'R':

Let us consider an R-L circuit with AC sinusoidal excitation in which the inductance is constant and resistance is variable as shown in figure.





Locus of Impedance

The impedance of the network is Z = R + jX_L, where X_L = Lω. The current in the network is

$$\bar{I} = \frac{V}{R + jX_L} = \frac{V}{R^2 + X_L^{\,2}}(R - jX_L) \qquad \text{Is in complex form}$$

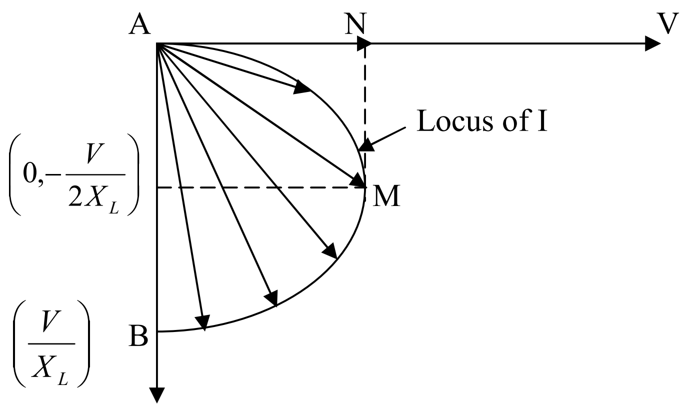
$$= \frac{VR}{R^2 + X_L^{\,2}} - j\frac{VX_L}{R^2 + X_L^{\,2}} = x - j\,y = \frac{V}{\sqrt{R^2 + X_L^{\,2}}}\cdot\frac{R}{\sqrt{R^2 + X_L^{\,2}}} - j\frac{V}{\sqrt{R^2 + X_L^{\,2}}}\cdot\frac{X_L}{\sqrt{R^2 + X_L^{\,2}}}$$

$$Where \qquad x = \frac{VR}{R^2 + X_L^{\,2}} = I\,Cos\phi \quad and \quad y = -\frac{VX_L}{R^2 + X_L^{\,2}} = I\,Sin\phi$$

$$x^2 + y^2 = I^2 = \frac{V^2}{R^2 + X_L^{\,2}} = \frac{V^2}{R^2 + X_L^{\,2}} * \frac{X_L}{X_L} = \frac{V}{X_L} * \frac{V\,X_L}{R^2 + X_L^{\,2}} = -\frac{V * y}{X_L}$$

$$\Rightarrow x^2 + y^2 = -\frac{V * y}{X_L} \Rightarrow (x - 0)^2 + \left(y + \frac{V}{2X_L}\right)^2 = \left(\frac{V}{2X_L}\right)^2$$

The above equation is a circle whose center is at $\left(0, -\frac{V}{2X_L}\right)$ with a radius of $\left(\frac{V}{2X_L}\right)$, where 'R' is varying from 0 to ∞. The current varies as a semicircle as shown in figure.



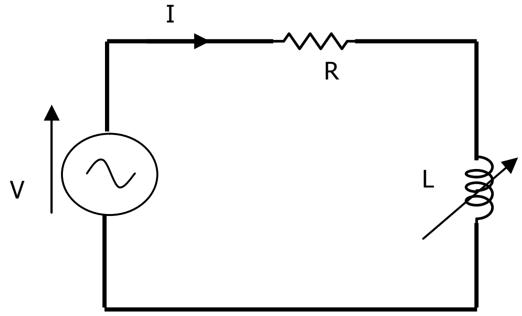
As the resistance is varied from 0 to ∞ the current is traversing along the semicircle AMB. The maximum horizontal component of I is $AN = \dfrac{V}{2X_L}$

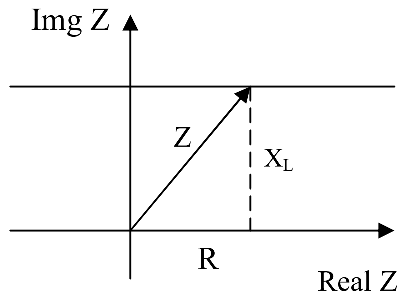
Power factor is Cos 45^0 = 0.707 lagging.

$$P_{\max} = V(I\cos\phi)_{\max} = V \times \frac{V}{2X_L} = \frac{V^2}{2X_L} \quad Watts.$$

2. SERIES R – L CIRCUIT WITH CONSTANT 'R' AND VARIABLE 'X_L':

Let us consider an R-L circuit with AC sinusoidal excitation in which the inductance is constant and resistance is variable as shown in figure.





Locus of Impedance

The impedance of the network is Z = R + jX_L, where X_L = Lω. The current in the network is

$$\bar{I} = \frac{V}{R + jX_L} = \frac{V}{R^2 + X_L^2}(R - jX_L) \qquad \text{Is in complex form}$$

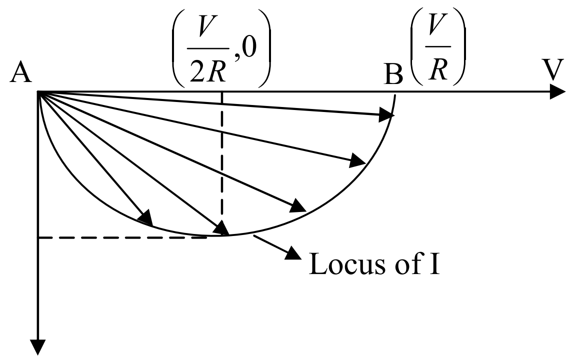
$$= \frac{VR}{R^2 + X_L^2} - j\frac{VX_L}{R^2 + X_L^2} = x - j\,y = \frac{V}{\sqrt{R^2 + X_L^2}} \cdot \frac{R}{\sqrt{R^2 + X_L^2}} - j\frac{V}{\sqrt{R^2 + X_L^2}} \cdot \frac{X_L}{\sqrt{R^2 + X_L^2}}$$

$$Where \qquad x = \frac{VR}{R^2 + X_L^2} = I\,Cos\phi \quad and \quad y = -\frac{VX_L}{R^2 + X_L^2} = I\,Sin\phi$$

$$x^2 + y^2 = I^2 = \frac{V^2}{R^2 + X_L^2} = \frac{V^2}{R^2 + X_L^2} * \frac{R}{R} = \frac{V}{R} * \frac{VR}{R^2 + X_L^2} = \frac{V * x}{R}$$

$$\Rightarrow x^2 + y^2 = \frac{V * x}{R} \Rightarrow \left(x - \frac{V}{2R}\right)^2 + (y - 0)^2 = \left(\frac{V}{2R}\right)^2$$

The above equation is a circle having radius $\left(\dfrac{V}{2R}\right)$ and centre $\left(\dfrac{V}{2R}, 0\right)$. The locus of the current when L is varying from 0 to ∞ is shown in figure.



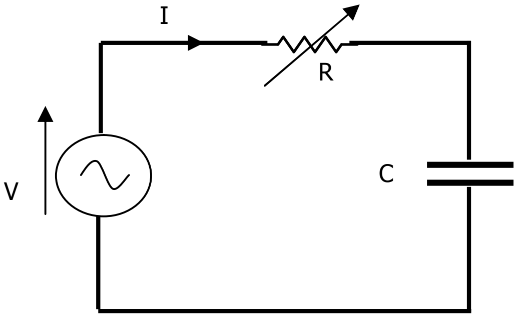
The current is maximum when the inductance is the circuit is zero. *Therefore* $I_{\max} = \dfrac{V}{R}$.

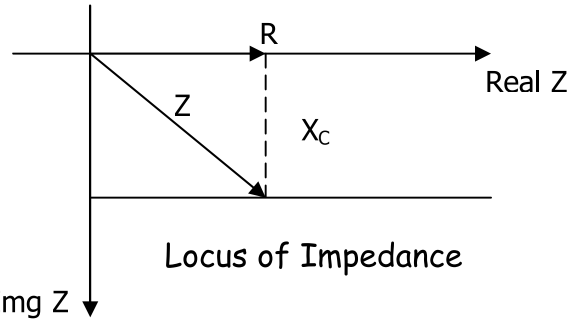
The current and the voltage are in phase. Then the power factor is unity.

$$P_{\max} = V(I\cos\phi)_{\max} = V * \frac{V}{R} = \frac{V^2}{R} \quad Watts .$$

3. SERIES R – C CIRCUIT WITH CONSTANT 'X_c' AND VARIABLE 'R':

Let us consider an R-C circuit with AC sinusoidal excitation in which the capacitance is constant and resistance is variable as shown in figure.





The impedance of the network is Z = R – jX_C, where $X_C = \frac{1}{C\omega}$. The current in the network is

$$\bar{I} = \frac{V}{R - jX_C} = \frac{V(R + jX_C)}{R^2 + X_C{}^2} = \frac{VR}{R^2 + X_C{}^2} + j\frac{VX_C}{R^2 + X_C{}^2}$$

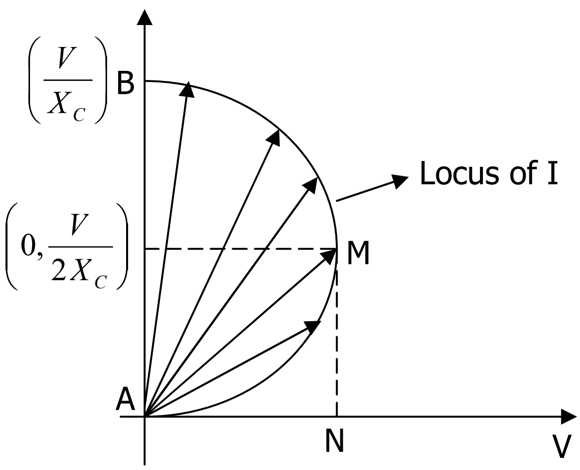
$$= \frac{V}{\sqrt{R^2 + X_C{}^2}} \cdot \frac{R}{\sqrt{R^2 + X_C{}^2}} + j\frac{V}{\sqrt{R^2 + X_C{}^2}} \cdot \frac{X_C}{\sqrt{R^2 + X_C{}^2}} = x + jy$$

$$Where \quad x = I\cos\phi = \frac{VR}{R^2 + X_C{}^2} \quad and \quad y = I\sin\phi = \frac{VX_C}{R^2 + X_C{}^2}$$

$$x^2 + y^2 = I^2 = \frac{V^2}{R^2 + X_C{}^2} = \frac{V^2}{R^2 + X_C{}^2} \cdot \frac{X_C}{X_C} = \frac{V}{X_C} * \frac{VX_C}{R^2 + X_C{}^2} = \frac{V * y}{X_C}$$

$$\Rightarrow \left(x^2 - 0\right) + \left(y - \frac{V}{2X_C}\right)^2 = \left(\frac{V}{2X_C}\right)^2$$

This is an equation of a circle with $\left(\frac{V}{2X_C}\right)$ as radius and $\left(0, \frac{V}{2X_C}\right)$ as the centre of the circle. The current locus diagram is as shown in figure.

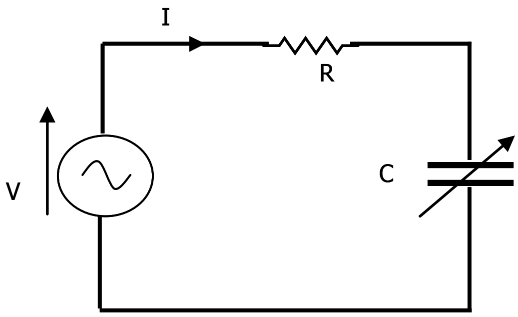


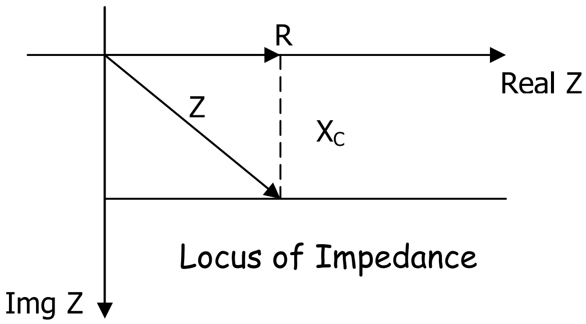
The maximum horizontal component is AN = $\frac{V}{2X_C}$ and this occurs at $\phi=45^0$ leading (Current leads the voltage).

$$P_{\max} = V(I\cos\phi)_{\max} = V * \frac{V}{2X_C} = \frac{V^2}{2X_C} \quad Watts.$$

4. SERIES R – C CIRCUIT WITH CONSTANT 'R' AND VARIABLE 'X_C':

Let us consider an R-C circuit with AC sinusoidal excitation in which the resistance is constant and capacitance is variable as shown in figure.





The impedance of the network is Z = R – jX_C, where $X_C = \frac{1}{C\omega}$. The current in the network is

$$\bar{I} = \frac{V}{R - jX_C} = \frac{V(R + jX_C)}{R^2 + X_C{}^2} = \frac{VR}{R^2 + X_C{}^2} + j\frac{VX_C}{R^2 + X_C{}^2}$$

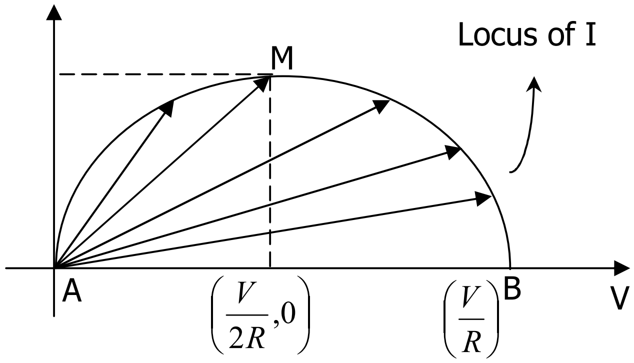
$$= \frac{V}{\sqrt{R^2 + X_C{}^2}} \cdot \frac{R}{\sqrt{R^2 + X_C{}^2}} + j\frac{V}{\sqrt{R^2 + X_C{}^2}} \cdot \frac{X_C}{\sqrt{R^2 + X_C{}^2}} = x + jy$$

$$Where \quad x = I\cos\phi = \frac{VR}{R^2 + X_C{}^2} \quad and \quad y = I\sin\phi = \frac{VX_C}{R^2 + X_C{}^2}$$

$$x^2 + y^2 = I^2 = \frac{V^2}{R^2 + X_C{}^2} = \frac{V^2}{R^2 + X_C{}^2} \cdot \frac{R}{R} = \frac{V}{R} * \frac{VR}{R^2 + X_C{}^2} = \frac{V * x}{R}$$

$$\Rightarrow \left(x - \frac{V}{2R}\right)^2 + (y - 0)^2 = \left(\frac{V}{2R}\right)^2$$

This is an equation of a circle having radius $\frac{V}{2R}$ and center at $\left(\frac{V}{2R}, 0\right)$. The current locus of a series RC circuit with variable C is as shown in figure since the current leads the voltage.



RESONANCE IN AC CIRCUITS

INTRODUCTION

If the voltage applied to a RLC circuit is sinusoidal and happens to have the same frequency as the natural frequency then a very sinusoidal current flows in the network, even with the small voltage applied. This phenomenon is called **RESONANCE**. Signals at hundreds of frequencies are almost available at different locations waiting to be picked up. It is the phenomenon of resonance through resonance networks which enables us to select the signal we want and discord others. Hence resonance is an important practical phenomenon. The use of resonance networks in Radio and communication systems is an established fact.

In general there are two types of resonance in electrical circuits:

- Series resonance and
- Parallel resonance

SERIES RESONANCE IN AC CIRCUITS

SERIES RLC RESONANCE IN AC CIRCUITS

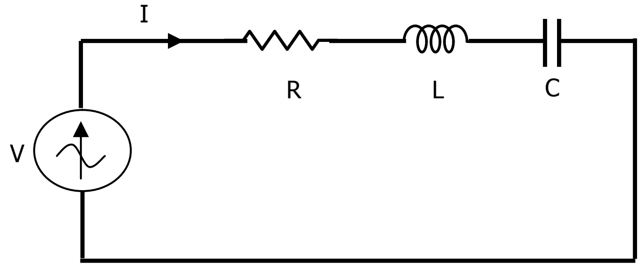


FIG (1): Series RLC Circuit

Let us consider a RLC series circuit with an AC sinusoidal excitation as shown in figure (1). An electrical circuit is said to undergo resonance when the net or total current is in phase with the applied voltage. A circuit at resonance exhibits certain characteristic properties.

A series RLC circuit is said to be in resonance when the net reactance is zero. By varying ω of the supply a Condition that the total reactance is zero can be achieved when

$$I X_C = I X_L$$

$$\frac{1}{C\omega_O} = L\omega_O$$

$$\omega_O = \sqrt{\frac{1}{LC}} \; Rad/\sec$$

$$\therefore the\ resonance\ frequency, f_0 = \frac{1}{2\pi\sqrt{LC}} \; Hz$$

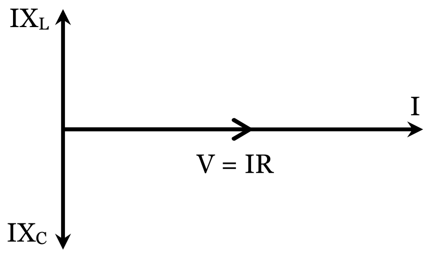


FIG (2): Phasor Diagram

The voltages at resonance condition are shown in figure (2). The impedance at resonance is $Z = R + j0$ is the minimum. The current at resonance is maximum and this value is given by $I_O = \frac{V}{R} \; Amps$.

IMPEDANCE OF RLC CIRCUIT AS FUNCTION OF 'ω'

As ω is varying from 0 to ∞ the impedance of a series RLC circuit is also varying.

$$Z = R + j(L\omega - \frac{1}{C\omega})(or)\, Z = R + j(X_L - X_C)$$

The variation of X_L, X_C and Z are depicted in figure (3).

At resonance frequency, $\omega = \omega_O = \sqrt{\frac{1}{LC}}$. The impedance is minimum and the admittance is maximum. At this instant Z = R and the voltage and current are in phase and the power factor is unity.

The current in RLC circuit is given by,

$$I = \frac{V}{\sqrt{R^2 + (X_L - X_C)^2}} = \frac{V}{\sqrt{R^2 + (L\omega - \frac{1}{LC})^2}}$$

The voltage across R is V_R then

$$V_R = \frac{VR}{\sqrt{R^2 + (L\omega - \frac{1}{C\omega})^2}} = IR$$

Similarly,

$$V_L = \frac{VX_L}{\sqrt{R^2 + (L\omega - \frac{1}{C\omega})^2}} = IX_L$$

$$V_C = \frac{VX_C}{\sqrt{R^2 + (L\omega - \frac{1}{C\omega})^2}} = IX_C$$

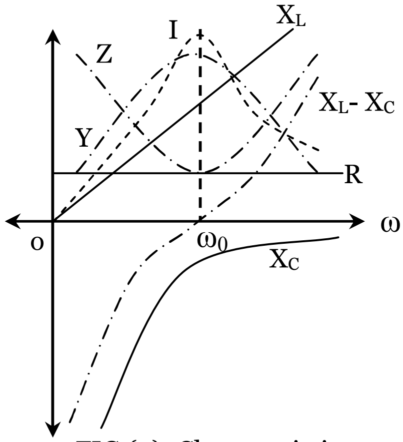


FIG (3): Characteristics

At resonance $V_L = V_C$ and $I_O = \frac{V}{R}$ (maximum resonant current) and for other frequencies 'I' will be less than 'I_O' as shown in figure (4). The peak of this current can be increased with decrease in R. Hence the small resistance is said to give sharp tuning and large resistance broad tuning. A desired frequency can be selected by decreasing the resistance. Effect of resistance on current variation is shown in figure (5). This selection of desired frequency with reduced value of R is called selectivity of RLC series circuit.

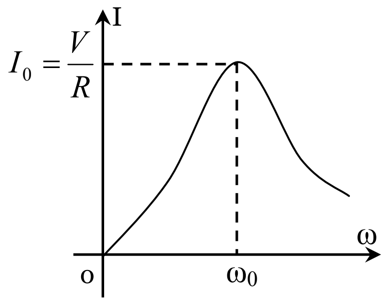


FIG (4): Current

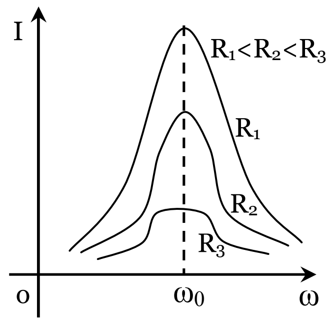


FIG (5): Different R's

QUALITY FACTOR or VOLTAGE MAGNIFICATION FACTOR

The quality factor Q_0 of the resonant circuit may also be called as figure of merit of the circuit (or) magnification factor. The value of Q_0 represents the quality of the circuit in terms of its voltage magnification. The ratio of the voltage developed across L or C to the applied voltage is termed as voltage magnification and is denoted by as 'Q'. We have

$$Q_0 = \frac{(Voltage\ across\ the\ inductor\ at\ resonance)}{Supply\ Voltage} OR \frac{(Voltage\ across\ the\ capacitor\ at\ resonance)}{Supply\ Voltage}$$

$$\therefore Q_0 = \left|\frac{I_0 X_L}{I_0 R}\right| OR \left|\frac{I_0 X_C}{I_0 R}\right| \Rightarrow Q_0 = \frac{L\omega_0}{R} OR \frac{1}{C\omega_0 R}$$

Puttiing $\omega_O = \sqrt{\frac{1}{LC}}$, we get

$$Q_0 = \frac{L}{R\sqrt{LC}} \text{ OR } \frac{1}{CR\left(\frac{1}{\sqrt{LC}}\right)} \Rightarrow Q_0 = \frac{1}{R}\sqrt{\frac{L}{C}}$$

CONDITION FOR MAXIMUM VALUE OF V_L BY VARIATION OF INDUCTANCE

By varying inductance the condition for the maximum voltage across the inductor and the corresponding frequency can be determined as given below,

The voltage across the inductor is

$$V_L = IX_L = \frac{VX_L}{\sqrt{R^2 + (X_L - X_C)^2}}$$

The voltage across the inductor is maximum when $\frac{dV_L}{dX_L} = 0$

$$\frac{dV_L}{dX_L} = \sqrt{R^2 + (X_L - X_C)^2} * V - \frac{VX_L}{2}\left[R^2 + (X_L - X_C)^2\right]^{-\frac{1}{2}} * 2(X_L - X_C) = 0$$

$$\Rightarrow R^2 + X_C{}^2 = X_L X_C$$

$$\Rightarrow X_L = \frac{R^2 + X_C{}^2}{X_C}$$

The frequency at which the voltage across the inductor is maximum is derived as shown below,

$$X_C{}^2 = X_L X_C - R^2$$

$$\Rightarrow \frac{1}{C^2\omega_L^2} = L\omega_L * \frac{1}{C\omega_L} - R^2$$

$$\Rightarrow \omega_L = \sqrt{\frac{1}{CL - C^2R^2}} \quad \text{rad/sec}$$

UNIT-4

NETWORK FUNCTIONS

INTRODUCTION: - A function relating currents (or) voltages at different ports of the network, called a transfer function is found to be mathematically similar to the transform impedance function. These functions are called **Network Functions**.

PORTS [or] TERMINAL PAIRS

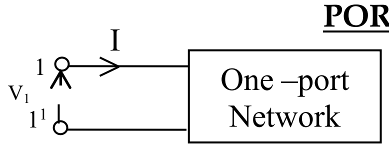


Fig (a)

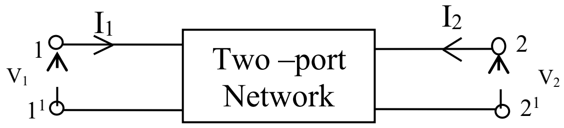


Fig (b)

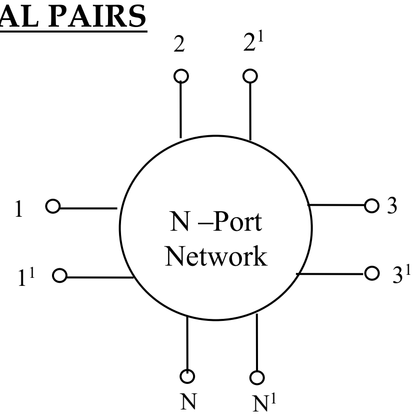


Fig (c)

Fig (a), (b) & (c): REPRESENTION OF ONE PORT, TWO PORT & N-PORT NETWORKS

In fig.(a) is shown a representation of a one-port network. The pair of terminals is connected to an energy source which is the driving force for the network. So that, the pair of terminals is known as the driving point of the network. Fig.(b) shows a two port network. The port $1\text{-}1^1$ is assumed to be connected to the driving force (as an input), and port $2\text{-}2^1$ is connected to a load (as an output). In fig.(c) shows a representation of an N-port network for the general case.

COMPLEX FREQUENCY

We considered only DC and pure sinusoidal as the forcing functions. However, circuits may be subjected to other types of forcing functions also as for example exponentially increasing (or) decreasing sinusoidal. In order to facilitate this study, a new concept of frequency called "complex frequency" is introduced. Thus complex frequencies is defined as $S = \sigma + j\omega$, and by setting σ or ω as both equal to zero, the functions are obtained.

Case (1) Let $\sigma = 0$: we have $S = j\omega$, we know that this is the frequency of a purely sinusoidal alternating function.

Case (2) Let $\omega = 0$; we have $S = \sigma$, a real number, either positive (or) negative. This gives us an exponentially varying function. [Exponentially decreasing if σ is negative]

Case (3) Let $\sigma = 0$ & $\omega = 0$, we have $S = 0$. This is representative of a direct function.

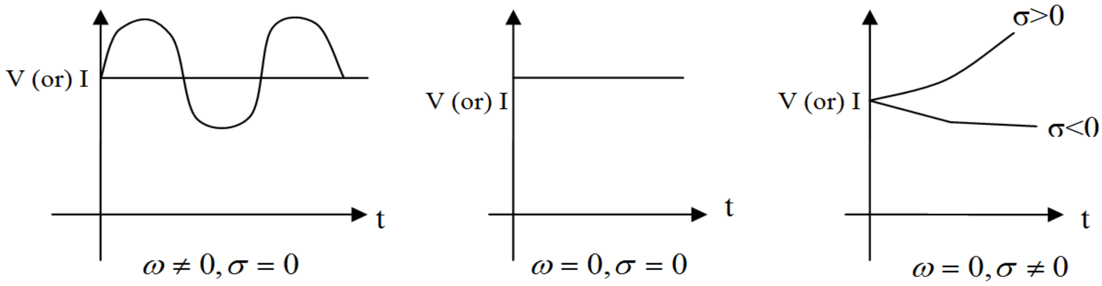


Fig (a), (b) & (c): REPRESENTION Sinusoidal, DC & Exponential excitations

Since 'S' is complex in nature and is also variable, it is termed as "complex frequency variable". Its real part 'σ' is termed as "Neper frequency" with units Nepers / Sec and its imaginary part 'w' is termed as "Radian frequency" with units Radians / Sec.

NETWORK FUNCTIONS

The transform impedance at a port has been defined as the ratio of voltage transform to current transform, with zero initial conditions with no internal voltage (or) current sources except controlled sources. Thus we write

$$Z(S) = \frac{V(S)}{I(S)}$$

Similarly, the transform admittance is defined as the ratio of current transform to voltage transform i.e.,

$$Y(S)=\frac{I(S)}{V(S)}=\frac{1}{Z(S)}$$

The transform impedance and transform admittance must relate to the source port $1-1^1$ (or) $2-2^1$. The impedance (or) admittance found at a given port is called a driving point impedance (or) admittance i.e., transform impedance (or) admittances of port $1-1^1$ and $2-2^1$ are also called as input and output driving point impedances (or) admittances respectively.

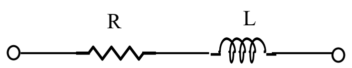
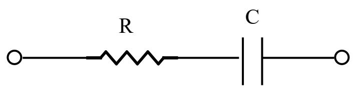
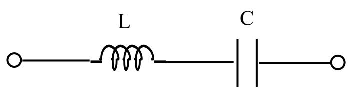
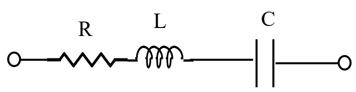
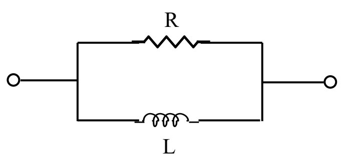
Because of the similarly of impedance & admittance, these two quantities are assigned one name "Immittance" function [combination of impedance and admittance]. An immittance function is thus an impedance function (or) admittance function.

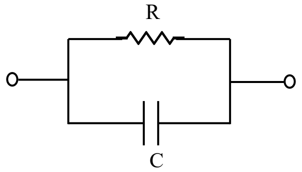
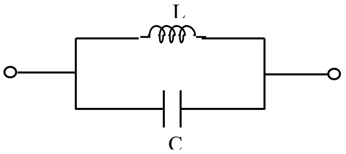
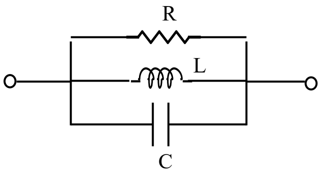
IMMITTANCE FUNCTIONS FOR CIRCUIT ELEMENTS

ELEMENTS	IMPEDANCE FUNCTION Z(S)	ADMITTANCE FUNCTION Y(S)
RESISTOR (R) in Ω	R	$\frac{1}{R}=G$
INDUCTOR (L) in H	SL	$\frac{1}{SL}$
CAPACITOR (C) in F	$\frac{1}{SC}$	CS

Therefore $Z(S)\equiv R\equiv SL\equiv\frac{1}{SC}$ & $Y(S)\equiv\frac{1}{R}=G\equiv\frac{1}{SL}\equiv SC$

IMMITTANC FUNCTION FOR SOME SIMPLE NETWORKS

NETWORKS	IMPEDANCE FUNCTION Z(S)	ADMITTANCE FUNCTION y(S)
	R+SL	$\frac{1}{R+SL}$
	$R+\frac{1}{SC}=\frac{SRC+1}{SC}$	$\frac{SC}{SRC+1}$
	$SL+\frac{1}{SC}=\frac{S^2LC+1}{SC}$	$\frac{SC}{S^2LC+1}$
	$R+SL+\frac{1}{SC}=\frac{SRC+S^2LC+1}{SC}$	$\frac{SC}{SRC+S^2LC+1}$
	$\frac{SRL}{R+SL}$	$\frac{R+SL}{SRL}$

	$\frac{R}{SRC+1}$	$\frac{SRC+1}{R}$
	$\frac{SL}{S^2LC+1}$	$\frac{S^2LC+1}{SL}$
	$\frac{SRL}{S^2RLC+SL+R}$	$\frac{S^2RLC+SL+R}{SRL}$

The transfer function is used to describe networks which have at least two ports. In general, the transfer function relates the transform of a quantity at one part to the transform of another quantity at another part. Thus transfer functions have the following possible forms:

- ✓ The ratio of one voltage to another current (or) one current to another voltage is Z(S) or Y(S).
- ✓ The ratio of one voltage to another voltage called voltage transfer function; G(S)
- ✓ The ratio of one current to another current called current transfer function; α (S)

It is conventional, t define transfer functions as the ratio of an output quantity. In terms of the two-port network, the output quantities are $V_2(S)$ & $I_2(S)$ and the input quantities are $V_1(S)$ and $I_1(S)$. Using this scheme, we have

1. Transfer, impedance function; $Z_{21}(S)=\frac{V_2(S)}{I_1(S)}$
2. Transfer Admittance function; $Y_{21}(S)=\frac{I_2(S)}{V_1(S)}$
3. Voltage transfer function; $G_{21}(S)=\frac{V_2(S)}{V_1(S)}$
4. Current transfer function; $\alpha_{21}(S)=\frac{I_1(S)}{I_2(S)}$

NOTE: - The ratio of an input Quantity to an output quantity is termed as the "Inverse transfer function"

5. Inverse Transfer impedance function; $Z_{12}(S)=\frac{V_1(S)}{I_2(S)}$
6. Inverse transfer Admittance function; $Y_{12}(S)=\frac{I_1(S)}{V_2(S)}$
7. Inverse Voltage transfer function; $G_{12}(S)=\frac{V_1(S)}{V_2(S)}$
8. Inverse current transfer function; $\alpha_{12}(S)=\frac{I_1(S)}{I_2(S)}$

POLES AND ZEROS OF NETWORK FUNCTIONS

If the network functions of a circuit (or) system are known, the behaviour of the circuit can be easily understood. Network functions are generally characterized by their poles and zeros. The poles and zeros of network impedance functions facilitate both network analysis and synthesis. We know that the driving point impedance Z(S) is given as a ratio of two polynomials in the complex frequencies considers the network function.

$$N(S)=\frac{p(S)}{q(S)}=\frac{a_0S^n+a_1S^{n-1}+.....+a_{n-1}S+a_n}{b_0S^m+b_1S^{m-1}+.....+b_{m-1}S+b_m}$$

Where a_0, a_1…, a_n & b_0, b_1, …, b_m are coefficients which are real & positive, if the network is passive and does not contain any dependant energy sources.

Let Z_0, Z_1, …, Z_n be the roots of the equation p(S) =0 and p_0, p_1, ….., p_m the roots of the equation q(S) =0. also let $\frac{z_0}{p_0}=H$. Where H is termed as "Scale factor".

The network function can be expressed in the form

$$N(S)=H\frac{(S-z_1)(S-z_2)......(S-z_n)}{(S-p_1)(S-p_2).....(S-p_m)}$$

By putting S = z_1, z_2, ..z_n, the network function becomes equal to zero. Hence the roots z_1, z_2, …, z_n are termed as "Zeros" of the function. Similarly by putting S = p_1, p_2, .., p_m the network function becomes infinite. Hence the roots p_1, p_2, …, p_m are termed as "poles" of the function.

All the roots are complex frequencies. If z_1, z_2, .. z_n are all unequal, the network function is said to have n distinct zeros. Like wise, if p_1, p_2,…,p_m are all unequal, the network function is said to process m distinct poles. If on the other n and, some of the zeros (or) poles are equal, the zero (or) pole is said to have repeated multiplicity. For example, if a zero repeats it self, say $z_1 = z_2$, it is said to be double (or) have multiplicity 2. Similarly if a pole repeats it self twice, say $p_5 = p_6 = p_7$, it is said to be triple (or) have multiple3.

The poles and zeros of a rational network function for S = 0 & S = ∞ are termed as "external poles & zeros", and all other poles are zeros are termed as "Internal poles & zeros". If both internal & External poles and zeros of a network function are taken into account, then, total no of poles = total no of zeros. A function has either a zeros or a pole at S = 0 and at S = ∞. The poles and zeros of a network function are usually plotted in a S-plane.

Example: - Consider a network function

$$N(S)=\frac{S(S+S)}{(S+2)(S+1+j1)(S+1-j1)}$$

By inspection, for expression N(S), has two zeros at S = 0 & S = -5 and has three poles S = -2, S = - (1+j1) & S = - (1-j1). These poles and zeros are shown in the S-plane. Zeros are shown as circles (or) simple '0' and poles are shown as crosses (or) simple 'X' by standard convention. Such as display of zeros & poles of a network function in the S^2 plane is termed as "pole - zero plots" (or) pole zero configuration". The pole zero plot is very useful in describing the behaviour of the network following fig.

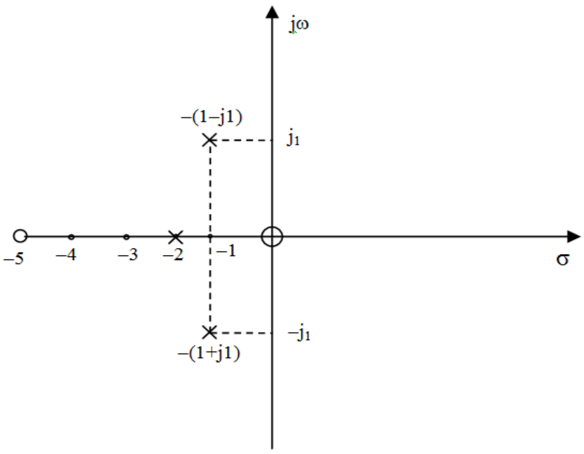


Fig.: Pole – zero plot

SIGNIFICANCE OF POLES & ZEROS

Consider the impedance function $Z_{11}(S)=\frac{V_1(S)}{I_1(S)}$

At every zero of the function, the network impedance is zero, and hence voltage would be zero. This represents a short circuit. Thus at critical frequencies represented by the zeros of the function, the network behaves as if it is short circuited. Also, at critical frequencies represented by the poles of the function, the impedance is infinite and hence current would be zero. This represents open circuit condition.

Similarly we consider the driving point admittance function $Y_{ii}(S)=\frac{I_1(S)}{V_1(S)}$, we see that at every zero of this function, the network admittance is zero and this represents an open–circuit condition. Also at critical frequencies represented by the poles of the function, the admittance of the circuit is infinite and this represents a short –circuit condition. With regard to transfer functions also, it can be shown that the zero's of such functions determine the magnitude of the response and the poles determine the time –domain behaviour of the network response.

NECESSARY CONDITIONS FOR DRIVING POINT FUNCTIONS

(With common factors is p(S) & q(S) cancelled)

1. The coefficients in the polynomials p(S) & q(S) must be real & positive.
2. Poles & zeros must be conjugate if imaginary (or) complex.
3. The real part of all poles & zeros must be negative (or) zero, if the real part is zero, then that pole (or) zero must be simple i.e., all the roots of p(S) = 0 & q(S) = 0 lie on the left half of S –plane and simple roots may lie on the imaginary (or) jω –axis.
4. The polynomials p(S) & q(S) may not have missing terms between those of highest and lowest degrees, unless all even (or) all odd terms are missing.
5. The highest degree of p(S) & q(S) may differ either zero (or) one only.
6. The lowest degree of p(S) & q(S) may differ either zero (or) one only.

NECESSARY CONDITIONS FOR TRANSFER FUNCTIONS

(With common factors in p(S) and q(S) cancelled)

1. The coefficient in the polynomials p(S) & q(S) of $N(S) = \frac{p(S)}{q(S)}$ must be real and those for q(S) must be positive
2. Poles & zeros must be conjugate if imaginary (or) complex.
3. The real part of poles must be negative (or) zero, if the real part is zero, then that pole must be simple. This includes the origin.
4. The polynomial q(S) may not have any missing terms between that of highest & lowest degrees, unless all even (or) all odd terms are missing.
5. The polynomial p(S) may have terms missing between the terms of lowest and highest degree and some of the coefficients may be negative.
6. The degree of p(S) may be as small as zero in dependant of the degree of q(S).
7. a). For G and α :- The maximum degree of p(S) is equal to the degree of q(S)
   b). For Z and Y :- The maximum degree of p(S) is equal to the degree of q(S) plus one.

UNIT-5

NETWORK SYNTHESIS

INTRODUCTION

Two important topics with in the domain of Electric Network Theory are Network Analysis and Network Synthesis. The difference between Network Synthesis and Network Analysis is as follows:

"If the network and excitation (input) are given and the response (output) is to be determine, the problem is defined as **Network Analysis**".

"When the excitation (input) and the response (output) are given and it is required to determine a network, the problem is defined as **Network Synthesis**".

There fore we can say Synthesis by the process of finding a network corresponding to a given Driving Point Impedance (or) Admittance. Such a Synthesis is called Driving Point Synthesis.

The starting point for any network synthesis problem is the network function N(S) which is the ratio of response (output) to the excitation (input). The first step in a network synthesis is to determine whether the network functions N(S) could be realized as a physical passive network.

One of the elements of realizability is **positive real function** which is important because it represents physically realizable passive driving point impedances ie., impedances and admittances. Another element of realizability is a class of polynomial known as **Hurwitz Polynomial**. It is infact the denominator polynomial of the network function satisfying certain conditions.

Basically, there are two methods of synthesizing One – Port network, first one is the partial fraction method, and the network obtained by this method of synthesis is called **FOSTER FORM** of network and the second method of synthesis is continued fraction expansion method, and network obtained by this method of synthesis is called **CAUER FORM** of network. Quite often the term canonic is used for foster and cauer form. Because **CANONIC** means the network contains the possible number of elements.

The elements required in synthesizing these networks are linear, passive and time invariant and are inherently causal ie., in such networks the response (Effect) can not precede the excitation (Cause) and therefore, given the transfer function we need not bother about the causality condition. We must test the stability of the network which indirectly means we are left with the checking of physical realizability of H(S) in terms of the following conditions:

- ✓ H(S) should not have poles in the right half of the S – plane.
- ✓ H(S) should not have multiple poles in the $j\omega$ - axis.
- ✓ The degree of the numerator of H(S) should not be more than unity with reference to the denominator. This means in order to test H(S) for its physical realizability we have to test the denominator polynomial of H(S). Infact the denominator polynomials of the transfer function belong to across of polynomials known as Hurwitz polynomials.

HURWITZ – POLYNOMIAL

A polynomial is said to be Hurwitz if the following conditions are satisfied.

$$N(S) = \frac{p(S)}{q(S)}$$

- ➢ p(S) is real when S is real.
- ➢ The roots of p(S) have real parts which are zero (or) negative which means these lie along $j\omega$ - axis as in the negative half of S – plane.

Properties:-

Suppose p(s) is given as

$$p(s) = a_n S^n + a_{n-1} S^{n-1} + \cdots + a_1 S + a_0$$

Then

- ➢ All the coefficients a_n, a_{n-1}, …,a_0 must be real and positive which means that between the highest order term in '*S*' and the lowest order term. None of the coefficients is zero unless of course the polynomial is even (or) odd.

- Hurwitz polynomial is that either odd or even parts have roots on the $j\omega$ - axis only. If we denote the odd and even parts of p(S) by n(S) and m(S) respectively then

$$p(S) = n(S) + m(S)$$

Then n(S) and m(S) both have roots on the $j\omega$ - axis only. Therefore, if p(S) is either even or odd its roots lies on jω - axis.
- The continued fraction expansion of the ratio of the odd to even parts (or) even to odd parts of p(S) yields all positive quotients.

CONTINUED FRACTION EXPANSION REQUIRES DIVISION AND INVERSION AND HAS THE FOLLOWING IMPORTANT CHARACTERISTICS

- ✓ The continued fraction expansion of even to odd parts (or) vice – versa of a polynomial must be finite in length ie., the process terminates and does not continue indefinitely.
- ✓ If the continued fraction expansion of odd to even parts (or) vice – versa of a polynomial yields positive quotient term, then the polynomial must be Hurwitz to with in a multiplicative factor W(S) ie., if we write

$$F(S) = W(S) \times F_1(S)$$

Then, F(S) is Hurwitz if W(S) and $F_1(S)$ are Hurwitz. Therefore, in order to check whether the given polynomial is Hurwitz (or) not following tests must be carried out.

- All the coefficients of the polynomial must be real and positive and none of them must be missing except if the polynomial has only even (or) odd order terms. This can be done by inspection of the polynomial, no calculation is required.
- The quotients of the continued fraction expansion of even to odd parts (or) vice – versa of the polynomial are positive. This is done by actual carrying out the continued fraction expansion. However, it is to be noted that if p(S) is completely even (or) completely odd then the second parts for continued fraction expansion are obtained by differentiating p(S) with reference to '*S*' ie., the continued fraction be begin with has the parts $\dfrac{p(S)}{p^1(S)}$. It can be seen that if p(S) is completely odd then $p^1(S)$ is completely even and vice – versa.

POSITIVE REAL FUNCTIONS

A very important class of functions is known as **POSITIVE REAL FUNCTIONS**. These functions are important in the sense, that if a function is positive real, this represents a physically realizable passive driving point Immitance.

Now, a function *F(S)* is said to be positive real if it satisfies the following conditions:

- *F(S)* is real for real '*S*' ie., $F(\sigma)$ is real (as S = σ+jω) we take only real part of '*S*'.
- $\text{Real } F(s) \geq 0 \text{ if } \text{Re}(S) \geq 0$

There fore, we provide here an alternative set of necessary and sufficient conditions for a rational function with real coefficient to be positive real. These are:

1. F(S) must have no poles in the right half of the S – plane.
2. F(S) may have only simple poles on the jω - axis with real and positive residues.
3. $\text{Re}\,F(j\omega) \geq 0$, For allω.

Let us understood one by one the implication of these three conditions.

The first one requires that we must test the denominator of F(S) whether it is Hurwitz (or) not which can be obtained by continued fraction expansion of the polynomials.

The second condition is tested by finding partial expansion of F(S) and checking whether the residues of the poles on the jω -axis are positive and real. Therefore if F(S) has a pair of poles at $S = \pm j\omega_1$ a partial fraction expansion gives terms of the form.

$$\frac{K_1}{s - j\omega_1} + \frac{K_2}{S + j\omega_1}$$

The residues of complex conjugate poles are conjugate themselves. If the residues are real which these should be for *F(S)* to be positive real then $K_1 = K_1^*$ so that

$$\frac{K_1}{S-j\omega_1}+\frac{K_1^*}{S+j\omega_1}=\frac{2K_1S}{S^2+\omega^2}$$

Therefore, if K_1 is found to be positive, then F(S) satisfies the second condition.

The third condition we first find the real part of F(jω) from the original function F(S) suppose,

$$F(s)=\frac{P(S)}{Q(S)}$$

Say M_1, M_2 and N_1, N_2 are the odd and even parts of P(S) and Q(S) respectively such that

$$F(S)=\frac{M_1(S)+N_1(S)}{M_2(S)+N_2(S)}$$

Let us obtain even and odd parts of F(S), multiply numerator and denominator of F(S) by $M_2(S)-N_2(S)$ we obtain

$$\frac{M_1(S)+N_1(S)}{M_2(S)+N_2(S)} * \frac{M_2(S)-N_2(S)}{M_2(S)-N_2(S)}$$

$$F(S)=\underbrace{\frac{M_1(S)M_2(S)-N_1(S)N_2(S)}{M_2^2(S)-N_2^2(S)}}_{\boxed{\text{Even function}}}+\underbrace{\frac{M_2(S)N_1(S)-M_1(S)N_2(S)}{M_2^2(S)-N_2^2(S)}}_{\boxed{\text{Odd function}}}$$

Note: - Multiplication of even with even (or) odd with odd gives even function where as multiplication of odd with even (or) Vice – versa gives an odd function.

If we substitute S = jω we find the even part of any polynomial is real where as the odd part is imaginary so that if F (jω) is written as

$$F(j\omega)=\mathrm{Re}[F(j\omega)]+j\,\mathrm{Im}[F(j\omega)]$$

Then

$$\mathrm{Re}\,[F(j\omega)]=\mathrm{Even}[F(S)]_{S=j\omega}$$

$$j\,\mathrm{Im}[F(j\omega)]=\mathrm{Odd}[F(S)]_{S=j\omega}$$

Therefore to test for the third condition for positive realness we determine the real part of F (jω) by finding out the even part F(S) and then substituting S = jω we then check to see whether $\mathrm{Re}[F(j\omega)]\geq 0$ for all ω.

The denominator of $\mathrm{Re}[F(j\omega)]$ is always a positive quantity because

$$M_2^2(j\omega)-N_2^2(j\omega)=M_2^2(j\omega)+N_2^2(j\omega)\geq 0$$

Since A (ω^2) represent an even polynomial and may be written as

$$A(\omega^2)=A_0\,\omega^{2r}+A_2\,\omega^{2r-2}+A_4\,\omega^{2r-4}+\cdots+A_{2r-2}\,\omega^2+A_{2r}$$

If all the coefficients of A (ω^2) are positive then A (ω^2) is positive for all values of ω between 0 and∞. However, if all the A – Coefficients are not positive then a test known as **STURM's** test is carried out which is explained as follows:

Let us put ω^2 = x and assume A (ω^2) = $P_0(x)$. The above equation reduced to

$$A(\omega^2)=P_0(x)=a_0\,x^r+a_1\,x^{r-1}+\cdots+a_{r-1}\,x+a_0$$

Here $P_0(x)$ is the first of a set of functions known as Sturm's functions. The second function is obtained by differentiating the first one, thus

$$P_1(x)=a_0\,r\,x^{r-1}+a_1\,(r-1)\,x^{r-2}+\cdots$$

Now if $P_0(x)$ is divided by $P_1(x)$, it gives a two – term quotient, the remainder is the negative of the next Sturm's function $P_2(x)$ ie.,

$$\frac{P_0(x)}{P_1(x)}=\alpha_1\,x+\alpha_2+\frac{-P_2(x)}{P_1(x)}$$

Here $P_2(x)$ is one degree lower than $P_1(x)$. The division is repeated to yield the next Sturm's function $P_3(x)$ ie.,

$$\frac{P_1(x)}{P_2(x)} = \alpha_3 x + \alpha_4 + \frac{-P_3(x)}{P_2(x)}$$

The procedure is continued till (i) the last Sturm's function P_r of degree zero is found (or) (ii) the remainder resulting from the division process is identically zero.

After we have obtained $P_0, P_1, \cdots\cdots, P_r$, Sturm's theorem states that the number of zeros of $P_0(x)$ in the interval $0 < x < \infty$ is equal to $S_\infty - S_0$ where S_∞ and S_0 are the number of sing changes in the set $(P_0, P_1, \cdots\cdots, P_r)$. Evaluate at x = 0 and x = ∞ respectively. If, however, there are no sign changes ie., $S_\infty - S_0 = 0$ then $A(\omega^2) \geq 0$, for all ω. It is to be noted that $\alpha_0, \alpha_1, \cdots\cdots, \alpha_r$ are of no consequence to find P_0 through P_r and are to be ignored.

PROPERTIES OF POSITIVE REAL FUNCTIONS

The coefficients of the numerator and denominator polynomials in $N(S) = \frac{P(S)}{q(S)}$ are real and positive, as follows

- ✓ N(S) is real when 'S' is real.
- ✓ Complex poles and zeros of N(S) occur in conjugate pairs.
- ✓ The scale factor $H = \frac{a_0}{b_0}$ is real and positive.
- ✓ The poles and zeros of N(S) have either negative or zero real parts.
- ✓ Pole of N(S) on the imaginary axis must be simple and their residues must be real and positive. The same statement applies to the poles of $\frac{1}{N(S)}$.
- ✓ The degrees of numerator and denominator polynomials in N(S) differ at most by 1. Thus the number of finite poles and finite zeros of N(S) differ at must be by 1.
- ✓ The terms of lowest degree in the numerator and denominator polynomials of N(S) differ in degree at most by 1. So, N(S) has neither multiple poles nor zeros at the origin.

The passive elements to be used for synthesizing a network the R, L and C and the impedances and / or admittances of these elements are rational function of 'S' as is seen here under…

- ✓ If F(S) = SL where L is a real positive number, it is a positive real by definition and L is an inductance if F(S) is impedance.
- ✓ F(S) = R where 'R' is a real positive number, here again F(S) is positive real by definition and if F(S) is an impedance 'R' is the resistance.
- ✓ $F(S) = \frac{K}{S}$ Where 'K' is a real positive, here F(S) is positive real by definition and $\frac{1}{K}$ is the capacitance. Here F(S) is positive real because when 'S' is real F(S) is real and when real part of $S > 0, \mathrm{Re}(S) = \sigma > 0$ then $\mathrm{Re}\left(\frac{K}{S}\right) = \frac{K\sigma}{\sigma^2 + \omega^2} > 0$

Therefore, F(S) is positive real. So, the passive impedances are positive real functions. Similarly, it is clear that the admittance.

$$Y(S) = KS; Y(S) = K \text{ and } Y(S) = \frac{K}{S}$$

are positive real if 'K' is real and positive.

LC – IMMITANCE FUNCTIONS

Let us consider the impedance of a passive one – port network.

$$Z(S) = \frac{M_1(S) + N_1(S)}{M_2(S) + N_2(S)} \qquad ---(1)$$

The average power dissipated by the one – port network having reactance elements L and C is zero which mean that the real part of $Z(j\omega)$ is zero ie.,

$$\mathrm{Re}[Z(j\omega)] = [Z(j\omega)]_{Even} = 0$$

$$[Z(j\omega)]_{Even} = \frac{M_1(S)M_2(S) - N_1(S)N_2(S)}{M_2^2(S) - N_2^2(S)} \quad ---(2)$$

which means $M_1(S)M_2(S) - N_1(S)N_2(S) = 0$.

For this situation to raise either of the following cases must hold

(a) $M_1 = 0 = N_2$

(b) $M_2 = 0 = N_1$

From equation (1), for (a) $Z(S) = \frac{N_1(S)}{M_2(S)} \quad ---(3)$ and (b) $Z(S) = \frac{M_1(S)}{N_2(S)} \quad ---(4)$

This means that the driving point Immitance

- Is a ratio of even to odd (or) odd to even polynomials.
- Has poles and zeros of Z(S) (or) Y(S) on the imaginary axis.
- The poles and zeros interlace on the jω – axis ie., the poles and zeros alternate on the jω – axis. This is known as separation property.

The separation property is given by $0 \le \omega_0 \le \omega_1 \le \omega_2 \le \cdots\cdots \le \infty$ where there frequencies in Z(S) are given as

$$Z(S) = \frac{K(S^2+\omega_1^2)(S^2+\omega_3^2)\cdots(S^2+\omega_{2n-1}^2)}{S(S^2+\omega_2^2)(S^2+\omega_4^2)\cdots(S^2+\omega_{2m-2}^2)} \quad ---(5)$$

Here frequencies $\omega_1, \omega_3, \cdots, \omega_{2n-1}$ are known as internal zeros and $\omega_2, \omega_4, \cdots, \omega_{2m-2}$ are known as internal poles. The critical frequencies at S = 0 and S = ∞ are called external critical frequencies, expanding x(S) in to partial fraction we have

$$Z(S) = \frac{K_0}{S} + \frac{2K_2 S}{S^2+\omega_2^2} + \frac{2K_4 S}{S^2+\omega_4^2} + \cdots\cdots + K_\infty S \quad ---(6)$$

Let S = jω we see that $Z(j\omega)$ has zero real part and can thus be rewritten as a pure reactance jx(ω)

$$Z(j\omega) = j\left[\frac{-K_0}{\omega} + \frac{2K_2\omega}{\omega_2^2-\omega^2} + \cdots\cdots + K_\infty\omega\right] \quad ---(7)$$

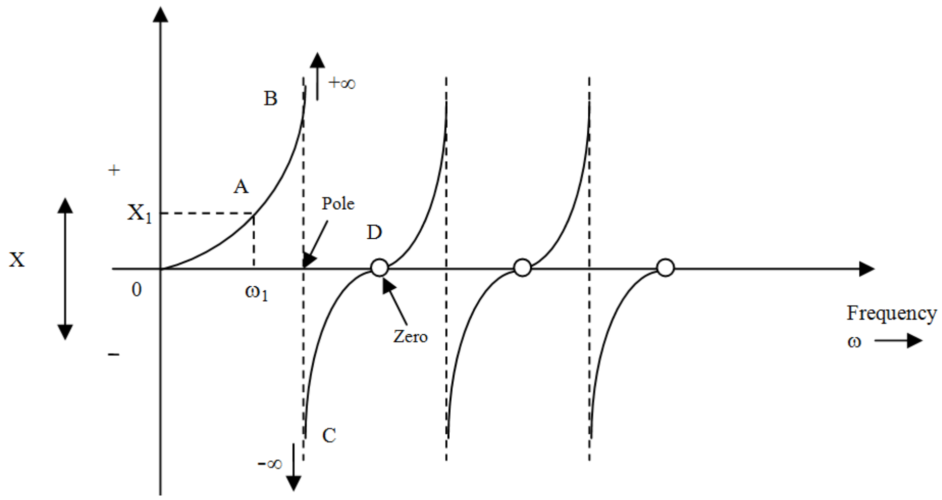
$$= jx(\omega)$$

Differentiating with respect to 'ω' we have

$$\frac{dx(\omega)}{d\omega} = \frac{K_0}{\omega^2} + K_\infty + \frac{2K_2(\omega^2+\omega_2^2)}{(\omega_2^2-\omega^2)^2} + \cdots\cdots \quad ---(8)$$

Since all the residues K_i are positive it is found that for an LC function $\frac{dx(\omega)}{d\omega} \ge 0$

Similarly for an admittance function it can be shown that $\frac{dB(\omega)}{d\omega} \ge 0$ where B (ω) is the susceptance of the LC function.



Thus on plotting X against ω, the slope of the curve is always positive ie., X always increases with the increase of ω. Thus as we increase the frequency from a value ω_1, the impedance X increases from X_1 reaching infinity at a certain higher values as shown. Under this conditions of frequency at infinite reactance, the sign of X changes ie., X(ω) becomes - ∞.

For higher values of ω , reactance X increases becoming less and less negative. Thus, the slope is again positive as given by curve CD in figure. At some frequency, X becomes zero and then beyond this frequency, X again becomes positive and ultimately reaches infinity at a higher frequency. Beyond this frequency, the cycle repeats. Frequencies at which X becomes zero are called **ZERO'S** while the frequencies at which X becomes infinite in magnitude are called **POLE'S**. The zero frequencies are called Anti – resonant frequencies.

Now in a reactive network since $\frac{dX}{d\omega}$ is always positive the poles and zeros must alternate, ie., zero must lie in between two poles and similarly a pole must lie in between two zeros. This property of a reactive network is referred as the separation property.

- The Highest (or) Lowest powers of numerator and denominator must differ by unity.
- With this we observe that at S = 0 at S = ∞; there is always a critical frequency whether zero (or) a pole.

RC – NETWORK FUNCTIONS

The fig.s (a) and (b) shows simple parallel RC and series RC networks. The impedance of the parallel network of fig. (a) is

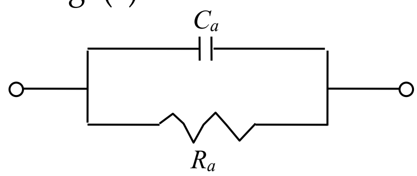


Fig.: (a)

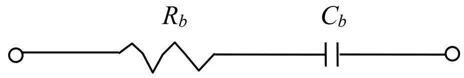


Fig.: (b)

$$Z_a(S) = R_a \| C_a = \frac{1}{C_a}\left(\frac{1}{S + \frac{1}{R_a C_a}}\right) \quad ---(1)$$

and the admittance of the network of figure (b) is $Y_b(S) = \frac{1}{R_b} * \frac{S}{\left(S + \frac{1}{R_b C_b}\right)} \quad ---(2)$

A comparison of equations (1) and (2) shows that both have their poles on the negative real axis in the S – plane, comparison also shows that Z(S) and $\frac{Y(S)}{S}$ have the same from for these two particular networks.

NOTE: - $\rightarrow$

$$\frac{1}{Z_a(S)} = \frac{1}{R_a} + \frac{1}{\frac{1}{C_a S}} = \frac{1}{R_a} + C_a S = \frac{1 + R_a C_a S}{R_a} \quad \text{(or)} \quad Z_a(S) = \frac{R_a}{1 + R_a C_a S} = \frac{1}{C_a\left(S + \frac{1}{R_a C_a}\right)}$$

$\leftarrow$

$\rightarrow$

$$Z_b(S) = R_b + \frac{1}{C_b S} = \frac{R_b C_b S + 1}{C_b S} \quad \text{(or)} \quad Y_b(S) = \frac{1}{Z_b(S)} = \frac{1}{R_b} * \frac{S}{\left(S + \frac{1}{R_b C_b}\right)}$$

$\leftarrow$

The general form of $Z_{RC}(S)$ is as follows

$$Z_{RC}(S) = H\frac{(S+\sigma_1)(S+\sigma_3)\cdots}{S(S+\sigma_2)\cdots} \quad ---(3)$$

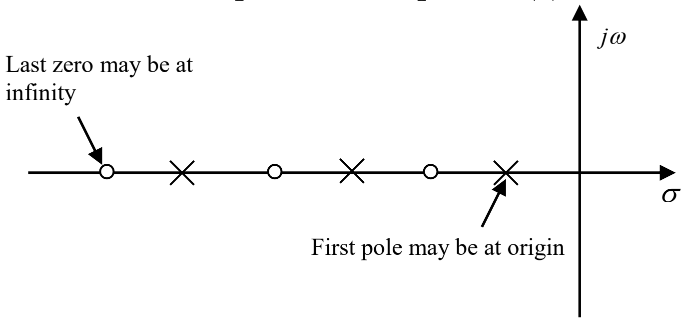
By partial fraction expansion the above equation can be reduced to

$$Z_{RC}(S) = \frac{K_0}{S} + \frac{K_2}{(S+\sigma_2)} + \frac{K_4}{(S+\sigma_4)} + \cdots + K_\infty \quad ---(4)$$

If zero at S = 0, then equation (3) becomes

$$Z_{RC}(S)=\frac{H(S+\sigma_2)(S+\sigma_4)\cdots}{(S+\sigma_1)(S+\sigma_3)\cdots}\quad ---(5)$$

Which has the same of partial fraction expansion as equation (4) but with $K_0 = 0$



From these results we may state the following properties of RC impedance functions:

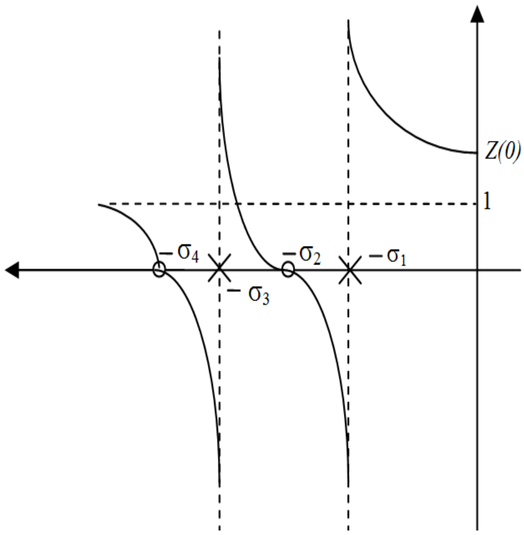
1. All poles and zeros are simple and are located on the negative real axis of the S – plane.
2. Poles and Zeros are interlaced.
3. The lowest critical frequency is a pole which is at the origin only if $K_0 \neq 0$.
4. The highest critical frequency is a zero which is at infinity only if $K_\infty = 0$.
5. The residues evaluated at the poles of $Z_{RC}(S)$ are real and positive.

Now that the poles and zeros of RC impedances are on the – σ axis, let us find the relative locations of the critical frequency, for which, we find out the slope of Z(S) at S = σ

$$\frac{dZ_{RC}(S)}{d\sigma}=\frac{K_0}{\sigma^2}+\frac{-K_1}{(\sigma+\sigma_1)^2}+\cdots+\frac{-K_i}{(\sigma+\sigma_i)^2}$$

It is clear that $\frac{dZ_{RC}(S)}{d\sigma}<0$ ie., it is negative.

Since the slope of $Z_{RC}(\sigma)$ is always negative as – σ decreases (or) σ increases, $Z_{RC}(\sigma)$ must increase until at $S=\sigma_1$, $Z(-\sigma_1)=\infty$. At $\sigma=-\sigma_1$, $Z(\sigma)$ changes sign and is negative because between $-\sigma_1$ & $-\sigma_2$, the term in the denominator $(\sigma+\sigma_1)$ becomes negative whereas the other three factors are positive and it continues until the next critical frequency $(-\sigma_2)$ is reached where the function is zero. Since $Z_{RC}(\sigma)$ increases for decreasing – σ, the third critical frequency must be a pole at $S=-\sigma_3$. As $Z_{RC}(\sigma)$ changes sign at $-\sigma_3$, the final critical frequency must be zero at $S=-\sigma_4$. Beyond $\sigma=-\sigma_4$ the curve becomes equal to Z(∞) = 1.



RL – NETWORK FUNCTIONS

The fig.s (a) & (b) shows simple parallel RL and series RL networks. The impedance of the parallel network of fig.(a) is

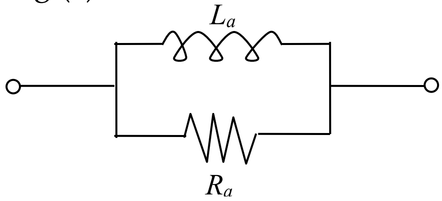


Fig.: (a)

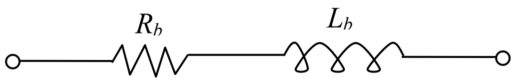


Fig.: (b)

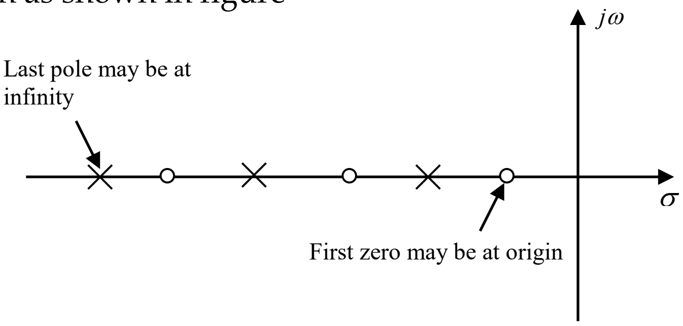
$$Z_a(S)=R_a \| L_a=\frac{SL_aR_a}{R_a+SL_a}=R_a\frac{S}{\left(S+\frac{R_a}{L_a}\right)}\quad ---(1)$$

and the admittance of the network of figure (b) is $Y_b(S) = \dfrac{\dfrac{1}{L_b}}{\left(S + \dfrac{R_b}{L_b}\right)} \quad ---(2)$

A comparison of equations (1) and (2) shows that both have their poles on the negative real axis in the S – plane, comparison also shows that $\dfrac{Z(S)}{S}$ and $Y(S)$ have the same form for these two particular networks. The general form of $Z_{RL}(S)$ is as follows

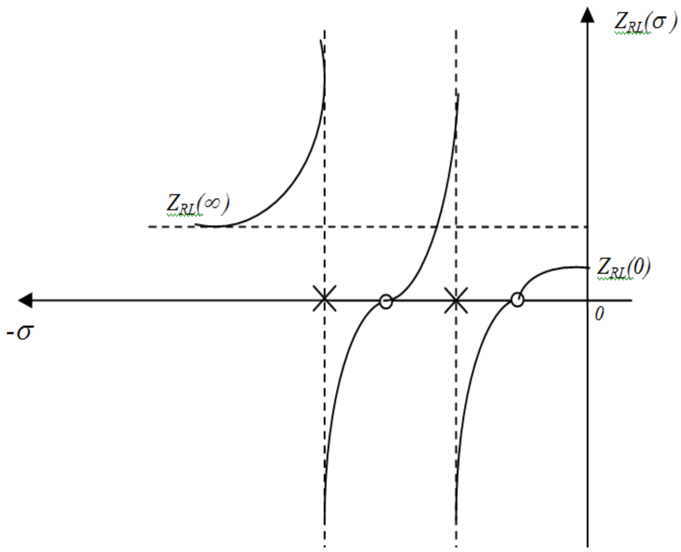
$$Z_{RL}(S) = K_0 + \frac{K_1 S}{S+\sigma_1} + \frac{K_2 S}{S+\sigma_2} + \cdots + \frac{K_i S}{S+\sigma_i} + \cdots + K_\infty S$$

The pole zero configuration as shown in figure



From these results we may state the following properties of RL impedance functions:

1. Poles and zeros are simple and are located on the negative real axis of the S – plane.
2. Poles and zeros are interlaced.
3. The singularity nearest to (or at) the origin is a zero. The singularity nearest to (or at) S = ∞ must be a pole.
4. The residues of $Z_{RL}(S)$ the poles must be real and negative however the residues of $\dfrac{Z_{RL}(S)}{S}$ are real and positive.
5. The slope of $Z_{RL}(S)$ is positive and $Z_{RL}(\infty) > Z_{RL}(0)$.
6. The plot of $Z_{RL}(S)$ as a function of σ are as shown in fig.



<u>Realization of canonic forms [Foster & Cauer Forms]</u>

Forster form– I : –

Since we know that the poles & zeros of an LC immittance function lie on the $j\omega-$axis, the partial fraction of the LC immittance function will, in general, be of the form

$$F(S) = \frac{K_0}{S} + \frac{2K_2 S}{S^2+\omega_2^2} + \frac{2K_4 S}{S^2+\omega_4^2} + \cdots\cdots + K_\infty S \qquad ---(1)$$

While synthesizing such a network each term in the partial fraction is associated with an element (or) a pair of elements and then these are to be connected in series. For example if F(S) is Z(S) then the term $\dfrac{K_0}{S}$ represents a capacitor of $\dfrac{1}{K_0}$ Farads, the term $K_\infty S$ an inductor of K_∞ Henry and the term $\dfrac{2K_2 S}{S^2+\omega_2^2}$ represents a parallel combination of L & C where C is $\dfrac{1}{2K_2}$ Farads and the inductor is of $\dfrac{2K_2}{\omega_2^2}$ Henry and the elements are connected as shown in Table.

It is to be noted that since all poles of LC immittance function lie on $j\omega-$axis, we can remove all the poles simultaneously using partial fraction. If Z(S) has no pole at the origin [and so it has a zero at origin] the first term in the partial fraction will be missing [C_0 will be absent]. Similarly, if there is a zero rather than a pole at infinity then the inductor will be absent.

Forster form – II : –

However if the function F(S) is given in admittance form Y(S), then the partial fraction expansion of Y(S) gives a circuit consisting of parallel branches. Let

$$Y(S)=\frac{K_0}{S}+\frac{2K_2S}{S^2+\omega_2^2}+\frac{2K_4S}{S^2+\omega_4^2}+\cdots\cdots+K_\infty S \qquad ---(2)$$

Each term here represents an admittance of an element (or) a pair of elements to be connected across a two port network. $\frac{K_0}{S}$ represent an inductor element of $\frac{1}{K_0}$ Henry and $K_\infty S$ represents capacitor of K_∞ Farads. Similarly, if L & C are in series, their admittance is $L=\frac{1}{2K_2}$ Henry & $C=\frac{2K_2}{\omega_2^2}$ Farads. Therefore, the network takes the shape as shown in Table.

Cauer form– I : –

Continued fraction expansion is used in cauer method. In first cauer from we arrange the polynomials in the numerator and denominator in the descending powers of S and we eliminate during every step a pole at infinity $(S=\infty)$. If the original function is Z(S) with the order of numerator is 2n, then that of the denominator is 2n–1, we eliminate a pole at infinity by diving numerator by the denominator, and thereby we get the quotient is $Z_1(S)$. Therefore, the remainder function $Z_2(S)$ is still LC immittance function and is

$$Z_2(S)=Z(S)-Z_1(S)$$

Now the denominator of $Z_2(S)$ is of the order 2n–1 where as the numerator becomes 2n–2 as the difference in power must be one. Since in cauer-I, we always eliminate a pole at infinity, therefore, we invert $Z_2(S)$ to obtain,

$$Y_2(S)=\frac{1}{Z_2(S)}$$

Again divide this numerator of $Y_2(S)$ by its denominator we have $Z_3(S)$ which will forms the series element. Next we again invert $Y_4(S)$ so that we can have a pole at infinity. The process is continued till the remainder is zero. In fact, this process is known as continued fraction expansion and is given as follows:

$$Z(S)=Z_1(S)+\cfrac{1}{Y_2(S)+\cfrac{1}{Z_3(S)+\cfrac{1}{Y_4(S)+\cfrac{1}{Z_5(S)+\cdots\cdots}}}}$$

Since the expansion of $Z(S)$ looks a ladder, the network so synthesized are known as ladder network.

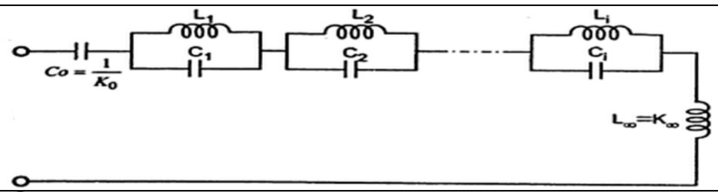
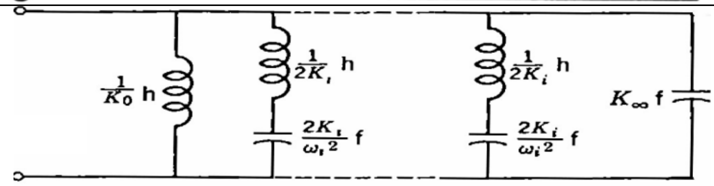
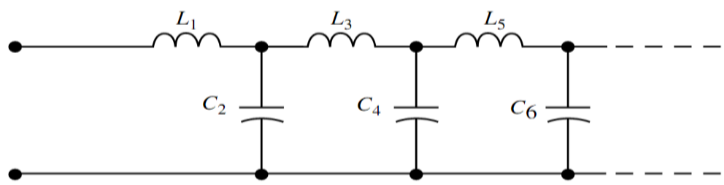
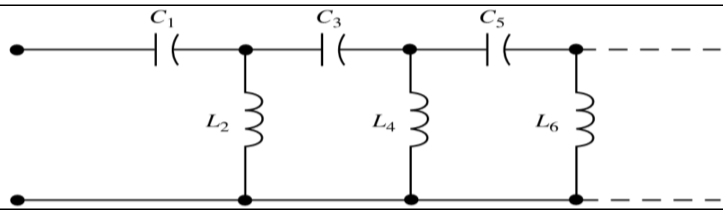
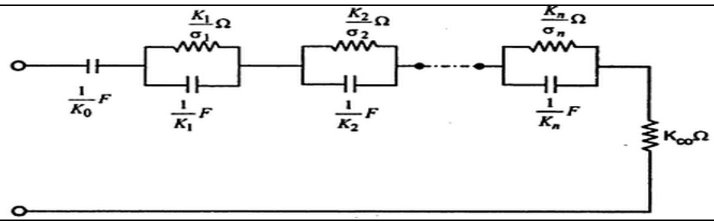
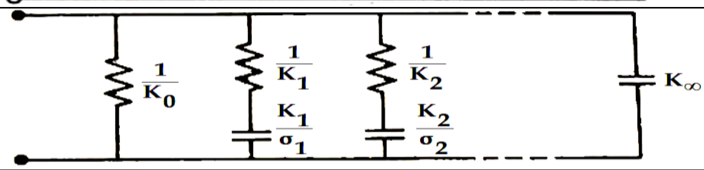
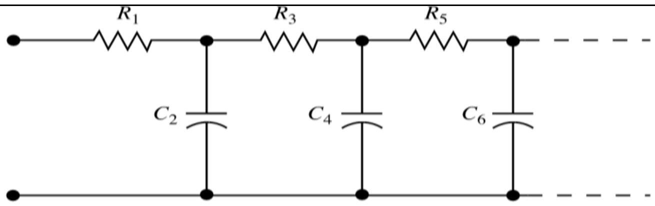
Whenever be the form of the original function whether $Z(S)$ (or) $Y(S)$, in first form of cauer a pole is eliminated at infinity $(S=\infty)$ at every step. Therefore, if the numerator of the original function has power of S smaller than that of the denominator, it should be inverted and the continued fraction expansion carried out. The final network synthesized is a ladder network whose series elements are inductor & shunt elements are capacitors shown in Table.

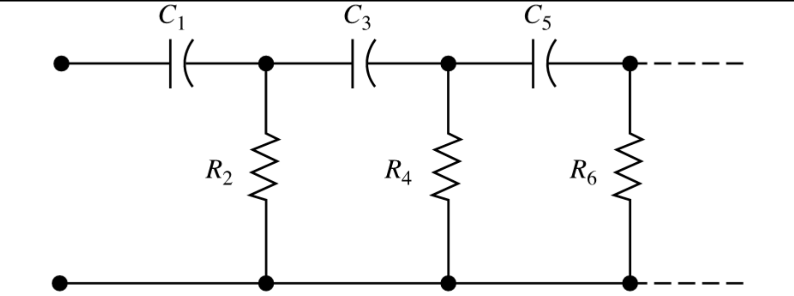
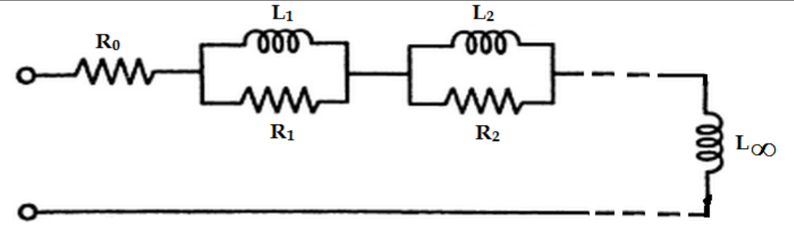
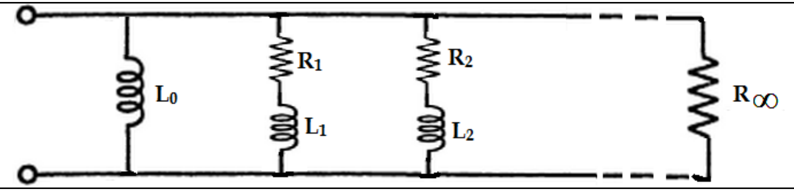
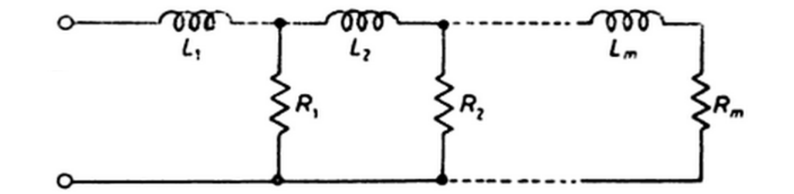
Cauer form– II : –

Here we arrange the given function polynomials (numerator as well as denominator) in the ascending order power of S and remove a pole at origin $(S=0)$ successively. Since the lowest degree of numerator & denominator of an LC admittance must differ by one, it follows that there must be a zero (or) a pole at $(S=0)$. After arranging the numerator & denominator in the ascending power of S we divide the lowest power of the numerator by the lowest power of denominator, and then we invert the remainder & divide again. With this we will have an alternate form of ladder network which will have series capacitors and shunt (reactors) inductors. Series capacitor impedance has a pole at origin whereas shunt reactor admittance has a pole at origin. A general network is shown in Table.

$$Z(S)=Z_1(S)+\cfrac{1}{Y_2(S)+\cfrac{1}{Z_3(S)+\cfrac{1}{Y_4(S)+\cfrac{1}{Z_5(S)+\cdots\cdots}}}}$$

Table: Foster forms – I & II and Cauer forms – I & II with LC, RC and RL Circuits

S. No.	Form	Description	Circuit
			LC CIRCUIT
1.	FOSTER FORM – I	Partial fraction expansion of Z(S)	
2.	FOSTER FORM – II	Partial fraction expansion of Y(S)	
3.	CAUER FORM – I	Continued fraction expansion of Z(S) (or) Y(S) about infinity	
4.	CAUER FORM – II	Continued fraction expansion of Z(S) (or) Y(S) about zero	
			RC CIRCUIT
1.	FOSTER FORM – I	Partial fraction expansion of Z(S)	
2.	FOSTER FORM – II	Partial fraction expansion of Y(S)	
3.	CAUER FORM – I	Continued fraction expansion of Z(S) (or) Y(S) about infinity	

4.	CAUER FORM – II	Continued fraction expansion of Z(S) (or) Y(S) about zero	
RL CIRCUIT			
1.	FOSTER FORM – I	Partial fraction expansion of Z(S)	
2.	FOSTER FORM – II	Partial fraction expansion of Y(S)	
3.	CAUER FORM – I	Continued fraction expansion of Z(S) (or) Y(S) about infinity	
4.	CAUER FORM – II	Continued fraction expansion of Z(S) (or) Y(S) about zero