LECTURE NOTES ON

NETWORK ANALYSIS
AND
TRANSMISSION LINES (R20A0262)

B.TECH -ECE -II-I

DEPARTMENT OF ELECTRONICS & COMMUNICATION ENGINEERING

MALLA REDDY COLLEGE OF ENGINEERING &TECHNOLOGY

(Affiliated to JNTU Hyderabad Approved by AICTE - Accredited by NBA & NAAC – 'A' Grade - ISO 9001:2015 Certified) Maisammaguda, Dhulapally (Post Via. Kompally), Secunderabad – 500100, Telangana State, India

MALLA REDDY COLLEGE OF ENGINEERING AND TECHNOLOGY

II Year B.Tech. ECE-I SEM **L T/P/D C**
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(R20A0262) NETWORK ANALYSIS & TRANSMISSION LINES

COURSE OBJECTIVES:

1. To understand the basic concepts of RLC circuits.
2. To know the behavior of the R,L,C in Steady state and Transient states
3. To understand the two port network parameters
4. To draw the Locus diagram of series and parallel RL,RC circuits and understand the concept of Resonance
5. To study the propagation, reflection and transmission of plane waves in bounded and Unbounded media

UNIT – I:

D.C.Transient Analysis (First and Second Order Circuits):

Introduction to transient response and steady state response, Transient response of series –RL, RC, RLC Circuits for D.C.excitation, Initial Conditions, Solution using Differential Equations approach and Laplace Transform method, Illustrative problems

UNIT – II:

Two Port Networks:

Impedance Parameters, Admittance Parameters, Hybrid Parameters, Transmission (ABCD) Parameters, Conversion of one parameter to another parameter, Conditions for Reciprocity and Symmetry, Interconnection of two port networks in Series, Parallel and Cascaded configurations, Illustrative problems.

UNIT-III:

Locus diagrams: Locus diagrams of Series and Parallel RL, RC, RLC circuits with variation of various parameters

Resonance: Resonance-Series and Parallel circuits, Concept of Bandwidth and Quality factor.

UNIT – IV:

Transmission Lines – I: Types, Parameters, Transmission Line Equations, Primary & Secondary Constants, Expressions for Characteristics Impedance, Propagation Constant, Phase and Group Velocities, Infinite Line Concepts, Losslessness/Low Loss Characterization, Distortion – Condition for Distortionlessness and Minimum Attenuation, Illustrative Problems.

UNIT V:

Transmission Lines – II: SC and OC Lines, Input Impedance Relations, Reflection Coefficient, VSWR, λ/4, λ 2, λ /8 Lines – Impedance Transformations, Significance of Zmin and Zmax, Smith Chart – Configuration and Applications, Single Stub Matching, Illustrative Problems.

TEXT BOOKS:

1. Electrical Circuits – A. Chakrabarhty, Dhanipat Rai & Sons.
2. Network Analysis – N.C Jagan and C. Lakhminarayana, BS publications.
3. A Text book of Electrical Technology by B.L Theraja and A.K Theraja, S.Chand publications
4. Basic Concepts of Electrical Engineering – PS Subramanyam, BS Publications.
5. Transmission Lines and Networks – Umesh Sinha, Satya prakashan, 2001, (Tech. India

Publications), New Delhi.

REFERENCE BOOKS:

1. Engineering Circuits Analysis – William Hayt and Jack E. Kemmerly, Mc Graw Hill Company, 7th Edition.
2. Basic Electrical Engineering – S.N. Singh PUI.
3. Electrical Circuits – David A. Bell, Oxford Printing Press.
4. Principles of Electrical Engineering by V.K Mehta, Rohit Mehta, S.Chand publications.
5. Electrical Circuit Analysis – K.S. Suresh Kumar, Pearson Education.

COURSE OUTCOMES:

- Upon successful completion of the course, students will be able to: Gain the knowledge on basic RLC circuits' behavior
- Analyze the Steady state and transient analysis of RLC Circuits
- Know the characteristics of two port network parameters.
- Determine the frequency response of RLC circuit using Locus diagram and apply the concept of Resonance in various circuits
- Analyze the transmission line parameters and configurations.

UNIT-I

DC TRANSIENTS

1. Introduction to Transient Response and Steady state Response
2. Transient Response series RL, RC, RLC circuits for D.C. Excitation
3. Initial Conditions
4. Solution using Differential Equation Approach and Laplace Transform Method
5. Illustrative Problems

Introduction:

Transients

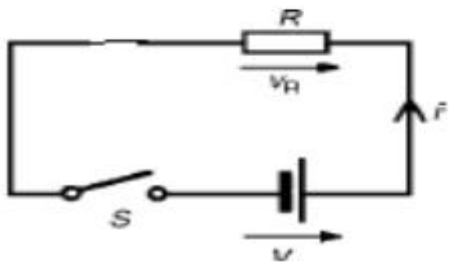
Whenever the electrical power supplied to a circuit changes momentarily over a short duration of time**, it is called transients.**

Transients invariably affect the voltage and current. AC and DC circuits are equally vulnerable to transients, and steady-state values are reached after the transient period.

For higher order differential equation, the number of arbitrary constants equals the order of the equation. If these unknowns are to be evaluated for particular solution, other conditions in network must be known. A set of simultaneous equations must be formed containing general solution and some other equations to match number of unknown with equations.

We assume that at reference time t=0, network condition is changed by switching action. Assume that switch operates in zero time. The network conditions at this instant are called initial conditions in network.

1. Resistor :

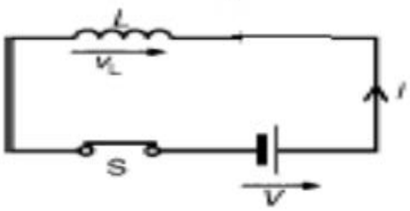


$$V = i \times R \longrightarrow (1)$$

Equ: 1 is linear and also time dependent. This indicates that current through resistor changes if applied voltage changes instantaneously. Thus in resistor, change in current is instantaneous as there is no storage of energy in it.

2.Inductor:

$$V_L = L \bullet \frac{diL}{dt} \longrightarrow (2)$$



If dc current flows through inductor, dil/dt becomes zero as dc current is constant with respect to time. Hence voltage across inductor, VL becomes zero. Thus, as for as dc quantities are considered, in steady stake, inductor acts as short circuit.

We can express inductor current in terms of voltage developed across it as

$$iL = \frac{1}{L}\int V_L dt$$

In above eqn. The limits of integration is from - ∞ to $t(0^-)$.

Assuming that switching takes place at t=0, we can split limits into two intervals as - ∞ to $t(0^-)$.

$$i_L = \frac{1}{L}\int_{-\infty}^{t} V_L dt$$

$$i_L = \frac{1}{L}\int_{-\infty}^{0-} V_L dt + \frac{1}{L}\int_{0^-}^{t} V_L dt$$

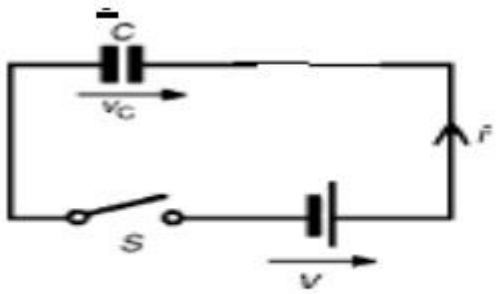
$$i_L = i_L(0^-) + \frac{1}{L}\int_{0^-}^{t} V_L dt$$

at $t = 0^t$ we can write $i_L(0^+) = i_L(0^-) + \frac{1}{L}\int V_L dt$

$i_L(0^+) = i_L(0^-)$

I through inductor cannot change instantaneously.

3.capacitor



$$i_C = C\frac{dV_C}{dt}$$

If dc voltage is applied to capacitor, *dVC / dt* becomes zero as dc voltage is constant with respect to time.

Hence the current through capacitor iC becomes zero, Thus as far as dc quantities are considered capacitor acts as open circuit.

$$V_C = \frac{1}{C}\int i_C dt$$

$$V_C = \frac{1}{C}\int_{-\infty}^{t} i_C dt$$

Splitting limits of integration

$$V_C = \frac{1}{C}\int_{-\infty}^{0-} i_C dt + \frac{1}{C}\int_{0^-}^{t} i_C dt$$

At $t(0^+)$, equation is given by

$$V_C(0^+) = V_C(0^-) + \frac{1}{C}\int_{0-}^{0+} i_C dt$$

$$V_C(0^+) = V_C(0^-)$$

Thus voltage across capacitor can not change instantaneously.

Let us consider the system as shown in figure.

Let (t)=Input or Excitation to the system y(t)Output or

Response from the system

Total response of the system=Transient Response+Steadystate response

$Y(t) = Y_{tr}(t) + Y_{ss}(t)$

$Y_{ss}(t) = \lim_{t\to\infty} (t) = (\infty)$

Y_{tr} (t) =Y (t)-Y_{ss} (t)

Problem: Y (t) =4+e^{-4t}.Find steady state response and transient response?

Solution:

$Y(t) = Y_{tr}(t) + Y_{ss}(t)$

Y (t) =4+e^{-4t}

$Y_{ss}(t) = \lim_{t\to\infty} (t) = (\infty)$

$Y_{ss}(t)$=4+$e^{-4\infty}$

$Y_{ss}(t)$=4+$e^{-\infty}$

$Y_{ss}(t)$=4+$e^{-\infty}$

$Y_{ss}(t)$=4+0

$Y_{ss}(t)$=4

Y_{tr} (t) =Y (t)-Y_{ss} (t)

Y_{tr} (t) =4+e^{-4t} −4

Y_{tr} (t) =e^{-4t}

Y (t) =4+e^{-4t}

- A Circuit having constant sources is said to be in steady state if current and voltages do not change with time.
- The amplitude and frequency of sinusoidal wave never changes in steady state circuit.
- At steady state $\frac{dV}{dt} = \mathbf{0}, \frac{dI}{dt} = \mathbf{0}$
- In a network containing energy storage elements, with change in excitation, the current and voltages change from one state to another state. The behavior of voltage and current when it is changed from one state to another state is called transient state.
- At Transient state $\frac{dV}{dt} \neq \mathbf{0}, \frac{dI}{dt} \neq \mathbf{0}$
- The time taken for the circuit to change from one steady state to another steady state is called transient time
- The application of KVL and KCL to the circuits containing energy storage results differential equations rather than algebraic equations
- When we consider a circuit containing storage elements which are independent of sources, the response depends upon the nature of circuit and is called natural response
- Storage elements deliver their energy to the resistances. Hence the response changes with time ,and is referred as transient response
- When we consider sources acting on a circuit, the response depends on the nature of the source or sources. This response is called forced response
- Total response of the system=Transient Response+Forced resonse
- Solution of differential equation=Complimentary function Particular solution
- The complimentary function dies out after short interval and is referred as transient response or source free response
- The particular solution is steady state response or forced response

- The first step in finding the complete solution to the circuit is to form a differential equation
- Several methods can be used to find out complete solution

DC response of Series RL circuit:

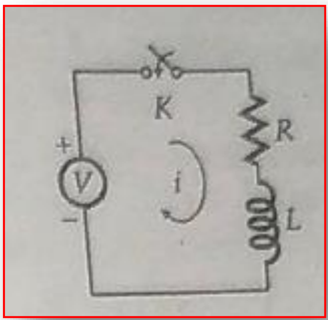


Fig. 1.1 Series RL Circuit

Assume inductor is initially uncharged

When switch is closed, we can find out complete solution for the current

$$V = iR + L\,di/dt$$

$$\frac{di}{dt} + \frac{R}{L}i = \frac{V}{L}$$

Comparing above differential equation with

$$\frac{dy}{dx} + Py = Q$$

Then: $y = i, x = t, P = \frac{R}{L}, Q = \frac{V}{L}$

Integrating Factor=I.F. $= e^{\int Pdx} = e^{P\int dx} = e^{Px}$

$$y \times I.F = \int I.F. \times Q dx$$

$$y \times e^{Px} = \int e^{Px} \times Q dx$$

$$y \times e^{Px} = Q \int e^{Px}\, dx$$

$$y \times e^{Px} = Q\frac{e^{Px}}{p} + K$$

$$y = Q\frac{e^{Px}}{p}e^{-Px} + Ke^{-Px}$$

$$y = \frac{Q}{p} + Ke^{-Px}$$

K=constant

$$y = i, x = t, P = \frac{R}{L}, Q = \frac{V}{L}$$

$$i = \frac{\frac{V}{L}}{\frac{R}{L}} + Ke^{-\frac{R}{L}t}$$

$$i = \frac{V}{R} + Ke^{-\frac{R}{L}t}$$

To get complete solution of i we need to find out k

The value of k can be calculated from initial conditions

At $t=0^-$ switch is open

At this condition i (t) =i (0^-) =0

As it is a series circuit $i = i_L$

i_L (0^-)=0

Inductor opposes sudden change in current

$i_L(0^-)= i_L(0)= i_L(0^+)=0$

Hence i_L (0) = 0

As it is a series circuit $i=i_L$

i (0)=0

$$i(t) = \frac{V}{R} + Ke^{-\frac{R}{L}t}$$

$$i(0) = \frac{V}{R} + Ke^{-\frac{R}{L}0}$$

$$0 = \frac{V}{R} + Ke^{-0}$$

$$0 = \frac{V}{R} + Ke^{0}$$

$$0 = \frac{V}{R} + (1)$$

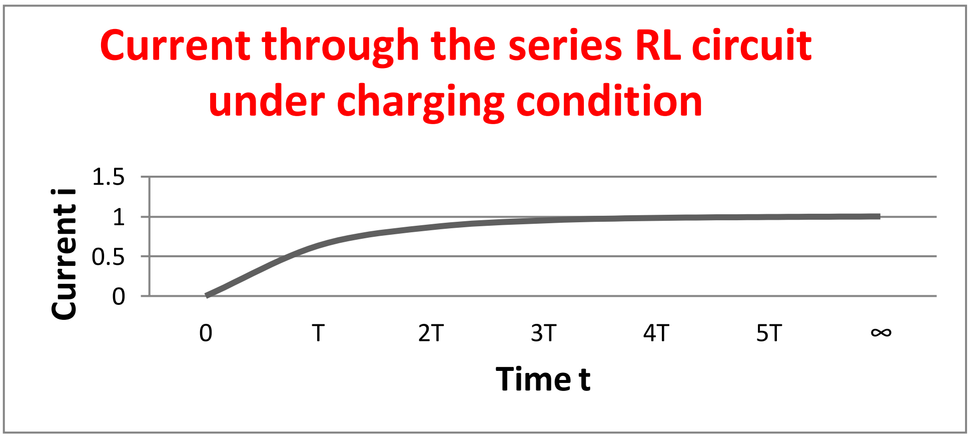
$$\boldsymbol{K = -\frac{V}{R}}$$

$$i(t) = \frac{V}{R} + Ke^{-\frac{R}{L}t}$$

$$i(t) = \frac{V}{R} - \frac{V}{R}e^{-\frac{R}{L}t}$$

$$\boldsymbol{i(t) = \frac{V}{R}(1 - e^{-\frac{R}{L}t})}$$

S.NO.	Time(t)	$i(t) = \frac{V}{R}(1 - e^{-\frac{R}{L}t})$
1	0	0
2	T	$0.6321\frac{V}{R}$
6	5T	$0.9932\frac{V}{R}$
7	∞	$\frac{V}{R}$



Time constant: For standard charging circuit it is the time at which response is 63.2% of its final value

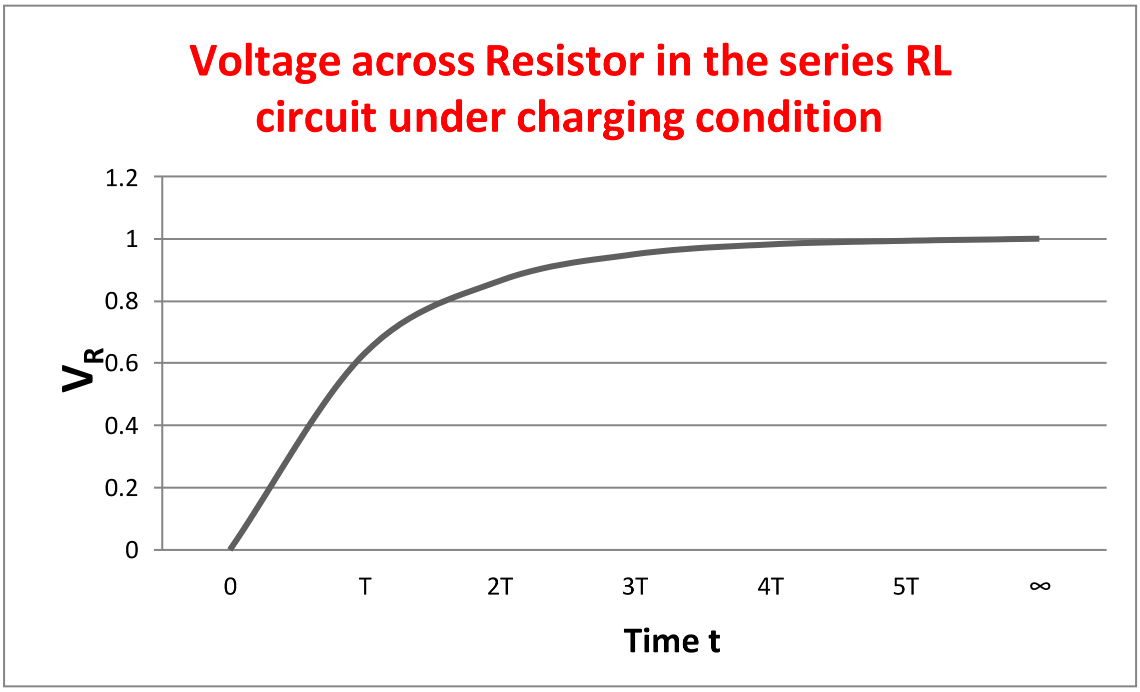
Settling Time: It is the time required by the circuit to reach steady state.

- Ideal settling time=∞
- If the error is 2% then settling time=4T, this concept is used in network analysis
- If the error is 1% then settling time=5T, this concept is used in control systems

Voltage across Resistor V_R=iR

$V_R = \frac{V}{R}(1 - e^{-\frac{R}{L}t})$R

$$V_R = V\,(1 - e^{-\frac{R}{L}t})$$



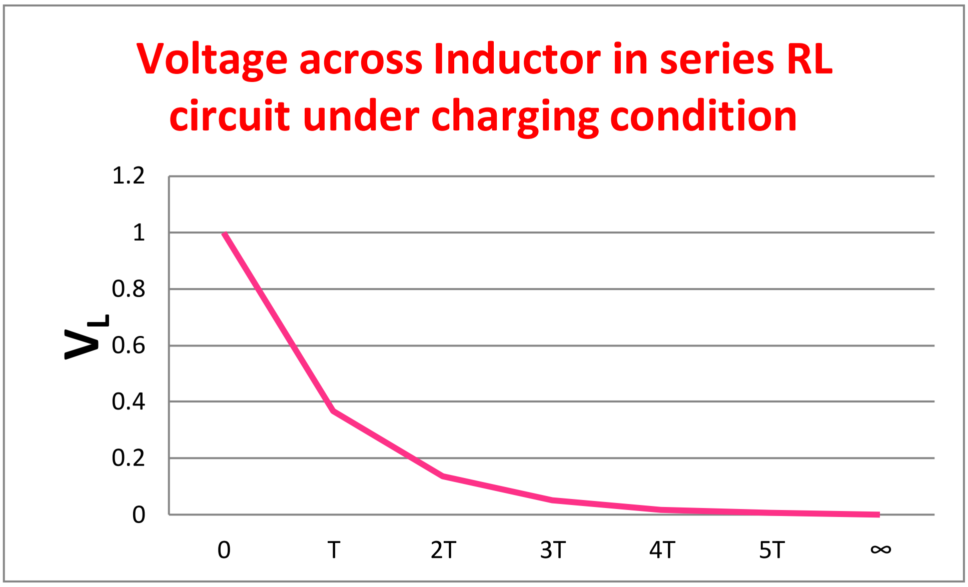
$V = V_R + V_L$

$V_L = V - V_R$

$V_L = V - \mathrm{V}\,(1 - \mathrm{e}^{-\frac{\mathrm{R}}{\mathrm{L}}\mathrm{t}})$

$$V_L = V - \mathrm{V} + \mathrm{V}e^{-\frac{\mathrm{R}}{\mathrm{L}}\mathrm{t}}$$

$$V_L = \mathrm{V}e^{-\frac{\mathrm{R}}{\mathrm{L}}\mathrm{t}}$$



Instantaneous power through the resistor $P_R = V_R \times i$

$$P_R = \boldsymbol{V}\,(\boldsymbol{1} - \boldsymbol{e}^{-\frac{\boldsymbol{R}}{\boldsymbol{L}}\boldsymbol{t}}) \times \frac{\boldsymbol{V}}{\boldsymbol{R}}(\boldsymbol{1} - \boldsymbol{e}^{-\frac{\boldsymbol{R}}{\boldsymbol{L}}\boldsymbol{t}})$$

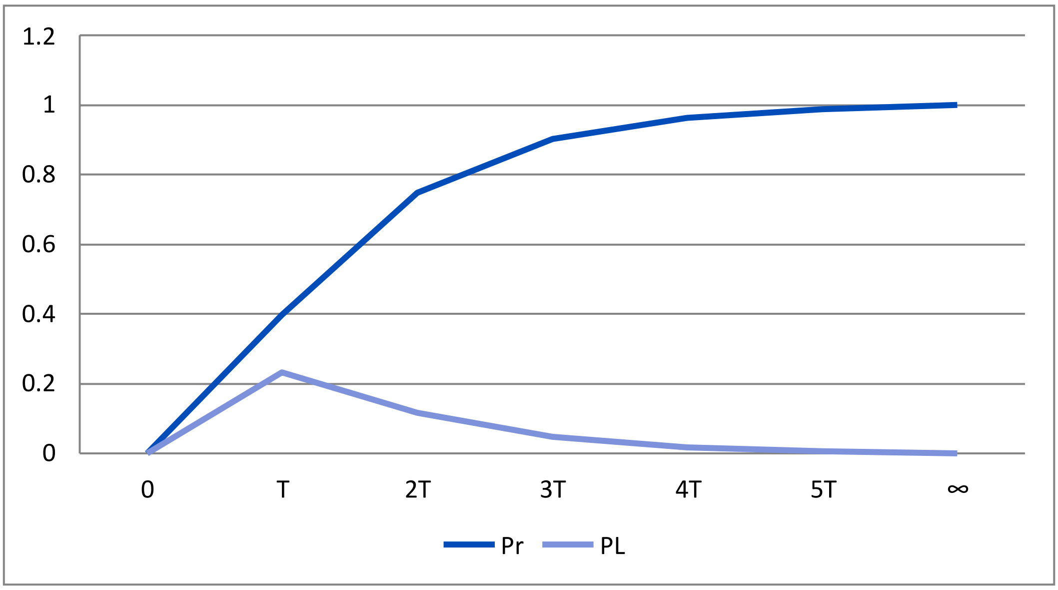
$$P_R = \frac{\boldsymbol{V}^2}{\boldsymbol{R}}(\boldsymbol{1} - \boldsymbol{e}^{-\frac{\boldsymbol{R}}{\boldsymbol{L}}\boldsymbol{t}})\,(\boldsymbol{1} - \boldsymbol{e}^{-\frac{\boldsymbol{R}}{\boldsymbol{L}}\boldsymbol{t}})$$

$$P_R = \frac{\boldsymbol{V}^2}{\boldsymbol{R}}(\boldsymbol{1} + \boldsymbol{e}^{-\frac{\boldsymbol{2R}}{\boldsymbol{L}}\boldsymbol{t}} - \boldsymbol{2e}^{-\frac{\boldsymbol{R}}{\boldsymbol{L}}\boldsymbol{t}})$$

Instantaneous power through the inductor $P_L = V_L \times i$

$$P_L = \mathbf{V}\mathbf{e}^{-\frac{\mathbf{R}}{\mathbf{L}}\mathbf{t}} \times \frac{\boldsymbol{V}}{\boldsymbol{R}}(\boldsymbol{1} - \boldsymbol{e}^{-\frac{\boldsymbol{R}}{\boldsymbol{L}}\boldsymbol{t}})$$

$$P_L = \frac{V^2}{R}\left(e^{-\frac{R}{L}t} - e^{-\frac{2R}{L}t}\right)$$



Discharge condition:

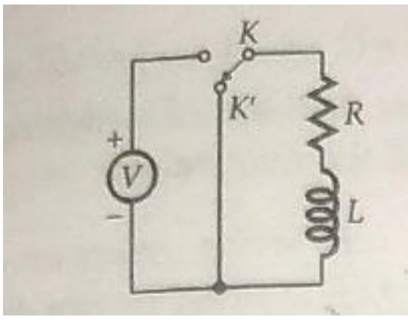


Fig. 1.2: R-L Discharging Condition

Switch is moved to K'

$$iR + L\frac{di}{dt} = 0$$

$$\frac{di}{dt} + \frac{R}{L}i = 0$$

The solution of above equation is

$$i = K'e^{-\frac{R}{L}t}$$

$K' is\ a\ constant$

This constant can be calculated from initial conditions

At t=0 the current $i = \frac{V}{R}$

$$\frac{V}{R} = K'e^{-\frac{R}{L}0}$$

$$K' = \frac{V}{R}$$

$$i = K'e^{-\frac{R}{L}t}$$

$$\boldsymbol{i = \frac{V}{R}e^{-\frac{R}{L}t}}$$

S.No.	Time	Current i
1	0	$\frac{V}{R}$
2	T	$\frac{0.368V}{R}$
3	2T	$\frac{0.135V}{R}$
4	3T	$\frac{0.049V}{R}$
5	4T	$\frac{0.018V}{R}$
6	5T	$\frac{0.00679V}{R}$
7	∞	0

Voltage across Resistor V_R=iR

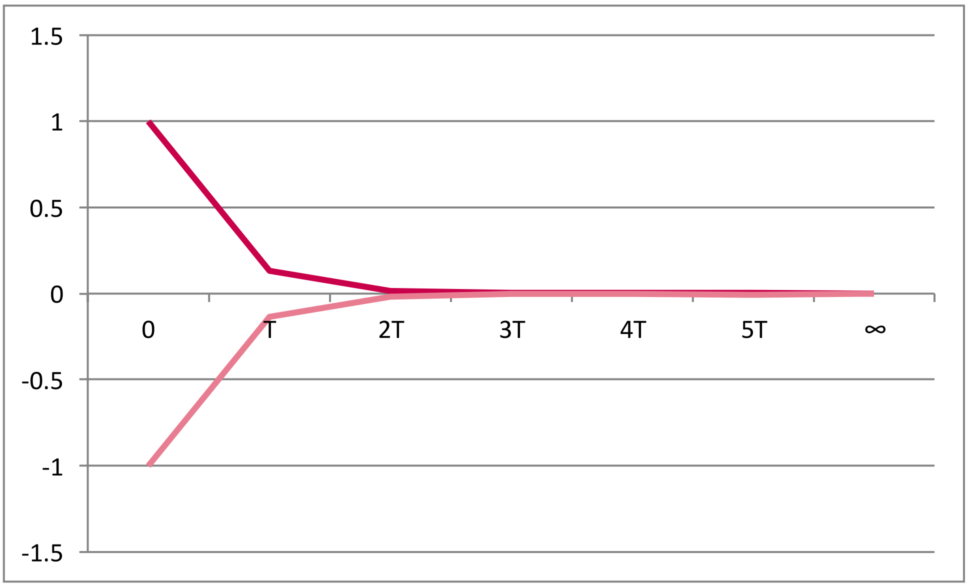
$V_R=\boldsymbol{\frac{V}{R}e^{-\frac{R}{L}t}} \times \mathrm{R}$

$V_R=\boldsymbol{Ve^{-\frac{R}{L}t}}$

$V_R+V_L=0$

Voltage across Inductor $V_L=-V_R$

$V_L = -Ve^{-\frac{R}{L}t}$



Hence during discharge condition Voltage polarity across inductor reverses but current direction remains same.

Power at the resistor $P_R = V_R \times i$

$$P_R = Ve^{-\frac{R}{L}t} \times \frac{V}{R}e^{-\frac{R}{L}t}$$

$$P_R = \frac{V^2}{R}e^{-\frac{2R}{L}t}$$

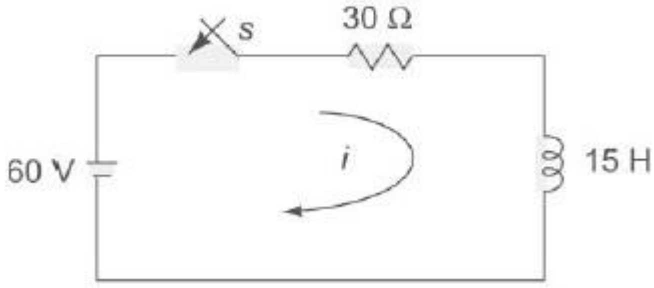
Power at the Inductor $P_L = V_L \times i$

$$P_L = -Ve^{-\frac{R}{L}t} \times \frac{V}{R}e^{-\frac{R}{L}t}$$

$$P_L = -\frac{V^2}{R}e^{-\frac{2R}{L}t}$$

Here Power at the inductor is negative. Hence during discharge condition inductor acts like a source delivers the power to the resistor.

Problem: Find the current, voltage across resistor, Voltage across inductor for the circuit shown



Solution:

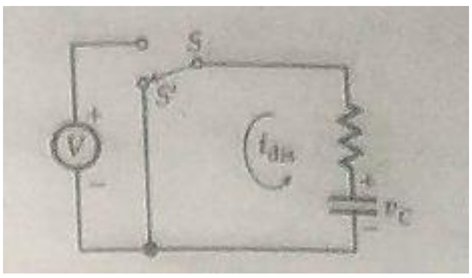
$i = \frac{V}{R}(1 - e^{-\frac{tR}{}})$=2$(1 - e^{-2t})$ Amps

$V_R = V\ (1 - e^{-\frac{tR}{}})$=60$(1 - e^{-2t})\ Volts$

$V_L = Ve^{-\frac{tR}{L}}$=60$e^{-2t}\ \ Volts$

D.C. Response of Series RC circuit:

During Charging:



Assume that capacitor is initially uncharged

$V_C(0^-) = 0$

At t=0 switch is closed

Apply KVL

$-V + V_R + \ = 0$

$V = V_R + V_C$

$$V = i(t) \times R + \frac{1}{C}\int i(t)\, dt$$

Differentiate above equation with respect to time

$$0 = R\frac{(t)}{dt} + \frac{1}{C}(t)$$

$$\frac{(t)}{dt} + \frac{1}{RC}(t) = 0$$

$$i(t) = k \times e^{-\frac{t}{RC}}$$

K=constant. This can be calculated from initial conditions

We know that $V_C(0^-) = 0$

According to capacitor property
$V_C(0^-) = V_C(0) = V_C(0+)$

$$V_C(0) = 0$$

Hence at t=0 capacitor acts like short circuit

$$(0) = \frac{V}{R}$$

$$i(t) = \frac{V}{R}e^{-\frac{t}{RC}}$$

$$V_R(t) = i(t) \times R$$

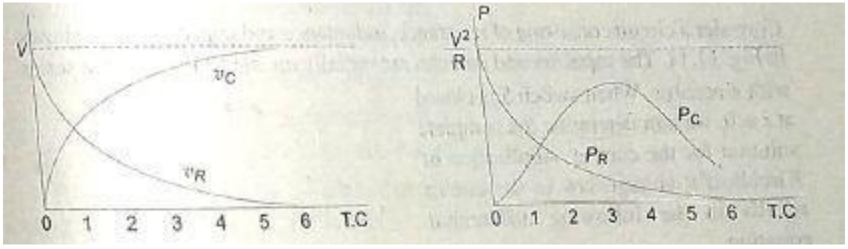
$$V_R(t) = \frac{V}{R}e^{-\frac{t}{RC}} \times R$$

$$V_R(t) = Ve^{-\frac{t}{RC}}$$

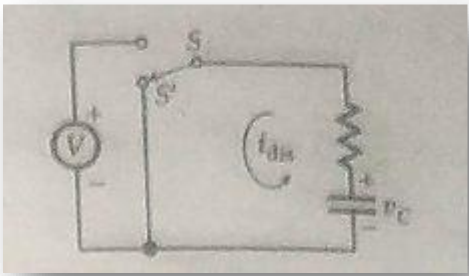
$$V_C = V - V_R$$

$$V_C = V - Ve^{-\frac{t}{RC}}$$

$$V_C = (1 - e^{-\frac{t}{RC}})$$



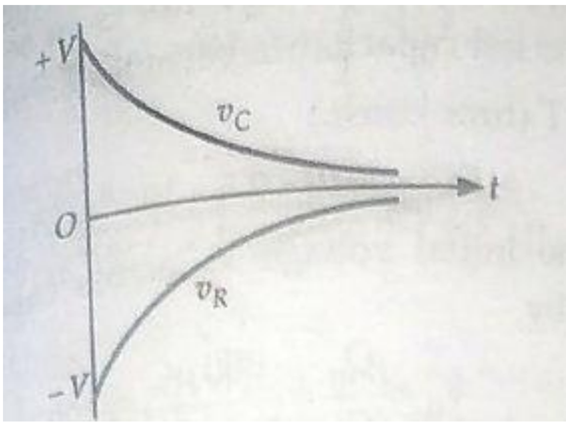
During Discharging:



$$i(t) = -\frac{V}{R}e^{-\frac{t}{RC}}$$

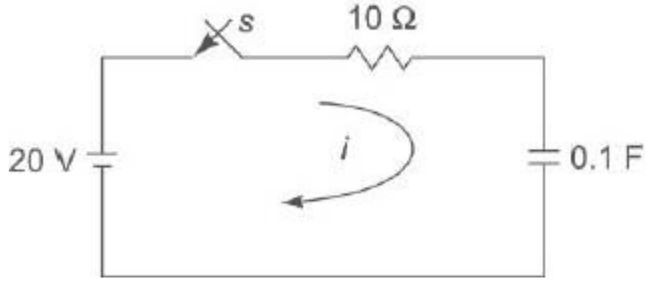
$$V_R(t) = -Ve^{-\frac{t}{RC}}$$

$$V_C(t) = Ve^{-\frac{t}{RC}}$$



Problem:

A series RC circuit consists of 10Ω Resistor and capacitor of 0.1F as shown in figure. A constant of 10V is applied at t=0Find the current, voltage across resistor, Voltage across capacitor



Solution:

$$i = \frac{V}{R} e^{-\frac{t}{RC}}$$

$$V_R = Ve^{-\frac{t}{RC}}$$

$$V_C = V\,(1 - e^{-\frac{t}{RC}})$$

By substituting V, R,C values

$i = 2e^{-t}$ Amps

$V_R = 20e^{-t}$ Volts

$V_C = 20(1 - e^{-t})$ Volts

DC response of series RLC circuit:

Let’s assume that capacitor and inductors are initially uncharged

(0^{-}) =0

$i_l(0^{-})$ =0

Lets Switch is closed at t=0

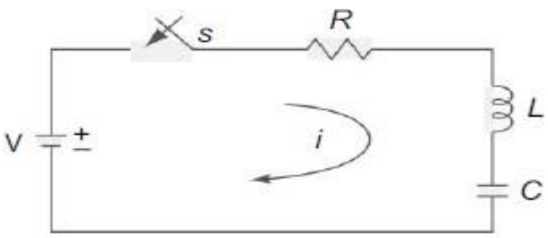


Fig: Series RLC Circuit

Apply KVL

$iR + L\frac{di}{dt} + \frac{1}{C}\int idt = 0$

Differentiate above equation with respect to't'

$R\frac{di}{dt} + L\frac{d^2i}{dt^2} + \frac{1}{C}i = 0$

$\frac{d^2i}{dt^2} + \frac{R}{L}\frac{di}{dt} + \frac{1}{LC}i = 0$

Let $\frac{di}{dt} = D$

$(D^2 + \frac{RD}{L} + \frac{1}{LC})$i=0

The above equation is second order linear differential equation, with only complimentary function .The particular solution is zero

Characteristic equation for above differential equation is

$(D^2 + \frac{RD}{L} + \frac{1}{LC}) = 0$

The roots of above equation are

$D_1, D_2 = \frac{-\frac{R}{L} \pm \sqrt{(\frac{R}{L})^2 - 4*1*\frac{1}{LC}}}{2*1}$

$D_1, D_2 = -\frac{R}{2L} \pm \sqrt{(\frac{R}{2L})^2 - \frac{1}{LC}}$

Assume $k_1 = -\frac{R}{2L}$

$K_2 = \sqrt{(\frac{R}{2L})^2 - \frac{1}{LC}}$

$D_1, D_2 = K_1 \pm K_2$

$D_1 = K_1 + K_2$

$D_2 = K_1 - K_2$

Here K_2 may be positive or zero or negative

Case (i):

$$(\frac{R}{2L})^2 > \frac{1}{LC}$$

K_2 is positive

Roots are real and unequal

The response is called over damped response

The solution of current equation is:

$$i = c_1^{(K_1+K_2)t} + c_1 e^{(K_1-K_2)t}$$

Case (ii):

$$(\frac{R}{2L})^2 < \frac{1}{LC}$$

K_2 is Negative

Roots are complex conjugate

The response is called under damped response

The solution of current equation is:

$$i = e^{K_1}(c_1 \cos K_2 t + c_2 \sin K_2 t)$$

Case (iii):

$$(\frac{R}{2L})^2 = \frac{1}{LC}$$

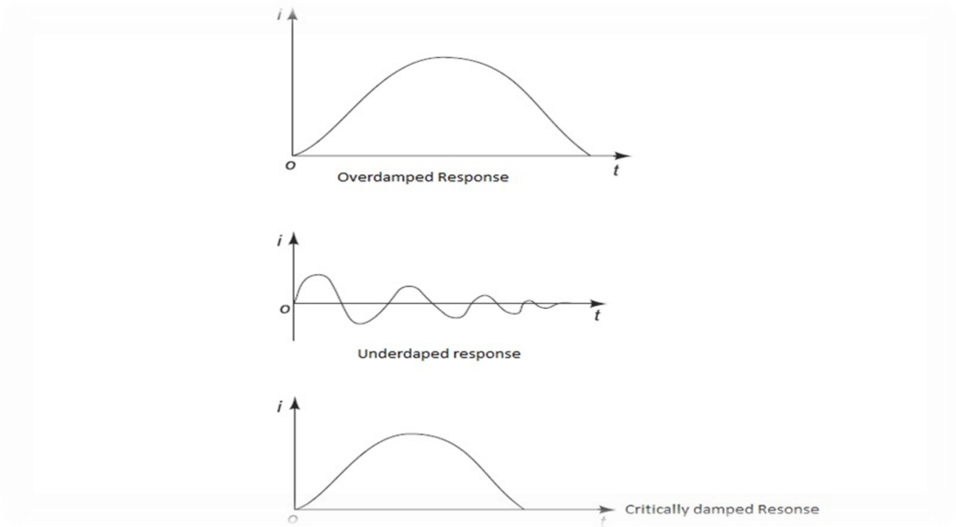
$K_2 = 0$

Roots are real and equal

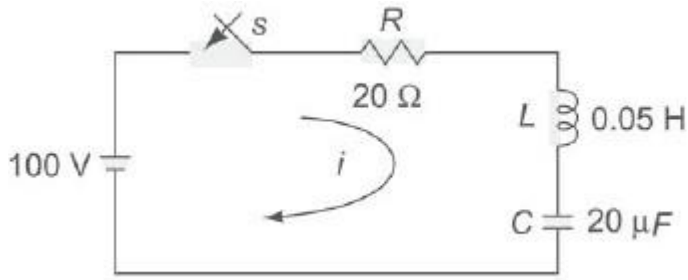
The response is called critically damped response

The solution of current equation is:

$$i = (c_1 + c_1 t)^{K_1 t}$$



Problem: The circuit as shown in figure consists of resistance, inductance and capacitance in series with a 100V constant source when switch is closed at t=0.Find current transient



Solution:

R=20Ω

L=0.05H

C=20μF=20×10^{-6}F

$$\left(\frac{R}{2L}\right)^2 = \left(\frac{20}{2*0.05}\right)^2 = 40000$$

$$\frac{1}{LC} = \frac{1}{0.05 \times 20 \times 10^{-6}} = 10^{+6}$$

$(\frac{R}{2L})^2 < \frac{1}{LC}$

Roots are complex conjugate

The response is called under damped response

The solution of current equation is:

$i = e^{K_1}(c_1 \cos K_2 t + c_2 \sin K_2 t)$

$k_1 = -\frac{R}{2L} = -200$

$K_2 = \sqrt{(\frac{R}{2L})^2 - \frac{1}{LC}} = \sqrt{40000 - 10^{+6}} = j979.8$

$i = e^{-200}(c_1 \cos 979.8t + c_2 \sin 979.8t)$

At t=0,i=0

$0 = c_1 \cos 979.8t$

$c_1 = 0$

$i = e^{-200}(c_2 \sin 979.8t)$

At t=0
$V_L = 100$

$L\frac{di}{dt} = 100$

$\frac{di}{dt} = \frac{100}{L} = 100/0.05 = 2000$

At t=0: $\frac{di}{dt} = 2000$

$979.8t c_2 \cos 979.8t = 2000$

$c_2 = \frac{2000}{979.8} = 2.04$

The current equation is

$i = e^{-200}(2.04 \sin 979.8t)$ Amps

S domain approach:

Advantages of S domain approach

1. All the elements behave as impedances in s domain so using various simplification techniques, the impedances can be combined easily to obtain simple form of the network.
2. The terms related to voltage drops across the elements are of simple form like [I(s) x Z(s)], according to ohm's law.
3. No integral or differential terms are present in the set of network equations.
4. From s domain network, the system function, resultant transform impedance etc. can be obtained. These concepts are important to analyse the network in s domain

Element	**S domain representation**
Resistor	R
Inductor	SL
Capacitor	$\frac{1}{SC}$

UNIT – II

TWO PORT NETWORKS:

- Impedance Parameters
- Admittance Parameters
- Hybrid Parameters
- Transmission (ABCD) Parameters
- Conversion of one parameter to another
- Conditions for Reciprocity and Symmetry
- Interconnection of two port networks in Series Parallel and Cascaded configurations
- Illustrative problems

UNIT-II

Two Port Networks

Port: Port is a pair of terminals at which the current enters or leave the network

At port we can measure the network variables.

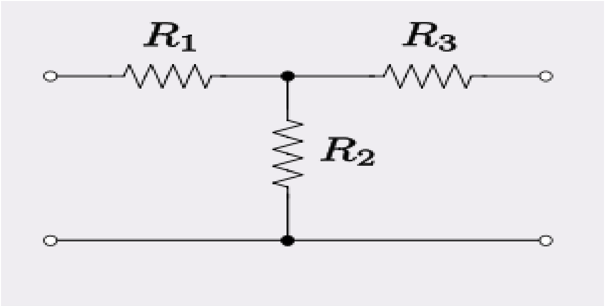
- Net current at the port=0

2 port network: In 2 port network we have 2 pairs of accessible terminals; usually one pair represent input port and other pair represent output port.

Symmetrical Network: If the network is divided in to 2 parts one part should be mirror image of another part

Reciprocal Network: obeys Reciprocity Therom

All passive networks are reciprocal networks



- In the figure
    - 1-1′ represents Input Port

- 2-2′ represents Output port
- We have 4 variables at 2 ports
  - V_1, I_1 voltage and current at port1
  - V_2, I_2 voltage and currents at port2
  - Out of 4 variables 2 are independent variables and 2 are dependent variables
  - So we have 4C2=6 Combinations

So we have 6 parameters for the 2 port network

They are

1. Z parameters or Open circuit parameters
2. Y parameters or Short circuit parameters
3. h parameters or hybrid parameters
4. g parameters or inverse hybrid parameters
5. ABCD parameters or Transmission parameters
6. Inverse ABCD parameters or Inverse Transmission parameters

Z parameters or Open circuit parameters:

$V_1=Z_{11}I_1+Z_{12}I_2$

$V_2=Z_{21}I_1+Z_{22}I_2$

V_1, V_2 Dependent Variables

I_1, I_2 Independent Variables

Case-I:

$I_2=0$(Port2 is open circuited)

$V_1=Z_{11}I_1$

$$Z_{11}=\left.\frac{V_1}{I_1}\right|_{I_2=0}$$

Z_{11}=Open Circuit Input Impedance in Ω

$V_2 = Z_{21} I_1$

$$Z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0}$$

Z_{21}=Open Circuit Forward Transfer Impedance in Ω

Case-II:

$I_1=0$(Port1 is open circuited)

$V_1 = Z_{12} I_2$

$$Z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0}$$

Z_{12}=Open Circuit Reverse Transfer Impedance in Ω

$V_2 = Z_{22} I_2$

$$Z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0}$$

Z_{22}=Open Circuit Output Impedance in Ω

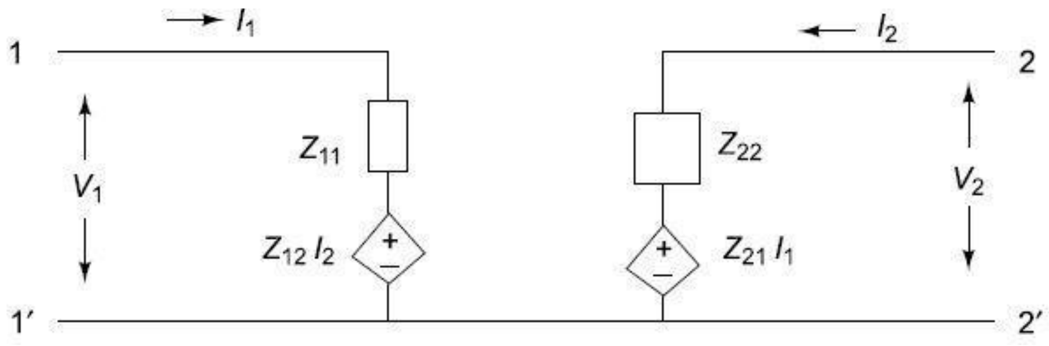
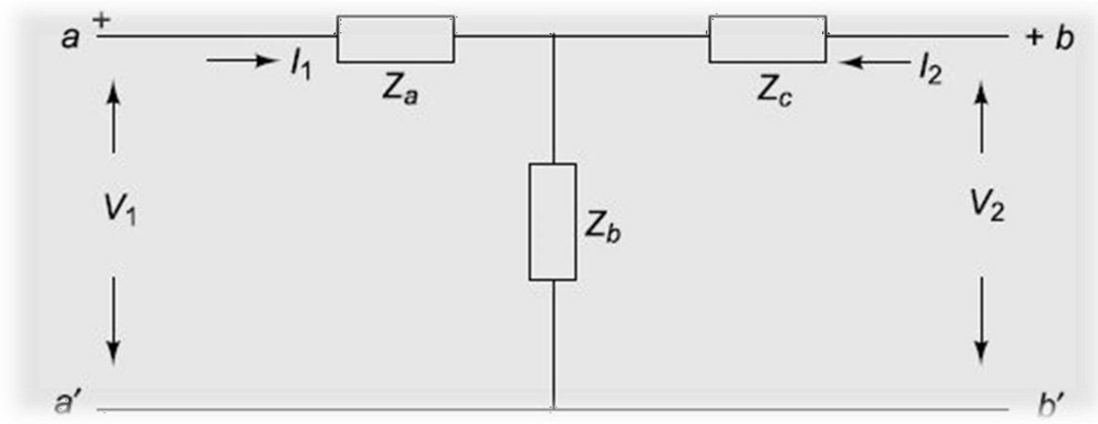


Fig. Equivalent Circuit Of 2port Network in terms of Z parameters

Condition for Symmetry: $Z_{11}=Z_{22}$

Condition for Reciprocity: Z_{12}=Z_{21}

Problem: Find Z parameters of for the circuit as shown



Solution:

In the circuit we have 2 loops

Apply KVL to loop1

$$I_1Z_a + (I_1 + I_2) - V_1 = 0$$

$$V_1 = (Z_a + Z_b)_1 + Z_bI_2$$

Apply KVL to loop2

$$I_2Z_C + (I_1 + I_2) - V_2 = 0$$

$$V_2 = Z_bI_1 + (Z_b + Z_c)I_2$$

Comparing above equations with Z parameter equations

V_1=$Z_{11}I_1$+$Z_{12}I_2$

V_2=$Z_{21}I_1$+$Z_{22}I_2$

Z_{11}= Z_a+Z_b

Z_{21}= Z_b

Z_{12}=Z_b

Z_{22}= Z_b+Z_c

The network given in the problem is known as T network

Yparameters or Short circuit parameters:

$I_1=Y_{11}V_1+Y_{12}V_2$

$I_2=Y_{21}V_1+Y_{22}V_2$

I_1, I_2 Dependent Variables

V_1, V_2 Independent Variables

Case1:

$V_2=0$(Port2 is short circuited):

$I_1=Y_{11}V_1$

Y_{11}=Short circuit Input Admittance in mhos or Siemens

$$Y_{11}=\left.\frac{I_1}{V_1}\right|_{V_2=0}$$

$I_2=Y_{21}V_1$

$$Y_{21}=\left.\frac{I_2}{V_1}\right|_{V_2=0}$$

Y_{21}=Short circuit Forward Transfer Admittance in mhos or Siemens

Case2:

$V_1=0$(Port1 is short circuited):

$I_1= Y_{12}V_2$

$$Y_{12}=\left.\frac{I_1}{V_2}\right|_{V_1=0}$$

Y_{12}= Short circuit Reverse Transfer Admittance in mhos or Siemens

$I_2 = Y_{22}V_2$

$$Y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1=0}$$

Y_{22}= Short circuit Output Admittance in mhos or Siemens

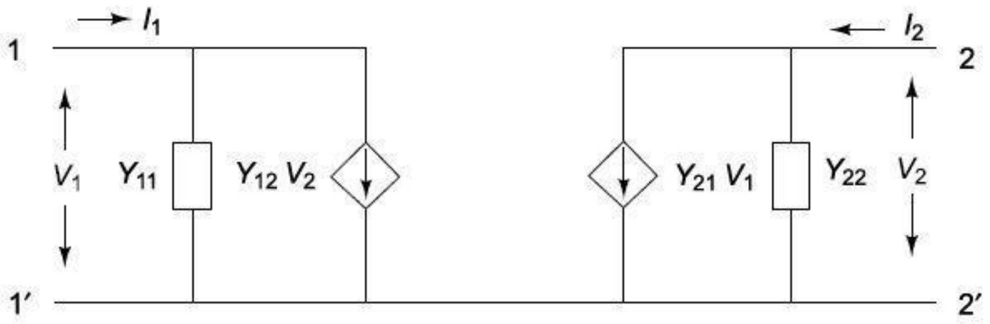
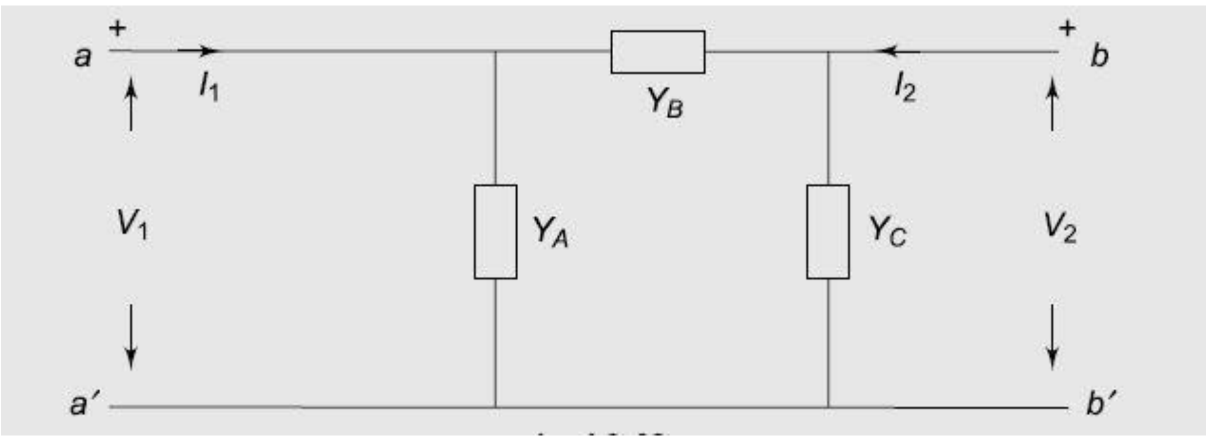


Fig. Equivalent Circuit Of 2port Network in terms of Y parameters

Condition for Symmetry: $Y_{11}=Y_{22}$

Condition for Reciprocity: $Y_{12}=Y_{21}$

Problem: Find Y parameters of for the circuit as shown



Solution:

Apply KCL

$-I_1 + V_1Y_A + (V_1 - V_2) = 0$

$I_1 = (Y_A + Y_B)_1$-Y_BV_2

$-I_2 + V_2 Y_C + (V_2 - V_1) = 0$

$I_2 = -Y_B V_1 + (Y_B + Y_C) V_2$

$Y_{11} = Y_A + Y_B$

$Y_{12} = Y_{21} = -Y_B$

$Y_{22} = Y_B + Y_C$

Comparing above equations with Y parameter equations

$I_1 = Y_{11} V_1 + Y_{12} V_2$

$I_2 = Y_{21} V_1 + Y_{22} V_2$

The network given in the problem is known as ∏ network

h parameters or hybrid parameters:

V_1, I_2 Dependent Variables

I_1, V_2 Independent Variables

$V_1 = h_{11} I_1 + h_{12} V_2$

$I_2 = h_{21} I_1 + h_{22} V_2$

When $V_2 = 0$, the port 2-2′ is short circuited.

Then $h_{11} = \left.\frac{V_1}{I_1}\right|_{V_2=0}$ Short circuit input impedance $\left(\frac{1}{Y_{11}}\right)$

$h_{21} = \left.\frac{I_2}{I_1}\right|_{V_2=0}$ Short circuit forward current gain $\left(\frac{Y_{21}}{Y_{11}}\right)$

Similarly, by letting port 1-1′ open, $I_1 = 0$

$h_{12} = \left.\frac{V_1}{V_2}\right|_{I_1=0}$ Open circuit reverse voltage gain $\left(\frac{Z_{12}}{Z_{22}}\right)$

$h_{22} = \left.\frac{I_2}{V_2}\right|_{I_1=0}$ Open circuit output admittance $\left(\frac{1}{Z_{22}}\right)$

Condition for Symmetry: $h_{11}h_{22} - h_{12}h_{21} = 1$

Condition for Reciprocity: $h_{12} = -h_{21}$

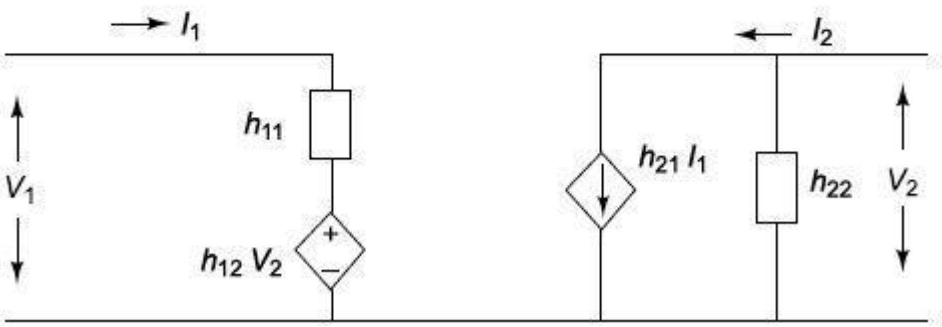
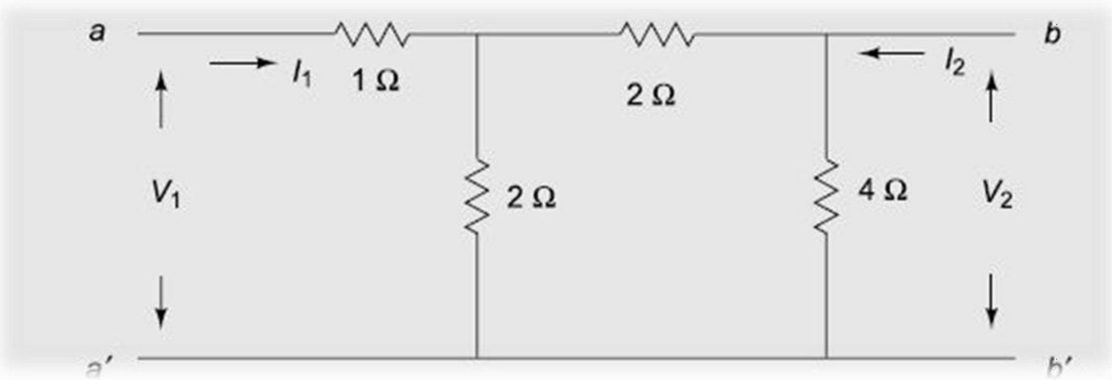


Fig. Equivalent Circuit Of 2port Network in terms of h parameters

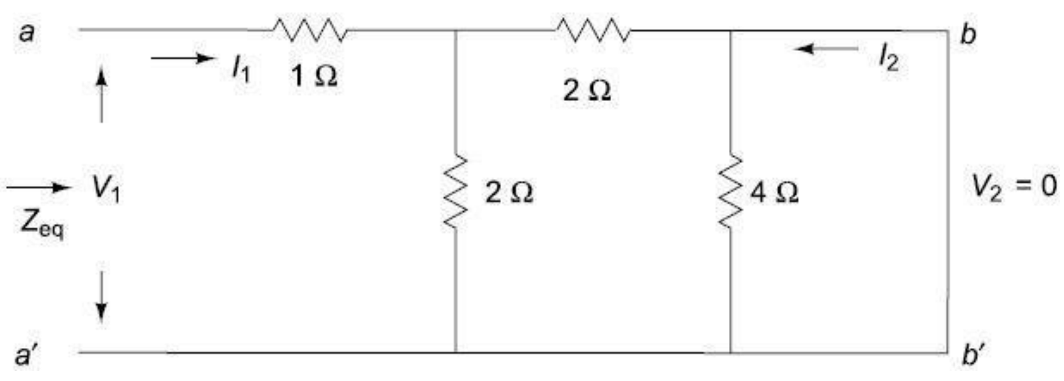
Problem: Find h parameters of for the circuit as shown



Solution:

$$h_{11} = \left.\frac{V_1}{I_1}\right|_{V_2=0} ; \quad h_{21} = \left.\frac{I_2}{I_1}\right|_{V_2=0} ; \quad h_{12} = \left.\frac{V_1}{V_2}\right|_{I_1=0} ; \quad h_{22} = \left.\frac{I_2}{V_2}\right|_{I_1=0}$$

V_2=0(Port2 is shorted):



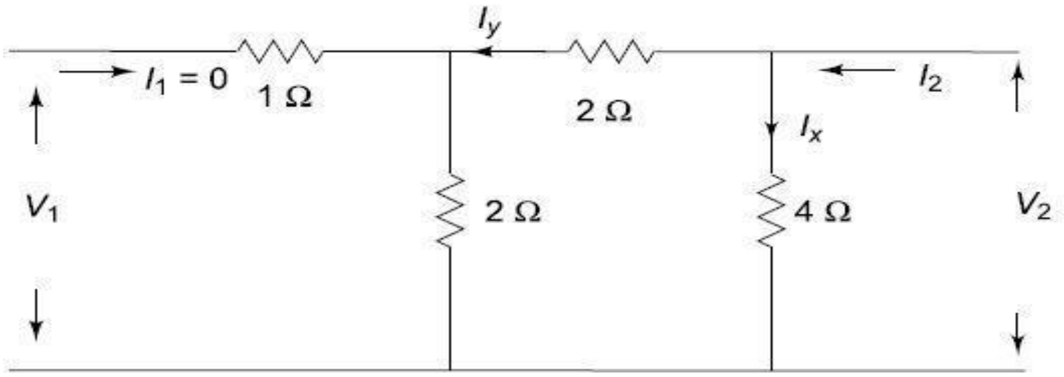
$$\therefore \quad V_1 = I_1 2\text{V}$$

$$h_{11} = \frac{V_1}{I_1} = 2\ \Omega$$

$$h_{21} = \left.\frac{I_2}{I_1}\right|_{V_2=0} \quad \text{when} \quad V_2 = 0; -I_2 = \frac{I_1}{2}$$

$$\therefore \quad h_{21} = -\frac{1}{2}$$

Port1 is open circuited:



$$h_{12} = \left.\frac{V_1}{V_2}\right|_{I_1=0}$$

$$V_1 = I_y 2;\ I_y = \frac{I_2}{2}$$

$$V_2 = I_X 4;\ I_X = \frac{I_2}{2}$$

$$\therefore \quad h_{12} = \left.\frac{V_1}{V_2}\right|_{I_1=0} = \frac{1}{2}$$

$$h_{22} = \left.\frac{I_2}{V_2}\right|_{I_1=0} = \frac{1}{2}\ \mho$$

g parameters or inverse hybrid parameters:

$I_1=g_{11}V_1+g_{12}I_2$

$V_2=g_{21}V_1+g_{22}I_2$

I_1, V_2 Dependent Variables

V_1, I_2 Independent Variables

Condition for Symmetry: $g_{11}g_{22}-g_{12}g_{21}=1$

Condition for Reciprocity: $g_{12}=-g_{21}$

ABCD parameters or Transmission parameters:

$V_1=AV_2-BI_2$

$I_1=C\,V_2-DI_2$

V_1, I_1 Dependent Variables

V_2, I_2 Independent Variables

$I_2=0$(Port2 open circuited):

$V_1=AV_2$

$I_1=C\,V_2$

$$A=\left.\frac{V_1}{V_2}\right|_{I_2=0} \text{ and } C=\left.\frac{I_1}{V_2}\right|_{I_2=0}$$

A=Open circuit Reverse Voltage Gain

C=Open circuit reverse Transfer admittance in mhos or Siemens

$V_2=0$(Port2 Short circuited):

$V_1= -BI_2$

$I_1 = -DI_2$

$$-B = \left.\frac{V_1}{I_2}\right|_{V_2=0} \quad \text{and} \quad -D = \left.\frac{I_1}{I_2}\right|_{V_2=0}$$

$$A = \left.\frac{V_1}{V_2}\right|_{I_2=0} \quad \text{and} \quad C = \left.\frac{I_1}{V_2}\right|_{I_2=0}$$

A=Open circuit Reverse Voltage Gain

B=Short circuit Reverse Transfer Impedance in ohms

C=Open circuit Reverse Transfer Admittance in mhos

D=Short circuit reverse current gain

Condition for Symmetry: A=D

Condition for Reciprocity: AD-BC=1

Inverse ABCD parameters or Inverse Transmission parameters:

$V_2 = A' V_1 - B' I_1$

$I_2 = C' V_1 - D' I_1$

V_2, I_2 Dependent Variables

V_1, I_1 Independent Variables

Condition for Symmetry: A′=D′

Condition for Reciprocity: A′D′-B′C′=1

Interconnection of 2 port networks:

1. Series Connection---------------Z matrices added
2. Parallel Connection--------------Y matrices added
3. Cascade Connection-------------ABCD matrices are multiplied

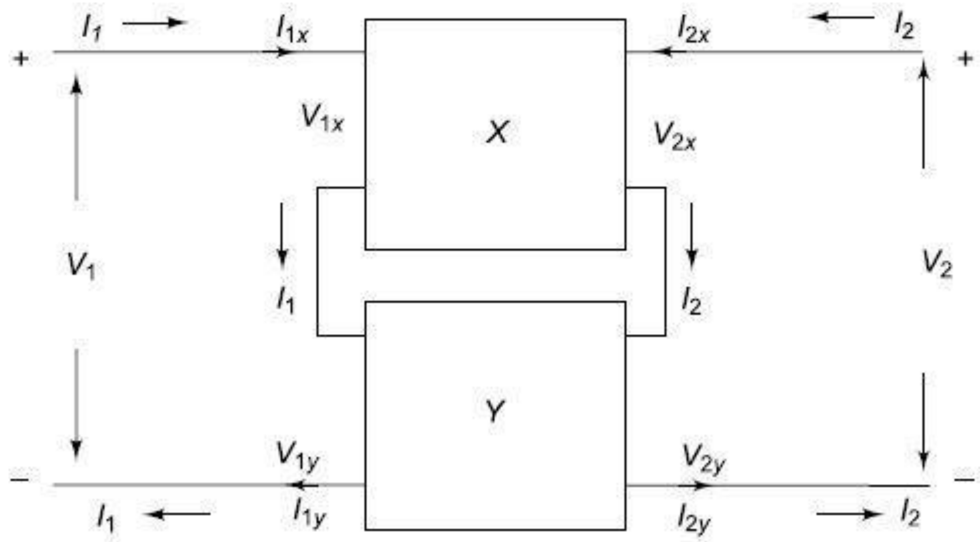


Fig.Series connection

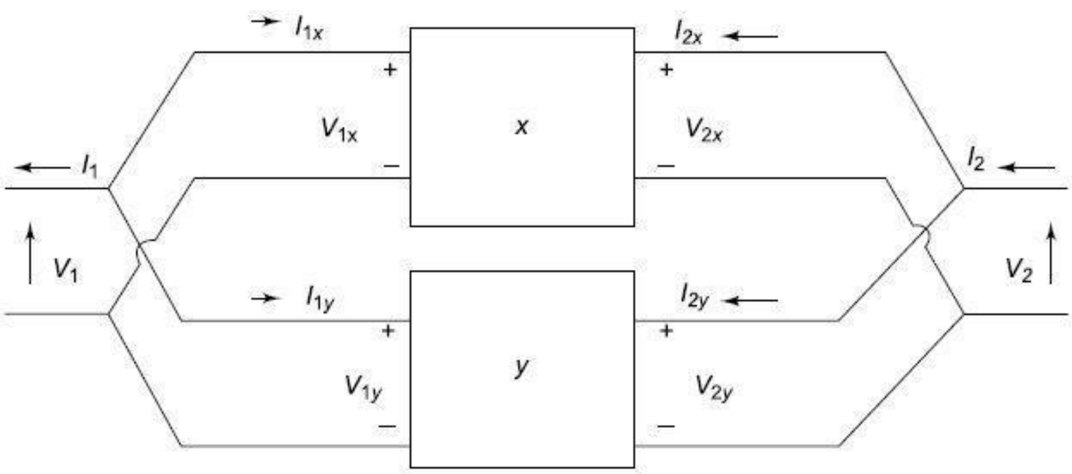


Fig:Parallel connection

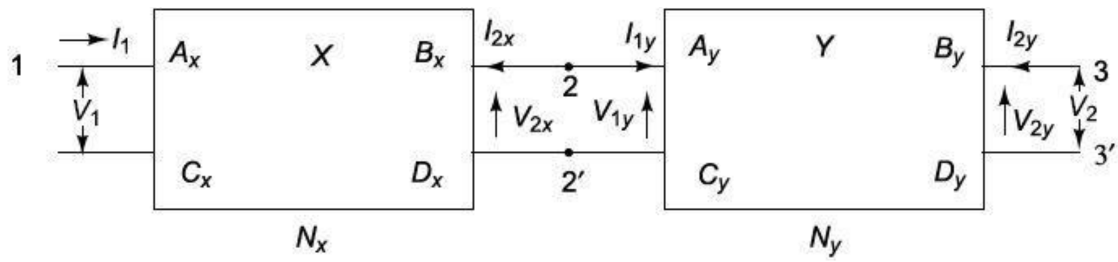


Fig:Cascade Connection

- If T network is given it is easier to calculate Z parameters
- If the ∏ network is given it is easier to calculate Y parameters

UNIT-III:
Locus diagrams:

Locus diagrams of Series and Parallel RL, RC, RLC circuits with variation of various parameters

Resonance:

Resonance-

Series and Parallel Circuits

Concept of Bandwidth

Quality factor

UNIT-3

LOCUS DIAGRAMS &RESONANCE

Locus diagram:

The path traced by tip of the phasor diagram is known as locus diagram

Applications of locus diagrams:

Locus diagrams useful in

1. Determining Frequency Response of RLC circuits
2. Circuit Analysis
3. Design of Electric wave filters
4. Analysis of operating characteristics of AC machines and Transmission Lines

Current Locus of series RL Circuit:

Here we have 2 conditions

1. Fixed Resistance(R), Variable Reactance(X_L)
2. Fixed Reactance(X_L), Variable Resistance(R)

Current Locus of series RL Circuit with Fixed Resistance(R), Variable Reactance (X_L):

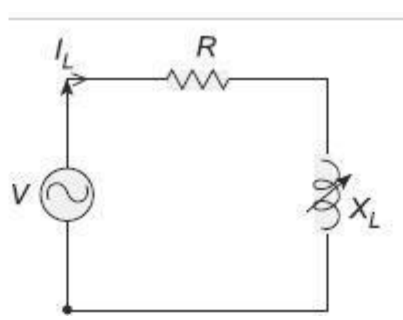


Fig: 3.1Series RL circuit (Fixed Resistance(R), Variable Reactance (X_L)) Excited with AC Supply

$$I = \frac{V}{R + jX_L} = \frac{V\angle 0°}{\sqrt{(R^2 + (X_L)^2)}\angle \tan^{-1}\left(\frac{X_L}{R}\right)}$$

$$I = \frac{V}{\sqrt{(R^2 + (X_L)^2)}}\left(\angle 0° - \tan^{-1}\left(\frac{X_L}{R}\right)\right)$$

S.No.	X_L	**I**
1	0	$\frac{V}{R}\angle 0°$
2	R	$\frac{V}{R\sqrt{2}}\angle -45°$
3	∞	$0\angle -90°$

Current locus is a semi circle of centre $(\frac{V}{2R}, 0)$ and radius $\frac{V}{2R}$

Current Locus of series RL Circuit with Fixed Reactance (X_L), Variable Resistance(R):

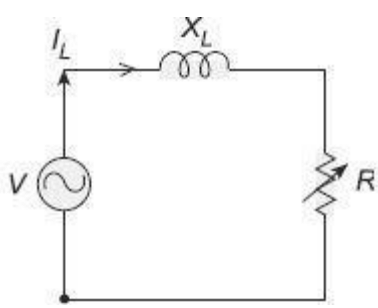


Fig: 3.2Series RL circuit (Fixed Reactance (X_L), Variable Resistance(R))Excited with AC Supply

$$\mathbf{I} = \frac{\mathbf{V}}{\mathbf{R} + \mathbf{j}\mathbf{X}_{\mathbf{L}}} = \frac{\mathbf{V}\angle\mathbf{0}^\circ}{\sqrt{(\mathbf{R}^2 + (\mathbf{X}_{\mathbf{L}})^2)}\angle \tan^{-1}\left(\frac{\mathbf{X}_{\mathbf{L}}}{\mathbf{R}}\right)}$$

$$\mathrm{I} = \frac{\mathbf{V}}{\sqrt{(\mathbf{R}^2 + (\mathbf{X}_{\mathbf{L}})^2)}}\left(\angle\mathbf{0}^\circ - \tan^{-1}\left(\frac{\mathbf{X}_{\mathbf{L}}}{\mathbf{R}}\right)\right)$$

S.No.	R	I
1	0	$\frac{V}{X_L}\angle -90°$
2	X_L	$\frac{V}{X_L\sqrt{2}}\angle -45°$
3	∞	$0\angle 0°$

Current locus is a semi circle of centre $(0, -\frac{V}{2X_L})$ and radius $\frac{\ }{2X_L}$

3.4Current Locus of series RC Circuit:

Here we have 2 conditions

1. Fixed Resistance(R), Variable Reactance(X_C)
2. Fixed Reactance(X_C), Variable Resistance(R)

Current Locus of series RC Circuit with Fixed Resistance(R), Variable Reactance (X_C):

$$\mathrm{I} = \frac{\mathrm{V}}{\mathrm{R} - \mathrm{j}\mathrm{X}_{\mathrm{C}}} = \frac{\mathbf{V}\angle\mathbf{0}^\circ}{\sqrt{(\mathbf{R}^2 + (\mathbf{X}_{\mathbf{C}})^2)}\angle -\tan^{-1}\left(\frac{\mathbf{X}_{\mathbf{C}}}{\mathbf{R}}\right)}$$

$$\mathrm{I} = \frac{\mathbf{V}}{\sqrt{(\mathbf{R}^2 + (\mathbf{X}_{\mathbf{C}})^2)}}\left(\angle\mathbf{0}^\circ + \tan^{-1}\left(\frac{\mathbf{X}_{\mathbf{C}}}{\mathbf{R}}\right)\right)$$

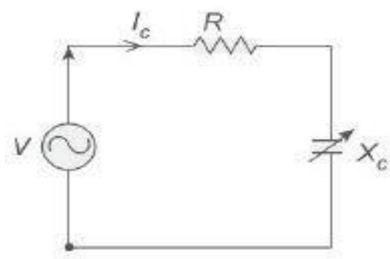


Fig: 3. 3Series RC circuit (Fixed Resistance(R), Variable Reactance (X_C)) Excited with AC Supply

S.No.	X_C	I
1	0	$\frac{V}{R}\angle 0°$
2	R	$\frac{V}{R\sqrt{2}}\angle +45°$
3	∞	$0\angle +90°$

Current locus is a semi circle of centre $(\frac{V}{2R},0)$ and radius $\frac{V}{2R}$

Current Locus of series RC Circuit with Fixed Reactance (X_C) Variable Resistance(R):

$$I = \frac{V}{R - jX_C} = \frac{\mathbf{V\angle 0°}}{\sqrt{(\mathbf{R^2} + (\mathbf{X_C})^2)}\angle -\tan^{-1}\left(\frac{\mathbf{X_C}}{\mathbf{R}}\right)}$$

$$I = \frac{\mathbf{V}}{\sqrt{(\mathbf{R^2} + (\mathbf{X_C})^2)}}\left(\angle \mathbf{0°} + \tan^{-1}\left(\frac{\mathbf{X_C}}{\mathbf{R}}\right)\right)$$

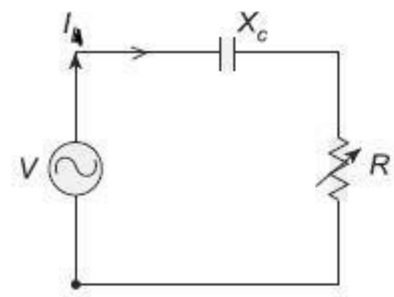
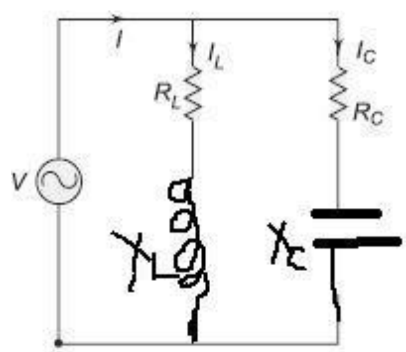


Fig: 3. 4Series RC circuit (Fixed Reactance (X_C), Variable Resistance(R)) Excited with AC Supply

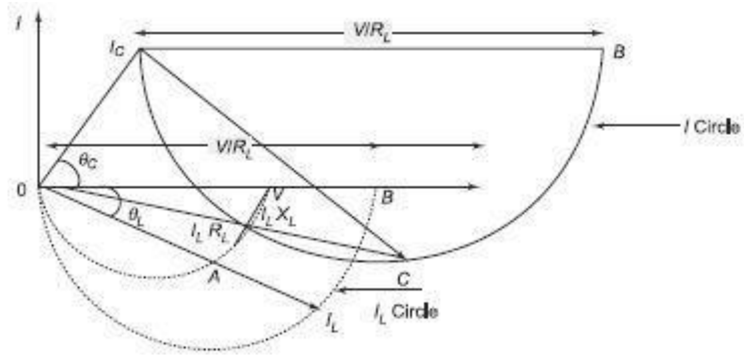
S.No.	R	I
1	0	$\frac{V}{2X_C}\angle 90°$
2	X_C	$\frac{V}{X_C\sqrt{2}}\angle +45°$
3	∞	$0\angle +0°$

Current locus is a semi circle of centre $(0,\frac{V}{2X_C})$ and radius $\frac{}{2X_C}$

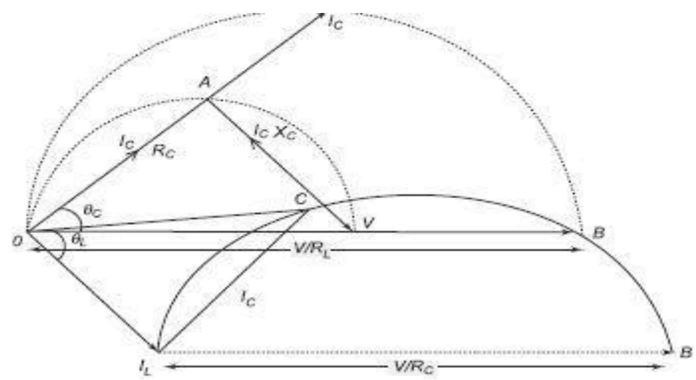
3.4 Locus Diagrams of Parallel Circuits:



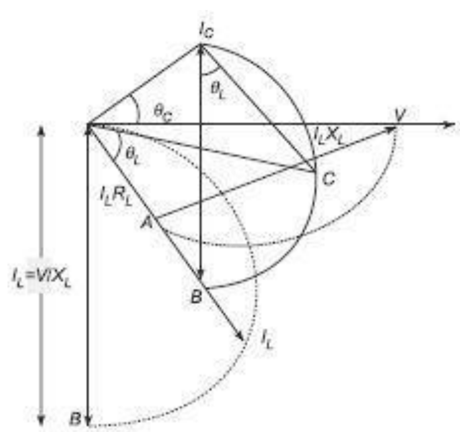
Variable X_L:



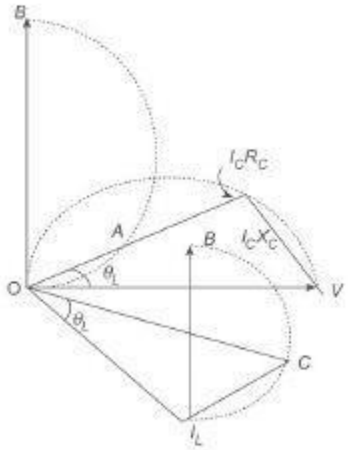
Variable X_c:



Variable R_L:



Variable R_C:

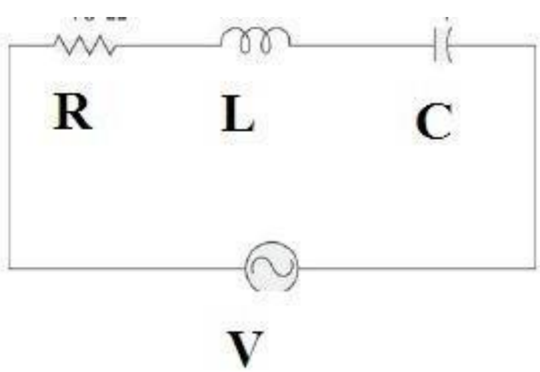


Resonance: If circuit has both inductor and capacitors resonance occurs

At resonance imaginary part of impedance or admittance is zero

The study of resonance is very useful, particularly in the area of Communications. For example, the ability of a radio receiver to select a certain frequency, transmitted by a station and to eliminate frequencies from other stations is based on the principle of resonance.

Series Resonance:



Current I= $\frac{V}{Z}$

Z=$R + (X_L - \ )$

$X_L = \omega L$

$X_C = \dfrac{1}{\omega C}$

At resonance:$(Z) = 0$

$(X_L - X_C) = 0$

$X_L = X_C$

$\omega_r L = \dfrac{1}{\omega_r C}$

${\omega_r}^2 = \dfrac{1}{LC}$

$\omega_r = \dfrac{1}{\sqrt{LC}}$

$2\pi f_r = \frac{1}{\sqrt{LC}}$

$f_r = \dfrac{1}{2\pi\sqrt{LC}}$

At series resonance Z=R=Minimum impedance=Z_{min}

Hence current is maximum

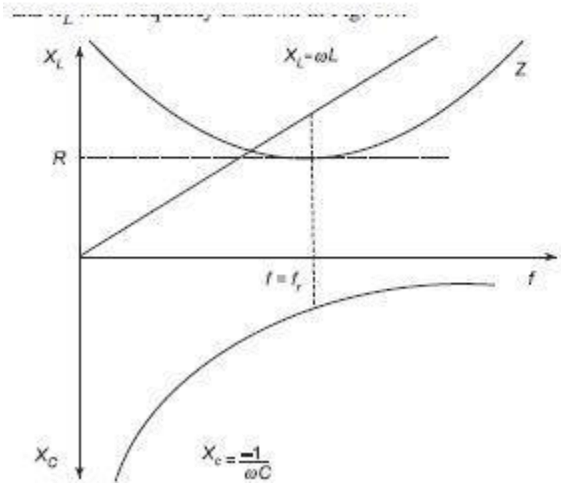
$I_{max=} \dfrac{V}{Z_{min}} = \dfrac{V}{R}$

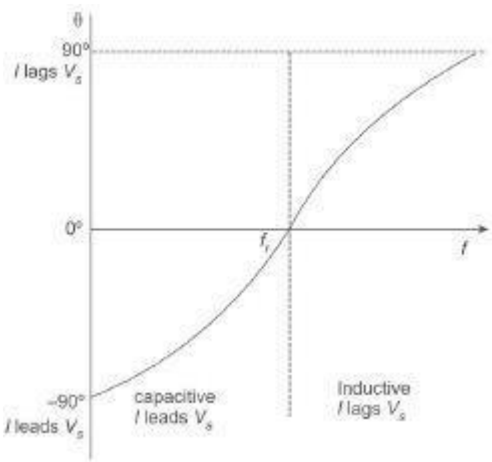
If$f < f_r$, $X_C > X_L \rightarrow$ Nature of circuit is Capacitive $\rightarrow$ I leads V

If$f = f_r$, $X_C = X_L \rightarrow$ Nature of circuit is Resistive $\rightarrow$ I in phase with V $\rightarrow$ Unity powerfactor

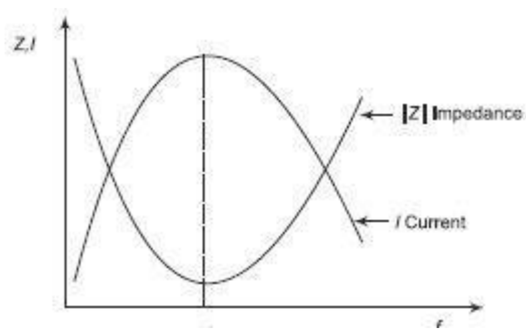
If$f > f_r$, $X_C < X_L \rightarrow$ Nature of circuit is Inductive $\rightarrow$ I in lags V

Impedance &Phase Angle at Resonance:

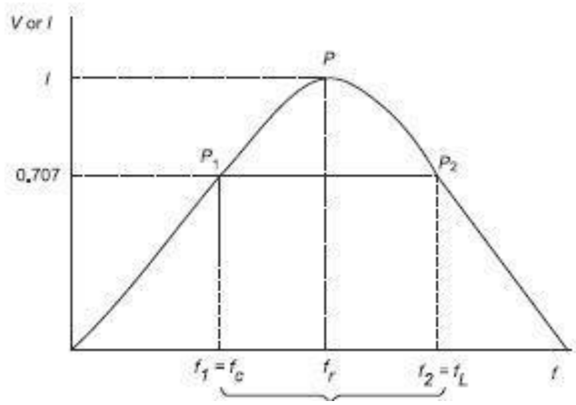




Voltage &currents at resonance:



Bandwidth of series RLC circuit:



It is the range of frequencies at which current will be 70.7% of maximum value

B.W=f_2-f_1

f_2=Upper cut-off frequency in HZ

ω_2=Upper cut-off frequency in rad/sec

f_1=Lower cut-off frequency in HZ

ω_1=Lowr cut-off frequency in rad/sec

$f_r = \sqrt{f_1 f_2}$ in HZ

ω_r=Resonant frequency in rad/sec

At f_1: I=0.707I_{max}

$\frac{V}{Z}$=0.707I_{max}

$\frac{V}{Z}=0.707\frac{V}{Z_{min}}$

$\frac{V}{Z}=0.707\frac{V}{R}$

$\frac{1}{Z}=0.707\frac{1}{R}$

Z=R/0.707

Z=$\sqrt{2}$R

Z^2=2R^2

$R^2 + (X_C - X_L)^2$=2R^2

$R^2 = (X_C - X_L)^2$

R=$X_C - X_L$

R=$\frac{1}{\omega_1 C} - \omega_1 L$------------- (1)

At f_2: I=0.707I_{max}

$\frac{V}{Z}$=0.707I_{max}

$\frac{V}{Z}$=0.707$\frac{V}{Z_{min}}$

$\frac{V}{Z}$=0.707$\frac{V}{R}$

$\frac{1}{Z}$=0.707$\frac{1}{R}$

Z=R/0.707

Z=$\sqrt{2}$R

Z^2=2R^2

$R^2 + (X_L - \ )^2$=2R^2

$R^2 = (X_L - X_C)^2$

R=$X_L - X_C$

R=$\omega_2 L - \frac{1}{\omega_2 C}$------------- (2)

From (1)&(2):

$$\frac{1}{\omega_1 C} - \omega_1 L = \omega_2 L - \frac{1}{\omega_2 C}$$

$$\frac{1}{\omega_1 C} + \frac{1}{\omega_2 C} = \omega_2 L + \omega_1 L$$

$$(\frac{1}{\omega_1} + \frac{1}{\omega_2})\frac{1}{C} = (\omega_1 + \omega_2)L$$

$$\frac{(\omega_1+\omega_2)}{C\omega_1\omega_2} = (\omega_1 + \omega_2)L$$

$$\omega_1\omega_2 = \frac{1}{LC} \text{-------------- (3)}$$

(1)+(2)→

$$R+R = \frac{1}{\omega_1 C} - \omega_1 L + \omega_2 L - \frac{1}{\omega_2 C}$$

$$2R = (\omega_2 - \omega_1) + \frac{1}{\omega_1 C} - \frac{1}{\omega_2 C}$$

$$2R = (\omega_2 - \omega_1) + \frac{C(\omega_2 - \omega_1)}{C^2\omega_1\omega_2}$$

$$2R = (\omega_2 - \omega_1)(L + \frac{1}{C\omega_1\omega_2}) \text{-------------- (4)}$$

From (3)

$$\omega_1\omega_2 = \frac{1}{LC}$$

$$\frac{1}{C\omega_1\omega_2} = L \text{---------------- (5)}$$

Substitute (5) in (4)

$$2R = (\omega_2 - \omega_1)(L + L)$$

$$\omega_2 - \omega_1 = \frac{R}{L} \text{-------------- (6)}$$

Equation (6) represents band width in rad/sec

Quality Factor:

$$Q = \frac{2\pi * \text{Maximum Energy Stored}}{\text{Energy Dissipated Per cycle}}$$

For Inductor:

Maximum Energy stored= $\frac{1}{2} L I_{max}^2 = \frac{1}{2} L (\frac{V_{max}}{R})^2$

Energy dissipated per cycle= $\frac{(\frac{V}{R})^2}{T}$

V= $\frac{V_{max}}{\sqrt{2}}$

Energy dissipated per cycle= $\frac{(\frac{V}{R})^2}{T} = f * \frac{1}{R^2} * V^2 = f * \frac{1}{R^2} * (\frac{V_{max}}{\sqrt{2}})^2 = \frac{fV_{max}^2}{2R^2}$

$$Q = \frac{2\pi * \text{Maximum Energy Stored}}{\text{Energy Dissipated Per cycle}}$$

$$Q = \frac{2\pi * \frac{1}{2} L (\frac{V_{max}}{R})^2}{\frac{fV_{max}^2}{2R^2}}$$

$$Q = \frac{2\pi fL}{R} = \frac{\omega L}{R} = \frac{X_L}{R}$$

For Capacitor:

$$Q = \frac{1}{2\pi fCR} = \frac{1}{\omega CR} = \frac{X_C}{R}$$

Relation between Q and B.W:

Q= $\frac{f_r}{B.W}$

A higher value of the circuit Q results in a smaller bandwidth. A lower value of Q causes a larger bandwidth

Voltage Magnification:

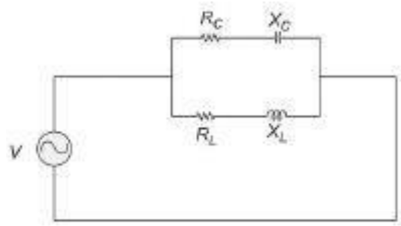
$V_R = V = Supply\ Voltage$

$V_L = jQV = |QV|\angle 90°$

$V_L = -jQV = |QV|\angle -90°$

In series resonance Voltage Magnification across L&C

Parallel Resonance:



Y=$Y_1 + Y_2$

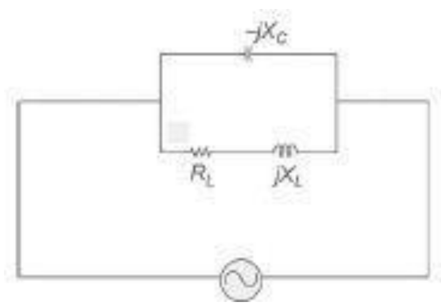
$$Y = \frac{1}{R_C - jX_C} + \frac{1}{R_L + jX_L}$$

Simplifying Making Imaginary part=0

$$\omega_r = \frac{1}{\sqrt{LC}}$$

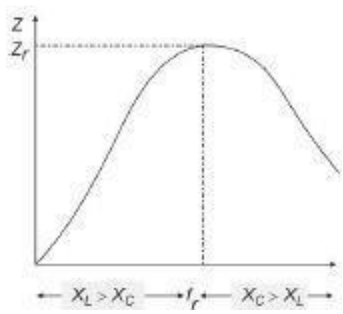
$$f_r = \frac{1}{2\pi\sqrt{LC}}$$

Resonant Frequency of a Tank Circuit:



$$f_r = \frac{1}{2\pi\sqrt{LC}}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$$

Variation of Z with frequency:

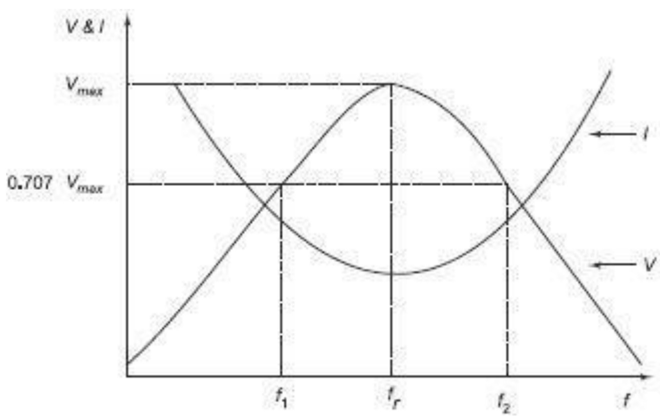


If $f < f_r$, $X_L > X_C$ → Nature of circuit is Inductive → I lags V

If $f = f_r$, $X_L = X_C$ → Nature of circuit is Resistive → I in phase with V → Unity powerfactor

If $f > f_r$, $X_L < X_{CL}$ → Nature of circuit is Capacitive → I in leads V

Bandwidth of Parallel RLC circuit:



BW=$\frac{1}{RC}$

Quality Factor:

$$Q = \frac{2\pi * \text{Maximum Energy Stored}}{\text{Energy Dissipated Per cycle}}$$

For Inductor:

$$Q = \frac{R}{2\pi f L} = \frac{R}{\omega L} = \frac{R}{X_L}$$

For Capacitor:

$$Q = R * 2\pi f C = \omega C R = \frac{R}{X_C}$$

Relation between Q and B.W:

$$Q = \frac{f_r}{B.W}$$

A higher value of the circuit Q results in a smaller bandwidth. A lower value of Q causes a larger bandwidth

Current Magnification:

$I_R = I = Supply\ Current$

$V_L = -jQI = |QI| \angle -90°$

$V_L = -jQI = |QI| \angle +90°$

In Parallel resonance Voltage Magnification across L&C

UNIT – IV:
Transmission Lines – I

- Types
- Parameters
- Transmission Line Equations
- Primary & Secondary Constants
- Expressions for Characteristic Impedance, Propagation Constant
- Phase and Group Velocities
- Infinite Line Concepts
- Losslessness/Low Loss Characterization
- Distortion
- Condition for Distortionlessness and Minimum Attenuation
- Illustrative Problems

UNIT-IV

TRANSMISSION LINES-I

Introduction:

Transmission Line: It is a medium used for transmitting electric waves.

Transmission line consists of two uniform wires

Examples of electric waves: Telephone messages, Electric Power signals

Sending End: The end of the transmission line to which Source is connected

Receiving End: The end of the transmission line to which Load is connected

Classification of Transmission Lines:

1) Open wire Lines: Air will acts like dielectric. Affected due to atmospheric conditions. High initial and maintenance cost. Advantage: Less capacitance
2) Cables: Used where the erection of open wire lines is not possible. We use dielectric between conductors .And also we use protective sheath. Fault detection is difficult
3) Co axial Cables: Consists of one hollow conductor and other conductor is coaxially placed on the hollow conductor. Between these two conductors a dielectric is used. Used for high voltage applications
4) Wave guides: Used for high frequency transmission. Transmits the power based on principle of total internal reflection

Constants of Transmission Lines:

1) Primary constants or Transmission line parameters:
Independent of frequency. They are R, L, C, and G. Here R and G are series parameters. C and G are shunt parameters

R=Resistance of the transmission line per 1 meter length measured in Ω/m. Due to resistance of the line we have voltage drop and power loss in the transmission line

L=Inductance of the transmission line per 1 meter length measured in H/m. Due to alternating current flowing through the transmission line alternating magnetic flux establishes. Inductance =Flux linkage/Current

C=Capacitance the transmission line per 1 meter length measured in F/m. Two conductors separated by dielectric forms capacitance

G=Conductance the transmission line per 1 meter length in Siemens/m. Due to imperfections in dielectrics and insulators in transmission system conductance exists. Leakage current flows due to conductance

2) Secondary constants: Depends on frequency. They are Z_0 and Z_0=Characteristic Impedance in Ω

γ =Propagation constant=$\alpha + j\beta$

α =Attenuation constant in nepers/m=Indicates reduction in the magnitude of wave

β =Phase constant in rad/m=Indicates phase lagging of wave

Transmission Line equations:

Let’s start the analysis of the transmission from receiving end.

Consider a section of Δx meters.

Apply KVL:

$$-V(x) + I(x)R\Delta x + I(x)j\omega L\Delta x + V(x + \Delta x) = 0$$

$$V(x+\Delta x)-V(x)=I(x)R\Delta x+I(x)j\omega L\Delta x$$

$$V(x+\Delta x)-V(x)=I(x)(R+j\omega L)\Delta x$$

$$\frac{V(x+\Delta x)-V(x)}{\Delta x}=I(x)Z$$

Z=$R+J\omega L$

$$\lim_{\Delta x\to\infty}\left(\frac{V(x+\Delta x)-V(x)}{\Delta x}\right)=\lim_{\Delta x\to\infty}(I(x)Z)$$

$$\frac{dV(x)}{dx}=I(x)Z$$

Apply KCL:

$$\frac{dI(x)}{dx}=V(x)Y$$

Y=$G+J\omega C$

$$\frac{d^2V(x)}{dx^2}=Z\frac{dI(x)}{dx}$$

$$\frac{d^2V(x)}{dx^2}=ZV(x)Y$$

$$\frac{d^2V(x)}{dx^2}=ZYV(x)$$

$$\frac{d^2V(x)}{dx^2}=\gamma^2V(x)$$

Here $\gamma^2=ZY$

$\gamma=\sqrt{ZY}$=Propagation Constant of transmission Line

$$\frac{d^2V(x)}{dx^2}-\gamma^2V(x)=0$$

Solution of above differential equation: $V(x)=K_1e^{\gamma x}+K_2e^{-\gamma x}$
K_1, K_2 Constants

$$\frac{d^2I(x)}{dx^2}=Y\frac{dV(x)}{dx}$$

$$\frac{d^2I(x)}{dx^2} = YI(x)Z$$

$$\frac{d^2I(x)}{dx^2} = ZYI(x)$$

$$\frac{d^2I(x)}{dx^2} = \gamma^2 I(x)$$

$$\frac{d^2I(x)}{dx^2} - \gamma^2 I(x) = 0$$

Solution of above differential equation: $I(x) = K_3 e^{\gamma x} + K_4 e^{-\gamma x}$

$K_3, K_4 = \text{Constants}$

At x=0: $V(x) = V_R$

$I(x) = I_R$

$V_R = K_1 e^{\gamma 0} + K_2 e^{-\gamma 0}$

$V_R = K_1 + K_2$

$I_R = K_3 e^{\gamma 0} + K_4 e^{-\gamma 0}$

$I_R = K_3 + K_4$

$V(x) = K_1 e^{\gamma x} + K_2 e^{-\gamma x}$

$$\frac{dV(x)}{dx} = K_1 \gamma e^{\gamma x} - K_2 \gamma e^{-\gamma x}$$

$I(x)Z = K_1 \gamma e^{\gamma x} - K_2 \gamma e^{-\gamma x}$

At x=0: $V(x) = V_R$

$I(x) = I_R$

$I_R Z = K_1 \gamma e^{\gamma 0} - K_2 \gamma e^{-\gamma 0}$

$I_R Z = \gamma (K_1 - K_2)$

$$(K_1 - K_2) = \frac{I_R Z}{\gamma}$$

$$(K_1 - K_2) = \frac{I_R Z}{\sqrt{ZY}}$$

$$(K_1 - K_2) = I_R \sqrt{\frac{Z}{Y}}$$

$$(K_1 - K_2) = I_R Z_0$$

$Z_0 = \sqrt{\frac{Z}{Y}}$ =Characteristic Impedance

$$I(x) = K_3 e^{\gamma x} + K_4 e^{-\gamma x}$$

$$\frac{dI(x)}{dx} = K_3 \gamma\, e^{\gamma x} - K_4 \gamma\, e^{-\gamma x}$$

$$V(x)Y = K_3 \gamma\, e^{\gamma x} - K_4 \gamma\, e^{-\gamma x}$$

At x=0: $V(x) = V_R$

$I(x) = I_R$

$$V_R Y = K_3 \gamma\, e^{\gamma 0} - K_4 \gamma\, e^{-\gamma 0}$$

$$V_R Y = \gamma\,(K_3 - K_4)$$

$$K_3 - K_4 = \frac{V_R Y}{\gamma}$$

$$K_3 - K_4 = \frac{V_R Y}{\sqrt{ZY}}$$

$$K_3 - K_4 = \frac{V_R}{\sqrt{Z/Y}}$$

$$K_3 - K_4 = \frac{V_R}{Z_0}$$

$$K_1 + K_2 = V_R$$

$$K_1 - K_2 = I_R Z_0$$

$$K_3 + K_4 = I_R$$

$$K_3 - K_4 = \frac{V_R}{Z_0}$$

Solving above equations:

$$K_1 = \frac{(V_R + I_R Z_0)}{2}$$

$$K_2 = \frac{(V_R - I_R Z_0)}{2}$$

$$K_3 = \frac{I_R + (\frac{V_R}{Z_0})}{2}$$

$$K_4 = \frac{I_R - (\frac{V_R}{Z_0})}{2}$$

$$V(x) = \frac{(V_R + I_R Z_0)}{2} e^{\gamma x} + \frac{(V_R - I_R Z_0)}{2} e^{-\gamma x}$$

$$V(x) = \frac{(e^{\gamma x} + e^{-\gamma x})}{2} V_R + \frac{(e^{\gamma x} - e^{-\gamma x})}{2} I_R Z_0$$

$$V(x) = V_R \cosh \gamma x + I_R Z_0 \sinh \gamma x$$

$$I(x) = \frac{I_R + (\frac{V_R}{Z_0})}{2} e^{\gamma x} + \frac{I_R - (\frac{V_R}{Z_0})}{2} e^{-\gamma x}$$

$$I(x) = \frac{V_R}{Z_0} \frac{(e^{\gamma x} - e^{-\gamma x})}{2} + \frac{(e^{\gamma x} + e^{-\gamma x})}{2} I_R$$

$$I(x) = \frac{V_R}{Z_0} \sinh \gamma x + I_R \cosh \gamma x$$

Transmission line equations in terms of Receiving end voltages and Currents:

$$V(x) = V_R \cosh \gamma x + I_R Z_0 \sinh \gamma x$$

$$I(x) = \frac{V_R}{Z_0} \sinh \gamma x + I_R \cosh \gamma x$$

Let l=Length of the transmission line

At x=l: $V(x) = V_S$

$I(x) = I_S$

$V_S = V_R \cosh \gamma l + I_R Z_0 \sinh \gamma l$

$I_S = \frac{V_R}{Z_0} \sinh \gamma l + I_R \cosh \gamma l$

Transmission line equations in terms of sending end voltage and Currents:

$V(x) = V_S \cosh \gamma x - I_S Z_0 \sinh \gamma x$

$I(x) = -\frac{V_S}{Z_0} \sinh \gamma x + I_S \cosh \gamma x$

Input impedance and Transfer impedance of the transmission line:

$V_S = V_R \cosh \gamma l + I_R Z_0 \sinh \gamma l$

$I_S = \frac{V_R}{Z_0} \sinh \gamma l + I_R \cosh \gamma l$

Load Impedance: $Z_R = \frac{V_R}{I_R}$

$V_R = I_R Z_R$

$V_S = V_R \cosh \gamma l + I_R Z_0 \sinh \gamma l$

$V_S = I_R Z_R \cosh \gamma l + I_R Z_0 \sinh \gamma l$

$V_S = I_R(Z_R \cosh \gamma l + Z_0 \sinh \gamma l)$

$\frac{V_S}{I_R} = Z_R \cosh \gamma l + Z_0 \sinh \gamma l$

$Z_T = Z_R \cosh \gamma l + Z_0 \sinh \gamma l$

Transfer Impedance: $Z_T = \frac{V_S}{I_R}$

$I_S = \frac{V_R}{Z_0} \sinh \gamma l + I_R \cosh \gamma l$

$I_S = \frac{I_R Z_R}{Z_0} \sinh \gamma l + I_R \cosh \gamma l$

$$I_S = \frac{(Z_R sinh\,\gamma l + Z_0 cosh\,\gamma l)}{Z_0}$$

$$Input\;\; Impedance: Z_{in} = \frac{V_S}{I_S}$$

$$Z_{in} = \frac{(Z_R\,cosh\,\gamma l + Z_0\,sinh\,\gamma l)}{\frac{(Z_R sinh\,\gamma l + Z_0 cosh\,\gamma l)}{Z_0}}$$

$$Z_{in} = \frac{Z_0(Z_R\,cosh\,\gamma l + Z_0\,sinh\,\gamma l)}{Z_0\,cosh\,\gamma l + Z_R\,sinh\,\gamma l}$$

$$Z_{in} = \frac{Z_0\,cosh\,\gamma l\,(Z_R + Z_0\,tanh\,\gamma l)}{cosh\,\gamma l\,(Z_0 + Z_R\,tanh\,\gamma l)}$$

$$Z_{in} = \frac{Z_0(Z_R + Z_0\,tanh\,\gamma l)}{(Z_0 + Z_R\,tanh\,\gamma l)}$$

For infinite line: l=∞

$$tanh\,\gamma l = 1$$

$$Z_{in} = \frac{Z_0(Z_R + Z_0\,tanh\,\gamma l)}{(Z_0 + Z_R\;\,tanh\,\gamma l)}$$

$$Z_{in} = \frac{Z_0(Z_R + Z_0 \times 1)}{(Z_0 + Z_R \times 1)}$$

$$Z_{in} = \frac{Z_0(Z_R + Z_0)}{(Z_0 + Z_R)}$$

$$Z_{in} = Z_0$$

Hence the input impedance of infinite line is characteristic impedance

For a line terminated with Z_0: Z_R=Z_0

$$Z_{in} = \frac{Z_0(Z_0 + Z_0\,tanh\,\gamma l)}{(Z_0 + Z_0\,tanh\,\gamma l)}$$

$$Z_{in} = Z_0$$

Hence a short line of finite length terminated with Z_0 *behaves like a infinte line*

For a line terminated with Open circuit: $Z_R = \infty$

$Z_{in} = Z_{OC}$

$$Z_{OC} = \frac{Z_0(Z_R + Z_0 \tanh \gamma l)}{(Z_0 + Z_R \tanh \gamma l)}$$

$$Z_{OC} = \frac{Z_0(1 + Z_0 \tanh \gamma l / Z_R)}{(Z_0/Z_R + \tanh \gamma l)}$$

$$Z_{OC} = \frac{Z_0(1 + Z_0 \tanh \gamma l/\infty)}{(Z_0/\infty + \tanh \gamma l)}$$

$$Z_{OC} = \frac{Z_0(1 + 0)}{(0 + \tanh \gamma l)}$$

$$Z_{OC} = \frac{Z_0}{\tanh \gamma l}$$

For a line terminated with Short circuit: $Z_R = 0$

$$Z_{SC} = \frac{Z_0(0 + Z_0 \tanh \gamma l)}{(Z_0 + 0 \tanh \gamma l)}$$

$$Z_{SC} = \frac{Z_0(Z_0 \tanh \gamma l)}{Z_0}$$

$$Z_{SC} = Z_0 tanh\gamma l$$

$$Z_{OC} Z_{SC} = \frac{Z_0}{\tanh \gamma l} Z_0 tanh\gamma l$$

$$Z_{OC} Z_{SC} = {Z_0}^2$$

$$Z_0 = \sqrt{Z_{OC} Z_{SC}}$$

$$\frac{Z_{SC}}{Z_{OC}} = \frac{Z_0 \tanh \gamma l}{\frac{Z_0}{\tanh \gamma l}}$$

$$\frac{Z_{SC}}{Z_{OC}} = (tanh\, \gamma l)^2$$

$$tanh\, \gamma l = \sqrt{\frac{Z_{SC}}{Z_{OC}}}$$

Wave length and velocity of propagation of transmission line:

$$\lambda = \frac{2\pi}{\beta}$$

$$v = \frac{\lambda}{T}$$

$$v = \lambda \times f$$

$$v = \frac{2\pi}{\beta} \times f$$

$$v = \frac{2\pi f}{\beta}$$

$$v = 2\pi f / \beta \omega$$

$$v = \omega / \beta$$

The above velocity also called as phase velocity

Group velocity:

$$V_g = \frac{d\omega}{d\beta}$$

This concept is used in case of wave guides where multiple waves are transmitted.

Attenuation constant(α) and phase constant (β):

Propagation constant= $\gamma = \alpha + j\beta$

$$\gamma = \sqrt{ZY} = \sqrt{(R + j\omega L)(G + j\omega C)}$$

$$\alpha + j\beta = \sqrt{(R + j\omega L)(G + j\omega C)}$$

$(\alpha + j\beta)^2 = (R + j\omega L)(G + j\omega C)$

$\alpha^2 + 2\,\alpha\, j\beta + (j\beta)^2 = RG + j\, R\omega C + j\omega LG - \omega^2 LC$

$\alpha^2 + 2\,\alpha\, j\beta - \beta^2 = RG - \omega^2 LC + j(R\omega C - \omega LG)$

$\alpha^2 - \beta^2 + j2\,\alpha\,\beta = RG - \omega^2 LC + j(R\omega C - \omega LG)$

Compare real parts: $\alpha^2 - \beta^2 = RG - \omega^2 LC$

Compare imaginary :2 α β= $R\omega C - \omega LG$

$$\alpha\,\beta = \frac{R\omega C - \omega LG}{2}$$

In polar form $|\gamma| = \sqrt{\alpha^2 + \beta^2} = \sqrt{\sqrt{(R^2 + \omega^2 L^2)(G^2 + \omega^2 C^2)}}$

Square on both sides

$\alpha^2 + \beta^2 = \sqrt{(R^2 + \omega^2 L^2)(G^2 + \omega^2 C^2}$

$\alpha^2 + \beta^2 + \alpha^2 - \beta^2 = \sqrt{(R^2 + \omega^2 L^2)(G^2 + \omega^2 C^2} + RG - \omega^2 LC$

$2\alpha^2 = \sqrt{(R^2 + \omega^2 L^2)(G^2 + \omega^2 C^2} + RG - \omega^2 LC$

$$\alpha^2 = \frac{1}{2}\{\sqrt{(R^2 + \omega^2 L^2)(G^2 + \omega^2 C^2} + RG - \omega^2 LC\}$$

$\alpha = \frac{1}{2}\{\sqrt{(R^2 + \omega^2 L^2)(G^2 + \omega^2 C^2} + RG - \omega^2 LC\}^{1/2}$

$\alpha^2 + \beta^2 - (\alpha^2 - \beta^2) = \sqrt{(R^2 + \omega^2 L^2)(G^2 + \omega^2 C^2} - (RG - \omega^2 LC)$

$2\beta^2 = \sqrt{(R^2 + \omega^2 L^2)(G^2 + \omega^2 C^2} - (RG - \omega^2 LC)$

$$\beta = \frac{1}{2}\{\sqrt{(R^2 + \omega^2 L^2)(G^2 + \omega^2 C^2} - (RG - \omega^2 LC)\}$$

Condition for minimum attenuation:

$\alpha = \frac{1}{2}\{\sqrt{(R^2 + \omega^2 L^2)(G^2 + \omega^2 C^2} + RG - \omega^2 LC\}^{1/2}$

The value of α *depends on* R, L, C, G

Case1 $\boldsymbol{L\ is\ variable}$:

For minimum attenuation: $\frac{d\alpha}{dL}$=0

$\frac{d}{dL}\frac{1}{2}\{\sqrt{(R^2+\omega^2 L^2)(G^2+\omega^2 C^2}+RG-\omega^2 LC\}^{1/2}$=0

$\frac{1}{2}\frac{d}{d}\{\sqrt{(R^2+\omega^2 L^2)(G^2+\omega^2 C^2}+RG-\omega^2 LC\}^{1/2}$=0

$\frac{d}{dL}\{\sqrt{(R^2+\omega^2 L^2)(G^2+\omega^2 C^2}+RG-\omega^2 LC\}^{1/2}$=0

$\frac{1}{2\sqrt{(R^2+\omega^2L^2)(G^2+\omega^2C^2}+(RG-\omega^2LC)}\times\{\frac{1}{2\sqrt{(R^2+\omega^2L^2)(G^2+\omega^2C^2)}}(2\omega^2L)(G^2+\omega^2C^2)-$

$\omega^2C\}$=0

$\frac{1}{2\sqrt{(R^2+\omega^2L^2)(G^2+\omega^2C^2)}}(2\omega^2L)(G^2+\omega^2C^2)-\omega^2C$=0

$L\sqrt{\frac{G^2+\omega^2C^2}{R^2+\omega^2L^2}}$=C

$L\sqrt{G^2+\omega^2C^2}$=C$\sqrt{R^2+\omega^2L^2}$

Square on both sides

$L^2(G^2+\omega^2C^2)=C^2(R^2+\omega^2L^2)$

$L^2G^2+L^2\omega^2C^2=C^2R^2+C^2\omega^2L^2$

$L^2G^2=C^2R^2$

LG=CR

$\frac{R}{G}=\frac{L}{C}$

$L=\frac{RC}{G}$

Case2: $\boldsymbol{C\ is\ variable}$:

For minimum attenuation: $\frac{d\alpha}{dC}$=0

$\frac{d}{dC}\frac{1}{2}\{\sqrt{(R^2+\omega^2L^2)(G^2+\omega^2C^2}+RG-\omega^2LC\}^{1/2}$=0

$\frac{1}{2}\frac{}{dC}\{\sqrt{(R^2+\omega^2L^2)(G^2+\omega^2C^2}+RG-\omega^2LC\}^{1/2}$=0

$\frac{d}{dC}\{\sqrt{(R^2+\omega^2L^2)(G^2+\omega^2C^2}+RG-\omega^2LC\}^{1/2}$=0

$\frac{1}{2\sqrt{(R^2+\omega^2L^2)(G^2+\omega^2C^2+(RG-\omega^2LC)}}\times\{\frac{1}{2\sqrt{(R^2+\omega^2L^2)(G^2+\omega^2C^2)}}(R^2+\omega^2L^2)(2\omega^2C)-\omega^2L\}$=0

$\{\frac{1}{2\sqrt{(R^2+\omega^2L^2)(G^2+\omega^2C^2)}}(R^2+\omega^2L^2)(2\omega^2C)-\omega^2L\}$=0

$C\sqrt{\frac{R^2+\omega^2L^2}{G^2+\omega^2C^2}}$=L

Square on both sides:

$C^2(R^2+\omega^2L^2)=L^2(G^2+\omega^2C^2)$

$C^2R^2+C^2\omega^2L^2=L^2G^2+L^2\omega^2C^2$

$C^2R^2=L^2G^2$

CR=LG

$\frac{R}{G}=\frac{L}{C}$

C= $\frac{LG}{R}$

Waveform Distortion:

If the receiving end signal is not exact replica of sending end signal it is said to have waveform distortion.

Causes of waveform distortion:

1) Change in Z_0 With respect to frequency
2) Change in α With respect frequency
3) Change in β With respect frequency

1) Change in Z_0 With respect to frequency:

Due to this selective power absorption takes place

To avoid this problem Z_0 should be independent of frequency

$$Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}}$$

$$Z_0 = \sqrt{\frac{R\,(1 + j\frac{\omega L}{R})}{G\,(1 + j\frac{\omega C}{G})}}$$

To make Z_0 indepndent of frequency: $1 + j\frac{\omega L}{R} = 1 + j\frac{\omega C}{G}$

$$j\frac{\omega L}{R} = j\frac{\omega C}{G}$$

$$\frac{L}{R} = \frac{C}{G}$$

$$\frac{R}{G} = \frac{L}{C}$$

$$Z_0 = \sqrt{\frac{R}{G}} < 0°$$

$$\frac{R}{G} = \frac{L}{C} \Rightarrow Z_0 = \sqrt{\frac{L}{C}} < 0°$$

2) Change in α With respect to frequency:

This type of distortion is called frequency distortion

$$\alpha = \frac{1}{2}\{\sqrt{(R^2 + \omega^2 L^2)(G^2 + \omega^2 C^2} + RG - \omega^2 LC\}^{1/2}$$

Due to frequency distortion, attenuation is different at different frequencies. Due to this, audio signals are more affected compared video signals. To avoid this problem α should be independent of frequency .We use equalizers to reduce frequency distortion

3) Change in β With respect to frequency:

This type of distortion is called phase distortion

$\beta = \frac{1}{2}\{\sqrt{(R^2 + \omega^2L^2)(G^2 + \omega^2C^2} - (RG - \omega^2LC)\}$

$V = \frac{\omega}{Q}$

Due to phase distortion, velocity of propagation is different at different frequencies. Due to this, video signals are more affected compared audio signals. To avoid this problem β should be independent of frequency. We use co axial cables to reduce phase distortion

---

Distortion less line:

If the line is free from frequency and phase distortion then such line is called as distortion less line.

It means α and velocity of propagation should be independent of frequency.

$\gamma = \sqrt{(R + j\omega L)(G + j\omega C)}$

$\gamma = \sqrt{RG + jR\omega C + j\omega LG + j^2\omega^2LC}$

$\gamma = \sqrt{RG + jR\omega C + j\omega LG - \omega^2LC}$

$\gamma = \sqrt{RG - \omega^2LC + (RC + LG)}$

$\frac{R}{G} = \frac{L}{C}$

RC=LG

$\gamma^2 = RG - \omega^2LC + (RC + LG)$

$(\alpha + j\beta)^2 = RG - \omega^2LC + j\omega(RC + RC)$

$(\alpha + j\beta)^2 = RG - \omega^2LC + j\omega 2RC$

$(\alpha + j\beta)^2 = RG - \omega^2LC + 2j\omega\sqrt{RCRC}$

$(\alpha + j\beta)^2 = RG - \omega^2LC + 2j\omega\sqrt{RCLG}$

$(\alpha + j\beta)^2 = (\sqrt{RG})^2 + (j\omega\sqrt{LC})^2 + 2\sqrt{RG}\, j\omega\sqrt{LC}$

$(\alpha + j\beta)^2 = (\sqrt{RG} + j\omega\sqrt{LC})^2$

$\alpha = \sqrt{RG}$

$\alpha\ is\ independent\ of\ frequency. Hence\ frequency\ distortion\ is\ zero$

$\beta = \omega\sqrt{LC}$

$$\nu = \frac{\omega}{\beta}$$

$$\nu = \frac{\omega}{\omega\sqrt{LC}}$$

$$\nu = \frac{1}{\sqrt{LC}}$$

Velocity is independent of frequency. Hence phase distortion is zero

---

Loss less line:

R=0, G=0

$$Z_0 = \sqrt{\frac{0 + j\omega L}{0 + j\omega C}}$$

$$Z_0 = \sqrt{\frac{j\omega L}{j\omega C}}$$

$$Z_0 = \sqrt{\frac{L}{C}}$$

$\gamma = \sqrt{ZY} = \sqrt{(0 + j\omega L)(0 + j\omega C)}$

$\gamma = j\omega\sqrt{LC}$

Hence $\alpha = 0$ and $\beta = \omega\sqrt{LC}$

Velocity of propagation $\nu = \frac{\omega}{\omega\sqrt{LC}} = \frac{1}{\sqrt{LC}}$

UNIT V

Transmission Lines – II

- SC and OC Lines
- Input Impedance Relations
- Reflection Coefficient
- VSWR
- λ/4, λ 2, λ /8 Lines – Impedance Transformations
- Significance of Zmin and Zmax
- Smith Chart – Configuration and Applications
- Single Stub Matching
- Illustrative Problems

Unit-V

Transmission Lines-II

Incident &Reflected Waves:

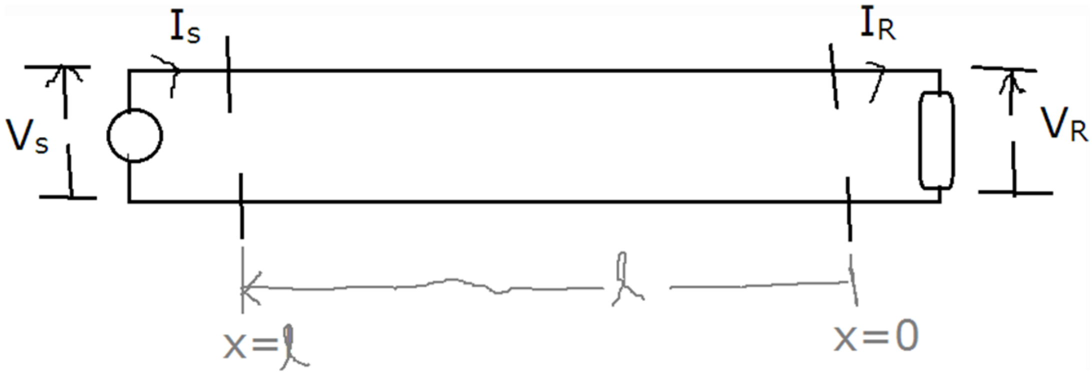


Fig: 5.1

Consider a transmission line as shown

The travelling waves travel from source to load

The impedance offered to these waves on the line is known as Characteristic Impedance (Z_0)

When wave reached receiving end impedance offered to the wave is Z_R

If $Z_R = Z_0$ there is no change in impedance hence there is no reflection of the waves.

If $Z_R \# Z_0$ there is change in impedance hence there is a reflection of the waves.

Let's starts analysis from receiving end

l= $length\ of\ transmission\ line$

At x=0, Receiving end

If we travel towards sending end x value increases

Maximum value of $x = l$

At $x = l$ sending end

$$V(x) = \frac{(V_R+I_R Z_0)}{2} e^{\gamma x} + \frac{(V_R-I_R Z_0)}{2} e^{-\gamma x}$$

$V(x) = V^+(x) + V^-(x)$

$I(x) = I^+(x) + I^-(x)$

$V^+(x) = \frac{(V_R+I_R Z_0)}{2} e^{\gamma x}$ =Incident Voltage wave Travel from sending end to receiving end

$V^-(x) = \frac{(VR-IRZ0)}{2} e^{-\gamma x}$=Reflected Voltage wave Travel from receiving end to sending end

$I^+(x) = \frac{(IR+VR/Z0)}{2} e^{\gamma x}$ =Incident Current wave Travel from sending end to receiving end

$I^-(x) = \frac{(IR-VR/Z0)}{2} e^{-\gamma x}$=Reflected Current wave Travel from receiving end to sending end

At any point on transmission line:

1. Transmission voltage is sum of incident and reflected voltages
2. Transmission Current is sum of incident and reflected currents

Reflection Coefficient (K):

Reflection Voltage Coefficient (K_V):

$$K_V = \frac{\text{Reflected voltage at Receiving End}}{\text{Incident Voltage at Receiving End}}$$

At receiving end: x=0

Reflected Voltage=$V^-(x) = \frac{(V_R-I_R Z_0)}{2} e^{-\gamma x}$

At receiving end: x=0

$$\text{Reflected voltage at Receiving End} = \frac{(V_R - I_R Z_0)}{2}$$

$$\text{Incident Voltage} = V^+(x) = \frac{(VR+IRZ0)}{2} e^{\gamma x}$$

$$\text{Incident Voltage at Receiving End} = \frac{(V_R + I_R Z_0)}{2}$$

$$K_V = \frac{(V_R - I_R Z_0)}{(V_R + I_R Z_0)}$$

$$K_V = \frac{(I_R Z_R - I_R Z_0)}{(I_R Z_R + I_R Z_0)}$$

$$K_V = \frac{(Z_R - Z_0)}{(Z_R + Z_0)}$$

Reflection Current Coefficient (K_I):

$$K_I = \frac{\text{Reflected Current at Receiving End}}{\text{Incident Current at Receiving End}}$$

At receiving end: x=0

$$\text{Reflected Current} = I^-(x) = \frac{(I_R - V_R/Z_0)}{2} e^{-\gamma x}$$

At receiving end: x=0

$$\text{Reflected current at Receiving End} = \frac{(I_R - V_R/Z_0)}{2}$$

$$\text{Incident Current} = I^+(x) = \frac{(IR+VR/Z0)}{2} e^{\gamma x}$$

$$\text{Incident Current at Receiving End} = \frac{(I_R + V_R/Z_0)}{2}$$

$$K_I = \frac{(I_R - V_R/Z_0)}{I_R + V_R/Z_0)}$$

$$K_I = \frac{(\frac{V_R}{Z_R} - \frac{V_R}{Z_0})}{\frac{V_R}{Z_R} + \frac{V_R}{Z_0})}$$

$$K_I = \frac{(Z_0 - Z_R)}{(Z_0 + Z_R)}$$

K_V, K_I are complex numbers

$K_I = -K_V$

If line is terminated with Open Circuit:

$Z_R = \infty$

$K_V = \frac{(Z_R - Z_0)}{(Z_R + Z_0)}$

$K_V = \frac{Z_R(1 - \frac{Z_0}{Z_R})}{Z_R(1 + \frac{Z_0}{Z_R})}$

$K_V = \frac{(1 - \frac{Z_0}{\infty})}{(1 + \frac{Z_0}{\infty})}$

$K_V = \frac{(1-0)}{(1+0)}$

$K_V = 1$

$K_I = -1$

If line is terminated with Short Circuit:

$Z_R = 0$

$K_V = \frac{(Z_R - Z_0)}{(Z_R + Z_0)}$

$K_V = \frac{(0 - Z_0)}{(0 + Z_0)}$

$K_V = -1$

$K_I = 1$

If line is terminated with impedance Z_0

$Z_R = Z_0$

$K_V = \frac{(Z_R - Z_0)}{(Z_R + Z_0)}$

$K_V = \frac{(Z_0 - Z_0)}{(Z_0 + Z_0)}$

$K_V = 0$

$K_I = 0$

Disadvantages of Reflection:

1. Due to reflection the part of Energy rejected by load.Hence output reduces
2. There is a reduction in efficiency
3. If the attenuation is not large ,reflected wave appears as echo at sending end
4. If the generator impedance at sending end is not Z_0 then reflected wave is again reflected again from sending end and becomes a new incident wave.The energy is transmitted back and forth till all the energy get dissipated in the line losses

Hence it is necessary that line must be terminated with Z_0 to avoid reflection.

Transmission line is operating at high frequency:

If the transmission line is operating at high frequency waves

1. Skin effect is considerable
2. At high frequency f is high hence ω is high hence $\omega L >> R$, $\omega C >> G$.Hence R and G can be neglected

Skin effect: The concentretion of alternating current near the surface of the conductor is known as skin effect

Characteristic Impedance $Z_0 = \sqrt{\frac{R + jWL}{G + jWC}}$

At high frequency:R,G can be neglected

$Z_0=\sqrt{\frac{jWL}{jWC}}$

$Z_0=\sqrt{\frac{L}{C}}$

Lossless line:

For loss less line R=0,G=0

Characteristic Impedance $Z_0=\sqrt{\frac{R+jWL}{G+jWC}}=\sqrt{\frac{0+jWL}{0+jWC}}=\sqrt{\frac{L}{C}}=R_0$(Because purely resistive)

Propagation constant

$y=\sqrt{(R+jWL)(G+jWC)}$

For lossless line R=0,G=0

$y=\sqrt{(0+jWL)(0+jWC)}$

$y=\sqrt{(jWL)(jWC)}$

$y=jW\sqrt{LC}$

$y=\alpha+j\beta$

$\alpha+j\beta=jW\sqrt{LC}$

$\alpha+j\beta=0+jW\sqrt{LC}$

Comparing real and imaginary parts

$\alpha=0$

$\beta=W\sqrt{LC}$

Velocity of propagation $P=\lambda f$

$\lambda=\frac{2\pi}{\beta}$

$\mathsf{P} = \frac{2\pi}{\beta} f$

$\mathsf{P} = \frac{\omega}{\beta}$

For lossless line β = W$\sqrt{LC}$

$\mathsf{P} = \frac{\omega}{W\sqrt{LC}}$

$\mathsf{P} = \frac{1}{\sqrt{LC}}$

<u>Voltage and Current equations for lossless line:</u>

$(x) = V_R \cosh \gamma x + I_R Z_0 \sinh \gamma x$

$(x) = \frac{V_R}{Z_0} \sinh \gamma x + I_R \cosh \gamma x$

At x=l:

$V(x) = V_S, I(x) = I_S$

For lossless line: γ=jβ=jW$\sqrt{LC}$ and $Z_0=R_0$

$(x) = V_R \cosh j\beta + I_R R_0 \sinh j\beta$

$(x) = V_R \cosh \beta + jI_R R_0 \sinh \beta$

$(x) = \frac{V_R}{R_0} \sinh j\beta + I_R \cosh j\beta$

$I(x) = I_R \cosh \beta + j \frac{V_R}{R_0} \sinh \beta$

For open circuit end $I_R = 0$

$(x) = V_R \cosh \beta$

$(x) = j \frac{V_R}{R_0} \sinh \beta$

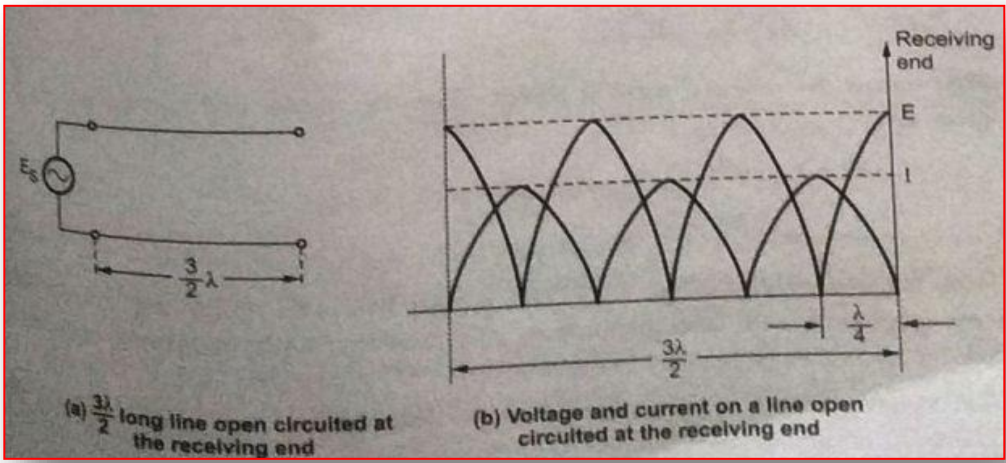


Fig: 5.2:OpenCircuit Line

For short circuit end: $V_R = 0$

$(x) = jI_R R_0 \sinh \beta$

$(x) = I_R \cosh \beta$

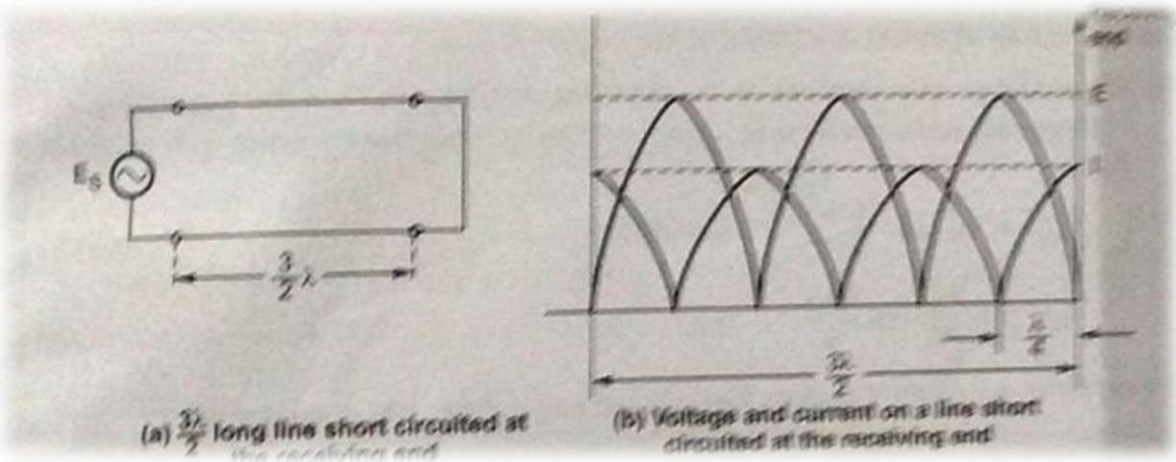


Fig: 5.3:Short Circuit Line

For a line terminated with R_0:

$(x) = V_R \cosh j\beta + I_R R_0 \sinh j\beta$

$$I_R = \frac{V_R}{R_0}$$

$$V(x) = V_R \cosh j\beta + \frac{V_R}{R_0} R_0 \sinh j\beta$$

$(x) = V_R \cosh j\beta + V_R \sinh j\beta$

V(x)= $V_R e^{j\beta}$

V(x)= $V_R \angle\beta$

$I(x) = I_R \cosh\beta + j \frac{V_R}{R_0} \sinh\beta$

$I_R = \frac{V_R}{R_0}$

$(x) = \frac{V_R}{R_0} \cosh\beta + j \frac{V_R}{R_0} \sinh\beta$

I(x)= $\frac{V_R}{R_0} \angle\beta$

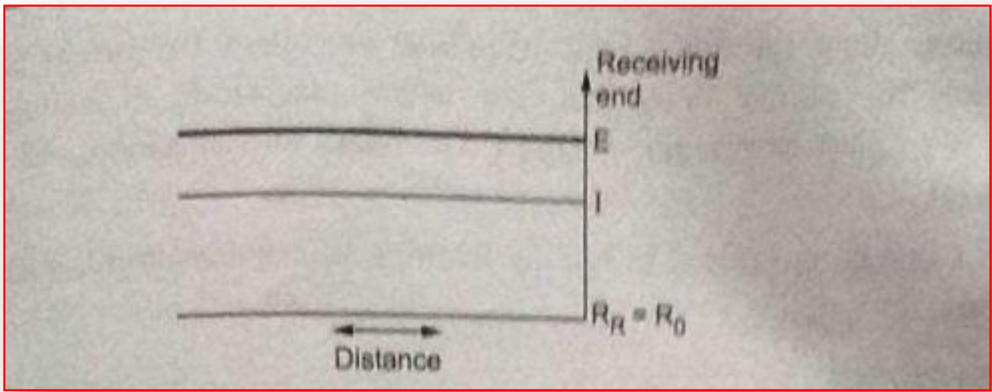


Fig: 5.4: Line Terminated with Z_R

Standing Waves:

Maximum point is called anti node

Minimum point is called node

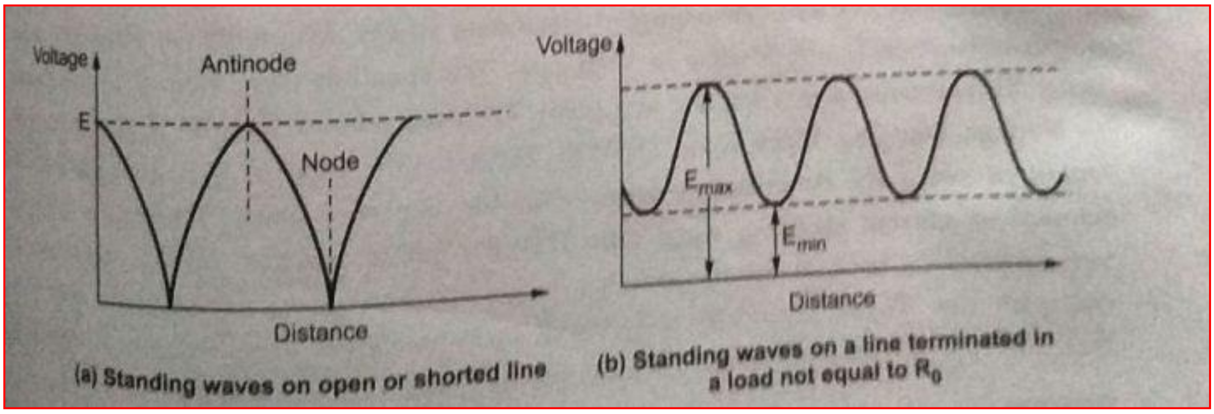


Fig: 5.5: Standing Waves

Standing wave ratio:

Voltage Standing Wave Ratio(VSWR)=$\frac{Maximum\ Voltage}{Minimum\ Voltage}$

VSWR=$\frac{|V_{max}|}{|V_{min}|}$

Current standing Wave Ratio(ISWR)=$\frac{Maximum\ Current}{Minimum\ Current}$

ISWR=$\frac{|I_{max}|}{|I_{min}|}$

The value of VSWR can be measured by connecting RF voltmeter across line at apoint

ISWR can be measured by connecting RF ammeter in series with line at a point

In practical we can’t measure ISWR as we have to cut the line and insert RF ammeter and Rejoin the line

Hence practically VSWR measurement is done

Hence standing wave ratio means VSWR

Relation between Standing wave ratio and magnitude of Reflection Coefficient:

Let $|K|$ = Magnitude of reflection coefficient

S=Standing wave ratio=$\frac{|V_{max}|}{|V_{min}|}$

S value is in between 1 to ∞

|K| value is in between 0 to 1

S=$\frac{|V_{max}|}{|V_{min}|}$

$|V_{max}| = |V|^+ + |V|^-$

$|V_{min}| = |V|^+ - |V|^-$

V^+=Incident Voltage

V^-=Reflected Voltage

S=$\frac{|V_{max}|}{|V_{min}|} = \frac{|V|^+ + |V|^-}{|V|^+ - |V|^-} = \frac{|V|^+(1+\frac{|V|^-}{|V|^+})}{|V|^+(1-\frac{|V|^-}{|V|^+})}$

$$|k| = \frac{|V|^-}{|V|^+}$$

S=$\frac{1+|k|}{1-|k|}$

$$\frac{s}{1} = \frac{1+|k|}{1-|k|}$$

$$\frac{1+s}{1-s} = \frac{1-|k|+1+|k|}{1-|k|-1-|k|}$$

$$\frac{1+s}{1-s} = \frac{2}{-2|k|}$$

$$|k| = \frac{s-1}{s+1}$$

$$|k| = \frac{\frac{|V_{max}|}{|V_{min}|} - 1}{\frac{|V_{max}|}{|V_{min}|} + 1}$$

$$|k| = \frac{|V_{max}| - |V_{min}|}{|V_{max}| + |V_{min}|}$$

In terms of currents:

$$\text{I.S.W.R} = \frac{|I_{max}|}{|I_{min}|} = \frac{|I|^{+} + |I|^{-}}{|I|^{+} - |I|^{-}} = \frac{|I|^{+}(1+\frac{|I|^{-}}{|I|^{+}})}{|I|^{+}(1-\frac{|I|^{-}}{|I|^{+}})} = \frac{1+|k|}{1-|k|}$$

$$|k| = \frac{ISWR-1}{ISWR+1} = \frac{|I_{max}|-|I_{min}|}{|I_{max}|+|I_{min}|}$$

$$|k| = \frac{Z_R - Z_0}{Z_R + Z_0}$$

$$S = \frac{1+\frac{Z_R-Z_0}{Z_R+Z_0}}{1-\frac{Z_R-Z_0}{Z_R+Z_0}}$$

$$S = \frac{2Z_R}{2Z_0} = \frac{Z_R}{Z_0}$$

$$Z_{min} = \frac{Z_0}{S}$$

$$Z_{max} = Z_0 S$$

The Voltage will be maximum if both incident and reflected voltages are in phase

The Voltage will be minimum if both incident and reflected voltages are in 180° out of phase

Impedance matching:

When high frequency line is terminated in its characteristic impedance R_0,it operates as a smooth line.Under such conditions reflections are absent.

But practically the load connected to the line may not equal to the R_0 giving rise to reflections

In order to overcome this difficulty impedance matching is required

Impedance matching with $\frac{\lambda}{8}, \frac{\lambda}{4}, \frac{\lambda}{2}$ lines:

Input Impedance of $\frac{\lambda}{8}$ line:

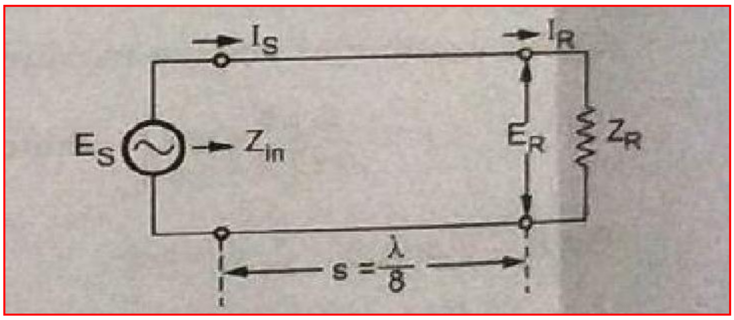


Fig5. 6The Eigth wave Line

$$Z_{in} = \frac{Z_0(Z_R + Z_0 \tanh \gamma l)}{Z_0 + Z_R \tanh \gamma l}$$

For lossless line $\alpha = 0$

$$\gamma = \alpha + j\beta = 0 + j\beta = j\beta$$

$$Z_{in} = \frac{Z_0(Z_R + Z_0 \tanh j\beta l)}{Z_0 + Z_R \tanh \gamma j\beta l}$$

$$Z_{in} = \frac{Z_0(Z_R + jZ_0 \tan \beta l)}{Z_0 + jZ_R \tan \beta l}$$

$$\beta = \frac{2\pi}{\lambda}$$

$$l = \frac{\lambda}{8}$$

$$Z_{in} = \frac{Z_0 (Z_R + jZ_0 \tan \frac{2\pi}{\lambda}\frac{\lambda}{8})}{Z_0 + jZ_R \tan \frac{2\pi}{\lambda}\frac{\lambda}{8}}$$

$$Z_{in} = \frac{Z_0 (Z_R + jZ_0 \tan \frac{\pi}{4})}{Z_0 + jZ_R \tan \frac{\pi}{4}}$$

$$Z_{in} = \frac{Z_0(Z_R + jZ_0)}{(Z_0 + jZ_R)}$$

$$|Z_{in}| = \frac{Z_0\,(\sqrt{(Z_R{}^2 + Z_0{}^2)})}{\sqrt{(Z_0{}^2 + Z_R{}^2)}}$$

$$|Z_{in}| = Z_0$$

$\frac{\lambda}{8}$ line converts any impedance into magnitude of impdance Z_0

Input Impedance of $\frac{\lambda}{4}$ line:

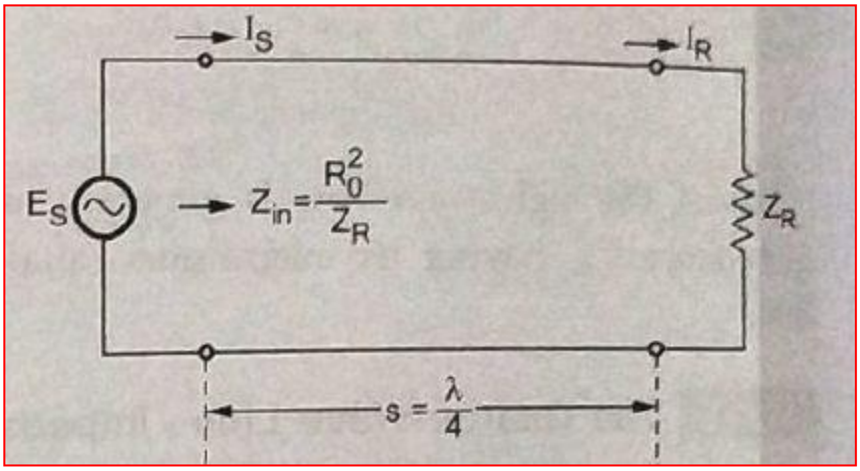


Fig5.7Quarter wave line

$$Z_{in} = \frac{Z_0(Z_R + Z_0 \tanh \gamma l)}{Z_0 + Z_R \tanh \gamma l}$$

For lossless line $\alpha = 0$

$$\gamma = \alpha + j\beta = 0 + j\beta = j\beta$$

$$Z_{in} = \frac{Z_0(Z_R + Z_0 \tanh j\beta l)}{Z_0 + Z_R \tanh \gamma j\beta l}$$

$$Z_{in} = \frac{Z_0(Z_R + jZ_0 \tan \beta l)}{Z_0 + jZ_R \tan \beta l}$$

$$\beta = \frac{2\pi}{\lambda}$$

$l = \lambda/4$

$$Z_{in} = \frac{Z_0\,(Z_R + jZ_0 \tan\frac{\pi}{2})}{Z_0 + jZ_R \tan\frac{\pi}{2}}$$

$$Z_{in} = \frac{Z_0\,(Z_R + jZ_0 \tan\frac{\pi}{2})}{Z_0 + jZ_R \tan\frac{\pi}{2}}$$

$$Z_{in} = \frac{Z_0 \tan\frac{\pi}{2}\left(\frac{Z_R}{\tan\frac{\pi}{2}} + \;\right)_0}{\tan\frac{\pi}{2}\left(\frac{Z_0}{\tan\frac{\pi}{2}} + jZ_R\right)}$$

$$Z_{in} = \frac{{Z_0}^2}{Z_R}$$

If $Z_R = \infty$(Open circuit line)

$Z_{in} = 0$

If $Z_R = 0$(Short circuit line)

$Z_{in} = \infty$

$\frac{\lambda}{4}$ line acts like impedance inverter

Input Impedance of $\frac{\lambda}{2}$ line:

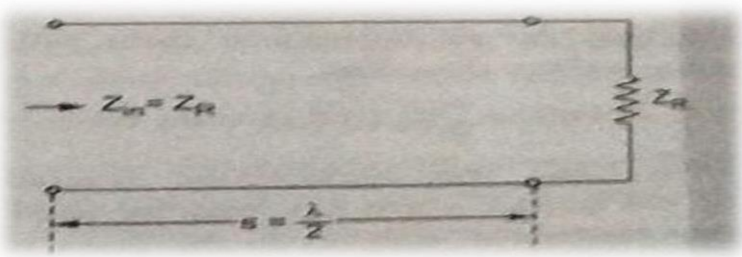


Fig.5.8 Halfwave line

$$Z_{in} = \frac{Z_0(Z_R + Z_0 \tanh \gamma l)}{Z_0 + Z_R \tanh \gamma l}$$

For lossless line $\alpha = 0$

$$\gamma = \alpha + j\beta = 0 + j\beta = j\beta$$

$$Z_{in} = \frac{Z_0(Z_R + Z_0 \tanh j\beta l)}{Z_0 + Z_R \tanh \gamma j\beta l}$$

$$Z_{in} = \frac{Z_0(Z_R + jZ_0 \tan \beta l)}{Z_0 + jZ_R \tan \beta l}$$

$$\beta = \frac{2\pi}{\lambda}$$

$$l = \frac{\lambda}{2}$$

$$Z_{in} = \frac{Z_0(Z_R + jZ_0 \tan \pi)}{Z_0 + jZ_R \tan \pi}$$

$$Z_{in} = \frac{Z_0(Z_R + j0)}{Z_0 + j0}$$

$$Z_{in} = Z_R$$

$\frac{\lambda}{2}$ line acts like one to one transformer

Smith chart :

P.H.Smith of Bell laboratorories developed smith chart.

Smith chart is graphical tool for solving radio frequency transmission line problems

Configuaration of smith chart:

Basic difference between circle diagram and smith chart is that values of resistive and reactive componenets are reprsented in rectangular form which are extended to infinity

But in smith chart infinite resistance and reactances are transferred to an area inside the circle

As reistive and reactive components are in circular form smith chart is also called as circular chart

Smith chart is basically a polar plot of the reflection coefficient k expressed in terms of normalised impedance

$$Z_{in} = \frac{R_0(1 + |k|\angle\emptyset - 2\beta s)}{(1 - |k|\angle\emptyset - 2\beta s)}$$

Normalized input impedance

$$\frac{Z_{in}}{R_0} = \frac{(1 + |k|\angle\emptyset - 2\beta s)}{(1 - |k|\angle\emptyset - 2\beta s)}$$

Normalized input impedance is a complex quantity expressed in rectangular form

$$\frac{Z_{in}}{R_0} = r_i + jx_i = \frac{(1+|k|\angle\emptyset-2\beta s)}{(1-|k|\angle\emptyset-2\beta s)}$$

$|k|\angle\emptyset - 2\beta s\ can\ be\ represented\ as\ u + jv$

$$r_i + jx_i = \frac{(1 + u + jv)}{(1 - (u + jv))}$$

Rearranging above equation

$$r_i = \frac{1-u^2-v^2}{1-u^2+v^2}$$

$$x_i = \frac{2v}{1-u^2+v^2}$$

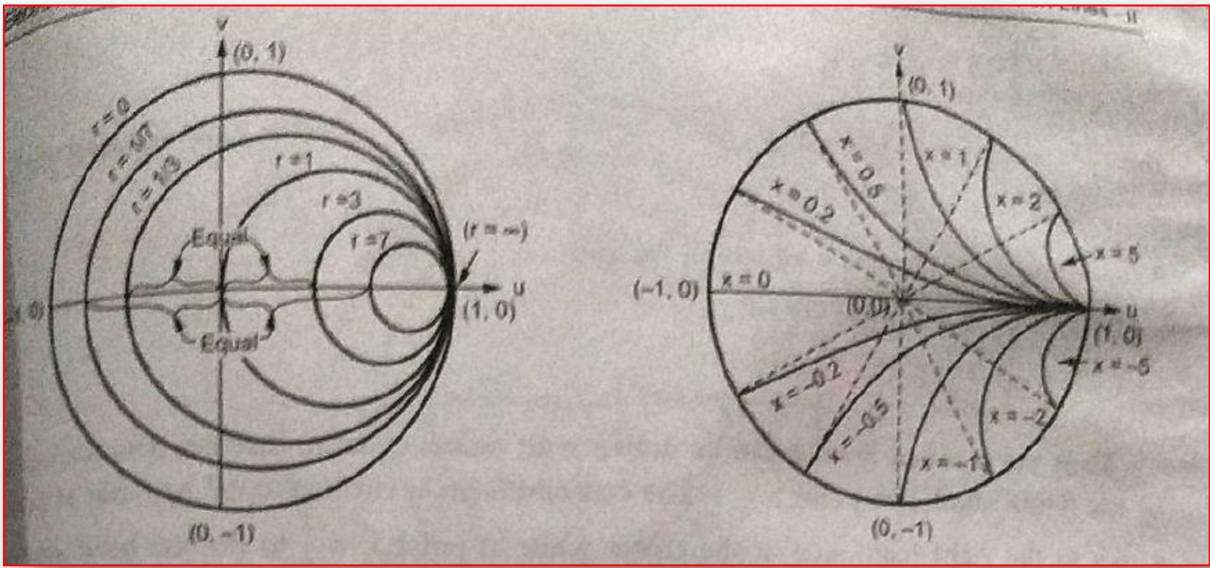


Fig5.9:Coonstant r and constant x circles

Properties of smith Chart:

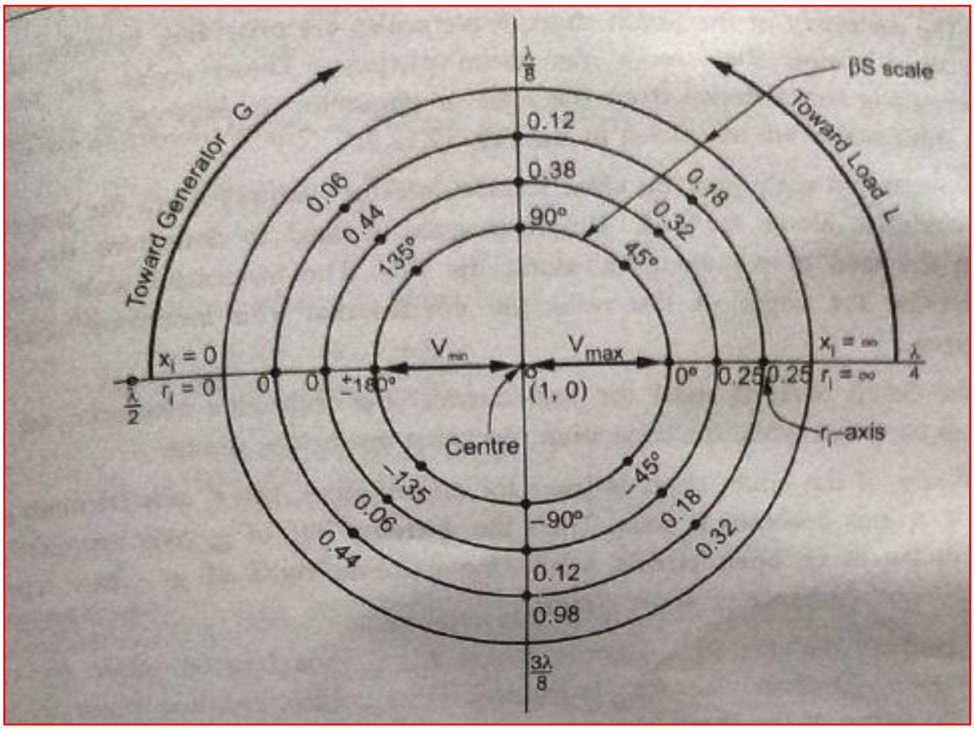


Fig 5.10 properties of smith chart

1. Smith chart may be used for impedance as well as admittances
2. The smith chart comnsists of constant r_i circles and constant x_i circles
   The values of r_i and x_i are normalized
   $r_i = \frac{R}{R_0}$
   $X_i = j\,\frac{X_R}{R_0}$
   The constant r_i circles have their centers on horizontal axis i.e.u axis and the constant X_i circles have their centers on vertical axis i.e.v axis
3. In smith chart it is possible to locate all the possible values of impedances inside the outer circle of unit radius
4. The impedance of a line of any length can be read at any point on the given S circle.For properly terminated line and any length, the impedance is represented by the point (1,0) which acts like a center of smith chart
5. The horizontal line passing through the center of the smith chart is real axis or r_i for impedance plot and g_i axis for admittance plot
   To the extreme left of the chart indicates zero impedances at load point or short circuit condition
   To the extreme right of the chart indicates infinite impedances at load point or opencircuit condition
6. The outer circle of the chart is called β circle of the chart which indicates electrical length of line
7. The distance corresponds to $\frac{\lambda}{2}$ $\text{indicates } 360°$ on the chart
   $\lambda = 720°$
   The clock wise movement along outer rim indicates travel towards generator from load and anti clockwise movement along rim indicated travel towards load from generator

8. On the periphery of the smith chart,three scales are are provided.The outermost scale is used to calculate distance from generator in wave lengths.The next scale is used to calculate distance from load in wavelength.The innermost scale is used to determine angle of reflection coefficients in degrees.
9. If the smith chart is used for impedances ,the inductive reactances are above the u axis and capacitive reactances are below the u axis.If the smith chart is used for admittances then r_i axis becomes g_i axis and x_i axis becomes b_i axis.Then extreme left of g_i represents zero conductance or open circuit .While the extreme right of g_i represents infinite conductance or short circuit
10. The voltage maxima occurs where $Z_{in(max)}$ located while voltage minima occurs where Z_{in} located.The voltage minima occur to the left of center of the chart,along r_i axis while voltage maxima occur to the right of centre of chart along r_i axis

Applications of smith chart:

1. It is used in the transmission line is used to calculate impedance provided at any load
2. The chart is even employed to calculate admittance values provided at any load
3. Used in the measurement of the length of the short-circuited section of the Tx.line in order to offer the required amount of inductive reactance of capacitance
4. Used for the purposes of impedance matching
5. Employed to know the value of VSWR amongst others.

Single stub matching:

In single stub matching technique a stub of suitable length is connected in parallel wirth a line at acertain distance from load

By using such stub Antiresonance is achieved.Hence at antiresonance impedance equal to R_0

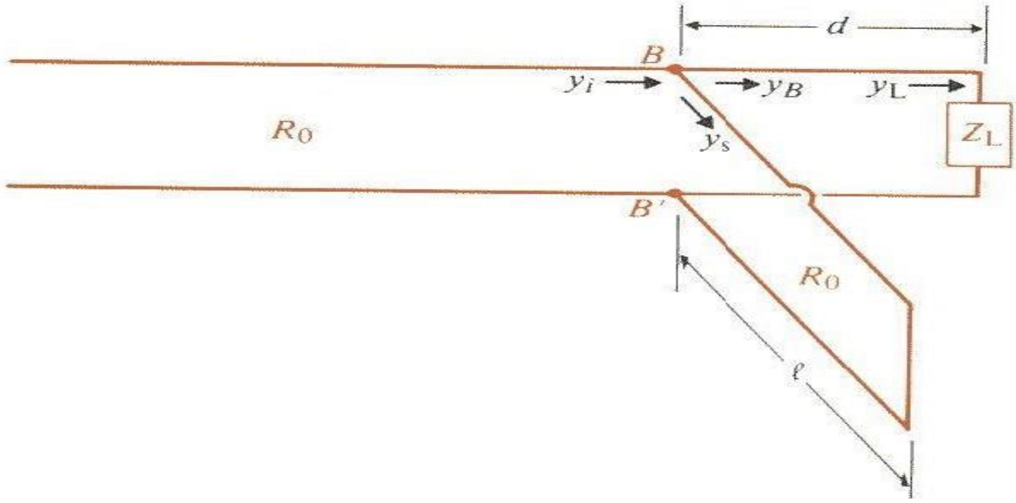


Fig.5.11 Single stub matching

Because of paralleling of the element it is convinient to work with admittances

The input admittance looking towards the load from any point on the transmission line

$Y_S = G_0 \pm jB$

This may be admittance at a point A before stub was connected

The point A is located such that at point A ,$G_0 = \frac{1}{R_0}$

Then at a point A ,a short vstub is connected

This line is selected such that its input susceptance is $\mp B$

This stub is connected across transmission line

Total admittance at input is given by

$Y_S = G_0 \pm jB \mp B =$ $G_0 = \frac{1}{R_0}$

$Z_S = R_0$

Thus the input impedance of the line pointing towards load is

$Z_S = \frac{1}{Y_S} = R_0$

Thus a line from source to the point A is then terminated with R_0

It acts like a smooth line

The reflection and hence standing waves occur in betweenthe portion of the line from point Ato the load.But making this distance less than wave length ,the losses can be minimized

For the impedance matching using single stub,it is very essential to know the exact point at which stub is connectedto the line and also exact length of the stub

For this two independent measurements must be made on the line

It is easy to measure standing wave ratio and voltage minimum nearest to the load

The measurement on the line is made for voltage minimum because voltage minimum can be mesured accurately rather than voltage maximum

The input impedance of the transmission line

$$Z_{in} = Z_S = \frac{R_0(1 + |K\angle\emptyset - 2\beta s|)}{(1 - |K\angle\emptyset - 2\beta s|)}$$

The input admittance of the transmission line

$$Y_{in} = \frac{1}{Z_S} = \frac{(1 - |K\angle\emptyset - 2\beta s|)}{R_0(1 + |K\angle\emptyset - 2\beta s|)}$$

Writing in terms of rectangular coordinates

$$Y_S = G_S + jB_S = G_0 \left[\frac{1 - |K|^2 - 2j|K|\sin(\emptyset - 2\beta s)}{1 + |K|^2 + 2|K|\cos(\emptyset - 2\beta s)}\right]$$

$$\frac{G_S + jB_S}{G_0} = \left[\frac{1 - |K|^2 - 2j|K|\sin(\emptyset - 2\beta s)}{1 + |K|^2 + 2|K|\cos(\emptyset - 2\beta s)}\right]$$

$$\frac{G_S}{G_0} = \frac{1-|K|^2}{1+|K|^2+2|K|\cos(\phi-2\beta s)}$$

$$\frac{B_S}{G_0} = \frac{-2|K|\sin(\phi-2\beta s)}{1+|K|^2+2|K|\cos(\phi-2\beta s)}$$

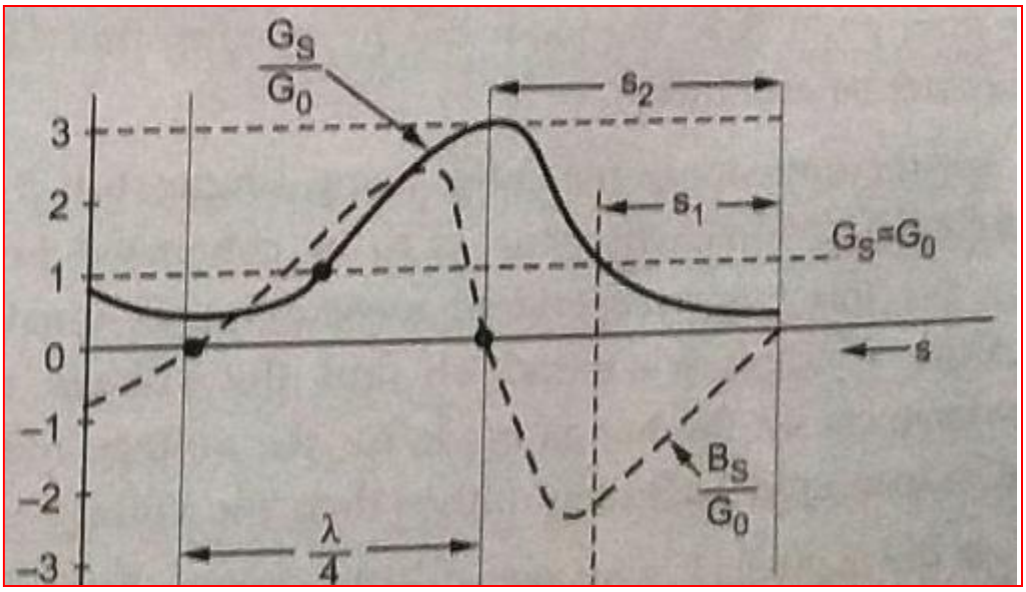


Fig5.11:Admittance Conditions on line

Location of short circuit stub=$\frac{\lambda}{4\pi}[\emptyset + \pi - \cos^{-1}|K|]$

Length of short circuit stub=L=$\frac{\lambda}{2\pi}\tan^{-1}\frac{\sqrt{1-|K|^2}}{2|K|}$

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Model Paper-1

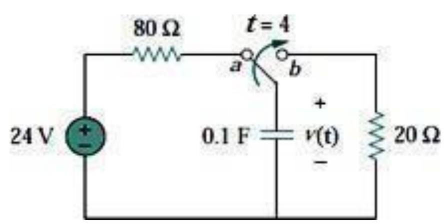
Time: 3 hours **Max. Marks: 70**

Note: This question paper Consists of 5 Sections. Answer **FIVE** Questions, Choosing ONE Question from each SECTION and each Question carries 14 marks.

SECTION-I

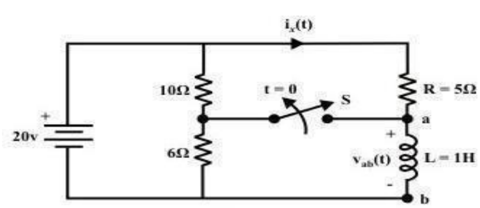
1. a.What are the initial conditions? Why are they needed? Explain [7M]

   b.The switch in the below figure has been in position *a* for a long time, At t = 4 s the Switch is moved to position *b* and left there. Determine *v(t)* at t = 10 s.[7M]

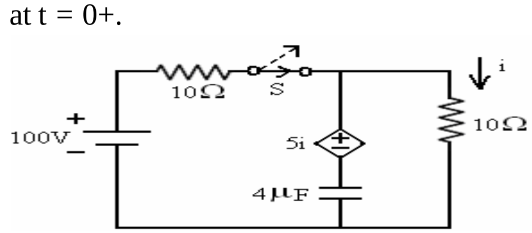


(OR)

2. a.In the given circuit the switch is opened at t=0.Find (i) $V_{ab}(0-)$ (ii) $i_x(0-)$ (iii) $i_x(0+)$ (i) $V_{ab}(0+)$ (v) $i_x(t=\infty)$ (vi) $i_x(t)$ for t>0. [7M]



   b.For the circuit shown below Figure, find the current equation when switch S is opened at t = 0+. [7M]



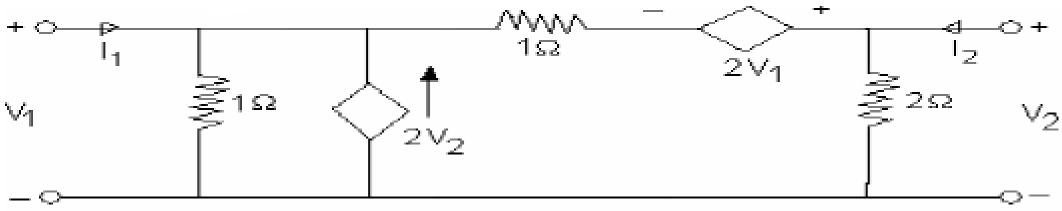
SECTION-II

3. a.Derive the relation between ABCD and 'Z'-parameters.[7M]

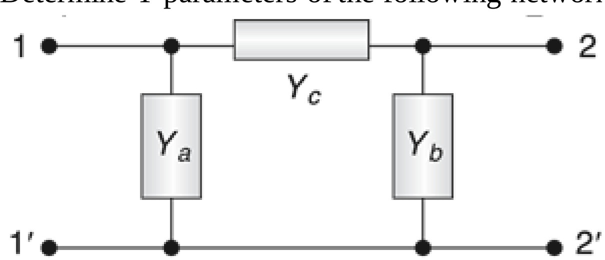
   b.Write the equations for Z, Y, ABCD, inverse ABCD, h, g parameters.
   A two port network has the following parameters: Z11 = 4 Ω, Z12 = 1 Ω, Z21 = 3 Ω and Z22 = 3 Ω. Calculate short circuit parameters. [7M]

(OR)

4. a)Determine h parameters of the following network[7M]



   b) Determine Y parameters of the following network[7M]



SECTION-III

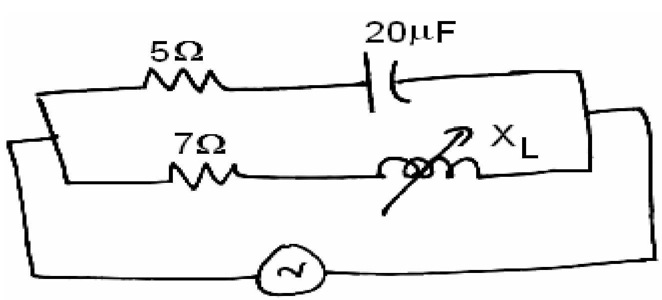
5. a) Explain the locus diagram of series R-L circuit when R is variable. [7M]

   b) Explain the locus diagram of series R-C circuit and when C is variable. [7M]

(OR)

6. a) Derive expression for half power frequencies of a R L C series network. [7M]
   b) Construct the admittance locus diagram and determine the variable inductance values so that the phase angle between the supply voltage and supply current is zero for the Fig.5.

   ω = 200 rad/s.[7M]



SECTION-IV

7. a) Derive the Condition for Distrotionless Transmission Line. [7]
   b) Measurements on a Transmission Line of length 120Km were made at frequency of 6000Hz.If Z_{OC}=520(-30deg) and Z_{SC}=640(43deg) find Z_o and P.[7]

OR

8. a. Explain the conditions which are used for minimum attenuation in transmission line [7]
b. The propagation constant of a lossy transmission line is $1+j2\ m^{-1}$ and its characteristic impedance is $20+j0\Omega$ at $\omega= 1$rad/s. Find R, C, L, G for the Line. [7]

SECTION-V

9. a)Derive the relation between reflection coefficient andcharacteristic impedance[7]
b)Write short notes on smith chart. [7]

OR

10. A transmission line of length 0.40λ has a characteristic impedance of 100Ω and is Terminated in a load impedance of $200 + j180\omega$. Find the [14]
    a. Voltage reflection coefficient
    b. Voltage standing waveratio
    c. Input impedance of the line.

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Model Paper-2

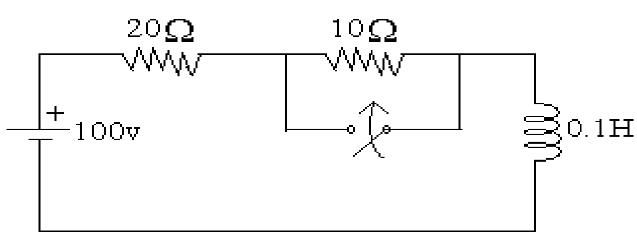
Time: 3 hours **Max. Marks: 70**

Note: This question paper Consists of 5 Sections. Answer **FIVE** Questions, Choosing ONE Question from each SECTION and each Question carries 14 marks.

SECTION-I

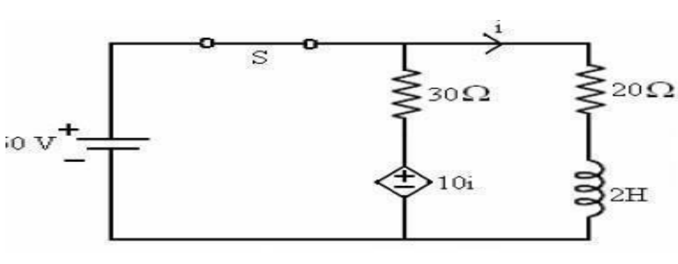
1. a) In a series RL circuit with R = 3 ohm and L = 1 H, a DC voltage of V = 50 V is applied at t = 0. Find the transient response of current and plot the response. [7M]

   b) A dc voltage of 100V is applied in the circuit shown in figure below and the switch is kept open. The switch K is closed at t = 0. Find the complete expression for the current. [7M]



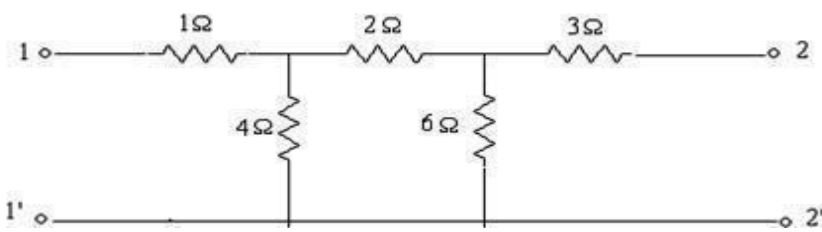
(OR)

2. For the below circuit (Fig. 1), find the current equation i(t), when the switch is opened at t = 0. [14M]



SECTION-II

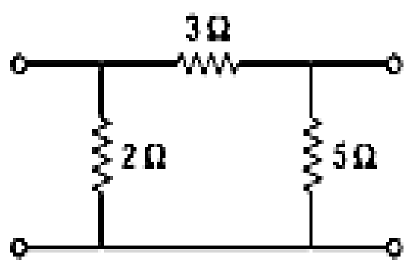
3. a) Obtain the transmission line parameters when the two transmission networks having the transmission parameters A1, B1, C1, D1 and A2, B2, C2, D2 are connected in cascade.[7M]



b) Obtain 'Y' – parameters for the given network shown in below figure. [7M]

(OR)

4. Determine the *g* parameters for the circuit shown in below figure. [14M]



SECTION-III

5. Show that the resonant frequency ω_0 of an RLC series circuit is the geometric mean of ω_1 and ω_2 the lower and upper half power frequencies respectively. [14M]

(OR)

6. A voltage V = 50∠0 V is applied to a series circuit consisting of fixed inductivereactance X = 5ohms and a variable resistance R. Sketch the admittance and current locus diagrams. [14M]

SECTION-IV

7. a) Derive The Expression for Transmission Line Equation.[7]

b) Given R = 10.4 Ω/mt L = 0.00367 H/mt

G = 0.8×10^{-4} mhos/mt C = 0.00835 μF/mt.

Calculate Z_0 and γ at1.0 KHz. [7]

OR

8. a)Derive the expression for α and β in terms of primary constants of a line[7]

b) Explain transmission line parameters in detail. [7]

SECTION-V

9.a) Establish the relations for Z_{sc} and Z_{oc} of rf lines and sketch their variation with βl. [7]

b) A 60ohm lossless line is 30m long and is terminated with a load of 75+j50ohms at 3MHz find its reflection coefficient, VSWR, if the line velocity is 60% of the velocity of light [7]

OR

10. a) Explain the principle of single stub matching.[7]

b) Calculate the skin depth for the following conditions. [7]

Copper $f=10^{10}$Hz, $\mu=\mu_0$, $\sigma=5.8\times10^7$s/m

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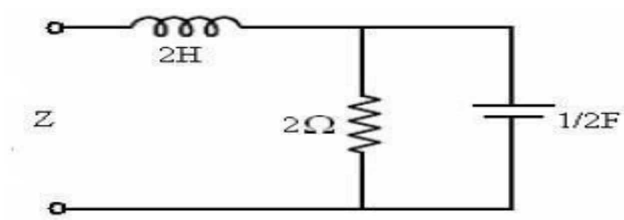
Model Paper-3

Time: 3 hours **Max. Marks: 70**

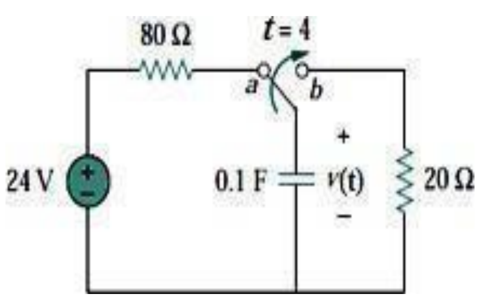
Note: This question paper Consists of 5 Sections. Answer **FIVE** Questions, Choosing ONE Question from each SECTION and each Question carries 14 marks. 5*14=70M

SECTION-I

1. a) Transform the below circuit in to 'S' domain and determine the Laplace transform impedance. [7M]

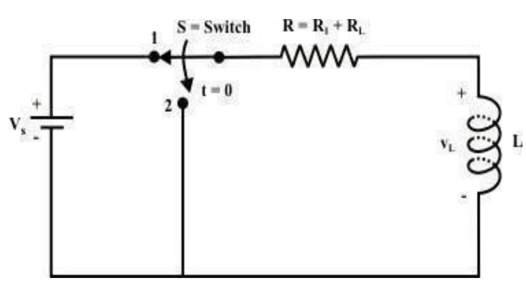


b) At t = 0, switch 1 in below figure is closed, and switch 2 is closed 4 s later. Find i(t) for t > 0. Calculate i for t = 2 s and t = 5 s. [7M]



(OR)

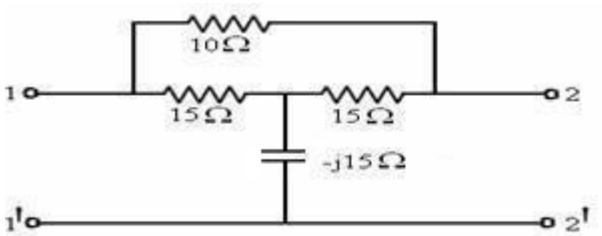
2. a) In the given circuit the switch is shifted from position 1 to 2 at t=0.Determine i(t) for t>0. [7M]



b) What are the initial conditions? Why are they needed? Explain. [7M]

SECTION-II

3. Determine the transmission parameter and hence determine the short circuit admittance parameters for the below circuit. [14M]



(OR)

4. Explain about the ABCD –parameters and derive the condition for symmetry and reciprocity. [14M]

SECTION-III

5. Explain the procedure to draw the locus diagram of R-L series circuit when L is varying. [14M]

(OR)

6. A series RLC circuit has to be designed so that it has a band width of 320 Hz and inductance of the coil is 0.2H. It is has to resonate at 350Hz, determine the resistance of coil and capacitance of condenser. If the applied voltage is 150V, determine the voltage across capacitor and coil. [14M]

SECTION-IV

7. a) Derive the attenuation constant and phase constant in terms of primary constants[7]

   b) Explain different types of loading for transmission lines.[7]

(OR)

8. a) Derive the characteristic impedance of a transmission line in terms of its line constants[7]

b) At 8MHz the characteristic impedance of a transmission line as 40 -j2ohms and the Propagation constant 0.01+j0.18 per meter. Find theprimary constant. [7]

SECTION-V

9. a) Explain the principal of singlestub matching [7]

   b) Write Short notes on Smith Chart [7]

OR

10.a) Derive the relation between reflection coefficient and characteristic impedance

   b) Write short notes on smith chart.[7+7]

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NETWORK ANALYSIS AND TRANSMISSION LINES

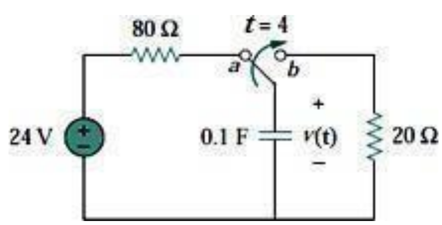
Model Paper-5

Time: 3 hours **Max. Marks: 70**

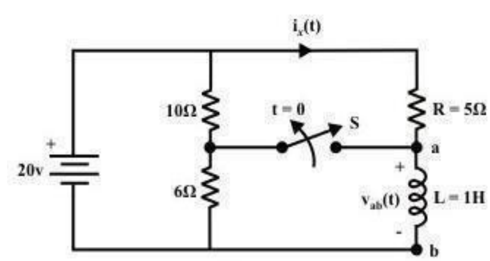
Note: This question paper Consists of 5 Sections. Answer **FIVE** Questions, Choosing ONE Question from each SECTION and each Question carries 14 marks. 5*14=70M

SECTION-I

1. a) The switch in the below figure has been in position *a* for a long time, At *t* = 4 s the switch is moved to position *b* and left there. Determine *v(t)* at *t* = 10 s. [7M]

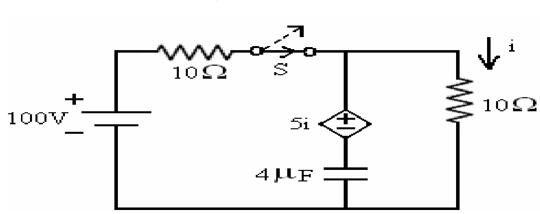


b) In the given circuit the switch is opened at t=0.Find (i) V_{ab} (0-) (ii) i_x (0-) (iii) i_x (0+) (iv) V_{ab} (0+) (v) i_x (t=∞) (VI) ix (t) for t>0. [7M]



(OR)

2. a)For the circuit shown below Figure, find the current equation when switch S is opened

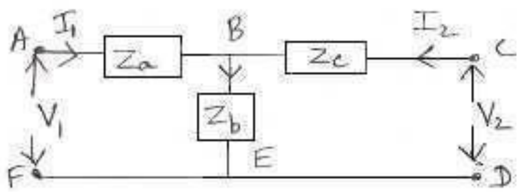


at t = 0.[7M]

b)In a series RL circuit with R = 3 ohm and L = 1 H, a DC voltage of V = 50 V is applied at t = 0. Find the transient response of current and plot the response. [7M]

SECTION-II

3. a)Find the Z parameters and Y parameters of the T- network shown in figure below. [7M]



b) Explain about driving point and transfer impedances. [7M]

(OR)

4. Draw the equivalent circuits of Z, Y, h, g parameters. [14M]

SECTION-III

5. Explain about the series resonance and derive the expression for resonant frequency. [14M]

(OR)

6. Define the bandwidth and derive the expressions for bandwidth of series resonating circuit and its relation with Q-factor. [14M]

SECTION-IV

7. Explain the conditions which are used for minimum attenuation in transmission lines. [14M]

(OR)

8. Derive the secondary conditions for loss less transmission line.[14M]

SECTION-V

9. a)Explain the principal of single stub matching [7]
 b)Write Short notes on Smith Chart [7]

(OR)

10. Derive the relation between reflection coefficient and characteristic impedance [14]