RADAR SYSTEMS

(Digital Notes)

BTECH

IV YEAR I SEMESTER

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By

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(Affiliated to JNTUH, Hyderabad, Approved by AICTE - Accredited by NBA & NAAC – ‘A’ Grade)

Maisammaguda, Dhulapally (Post Via. Kompally), Secunderabad – 500100, Telangana State, India
CORE ELECTIVE – V
(R18A0419) RADAR SYSTEMS

COURSE OBJECTIVES
1. To learn Radar Fundamentals like Radar Equation, Operating frequencies & Applications.
2. To understand the basic concepts of different types of Radars for surveillance & Tracking.
3. To know the various types of tracking techniques involved.
4. To understand Radar Receivers, MTI filters, displays and antennas.

UNIT I

Radar Equation: SNR, Envelope Detector-False Alarm Time and Probability, Integration of Radar Pulses, Radar Cross Section of Targets (simple targets - sphere, cone-sphere), Transmitter Power, PRF and Range Ambiguities, System Losses (qualitative treatment), Related Problems.

UNIT-II


UNIT-III

Tracking Radar: Tracking with Radar, Sequential Lobing, Conical Scan, Monopulse Tracking Radar – Amplitude Comparison Monopulse (one- and two- coordinates), Angular Accuracy, Tracking in Range, Acquisition and Scanning Patterns, Comparison of Trackers.

UNIT-IV
UNIT-V

Radar Receivers: Noise Figure and Noise Temperature, Displays – types, Introduction to Phased Array Antennas –Basic Concepts, Radiation Pattern, Beam Steering and Beam Width changes, Applications, Advantages and Limitations.

Electronic Warfare: Introduction to ESM, ECM and ECCM systems.

TEXT BOOK:

REFERENCES:

COURSE OUTCOMES
1. Demonstrate an understanding of the factors affecting the radar performance using Radar Range Equation.
2. Analyze the principle of FM-CW radar and apply it in FM-CW Altimeter.
3. Differentiate between a MTI Radar and a Pulse Doppler Radar based on their working principle.
4. Demonstrate an understanding of the importance of Matched Filter Receivers in Radars.
5. Familiarize with the different types of Radar Displays and their application in real time scenario.
UNIT-1

BASICS OF RADAR

• Introduction
• Maximum Unambiguous Range
• Simple form of Radar Equation
• Radar Block Diagram and Operation
• Radar Frequencies and Applications
• Prediction of Range Performance
• Minimum Detectable Signal
• Receiver Noise
• Modified Radar Range Equation

RADAR EQUATION

• SNR
• Envelop Detector
• False Alarm time and Probability
• Integration of Radar Pulses
• Radar Cross Section of Targets (simple targets: sphere and cone sphere)
• Transmitter Power
• PRF and Range Ambiguities
• System Losses (qualitative treatment)
  • Important Formulae
  • Illustrative Problems
  • Questions from Previous Year Examinations
BASICS OF RADAR

Introduction:

Basic principles and features:

- Radar is a contraction of the words RA dio Detection And Ranging. Radar is an electromagnetic system for the detection and location of objects. It operates by transmitting a particular type of waveform, a pulse-modulated sine wave for example, and detects the nature of the echo signal.

- Radar can see through conditions such as darkness, haze, fog, rain, and snow which is not possible for human vision. In addition, radar has the advantage that it can measure the distance or range to the object.

- An elementary form of radar consists of a transmitting antenna emitting electromagnetic Radiation generated by an oscillator of some sort, a receiving antenna, and an energy-detecting device or receiver. A portion of the transmitted signal is intercepted by a reflecting object (target) and is reradiated in all directions. The receiving antenna collects the returned energy and delivers it to a receiver, where it is processed to detect the presence of the target and to extract its location and relative velocity. The distance to the target is determined by measuring the time taken for the radar signal to travel to the target and back. The direction, or angular position, of the target is determined from the direction of arrival of the reflected wave front. The usual method of measuring the direction of arrival is with narrow antenna beams.

- If relative motion exists between target and radar, the shift in the carrier frequency of the reflected wave (Doppler Effect) is a measure of the target's relative (radial) velocity and may be used to distinguish moving targets from stationary objects. In radars which continuously track the movement of a target, a continuous indication of the rate of change of target position is also available.

- It was first developed as a detection device to warn the approach of hostile aircraft and for directing antiaircraft weapons. A well-designed modern radar can extract more information from the target signal than merely range.

Measurement of Range:

- The most common radar waveform is a train of narrow, rectangular-shape pulses modulating a sine wave carrier.

- The distance, or range, to the target is determined by measuring the time \( T_R \) taken by the pulse to travel to the target and return.

- Since electromagnetic energy propagates at the speed of light \( c (3 \times 10^8 \text{ m/s}) \) the range \( R \) is given by \( R = cT_R / 2 \)

- The factor 2 appears in the denominator because of the two-way propagation of radar. With the range \( R \) in kilometers or nautical miles, and \( T_R \) in microseconds, the above relation becomes: \( R(\text{km}) = 0.15 \times T_R (\mu\text{s}) \) or \( R(\text{nmi}) = 0.081 \times T_R (\mu\text{s}) \)

Each microsecond of round-trip travel time corresponds to a distance of 0.081 nautical mile, 0.093 statute mile, 150 meters, 164 yards, or 492 feet.
(1 mile = 0.8689 nautical mile or 1.6 km
1 nautical mile = 1.15078 miles or 1.8412 km)

**Maximum unambiguous range:**

Once the transmitter pulse is emitted by the radar, sufficient time must elapse to allow any echo signals to return and be detected before the next pulse is transmitted. Therefore, the rate at which the pulses may be transmitted is determined by the longest range at which targets are expected. If the pulse repetition frequency is too high, echo signals from some targets might arrive after the transmission of the next pulse, and ambiguities in measuring range might result. Echoes that arrive after the transmission of the next pulse are called second-time-around (or multiple-time-around) echoes. Such an echo would appear to be at a much shorter range than the actual. The range beyond which targets appear as second-time-around echoes (or the farthest target range that can be detected by a Radar without ambiguity) is called the **maximum unambiguous range** and is given by: \( R_{unambig} = \frac{C}{2f_p} \)

where \( f_p \) = pulse repetition frequency, in Hz. A plot of the maximum unambiguous range as a function of pulse repetition frequency is shown in the figure below.

*This can also be explained with the following simple relations.*

- \( T_R \) is the time elapsed between transmission pulse and Echo pulse.

- \( T_R = \frac{2R}{C} \) \( \) where \( R \) = Range of target

- \( T_R \) increases with Range \( R \) and in extreme case Echo pulse merges with next Transmitted Pulse. Then \( T_R \) becomes equal to \( T_p \) Where \( T_p \) = Pulse repetition period

- \( T_{R_{max}} = T_p = \frac{2R_{max}}{C} \) and so \( R_{max} = \frac{CT_p}{2} = \frac{C}{2f_p} = R_{unambig} \)

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**Fig 1.1:** Plot of maximum unambiguous range as a function of the pulse repetition frequency.
Simple form of Radar Equation:

The radar equation
- Relates the range of a Radar to the characteristics of the transmitter, receiver, antenna, target, and environment.
- Useful as a means for determining the maximum measurable distance from the radar to the target
- It serves both as a tool for understanding radar operation and as a basis for radar design.

Derivation of the simple form of radar equation:
- If the power of the radar transmitter is denoted by \( P_t \) and if an isotropic antenna is used (one which radiates uniformly in all directions) the power density (watts per unit area) at a distance \( R \) from the radar is equal to the transmitter power divided by the surface area \( 4\pi R^2 \) of an imaginary sphere of radius \( R \) with radar at its centre, or
\[
\text{Power density from anisotropic antenna} = \frac{P_t}{4\pi R^2}
\]

- Radars employ directive antennas to direct the radiated power \( P_t \) into some particular direction. The gain \( G \) of an antenna is a measure of the increased power radiated in the direction of the target as compared with the power that would have been radiated from an isotropic antenna. It may be defined as the ratio of the maximum radiation intensity from the given antenna to the radiation intensity from a lossless, isotropic antenna with the same power input. (The radiation intensity is the power radiated per unit solid angle in a given direction.) Then the power density at the target from an antenna with a transmitting gain \( G \) is given by
\[
\text{Power density from directive antenna} = \frac{P_t \cdot G}{4\pi R^2}
\]

- The target intercepts a portion of the incident power and reradiates it in various directions. The measure of the amount of incident power intercepted by the target and reradiated back in the direction of the radar is denoted as the radar cross section \( \sigma \), and is defined by the relation
\[
\text{Power density of echo signal at radar} = \left( \frac{P_t \cdot G}{4\pi R^2} \right) \cdot \frac{\sigma}{4\pi R^2}
\]

- The radar cross section \( \sigma \) has units of area. It is a characteristic of the particular target and is a measure of its size as seen by the radar. The radar antenna captures a portion of the echo power. If the effective area of the receiving antenna is denoted \( A_e \), then the power \( P_r \) received by the radar is given by
\[
P_r = \left( \frac{P_t \cdot G}{4\pi R^2} \right) \cdot \frac{\sigma}{4\pi R^2} \cdot A_e
\]
\[
= \left( P_t \cdot G \cdot A_e \cdot \sigma \right) / (4\pi R^2)
\]
• The maximum radar range $R_{\text{max}}$ is the distance beyond which the target cannot be detected. It occurs when the received echo signal power $P_r$ just equals the minimum detectable signal $S_{\text{min}}$.

Therefore

$$R_{\text{max}} = \left[ \frac{(Pt \cdot G \cdot A_e \cdot \sigma)}{(4\pi)^2 \cdot S_{\text{min}}} \right]^{1/4} \quad \ldots \ldots (1)$$

This is the fundamental form of the radar equation. Note that the important antenna parameters are the transmitting gain and the receiving effective area.

Antenna theory gives the relationship between the transmitting gain and the receiving effective area of an antenna as:

$$G = 4\pi A_e / \lambda^2$$

Since radars generally use the same antenna for both transmission and reception, the above relation between gain $G$ and effective aperture area $A_e$ can be substituted into the above equation, first for $A_e$ and then for $G$, to give two other forms of the radar equation.

$$R_{\text{max}} = \left[ \frac{(Pt \cdot G^2 \cdot \lambda^2 \cdot \sigma)}{(4\pi)^3 \cdot S_{\text{min}}} \right]^{1/4} \quad \ldots \ldots (2)$$

$$R_{\text{max}} = \left[ \frac{(Pt \cdot A_e^2 \cdot \sigma)}{4\pi \cdot \lambda^2 \cdot S_{\text{min}}} \right]^{1/4} \quad \ldots \ldots (3)$$

These three forms (eqs. 1, 2, and 3) illustrate the need to be careful in the interpretation of the radar equation. For example, from Eq. (2) it might be thought that the range of radar varies as $\lambda^{1/2}$, but Eq. (3) indicates a $\lambda^{-1/2}$ relationship, and Eq. (1) shows the range to be independent of $\lambda$.

The correct relationship depends on whether it is assumed the gain is constant or the effective area is constant with wavelength.

**Limitations of the simple form of the Limitations of Simple Radar equation:**

- Does not adequately describe the performance of practical radar.
- Many important factors that affect range are not explicitly included.
- In practice, the observed maximum radar ranges are usually much smaller than what would be predicted by the above equations, sometimes by as much as a factor of two.

There are many reasons for the failure of the simple radar equation to correlate with actual performance and these will be explained subsequently in the modified Radar range equation.

**Radar block diagram and operation:**

The operation of a typical pulse radar is described with the help of a simple block diagram shown in the figure below. The transmitter is an oscillator, such as a magnetron, that is “pulsed” (turned on and off) by the modulator to generate a repetitive train of pulses. The magnetron has been the most widely used of the various microwave generators for radar. A typical radar for the detection of aircraft at ranges of 100 or 200 nm is employs a peak power of the order of one megawatt, an average power of several kilowatts, a pulse width of several microseconds, and a
pulse repetition frequency of several hundred pulses per second. The waveform generated by the transmitter travels via a transmission line to the antenna, where it is radiated into space.

A single antenna is generally used for both transmitting and receiving. The receiver must be protected from damage caused by the high power of the transmitter. This is the function of the duplexer. The duplexer also serves to channel the returned echo signals to the receiver and not to the transmitter. The duplexer consists of two gas-discharge devices, one known as a TR (transmit-receive) and the other as ATR (anti-transmit-receive). The TR protects the receiver during transmission and the ATR directs the echo signal to the receiver during reception. Solid-state ferrite circulators and receiver protectors with gas-plasma TR devices and/or diode limiters are also employed as duplexers. The receiver is usually of the super heterodyne type. The first stage normally is a low-noise RF amplifier, such as a parametric amplifier or a low-noise transistor. The mixer and local oscillator (LO) convert the RF signal to an intermediate frequency IF. Typical IF amplifier center frequencies are 30 or 60 MHz and will have a bandwidth of the order of one megahertz.

The IF amplifier should be designed as a matched filter i.e., its frequency-response function \( H(f) \) should maximize the peak-signal-to-mean-noise-power ratio at the output. This occurs when the magnitude of the frequency-response function \( |H(f)| \) is equal to the magnitude of the echo signal spectrum \( |S(f)| \), and the phase spectrum of the matched filter is the negative of the phase spectrum of the echo signal. In a radar whose signal waveform approximates a rectangular pulse, the conventional IF filter band pass characteristic approximates a matched filter when the product of the IF bandwidth \( B \) and the pulse width \( \tau \) is of the order of unity, that is, \( B\tau = 1 \).

![Block diagram of a pulse radar.](image)

After maximizing the signal-to-noise ratio in the IF amplifier, the pulse modulation is extracted by the second detector and amplified by the video amplifier to a level where it can be properly displayed, usually on a cathode-ray tube (CRT). Timing signals are also supplied to the indicator to provide the range zero. Angle information is obtained from the pointing direction of the antenna.
Fig 1.3(a) PPI presentation displaying Range vs. Angle (intensity modulation)
(b) A-scope presentation displaying Amplitude vs. Range (deflection modulation)

The most common form of cathode-ray tube display is the Plan Position Indicator, or PPI (Fig. a) which maps in polar coordinates the location of the target in azimuth and range. This is an intensity-modulated display in which the amplitude of the receiver output modulates the electron-beam intensity (z axis) as the electron beam is made to sweep outward from the center of the tube. The beam rotates in angle in response to the antenna position. A B-scope display is similar to the PPI except that it utilizes rectangular, rather than polar, coordinates to display range vs. angle. Both the B-scope and the PPI, being intensity modulated, have limited dynamic range. Another form of display is the A-scope, shown in Fig. b, which plots target amplitude (y axis) vs. range (x axis), for some fixed direction. This is a deflection-modulated display. It is more suited for tracking-radar application than for surveillance radar.

A common form of radar antenna is a reflector with a parabolic shape, fed (illuminated) from a point source at its focus. The parabolic reflector focuses the energy into a narrow beam, just as a searchlight or an automobile headlamp. The beam is scanned in space by mechanical pointing of the antenna.

Radar frequencies and applications:
Radar frequencies:
Conventional radars are operated at frequencies extending from about 220 MHz to 35 GHz, a spread of more than seven octaves. These are not necessarily the limits, since radars can be, and have been, operated at frequencies outside either end of this range.
Sky wave HF over-the-horizon (OTH) radar might be at frequencies as low as 4 or 5 MHz, and ground wave HF radars as low as 2 MHz. At the other end of the spectrum, millimeter radars have operated at 94 GHz. Laser radars operate at even higher frequencies. The place of radar frequencies in the electromagnetic spectrum is shown in the figure below. Some of the nomenclature employed to designate the various frequency regions is also shown in this figure.

**ELECTROMAGNETIC SPECTRUM**

![Electromagnetic Spectrum Diagram](image)

**Letter code designation of Radar frequencies:**

Early in the development of radar, a letter code such as S, X, L, etc., was employed to designate Radar frequency bands. Although its original purpose was to guard military secrecy, the designations were maintained, probably out of habit as well as the need for some convenient short nomenclature. This usage has continued and is now an accepted practice of radar engineers. The table below lists the radar-frequency letter-band nomenclature adopted by the IEEE. These are related to the specific bands assigned by the International Telecommunications Union for radar. For example, although the nominal frequency range for L band is 1000 to 2000 MHz, a L-band radar is thought of as being confined within the region from 1215 to 1400 MHz since that is the extent of the assigned band. Letter-band nomenclature is not a substitute for the actual numerical frequency limits of radars. The specific numerical frequency limits should
be used whenever appropriate, but the letter designations of this Table may be used whenever a short notation is desired.

**Table 1.1: Standard radar-frequency letter-band nomenclature**

<table>
<thead>
<tr>
<th>Band designation</th>
<th>Nominal frequency range</th>
<th>Specific radiolocation (radar) bands based on ITU assignments for region 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>HF</td>
<td>3-30 MHz</td>
<td>138-144 MHz</td>
</tr>
<tr>
<td>VHF</td>
<td>30-300 MHz</td>
<td>216-225</td>
</tr>
<tr>
<td>UHF</td>
<td>300-1000 MHz</td>
<td>420-450 MHz</td>
</tr>
<tr>
<td></td>
<td></td>
<td>890-942</td>
</tr>
<tr>
<td>L</td>
<td>1000-2000 MHz</td>
<td>1215-1400 MHz</td>
</tr>
<tr>
<td>S</td>
<td>2000-4000 MHz</td>
<td>2300-2500 MHz</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2700-3700</td>
</tr>
<tr>
<td>C</td>
<td>4000-8000 MHz</td>
<td>5250-5925 MHz</td>
</tr>
<tr>
<td>X</td>
<td>8000-12,000 MHz</td>
<td>8500-10,680 MHz</td>
</tr>
<tr>
<td>$K_w$</td>
<td>12.0-18 GHz</td>
<td>13.4-14.0 GHz</td>
</tr>
<tr>
<td></td>
<td></td>
<td>15.7-17.7</td>
</tr>
<tr>
<td>$K$</td>
<td>18-27 GHz</td>
<td>24.05-24.25 GHz</td>
</tr>
<tr>
<td>$K_a$</td>
<td>27-40 GHz</td>
<td>33.4-36.0 GHz</td>
</tr>
<tr>
<td>mm</td>
<td>40-300 GHz</td>
<td></td>
</tr>
</tbody>
</table>

Applications of Radar:

1. **Military Use:**
   - Military is the initial and important user of Radar
   - (i) Early warning of intruding enemy aircrafts & missiles
   - (ii) Tracking hostile targets and providing location information to Air Defense systems
   - Consisting of Tracking Radars controlling guns and missiles.
   - (iii) Battle field surveillance
   - (iv) Information Friend or Foe IFF
   - (v) Navigation of ships, aircrafts, helicopter etc.

2. **Civilian Use:**
   - (i) Air Traffic Control (ATC)
   - All airports equipped with ATC Radars, for safe landing and take-off and guiding of aircrafts in bad weather and poor visibility conditions.
   - (ii) Aircraft Navigation
     - (a) All aircrafts fitted with weather avoidance radars. These Radars give warning information to pilot about storms, snow precipitation etc. lying ahead of aircraft’s path.
     - (b) Radar is used as an altimeter to indicate the height of the aircraft or helicopter.

3. **Maritime ship’s safety and Navigation:**
   - (i) Radar used to avoid collision of ships during poor visibility conditions (storms, cyclones etc.)
   - (ii) Guide ships into seaports safely.
4. Meteorological Radar:
Used for weather warnings and forecasting. Provides sufficient advance information to civilian administration for evacuation of population in times cyclones, storms etc.

Prediction of Range Performance:

The simple form of Radar equation derived earlier expresses the maximum radar range \( R_{\text{max}} \) in terms of radar and target parameters:

\[
R_{\text{max}} = \left[ \frac{(Pt \cdot G \cdot A_e \cdot \sigma)}{(4\pi)^2 \cdot S_{\text{min}}} \right]^{1/4}
\]

Where
- \( P_t \) = transmitted power, watts
- \( G \) = antenna gain
- \( A_e \) = antenna effective aperture, \( m^2 \)
- \( \Sigma \) = radar cross section, \( m^2 \)
- \( S_{\text{min}} \) = minimum detectable signal, watts

All the parameters are to some extent under the control of the radar designer, except for the target cross section \( \sigma \). The radar equation states that if long ranges are desired,
- The transmitted power must be large,
- The radiated energy must be concentrated into a narrow beam (high transmitting antenna gain),
- The received echo energy must be collected with a large antenna aperture (also synonymous with high gain) and
- The receiver must be sensitive to weak signals.

In practice, however, the simple radar equation does not predict the range performance of actual radar equipment to a satisfactory degree of accuracy. The predicted values of radar range are usually optimistic. In some cases, the actual range might be only half of that is predicted. Part of this discrepancy is due to
- The failure of the above equation to explicitly include the various losses that can occur throughout the system or
- The loss in performance usually experienced when electronic equipment is operated in the field rather than under laboratory-type conditions and
- Another important factor i.e the statistical or unpredictable nature of several of the parameters in the radar equation.

The minimum detectable signal \( S_{\text{min}} \) and the target cross section \( \sigma \) are both statistical in nature and must be expressed in statistical terms.
- Other statistical factors which do not appear explicitly in the simple radar equation but which have an effect on the radar performance are the meteorological conditions along the propagation path and the performance of the radar operator, if one is employed.

The statistical nature of these several parameters does not allow the maximum radar range to be described by a single number. Its specification must include a statement of the probability that the radar will detect a certain type of target at a particular range.
• Hence in order to cover these aspects, the simple radar equation will be modified to include most of the important factors that influence radar range performance.

Minimum detectable signal:

• The ability of a radar receiver to detect a weak echo signal is limited by the noise energy that occupies the same portion of the frequency spectrum as does the signal energy and accompanies the signal.
• The weakest signal the receiver can detect is called the minimum detectable signal. It is difficult to define minimum detectable signal (MDS) because of its statistical nature and because the criterion for deciding whether a target is present or not is not too well defined.
• Detection is normally based on establishing a threshold level at the output of the receiver (as shown by the dotted line in the figure below.) Whenever Rx output signal which is a mixture of echo and noise crosses this threshold, then it is detected as a target. This is called threshold detection.
• Consider the output of a typical radar receiver as a function of time as shown in the figure below which typically represents one sweep of the video output displayed on an A-scope.

![Diagram showing threshold level and rms value of noise over time](image)

Fig 1.4: Typical envelope of the radar receiver output as a function of time. A, B, and C are three targets representing signal plus noise. A and B are valid detections, but C is a missed detection.

• The envelope has a fluctuating appearance due to the random nature of noise and consists of three targets A, B and C of different signal amplitudes.
• The voltage envelope shown above is assumed to be from a matched-filter receiver. Receiver echoes no more look like rectangular pulses. They appear as triangular as shown in the figure above.
• If the threshold level were set sufficiently high, the envelope would not generally exceed the threshold if noise alone were present, but would exceed it if a strong signal were present. If the signal were small, however, it would be more difficult to recognize its
presence. The threshold level must be low if weak signals are to be detected, but it cannot be so low that noise peaks cross the threshold and give a false indication of the presence of targets.

- The signal at A is large which has much larger amplitude than the noise. Hence target detection is possible without any difficulty and ambiguity.
- Next consider the two signals at B and C, representing target echoes of equal amplitude. The noise voltage accompanying the signal at B is large enough so that the combination of signal plus noise exceeds the threshold and target detection is still possible.
- But for the target C the noise is not as large and the resultant signal plus noise does not cross the threshold and hence target is not detected.
- **Threshold Level setting**: Weak signals such as C would not be lost if the threshold level were lower. But too low a threshold increases the likelihood that noise alone will rise above the threshold and is taken as target. Such an occurrence is called a **false alarm**. Therefore, if the threshold is set too low, false target indications are obtained, but if it is set too high, targets might be missed. The selection of the proper threshold level is a compromise that depends upon how important it is if a mistake is made either by 1. Failing to recognize a signal that is present (**probability of a miss**) or by 2. Falsely indicating the presence of a signal when it does not exist (**probability of a false alarm**)
- The signal-to-noise ratio necessary to provide adequate detection is one of the important parameters that must be determined in order to compute the minimum detectable signal.
- Although the detection decision is usually based on measurements at the video output, it is easier to consider maximizing the signal-to-noise ratio at the output of the IF amplifier rather than in the video. The receiver may be considered linear up to the output of the IF. It is shown that maximizing the signal-to-noise ratio at the output of the IF is equivalent to maximizing the video output. The advantage of considering the signal-to-noise ratio at the IF is that the assumption of linearity may be made. It is also assumed that the IF filter characteristic approximates the matched filter, so that the output signal-to-noise ratio is maximized.

**Receiver noise:**

- Noise is unwanted electromagnetic energy which interferes with the ability of the receiver to detect the wanted signal thus limiting the receiver sensitivity.

It may originate within the receiver itself, or it may enter via the receiving antenna along with the desired signal. If the radar were to operate in a perfectly noise-free environment so that no external sources of noise accompanied the desired signal, and if the receiver itself were so perfect that it did not generate any excess noise, *there would still exist an unavoidable component of noise generated by the thermal motion of the conduction electrons in the ohmic portions of the receiver input stages. This is called thermal noise, or Johnson’s noise, and is directly proportional to the temperature of the ohmic portions of the circuit and the receiver band*
**width.** The available noise power generated by a receiver of bandwidth $B_n$ (in hertz) at a temperature $T$ (degrees Kelvin) is given by: \[ \text{Available thermal-noise power} = kT B_n \]
where $k$ = Boltzmann's constant = $1.38 \times 10^{-23}$ J/deg. If the temperature $T$ is taken to be 290 K, which corresponds approximately to room temperature (62°F), the factor $kT$ is $4 \times 10^{-21}$ W/Hz of bandwidth. If the receiver circuitry were at some other temperature, the thermal-noise power would be correspondingly different.

- A receiver with a reactance input such as a parametric amplifier need not have any significant ohmic loss. The limitation in this case is the thermal noise seen by the antenna and the ohmic losses in the transmission line.
- For radar receivers of the super heterodyne type (the type of receiver used for most radar applications), the receiver bandwidth is approximately that of the intermediate-frequency stages. It should be cautioned that the bandwidth $B_n$ mentioned above is not the 3-dB, or half-power, bandwidth commonly employed by electronic engineers. It is an integrated bandwidth and is given by:

$$B_n = \frac{\int_{-\infty}^{\infty} [H(f)]^2 \ df}{[H(0)]^2}$$

where $H(f)$ = frequency-response characteristic of IF amplifier (filter) and $f_0$ = frequency of maximum response (usually occurs at mid band).

- **The bandwidth $B_n$ is called the noise bandwidth and is the bandwidth of an equivalent rectangular filter whose noise-power output is same as the filter with characteristic $H(f)$.** It is not theoretically same as the 3-dB bandwidth. The 3-dB bandwidth is widely used since it is easy to measure. The measurement of noise bandwidth however involves a complete knowledge of the response characteristic $H(f)$. The frequency-response characteristics of many practical radar receivers are such that the 3 dB and the noise bandwidths do not differ appreciably. Therefore the 3-dB bandwidth may be used in many cases as an approximation to the noise bandwidth.

- The noise power in practical receivers is often greater than can be accounted for by thermal noise alone and is due to mechanisms other than the thermal agitation of the conduction electrons. The exact origin of the extra noise components is not important except to know that it exists. Whether the noise is generated by a thermal mechanism or by some other mechanism the total noise at the output of the receiver may be considered to be equal to the thermal-noise power obtained from an "ideal" receiver multiplied by a factor called the **noise figure**.

- The noise figure $F_n$ of a receiver is defined by the equation:

$$F_n = N_o / kT_0 B_n G_o$$

= (Noise output of practical receiver) / (Noise output of ideal receiver at std. temp $T_0$)

Where $N_o$ = noise output from receiver, and $G_o$ = available gain. The standard temperature $T_0$ is taken to be 290 K, according to the Institute of Electrical and Electronics Engineers definition. The noise $N_o$ is measured over the linear portion of the receiver input-output characteristic, usually at the output of the IF amplifier before the nonlinear second detector. The receiver bandwidth $B_n$ is that of the IF amplifier in most receivers.
available gain $G_a$ is the ratio of the signal out $S_o$ to the signal in $S_i$ and $kT_o B_o$ is the input noise $N_o$ in an ideal receiver. The above equation may be rewritten as:

$$F_n = \frac{S_i/N_i}{S_o/N_o}$$

Therefore, the noise figure may be interpreted, as a measure of the degradation of signal-to-noise-ratio as the signal passes through the receiver.

**Modified radar equation:**

Rearranging the above two equations for $F_n$, the input signal may be expressed as

$$S_i = \frac{kT_o B_o F_n S_o}{N_o}$$

If the minimum detectable signal $S_{min}$ is that value of $S_i$ corresponding to the minimum ratio of output (IF) signal-to-noise ratio $(S_o/N_o)_{min}$ necessary for detection, then

$$S_{min} = kT_o B_o F_n \left(\frac{S_o}{N_o}\right)_{min}$$

Substituting this expression for $S_{min}$ into the earlier basic Radar equation results in the following form of the modified radar equation:

$$R^4 = \frac{P_t G A_o \sigma}{(4\pi)^2 kT_o B_o F_n (S_o/N_o)_{min}} \quad \quad \quad \quad (4)$$

**RADAR EQUATION**

For the study of SNR, false alarm probability and probability of detection etc an introduction to the basics of probability and probability density functions is necessary and they are briefly explained here.

**Introduction to Probability-density functions:**

Probability is a measure of the likelihood of occurrence of an event. The scale of probability ranges from 0 to 1. An event which is certain is assigned the probability 1. An impossible event is assigned the probability 0. The intermediate probabilities are assigned so that the more likely an event, the greater is its probability.

One of the more useful concepts of probability theory needed to analyze the detection of signals in noise is the **probability-density function**. Consider the variable $x$ as representing atypical measured value of a random process such as a noise voltage or current. Imagine each $x$ to define a point on a straight line corresponding to the distance from a fixed reference point. The distance of $x$ from the reference point might represent the value of the noise current or the noise voltage. Divide the line into small equal
segments of length $\Delta x$ and count the number of times that $x$ falls in each interval. The probability-density function $p(x)$ is then defined as

$$p(x) = \lim_{\Delta x \to 0} \frac{\text{number of values in range } \Delta x \text{ at } x}{\text{total number of values } = N} \Delta x$$

Then, the probability that a particular measured value lies within the infinitesimal width $dx$ centered at $x$ is simply $p(x) \, dx$. The probability that the value of $x$ lies within the finite range from $x_1$ to $x_2$ is found by integrating $p(x)$ over the range of interest, or

$$\text{Probability } (x_1 < x < x_2) = \int_{x_1}^{x_2} p(x) \, dx$$

By definition, the probability-density function is positive. Since every measurement must yield some value, the integral of the probability density over all values of $x$ must be equal to unity. That is,

$$\int_{-\infty}^{\infty} p(x) \, dx = 1 \quad \text{(5)}$$

The average value of a variable function $\phi(x)$ that is described by the probability-density function $p(x)$ is,

$$\langle \phi(x) \rangle_{av} = \int_{-\infty}^{\infty} \phi(x)p(x) \, dx$$

This follows from the definition of an average value and the probability-density function. The mean or average value of $x$ is

$$\langle x \rangle_{av} = m_1 = \int_{-\infty}^{\infty} xp(x) \, dx$$

and the mean square value is

$$\langle x^2 \rangle_{av} = m_2 = \int_{-\infty}^{\infty} x^2 p(x) \, dx$$

The quantities $m_1$ and $m_2$ are sometimes called the first and second moments of the random variable $x$. If $x$ represents an electric voltage or current, $m_1$ is the d-c component. It is the value read by a direct-current voltmeter or ammeter. The mean square value $m_2$ of the current when multiplied by the resistance gives the mean power. The mean square value of voltage times the conductance is also the mean power. The variance is defined as

$$\mu_2 = \sigma^2 = \langle (x - m_1)^2 \rangle_{av} = \int_{-\infty}^{\infty} (x - m_1)^2 p(x) \, dx = m_2 - m_1^2 = \langle x^2 \rangle_{av} - \langle x \rangle_{av}^2$$

The variance is the mean square deviation of $x$ about its mean and is sometimes called the second central moment. If the random variable is a noise current, the product of the variance $\sigma$ and resistance gives the mean power of the a-c component. The square root of the variance $\sigma$ is called the standard deviation and is the root-mean-square (rms) value of the a-c component.
We shall explain briefly two PDFs that are required for our study of Signal to Noise Ratio (SNR):

1. The **Gaussian or normal probability density** is one of the most important in noise theory, since many sources of noise such as thermal noise or shot noise may be represented by Gaussian statistics. Also, a Gaussian representation is often more convenient to manipulate mathematically. The Gaussian density function shown in the fig 1.5(a) below has a bell-shaped appearance and is defined by:

\[
p(x) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{(x-x_0)^2}{2\sigma^2}\right)
\]

where \(\exp\) [ ] is the exponential function and the parameters have been adjusted to satisfy the normalizing condition of Eq. 5. It can be shown that

\[
m_1 = \int_{-\infty}^{\infty} xp(x) \, dx = x_0 \quad m_2 = \int_{-\infty}^{\infty} x^2 p(x) \, dx = x_0^2 + \sigma^2 \quad \mu_2 = m_2 - m_1^2 = \sigma^2
\]

The probability density of the sum of a large number of independently distributed quantities approaches the Gaussian probability-density function no matter what the individual distributions may be, provided that the contribution of any one quantity is not comparable with the resultant of all others. This is the **central limit theorem**. Another property of the Gaussian distribution is that no matter how large a value \(x\) we may choose, there is always some finite probability of finding a greater value. If the noise at the input of the threshold detector were truly Gaussian, then no matter how high the threshold were set, there would always be a chance that it would be exceeded by noise and appear as a false alarm. However, the probability diminishes rapidly with increasing \(x\), and for all practical purposes the probability of obtaining an exceedingly high value of \(x\), is negligibly small.

2. The **Rayleigh probability density function** is also of special interest to the radar systems engineer. It describes the envelop of the noise output from a narrow band filter (such as the IF filter in super heterodyne receiver). The cross-section fluctuations of certain types of complex radar targets and many kinds of clutter and weather echoes. The Rayleigh density function is shown in the fig. 1.5 (b) below and is given by:
Fig 1.5 (b): Rayleigh probability density function
\[ p(x) = \frac{2x}{\langle x^2 \rangle_{av}} \exp \left( -\frac{x^2}{\langle x^2 \rangle_{av}} \right) \quad x \geq 0 \]

The parameter \( x \) might represent a voltage and \( \langle x^2 \rangle_{av} \) the mean, or average, value of the voltage squared.

**Signal to Noise Ratio (SNR):**

The results of statistical noise theory will be applied to obtain:

- *The signal-to-noise ratio at the output of the IF amplifier necessary to achieve a specified probability of detection without exceeding a specified probability of false alarm.*

The output signal-to-noise ratio thus obtained is substituted into the final modified radar equation, we have obtained earlier.

**The details of system that is considered:**

- IF amplifier with bandwidth \( B_{IF} \) followed by a second detector and a video amplifier with bandwidth \( B_v \) as shown in the figure below.
- The second detector and video amplifier are assumed to form an envelope detector, that is, one which rejects the carrier frequency but passes the modulation envelope.
- To extract the modulation envelope, the video bandwidth must be wide enough to pass the low-frequency components generated by the second detector, but not so wide as to pass the high-frequency components at or near the intermediate frequency.
- The video bandwidth \( B_v \) must be greater than \( B_{IF}/2 \) in order to pass all the video modulation.

![Diagram](image_url)

**Figure 1.6: Envelope detector.**

**Step 1:** To determine the Probability of false alarm when noise alone is assumed to be present as input to the receiver:
The noise entering the IF filter (the terms filter and amplifier are used interchangeably) is assumed to be Gaussian, with probability-density function given by

\[ p(v) = \frac{1}{\sqrt{2\pi}\psi_0} \exp \left( -\frac{v^2}{2\psi_0} \right) \]

Where:
- \( p(v) \, dv \) is the probability of finding the noise voltage \( v \) between the values of \( v \) and \( v + dv \)
- \( \psi_0 \) is the variance, or mean-square value of the noise voltage, and the mean value of \( v \) is taken to be zero.

(Compare this with the Standard Probability density function of Gaussian noise)

\[ p(x) = \frac{1}{\sqrt{2\pi}\sigma^2} \exp \left( -\frac{(x - x_0)^2}{2\sigma^2} \right) \]

With \( \sigma^2 \) replaced by \( \psi_0 \) and \( (x-x_0) \) replaced by \( v \) with mean value of zero.

If Gaussian noise were passed through a narrowband IF filter whose Bandwidth is small compared with its mid band frequency-the probability density of the envelope of the noise voltage output is shown by Rice to be of the form of Rayleigh probability-density function

\[ p(R) = \frac{R}{\psi_0} \exp \left( -\frac{R^2}{2\psi_0} \right) \]

where \( R \) is the amplitude of the envelope of the filter output. The probability that the envelope of the noise voltage will lie between the values of \( V_1 \) and \( V_2 \) is

\[
\text{Probability } (V_1 < R < V_2) = \int_{V_1}^{V_2} \frac{R}{\psi_0} \exp \left( -\frac{R^2}{2\psi_0} \right) \, dR
\]

The probability that the noise voltage envelope will exceed the voltage threshold \( V_T \) is

\[
\text{Probability } (V_T < R < \infty) = \int_{V_T}^{\infty} \frac{R}{\psi_0} \exp \left( -\frac{R^2}{2\psi_0} \right) \, dR = \exp \left( -\frac{V_T^2}{2\psi_0} \right) = P_{fa} \]

Whenever the voltage envelope exceeds the threshold \( V_T \), a target is considered to have been detected. Since the probability of a false alarm is the probability that noise will cross the threshold, the above equation gives the probability of a false alarm, denoted by \( P_{fa} \).

The probability of false alarm as given above by itself does not indicate that Radar is troubled by the false indications of Target. The time between the false alarms \( T_{fa} \) is a better measure of the effect of Noise on the Radar performance. (Explained with reference to the figure below)
The average time interval between crossings of the threshold by noise alone is defined as the false-alarm time $T_{fa}$

$$T_{fa} = \lim_{N \to \infty} \frac{1}{N} \sum_{k=1}^{N} T_k$$

where $T_k$ is the time between crossings of the threshold $V_T$ by the noise envelope, when the slope of the crossing is positive.

The false-alarm probability may also be defined as the ratio of the duration of time the envelope is actually above the threshold to the total time it could have been above the threshold, i.e.

$$P_{fa} = \frac{\sum_{k=1}^{N} t_k}{\sum_{k=1}^{N} T_k} = \frac{\langle t_k \rangle_{av}}{\langle T_k \rangle_{av}} = \frac{1}{T_{fa} B}$$

---------- (8)

![Diagram](image)

**Fig 1.7: Envelope of receiver output illustrating false alarms due to noise.**

where $t_k$ and $T_k$ are shown in the Figure above. The average duration of a noise pulse is approximately the reciprocal of the bandwidth $B$, which in the case of the envelope detector is $B_W$. Equating eqs.7 and 8 we get

$$T_{fa} = \frac{1}{B_{IF}} \exp \left( \frac{V_T^2}{2\psi_0} \right)$$

---------- (9)

A plot of the above equation is shown in the figure below with $(V_T^2/2\psi_0)$ as the abscissa.
Fig. 1.8: Average time between false alarms as a function of the threshold level $V_t$ and the receiver band width B. $\Psi_0$ is the mean square noise voltage.

Step 2:
To determine Probability of detection when a sine wave signal is present along with noise:
Thus far, a receiver with only a noise input was discussed. Next, consider a sine-wave signal of amplitude A to be present along with noise at the input to the IF filters. The frequency of the signal is the same as the IF mid band frequency $f_w$. The output of the envelope detector has a probability-density function given by

$$p_s(R) = \frac{R}{\Psi_0} \exp \left(-\frac{R^2 + A^2}{2\Psi_0}\right) I_0 \left(\frac{RA}{\Psi_0}\right) \quad \cdots \cdots (10)$$

where $I_0(Z)$ is the modified Bessel function of zero order and argument Z.
When the signal is absent, \( A = 0 \) and the above equation for PDF for signal plus noise reduces to the probability-density function for noise alone. This equation is sometimes called the Rice probability-density function.

The probability that the signal will be detected (which is the probability of detection) is the same as the probability that the envelope \( R \) will exceed the predetermined threshold \( V_T \). The probability of detection \( P_d \) is therefore:

\[
P_d = \int_{V_T}^{\infty} p_s(R) \, dR = \int_{V_T}^{\infty} \frac{R}{\psi_0} \exp\left( -\frac{R^2 + A^2}{2\psi_0} \right) I_0\left( \frac{RA}{\psi_0} \right) \, dR
\]

When the expression of PDF for \( P_s(R)[Eq. 10] \) is substituted in the above equation for probability of detection [eqn.11] it cannot be evaluated by simple means, and numerical & empirical techniques or a series approximation must be used.

The expression for \( P_d \) given by equation (11) after series expansion is a function of the signal amplitude \( A \), threshold voltage \( V_T \), and mean noise power \( \psi_0 \). In Radar systems analysis, it is more convenient to use Signal to Noise power ratio (S/N) rather than signal to noise voltage ratio \( A/\psi_0^2 \). These are related by:

\[
A/\psi_0^2 = \frac{\text{Signal Amplitude/rms noise voltage}}{\text{rms signal voltage}} = \sqrt{2} \times \frac{\text{rms signal voltage}}{\text{rms noise voltage}}
\]

\[
= \left( \frac{2 \times \text{signal power/noise power}}{\text{signal power/noise power}} \right)^{1/2} = \left( \frac{2}{\text{S/N}} \right)^{1/2}
\]

The probability of detection \( P_d \) can then be expressed in terms of \( S/N \), and Threshold-Noise ratio \( V_T^2/2\psi_0 \). The probability of false alarm is also a function of \( V_T^2/2\psi_0 \) as given by: \( P_{fa} = \exp\left( -V_T^2/2\psi_0 \right) \).

The two expressions for \( P_d \) and \( P_{fa} \) can now be combined by eliminating the Threshold-Noise ratio \( V_T^2/2\psi_0 \) that is common in both expressions so as to get a single expression relating the probability of detection \( P_d \), probability of false alarm \( P_{fa} \), and signal to Noise ratio \( S/N \). The result is plotted in the figure below.

A much easier empirical formula developed by Albersheim for the relation ship between \( S/N, P_{fa} \), and \( P_d \) is also given below:

\[
S/N = A + 0.12AB + 1.7B
\]

Where \( A = \ln \left( \frac{0.62}{P_{fa}} \right) \) and \( B = \ln \left( \frac{P_d}{(1-P_d)} \right) \).
Fig. 1.9: Probability of detection for a sine wave in noise as a function of the signal-to-noise (power) ratio and the probability of false alarm

System design sequence:

- Both the false-alarm time $T_{fa}$ and the detection probability $P_d$ are specified by the system requirements.
- The radar designer computes the probability of the false alarm using the above $T_{fa}$ & the relation
  \[ P_{fa} = \frac{1}{T_{fa}} \cdot B \]
- Then from the figure above or using the Albersheim's empirical equation given above the required signal-to-noise ratio to achieve the above $P_{fa}$ & $P_d$ is determined.

For example, suppose that the desired false-alarm time was 15 min and the IF bandwidth was 1 MHz. This gives a false-alarm probability of $1.11 \times 10^{-7}$. Figure above indicates that a signal-to-noise ratio of 13.1 dB is required to yield a 0.50 probability of detection, 14.7 dB for 0.90, and 16.5 dB for 0.999.
This is the signal-to-noise ratio that is to be used in the final modified Radar Equation we have obtained earlier.

$$R_{\text{max}}^4 = \frac{P_r G A_e \sigma}{(4\pi)^2 k T_0 B_o F_n (S/N)_\text{min}}$$

**Integration of Radar Pulses:**

The relation between the signal to noise ratio, the probability of detection and the probability of false alarm as shown in the figure or as obtained using the Albersheim's empirical equation applies for a single pulse only. However, many pulses are usually returned from any target on each radar scan and can be used to improve detection. The number of pulses $n_b$ returned from a point target as the radar antenna scans through its beam width is

$$n_b = \theta_b \cdot f_p / \theta'_s = \theta_b \cdot f_p / 6 \omega_m$$

where

- $\theta_b$ = antenna beam width, deg
- $f_p$ = pulse repetition frequency, Hz
- $\theta'_s$ = antenna scanning rate, deg/s
- $\omega_m$ = antenna scan rate, rpm

The process of summing all the radar echo pulses for the purpose of improving detection is called integration.

Integration may be accomplished in the radar receiver either before the second detector (in the IF) or after the second detector (in the video).

- Integration before the detector is called pre detection or coherent integration. In this the phase of the echo signal is to be preserved if full benefit is to be obtained from the summing process
- Integration after the detector is called post detection or non coherent integration. In this phase information is destroyed by the second detector. Hence post detection integration is not concerned with preserving RF phase. Due to this simplicity it is easier to implement in most applications, but is not as efficient as pre detection integration.

If $n$ pulses, all of the same signal-to-noise ratio, were integrated by an ideal pre detection integrator, the resultant or integrated signal-to-noise (power) ratio would be exactly $n$ times that of a single pulse. If the same $n$ pulses were integrated by an ideal post detection device, the resultant signal-to-noise ratio would be less than $n$ times that of a single pulse. This loss in integration efficiency is caused by the nonlinear action of the second detector, which converts some of the signal energy to noise energy in the rectification process.

Due to its simplicity, Post detection integration is preferred many a times even though the integrated signal-to-noise ratio may not be as high as that of Pre-detection. An alert, trained operator viewing a properly designed cathode-ray tube display is a close approximation to the theoretical post detection integrator.

The efficiency of post detection integration relative to ideal pre-detection integration has been computed by Marcum when all pulses are of equal amplitude. The integration efficiency may be defined as follows:

$$E_i(n) = \frac{(S/N)_i}{n(S/N)_o}$$
where \( n \) = number of pulses integrated

\[
(S/N)_1 = \text{value of signal-to-noise ratio of a single pulse required to produce a given probability of detection (for }\ n = 1)\n\]

\[
(S/N)_n = \text{value of signal-to-noise ratio per pulse required to produce the same probability of detection when }\ n \text{ pulses (of equal amplitude) are integrated}\n\]

The improvement in the signal-to-noise ratio when \( n \) pulses are integrated post detection is \( n E_1(n) \) and is the integration-improvement factor. It may also be thought of as the effective number of pulses integrated by the post detection integrator. The improvement with ideal pre detection integration would be equal to \( n \). Integration loss in decibels is defined as \( \text{L}(n) = 10 \log_1 (1/E_1(n)) \).

The integration-improvement factor (or the integration loss) is not a sensitive function of either the probability of detection or the probability of false alarm.

The radar equation with \( n \) pulses integrated can be written

\[
R_{\text{max}}^n = \frac{P_i G A_e \sigma}{(4\pi)^2 k T_0 B_n F_n (S/N)_n^\dagger}\n\]

where the parameters are the same as in the earlier Radar equation except that \( (S/N)_n \) is the signal-to-noise ratio of one of the \( n \) equal pulses that are integrated to produce the required probability of detection for a specified probability of false alarm. Substituting the equation for integration efficiency

\[
E_i(n) = \frac{(S/N)_1}{n(S/N)_n}\n\]

into the above Radar equation gives the final modified Radar equation including integration efficiency.

\[
R_{\text{max}}^n = \frac{P_i G A_e \sigma n E_i(n)}{(4\pi)^2 k T_0 B_n F_n (S/N)_1^\dagger}\n\]

Radar Cross Section of Targets:

The radar cross section of a target is the (fictional) area intercepting that amount of power which when scattered equally in all directions, produces an echo at the radar equal to that from the target. Or in other terms

\[
\sigma = \frac{\text{power reflected toward source/unit solid angle}}{\text{incident power density/4}\pi}\n\]

\[
= \lim_{R \to \infty} 4\pi R^2 \left| \frac{E_r}{E_i} \right|^2\n\]
Where \( R \) = distance between radar and target
\( E_r \) = strength of reflected field at radar
\( E_t \) = strength of incident field at target

This equation is equivalent to the radar range equation of Sec. 1.2. For most common types of radar targets such as aircraft, ships, and terrain, the radar cross section does not necessarily bear a simple relationship to the physical area, except that the larger the target size, the larger will be the cross section.

*Scattering* and *diffraction*: are variations of the same physical process. When an object scatters an electromagnetic wave, the scattered field is defined as the difference between the total field in the presence of the object and the field that would exist if the object were absent (but with the sources unchanged). On the other hand, the diffracted field is the total field in the presence of the object. With radar backscatter, the two fields are the same, and one may talk about scattering and diffraction interchangeably.

**Radar cross section of a simple sphere:** is shown in the figure below as a function of its circumference measured in wavelengths.\((2\pi a/\lambda \text{ where } a \text{ is the radius of the sphere and } \lambda \text{ is the wavelength})\). The plot consists of three regions.

1. **Rayleigh Region:**
   - The region where the size of the sphere is small compared with the wavelength \((2\pi a/\lambda \ll 1)\) is called the Rayleigh region.
   - Named after Lord Rayleigh who, in the early 1870s first studied scattering by small particles. Lord Rayleigh was interested in the scattering of light by microscopic particles, rather than in radar. His work preceded the original electromagnetic echo experiments of Hertz by about fifteen years.
   - The Rayleigh scattering region is of interest to the radar engineer because the cross sections of raindrops and other meteorological particles fall within this region at the usual radar frequencies. Since the cross section of objects within the Rayleigh region varies as \(\lambda^4\), rain and clouds are essentially invisible to radars which operate at relatively long wavelengths (low frequencies). The usual radar targets are much larger than raindrops or cloud particles, and lowering the radar frequency to the point where rain or cloud echoes are negligibly small will not seriously reduce the cross section of the larger desired targets. On the other hand, if it were desired to actually observe, rather than eliminate, raindrop echoes, as in meteorological or weather-observing radar, the higher radar frequencies would be preferred.

2. **Optical Region:**
   - It is at the other extreme from the Rayleigh region where the dimensions of the sphere are large compared with the wavelength \((2\pi a/\lambda \gg 1)\). For large \(2\pi a/\lambda\), the radar cross section approaches the optical cross section \(\pi a^2\).

3. **Mie or Resonance Region:**
   - Between the optical and the Rayleigh region is the Mie, or resonance, region. The cross section is oscillatory with frequency within this region. The maximum value is 5.6 dB greater than the optical value, while the value of the first null is 5.5 dB below the optical value. (The theoretical values of the maxima and minima may vary according to the method of calculation employed.)
Figure 1.10: Radar cross section of the sphere. \( a = \text{radius}; \lambda = \text{wavelength.} \)

Since the sphere is a sphere no matter from what aspect it is viewed, its cross section will not be aspect-sensitive. The cross section of other objects, however, will depend upon the direction as viewed by the radar. (Aspect angle)

Radar cross section of a cone-sphere:

- An interesting radar scattering object is the cone-sphere, a cone whose base is capped with a sphere such that the first derivatives of the contours of the cone and sphere are equal at the joint. Figure below is a plot of the nose-on radar cross section. The next Figure is a plot as a function of aspect angle. The cross section of the cone-sphere from the vicinity of the nose-on direction is quite low.
- Scattering from any object occurs from discontinuities. The discontinuities, and hence the backscattering, of the cone-sphere are from the tip and from the join between the cone and the sphere.
- The nose-on radar cross section is small and decreases as the square of the wavelength. The cross section is small over a relatively large angular region. A large specular (having qualities of a mirror) return is obtained when the cone-sphere is viewed at near perpendicular incidence to the cone surface, i.e., when \( \theta = 90 - \alpha \), where \( \alpha \) = cone half angle. From the rear half of the cone-sphere, the radar cross section is approximately that of the sphere.
The nose-on cross section of the cone-sphere varies, but its maximum value is approximately \(0.4\lambda^2\) and its minimum is \(0.01\lambda^2\) for a wide range of half-angles for frequencies above the Rayleigh region. The null spacing is also relatively insensitive to the cone half-angle.

![Graph showing radar cross section](image)

**Figure 1.11**: Radar cross section of a cone sphere with 15° half angle as a function of the diameter in wavelengths.

- In order to realize in practice the very low theoretical values of the radar cross section for a cone sphere, the tip of the cone must be sharp and not rounded, the surface must be smooth (roughness small compared to a wavelength), the join between the cone and the sphere must have a continuous first derivative, and there must be no holes, windows, or protuberances on the surface.

- Shaping of the target, as with the cone-sphere, is a good method for reducing the radar cross section. Materials such as carbon-fiber composites, which are sometimes used in aerospace applications, can further reduce the radar cross section of targets as compared with that produced by highly reflecting metallic materials.

**Transmitter Power:**

The peak power: The power \(P_i\) in the radar equation is called the peak power. This is not the instantaneous peak power of a sine wave. It is the power averaged over that carrier-frequency cycle which occurs at the maximum power of the pulse.
The average radar power $P_{av}$ is defined as the average transmitter power over the pulse-repetition period. If the transmitted waveform is a train of rectangular pulses of width $\tau$ and pulse-repetition period $T_p = 1/f_p$, then the average power is related to the peak power by

$$P_{av} = \frac{P_r \tau}{T_p} = P_r \tau f_p$$

Duty cycle: The ratio $P_{av}/P_0 = \tau/T_p$, or $\tau f_p$, is called the duty cycle of the radar. A pulse radar for detection of aircraft might have typically a duty cycle of 0.001, while a CW radar which transmits continuously has a duty cycle of unity.

Writing the radar equation in terms of the average power rather than the peak power, we get

$$R_{\text{max}}^4 = \frac{P_{av} G A_e \sigma n E_s(n)}{(4\pi)^2 k T_0 F_n(B_s \tau)(S/N)^1 f_p}$$

The bandwidth and the pulse width are grouped together since the product of the two is usually of the order of unity in most pulse-radar applications.

**Pulse Repetition Frequencies and Range Ambiguities:**

- The pulse repetition frequency (prf) is determined primarily by the maximum range at which targets are expected. If the prf is made too high, the likelihood of obtaining target echoes from the wrong pulse transmission is increased. Echo signals received after an interval exceeding the pulse-repetition period are called multiple time around echoes.

- Consider the three targets labeled A, B, and C in the figure below. Target A is located within the maximum unambiguous range $R_{\text{unamb}}$ of the radar, target B is at a distance greater than $R_{\text{unamb}}$ but less than $2R_{\text{unamb}}$ and the target C is greater than $2R_{\text{unamb}}$ but less than $3R_{\text{unamb}}$. The appearance of the three targets on an A-scope is shown in the figure below. The multiple-time-around echoes on the A-scope cannot be distinguished from proper target echoes actually within the maximum unambiguous range. Only the range measured for target A is correct; those for B and C are not.

- One method of distinguishing multiple-time-around echoes from unambiguous echoes is to operate with a varying pulse repetition frequency. The echo signal from an unambiguous range target will appear at the same place on the A-scope on each sweep no matter whether the prf is modulated or not. However, echoes from multiple-time-around targets will be spread over a finite range as shown in the figure below. The prf may be changed continuously within prescribed limits, or it may be changed discretely among several predetermined values. The number of separate pulse repetition frequencies will depend upon the degree of the multiple time targets. Second-time targets need only two separate repetition frequencies in order to be resolved.
Radar Systems

Lecture Notes (K.Subhas)

Unit 1: Basics of Radar and Radar equation

Fig. 1.12: Multiple-time-around echoes that give rise to ambiguities in range. (a) Three targets A, B and C, where A is within \( R_{\text{range}} \) and B and C are multiple-time-around targets (b) the appearance of the three targets on the A-scope (c) appearance of the three targets on the A-scope with a changing prf.

System Losses:

- The losses in a radar system reduce the signal-to-noise ratio at the receiver output. They are two kinds, predictable with certain precision beforehand and unpredictable. The antenna beam-shape loss, collapsing loss, and losses in the microwave plumbing are examples of losses which are predictable if the system configuration is known. These losses are real and cannot be ignored.
- Losses not readily subject to calculation and which are less predictable include those due to field degradation and to operator fatigue or lack of operator motivation. They are subject to considerable variation and uncertainty.
- Although the loss associated with any one factor may be small, there are many possible loss mechanisms in a complete radar system, and their sum total will be significant.
- In this section, loss (number greater than unity) and efficiency (number less than unity) are used interchangeably. One is simply the reciprocal of the other.

Plumbing loss: This is loss in the transmission lines which connect the transmitter output to the antenna. (cables and waveguides) The losses in decibels per 100 ft for radar transmission lines are shown in the figure below. At the lower radar frequencies the transmission line introduces little loss, unless its length is exceptionally long. At higher radar frequencies, loss/attenuation will not be small and has to be taken into account.

Connector losses: In addition to the losses in the transmission line itself, an additional loss occurs at each connection or bends in the line and at the antenna rotary joint if used. Connector losses are usually small, but if the connection is poorly made, it can contribute significant attenuation. Since the same transmission line is generally used for both receiving and transmission, the loss to be inserted in the radar equation is twice the one-way loss.
Duplexer loss: The signal suffers attenuation as it passes through the duplexer. Generally, the greater the isolation required from the duplexer on transmission, the larger will be the insertion loss. Insertion loss means the loss introduced when the component is inserted into the transmission line. The precise value of the insertion loss depends to a large extent on the particular design. For a typical duplexer it might be of the order of 1 dB.

![Graph showing theoretical attenuation of RF transmission lines.](image)

Fig.1.13: Theoretical (one-way) attenuation of RF transmission lines. Waveguide sizes are inches and are the inside dimensions. *(Data, from Armed Services Index of R.F. Transmission Lines and Fittings, ASEA. 49-B)*

In S-band (3000 MHz) radar, for example, the typical plumbing losses will be as follows:

- **100 ft of RG-113/U A1 waveguide transmission line (two-way):** 1.0 dB
- **Loss due to poor connections (estimate):** 0.5 dB
- **Rotary-joint loss:** 0.4 dB
- **Duplexer loss:** 1.5 dB
- **Total plumbing loss:** 3.4 dB

Beam-shape loss: The antenna gain that appears in the radar equation was assumed to be a constant equal to the maximum value. But in reality the train of pulses returned from a target with scanning radar is modulated in amplitude by the shape of the antenna beam. To properly take into account the pulse-train modulation caused by the beam shape, the computations of the probability of detection (as explained earlier) would have to be performed assuming a modulated train of pulses rather than constant-amplitude pulses. But since this computation is difficult, in this text, this approach is not used. Instead a
beam-shape loss is added to the radar equation and a maximum gain is employed in the radar equation rather than a gain that changes pulse to pulse. This is a simpler, though less accurate method. 

**Scanning loss:** When the antenna scans rapidly enough that the gain on transmit is not the same as the gain on receive. An additional loss has to be computed, called the scanning loss. The technique for computing scanning loss is similar in principle to that for computing beam-shape loss. Scanning loss is important for rapid-scan antennas or for very long range radars such as those designed to view extraterrestrial objects.

**Limiting loss:** Limiting in the radar receiver lowers the probability of detection. Although a well-designed and engineered receiver will not limit the received signal under normal circumstances, intensity-modulated CRT displays such as the PPI or the B-scope have limited dynamic range and may limit. Some receivers, however, might employ limiting for some special purpose like pulse compression. Limiting results in a loss of only a fraction of a decibel for a large number of pulses integrated, provided the limiting ratio (ratio of video limit level to rms noise level) is as large as 2 or 3.

**Collapsing loss:** If the radar were to integrate additional noise samples along with the wanted signal-to-noise pulses, the added noise results in degradation called the **collapsing loss**. It can occur:

- In displays which collapse the range information, such as the C-scope which displays elevation vs. azimuth angle, the echo signal from a particular range interval must compete in a collapsed-range C-scope display, not only with the noise energy contained within that range interval, but with the noise energy from all other range intervals at the same elevation and azimuth.

- In some 3D radars (range, azimuth, and elevation) that display the outputs at all elevations on a single PPI (range, azimuth) display, the collapsing of the 3D radar information on to a 2D display results in a loss.

The mathematical derivation of the collapsing loss, assuming a square-law detector carried out by Marcum shows that the integration of \(m\) noise pulses along with \(n\) signal-plus-noise pulses with signal-to-noise ratio per pulse \((S/N)_m\) is equivalent to the integration of \(m+n\) signal-to-noise pulses each with signal-to-noise ratio \(n(S/N)_n/(m+n)\). The collapsing loss in this case is equal to the ratio of the integration loss \(L_i\) for \((m+n)\) pulses to the integration loss for \(n\) pulses, or

\[
L_i(m, n) = \frac{L_i(m + n)}{L_i(n)}
\]

For example, assume that 10 signal-plus-noise pulses are integrated along with 30 noise pulses and that \(P_s = 0.90\) and \(n = 10^4\). From the published data on integration loss we have \(L_i(40) = 3.5\) dB and \(L_i(10) = 1.7\) dB, so that the collapsing loss is \(1.8\) dB. It is also possible to account for the collapsing loss by substituting into the radar equation the parameter \(E(m + n)\) for \(E(n)\), since \(E(n) = 1/L_i(n)\).

**Non ideal equipment:** The **transmitter power** in the radar equation was assumed to be the specified output power (either peak or average). However, all transmitting tubes are not uniform in quality, and even any individual tube cannot be expected to remain at the same level of performance throughout its useful life. Also, the power is usually not uniform over the operating band of frequencies. Thus, for one reason or another, the transmitted power may be other than the design value. To allow for this variation, a loss factor has to be introduced. This factor can vary with the application, but in the absence of a correct number, a loss of about 2 dB might be used as an approximation.

Variations in the **receiver noise figure** over the operating frequency band also are to be expected. Thus, if the best noise figure over the band is used in the radar equation, a loss factor has to be introduced to account for its poorer value elsewhere in the frequency band.
If the receiver is not the exact matched filter for the transmitted waveform, a loss in Signal-to-noise ratio will occur.

A typical value of loss for anon-matched receiver might be about 1 dB. Because of the exponential relation between the false-alarm time and the threshold level a slight change in the threshold can cause a significant change in the false alarm time. In practice, therefore, it may be necessary to set the threshold level slightly higher than calculated so as to insure a tolerable false alarm time in the event of circuit instabilities. This increase in the threshold is equivalent to a loss.

**Operator loss:** An alert, motivated, and well-trained operator performs as described by theory. However, when distracted, tired, overloaded, or not properly trained, operator performance will decrease. The resulting loss in system performance is called operator loss. There is little guidance available on how to account for the performance of an operator. Hence normally it is better to take steps to avoid loss due to operator performance rather than tolerate it by including it as a loss factor in the radar equation.

**Field degradation:** When a radar system is operated under laboratory conditions by engineering personnel and experienced technicians, the inclusion of the above losses into the radar equation gives a realistic description of the performance of the radar under normal conditions (ignoring anomalous propagation effects). However, when a radar is operated under field conditions the performance usually deteriorates even more than that can be accounted for by the above losses, especially when the equipment is operated and maintained by inexperienced or unmotivated personnel. It may even apply, to some extent, to equipment operated by professional engineers under adverse field conditions. Factors which contribute to field degradation are poor tuning, weak tubes, water in the transmission lines, incorrect mixer-crystal current, deterioration of receiver noise figure, poor TR tube recovery, loose cable connections etc.

**To minimize field degradation** Radars should be designed with built-in automatic performance-monitoring equipment. Careful observation of performance-monitoring instruments and timely preventative maintenance will keep the radar performance up to design level. Radar characteristics that might be monitored include transmitter power, receiver noise figure, the spectrum and/or shape of the transmitted pulse, and the decay time of the TR tube.

A good estimate of the field degradation is difficult to obtain since it cannot be predicted and is dependent upon the particular radar design and the conditions under which it is operating. A degradation of 3 dB is sometimes assumed when no other information is available.

**Other loss factors:**

- A Radar designed to discriminate between moving targets and stationary objects (MTI radar) introduces an additional loss. The MTI discrimination technique results in complete loss of sensitivity for certain values of target velocity relative to the radar. These are called **blind speeds**.
- In a radar with overlapping range gates, the gates may be wider than optimum for practical reasons. The additional noise introduced by the non optimum gate width will result in some degradation.
- The straddling loss accounts for the loss in signal-to-noise ratio for targets not at the center of the range gate or at the center of the filter in a multiple-filter-bank processor.
- Another factor that has a profound effect on the radar range performance is the propagation medium.

**There are many causes of loss and inefficiency in a Radar. Although each of them may be small, the sum total can result in a significant reduction in radar performance. It is important to understand the origins of these losses, not only for better predictions of radar range, but also for the purpose of keeping them to a minimum by careful radar design.**
Important formulae:

- Range of a Radar \( R = \frac{cT_R}{2} \)
- Maximum unambiguous Range \( R_{\text{unamb}} = \frac{C}{2f_p} \)
- Basic Radar equation: \( R_{\text{max}} = \left[ \frac{(Pt \cdot G \cdot A_o \cdot \sigma)}{(4\pi)^2 \cdot S_{\text{min}}} \right]^{1/4} \)
- Gain of an antenna \( G = 4\pi A_o / \lambda^2 \)
- Noise figure of a receiver \( F_n = \frac{S_o}{N_o} / kT_0 B_o G_a \)
  Also given by \( F_n = \frac{S_o/N_o}{N_a/N_o} \)
- If the minimum detectable signal \( S_{\text{min}} \) is that value of \( S_o \) corresponding to the minimum ratio of output (IF) signal-to-noise ratio \( (S_o/N_o)_{\text{min}} \) necessary for detection, then

\[
S_{\text{min}} = kT_0 B_o F_n \left( \frac{S_o}{N_a} \right)_{\text{min}}
\]

- And modified Maximum possible range in terms of the IF amplifier output signal to noise ratio and noise figure

\[
R_{\text{max}}^4 = \frac{P_t G A_o \sigma}{(4\pi)^2 k T_0 B_o F_n (S_o/N_o)_{\text{min}}}
\]

- Relationships between Probability of false alarm, Probability of detection and \( S/N \):
  - \( P_{\text{fa}} = 1 / T_{\text{fa}} \cdot B \)
  - \( P_{\text{fa}} = \exp(-V^2 / 2\psi_0) \)
  - \( T_{\text{fa}} = \frac{1}{B_{\text{IF}}} \exp \left( \frac{V^2}{2\psi_0} \right) \)
- **Albersheim** empirical relationship between \( S/N, P_{\text{fa}} \) and \( P_d \):

\[
S/N = A + 0.12AB + 1.7 B
\]

Where \( A = \ln \left( 0.62 / P_{\text{fa}} \right) \) and \( B = \ln \left( P_d / (1-P_d) \right) \)

- The radar equation with \( n \) pulses integrated:

\[
R_{\text{max}}^4 = \frac{P_t G A_o \sigma}{(4\pi)^2 k T_0 B_o F_n (S/N)_n}
\]

- Integration efficiency:

\[
E_i(n) = \frac{(S/N)_1}{n(S/N)_n}
\]

- Final Radar equation including Integration efficiency with \( n \) pulses integrated:
\[ R_{\text{max}}^4 = \frac{P_I G A e \sigma n E_i(n)}{(4\pi)^2 k T_0 B_n F_a (S/N)_i} \]

**Illustrative Problems:**

**Example 1:** A certain Radar has PRF of 1250 pulses per second. What is the maximum unambiguous range? Max. Unambiguous Range is given by

\[ R_{\text{unambig.}} = \frac{C}{2f_p} \]

\[ R_{\text{unambig.}} = 3\times10^5/2\times1250 \text{ mtrs} = 120\times10^3 \text{ mts} = 120 \text{ Kms} \]

**Example 2:** A shipboard radar has 0.9 micro sec transmitted pulse width. Two small boats in the same direction are separated in range by 150 mts. Will the radar detect the two boats as two targets?

*Radar Range Resolution: The range resolution of a Radar is its ability to distinguish two closely spaced targets along the same line of sight (LOS). The Range resolution is a function of the pulse length, where the pulse length \( L_p = c \times \tau/2 \) (Two way range corresponding to the pulse width)*

Radar Range resolution \( = 3\times10^5 \times 0.9 \times 10^{-6}/2 = 135 \text{ mtrs.} \)

Since the boats are at 150 Mts. apart, and the range Resolution is 135mtrs., the radar can detect the 2 boats as 2 separate targets.

**Example 3:** A Pulse Radar transmits a peak power of 1 Mega Watt. It has a PRT equal to 1000 micro sec and the transmitted pulse width is 1 micro sec. Calculate (i) Maximum unambiguous range (ii) Average Power (iii) Duty Cycle (iv) Energy transmitted & (v) Bandwidth

(i) Maximum unambiguous range \( = cT_p/2 = 3\times10^8\times1000\times10^{-6}/2 = 150\times10^3 \text{ mtrs} = 150 \text{ Kms} \)

(ii) Average Power \( = P_p \times \tau / T_p = 1\times10^6 \times 1\times10^{-6} / 1000\times10^{-6} = 1000 \text{ watts} = 1 \text{ kw} \)

(iii) Duty Cycle \( = \tau / T_p = 1\times10^{-6} / 1000\times10^{-6} = 0.001 \)

(iv) Energy transmitted \( = P_p \times \tau \times \text{(Peak power x Time)} = 1\times10^6 \times 1\times10^{-6} = 1 \text{ Joule} \)

(v) Bandwidth \( = 1/\tau = 1/10^{-6} = 1 \text{ Mhz} \)

**Example 4:** The Bandwidth of I.F. Amplifier in a Radar Receiver is 1 MHz. If the Threshold to noise ratio is 12.8 dB determine the False Alarm Time.
Radar Systems 

Lecture Notes (K. Subhas)

Unit 1: Basics of Radar and Radar equation

\[ T_{fa} = \text{False Alarm Time } T_{fa} = \left[ \frac{1}{B_{if}} \right] \exp \frac{V_{t}^2}{2\psi_0} \] where \( B_{if} = 1 \times 10^5 \text{ Hz} \)

Threshold to Noise Ratio = 12.8 dB

\[ \text{i.e. } 10 \log_{10} \left( \frac{V_{t}^2}{2\psi_0} \right) = 12.8 \text{db} \]

\[ \therefore \frac{V_{t}^2}{2\psi_0} = \text{Antilog}_{10} [12.8/10] = 19.05 \]

\[ \therefore T_{fa} = \frac{1}{(1 \times 10^5)} \exp 19.05 = \frac{187633284}{10^5} = 187.6 \text{ Seconds} \]

Example 5: The probability density of the envelope of the noise voltage output is given by the Rayleigh probability-density function

\[ p(R) = \frac{R}{\psi_0} \exp \left( -\frac{R^2}{2\psi_0} \right) \]

where \( R \) is the amplitude of the envelope of the filter output for \( R \geq 0 \). If \( P_{fa} \) needed is \( \leq 10^{-5} \), determine the threshold level.

The probability of false alarm \( P_{fa} \) in terms of the threshold voltage level is given by:

\[ P_{fa} = \exp \left( -\frac{V_{t}^2}{2\psi_0} \right) = 10^{-5} \]

Taking logarithms on both sides we get

\[ -5 \log_{10} = -\frac{V_{t}^2}{2\psi_0} \]

\[ 5 \times 2.3026 = \frac{V_{t}^2}{2\psi_0} \]

\[ V_{t}^2 = 11.5 \times 2 \psi_0 \]

\[ V_{t} = \sqrt{23 \times \sqrt{\psi_0}} = 4.8 \times \sqrt{\psi_0} \]

Example 6: The bandwidth of an IF amplifier is 1 MHz and the average false-alarm time that could be tolerated is 15 min. Find the probability of a false alarm.

The relationship between average false-alarm time \( T_{fa} \), probability of a false alarm \( P_{fa} \) and the IF bandwidth \( B \) is given by:

\[ P_{fa} = \frac{1}{T_{fa} \times B} \]

Substituting \( B = 1 \text{ MHz} \) le \( 10^6 \) and \( T_{fa} = 15 \text{ mnts. i.e. } 900 \text{ secs.} \) we get \( P_{fa} = 1.11 \times 10^{-9} \)

Example 7: What is the ratio of threshold voltage to the rms value of the noise voltage necessary to achieve this false-alarm time?

This is found out using the relationship \( P_{fa} = \exp \left( -\frac{V_{t}^2}{2\psi_0} \right) \)

from which the ratio of Threshold voltage to rms value of the noise voltage is given by

\[ \frac{V_{t}}{\sqrt{\psi_0}} = \sqrt{2 \ln \left( \frac{1}{P_{fa}} \right)} = \sqrt{2 \ln 9 \times 10^8} = 6.45 = 16.2 \text{ dB} \]
Example 8: Typical parameters for a ground-based search radar are: 1. Pulse repetition frequency: 300 Hz, 2. Beam width: 1.5°, and 3. Antenna scan rate: 5 rpm (30°/s). Find out the number of pulses returned from a point target on each scan.
Solution: The number of pulses returned from a point target on each scan is given by:
\[ n_r = \theta_a \cdot f_p / \theta's = \theta_a \cdot f_p / 6 \omega_m \]
Substituting the above values we get: \[ n_r = 1.5 \times 300 / 30 = 15 \]

Questions from Previous Year Examinations:

1.(a) Derive Radar range equation in terms of MDS (minimum detectable signal) (b) What is maximum unambiguous range? How is it related with PRF?

2.(a) Explain the various system losses in a Radar (b) The bandwidth of the IF amplifier in a Radar is 1 Mhz and the threshold noise ratio is 13 db. Determine the false alarm time.

3.(a) Explain the basic principles of Radar and discuss about various parameters which improve the performance of the Radar (b) Discuss about Radar frequencies and list out the Applications of Radars.

4.(a) In a Radar receiver the mean noise voltage is 80 mv and the IF BW is 1 Mhz. If the tolerable false alarm time is 25 mnts., calculate the threshold voltage level and the probability of false alarm. (b) Bring out the advantages of Integration of Radar pulses.

5 (a) Discuss about the factors that influence the prediction of Radar range. (b) Define noise bandwidth of a radar receiver. How does it differ from 3-dB bandwidth? Obtain the expression for minimum detectable signal in terms of noise bandwidth, noise figure and other relevant parameters. [8+8]

6. (a) Write the simplified version of radar range equation and explain how this equation does not adequately describe the performance of practical radar? (b) What are the specific bands assigned by the ITU for the Radar? What are the corresponding frequencies? [8+8]

7. (a) Explain how the Radar is used to measure the range of a target? (b) Draw the block diagram of the pulse radar and explain the function of each block. [8+8]

8. (a) A low power, short range radar is solid-state throughout, including a low-noise RF amplifier which gives it an overall noise figure of 4.77dB. If the antenna diameter is 1m, the IF bandwidth is 500kHz, the operating frequency is 8 GHz and the radar set is supposed to be capable of detecting targets of 5m² cross sectional area at a maximum distance of 12 km, what must be the peak transmitted pulse power? (b) The average false alarm time is a more
significant parameter than the false alarm probability. Give the reasons. (c) Why post detection integration is not as efficient as pre-detection integration of radar pulses? [8+4+4]

9. (a) Explain the functioning and characteristics of PPI display and A-Scope. [8]

10. (a) Explain how the Radar is used to measure the direction and position of target? (b) What are the peak power and duty cycle of a radar whose average transmitter power is 200W, pulse width of 1μs and a pulse repetition frequency of 1000Hz? [8+8]

11. (a) Explain how a threshold level is selected in threshold detection? (b) How to find the number of pulses that returned from a point target as the radar antenna scans through its beam width? (c) Why most of the radar receivers are considered as envelop detectors while calculating the SNR? [6+4+6]

12. (a) Obtain the SNR at the output of IF amplifier of Radar Receiver for a specified probability of detection without exceeding a specified probability of false alarm. (b) Explain how system losses will affect on the Radar Range? [8+8]

13. (a) What are the different range of frequencies that a radar can operate and give their applications? (b) What are the basic functions of radar? In indicating the position of a target, what is the difference between azimuth and elevation? [8+8]

14. (a) Describe how pulse repetition frequency of a Radar system controls the range of it’s detection? (b) Explain how the Transmitted power affects the range. [8+8]

15. (a) Draw the block diagram of a pulsed radar and explain it’s operation. (b) What are the desirable pulse characteristics and the factors that govern them in a Radar system? [10+6]

16. (a) Explain the radar cross section of the sphere. (b) Discuss in brief about pulse repetition frequency and range ambiguities.

17. (a) Define Range resolution and explain the parameters which affect the range resolution. (b) Distinguish between Monostatic and Bistatic Radars (c) Explain RCS of target. [6+5+4]
UNIT-2

CW AND FREQUENCY MODULATED RADAR

- Doppler effect
- CW radar block diagram
- Isolation between Transmitter and receiver
- Nonzero IF receiver
- Receiver Bandwidth requirements
- Applications of CW Radar
- Illustrative problems

FM-CW RADAR

- Introduction
- Range and Doppler Measurement
- Block Diagram and characteristics (Approaching and Receding targets)
- FM-CW Altimeter
- Multiple frequency CW Radar
  - Important Formulae
  - Illustrative Problems
  - Questions from Previous Year Examinations
CW AND FREQUENCY MODULATED RADAR

Doppler Effect:
A technique for separating the received signal from the transmitted signal when there is relative motion between radar and target is based on recognizing the change in the echo-signal frequency caused by the Doppler effect.
It is well known in the fields of optics and acoustics that if either the source of oscillation or the observer of the oscillation is in motion, an apparent shift in frequency will result. This is the **Doppler effect** and is the basis of CW radar. If \( R \) is the distance from the radar to target, the total number of wavelengths \( \lambda \) contained in the two-way path between the radar and the target are \( 2R/\lambda \). The distance \( R \) and the wavelength \( \lambda \) are assumed to be measured in the same units.
Since one wavelength corresponds to an phase angle excursion of \( 2\pi \) radians, the total phase angle excursion \( \phi \) made by the electromagnetic wave during its transit to and from the target is \( 4\pi R/\lambda \) radians. If the target is in motion, \( R \) and the phase \( \phi \) are continually changing. A change in \( \phi \) with respect to time is equal to frequency. This is the Doppler angular frequency \( \omega_d \) and is given by:

\[
\omega_d = 2\pi f_d = \frac{d\phi}{dt} = \frac{d(4\pi R/\lambda)}{dt} = \frac{(4\pi/\lambda) \cdot dR}{dt} = \frac{(4\pi/\lambda) \cdot V_r}{\lambda} = \frac{4\pi V_r}{\lambda}
\]

where \( f_d \) is the Doppler frequency shift in Hz, and \( V_r \) = relative velocity of the target with respect to the Radar. The Doppler frequency shift \( f_d \) is given by

\[
f_d = \frac{2V_r}{\lambda} = 2V_c f_0 / c
\]

where \( f_0 \) is the transmitted frequency and \( c \) is the velocity of propagation of the electromagnetic waves (same as that of light) = \( 3 \times 10^8 \text{ m/s} \). If \( f_0 \) is in hertz, \( V_r \) in knots, and \( \lambda \) in meters then the Doppler frequency \( f_d \) is given by

\[
f_d = 1.03 \frac{V_r}{\lambda}
\]

A plot of this equation is shown in the figure below.

![Doppler frequency vs. Radar frequency and target relative velocity.](image)

*Figure: Doppler frequency \( f_d \) as a function of radar frequency and target relative velocity.*
The relative velocity may be written as \( V_r = V \cdot \cos \theta \) where \( V \) is the target speed and \( \theta \) is angle made by the target trajectory and the line joining radar and target. When \( \theta = 0 \) the Doppler frequency is maximum. The Doppler is zero when the trajectory is perpendicular to the radar line of sight (\( \theta = 90^\circ \)).

The CW radar is of interest not only because of its many applications, but its study also serves as a means for better understanding the nature and use of the Doppler information contained in the echo signal, whether in a CW or a pulse radar (MTI) application. In addition to allowing the received signal to be separated from the transmitted signal, the CW radar provides a measurement of relative velocity which may be used to distinguish moving targets from stationary objects or clutter.

**CW radar:**

Consider the simple CW radar as illustrated by the block diagram of Figure below. The transmitter generates a continuous (unmodulated) oscillation of frequency \( f_0 \), which is radiated by the antenna. A portion of the radiated energy is intercepted by the target and is scattered, some of it in the direction of the radar, where it is collected by the receiving antenna. If the target is in motion with a velocity \( V_r \) relative to the radar, the received signal will be shifted in frequency from the transmitted frequency \( f_0 \) by an amount \( +f_d \) as given by the equation:

\[
 f_d = 2V_r / \lambda = 2V_r f_0 / c.
\]

The plus sign associated with the Doppler frequency applies if the distance between target and radar is decreasing (approaching target) that is, when the received signal frequency is greater than the transmitted signal frequency. The minus sign applies if the distance is increasing (receding target). The received echo signal at a frequency \( f_o +/ - f_d \) enters the radar via the antenna and is heterodyned in the detector (mixer) with a portion of the transmitter signal \( f_0 \) to produce a Doppler beat note of frequency \( f_d \). The sign of \( f_d \) is lost in this process.

The purpose of the Doppler amplifier is to eliminate echoes from stationary targets and to amplify the Doppler echo signal to a level where it can operate an indicating device. It’s frequency response characteristic is shown in the figure (b) below. The low-frequency cutoff must be high enough to reject the d-c component caused by stationary targets, but yet it must be low enough to pass the smallest Doppler frequency expected. Sometimes both conditions cannot be met simultaneously and a compromise is necessary. The upper cutoff frequency is selected to pass the highest Doppler frequency expected.
Isolation between transmitter and receiver:

Isolation between transmitter and receiver is an important aspect to be studied and addressed in simple CW radars where a single antenna serves the purpose of both transmission and reception as described above. The related important aspects are explained below.

- In principle, a single antenna may be employed since the necessary isolation between the transmitted and the received signals is achieved via separation in frequency as a result of the Doppler Effect. In practice, it is not possible to eliminate completely the transmitter leakage. However, transmitter leakage is neither always undesirable. A moderate amount of leakage entering the receiver along with the echo signal supplies the reference necessary for the detection of the Doppler frequency shift. If a leakage signal of sufficient magnitude were not present, a sample of the transmitted signal has to be deliberately introduced into the receiver to provide the necessary reference frequency.

- There are two practical effects which limit the amount of transmitter leakage power which can be tolerated at the receiver. These are:
  1. The maximum amount of power the receiver input circuitry can withstand before it is physically damaged or its sensitivity reduced (burnout) and
The amount of transmitter noise due to hum, microphonics, stray pick-up
&Instability which enters the receiver from the transmitter and affects the
receiver sensitivity.
Hence additional isolation is usually required between the transmitter and the receiver if the
sensitivity is not to be degraded either by burnout or by excessive noise. The amount of
isolation required depends on the transmitter power and the accompanying transmitter noise
as well as the ruggedness and the sensitivity of the receiver. For example, if the safe value of
power which might be applied to a receiver is 10 mW and if the transmitter power is 1 kW, the
isolation between transmitter and receiver must be at least 50 dB.

- The amount of isolation needed in a long-range CW radar is more often determined by
the noise that accompanies the transmitter leakage signal rather than by any damage
cau sed by high power. For example, suppose the isolation between the transmitter and
receiver is such that 10 mW of leakage signal appeared at the receiver. If the minimum
detectable signal is $10^{-18}$ watt (100 dB below 1 mW), the transmitter noise must be at
least 110 dB (preferably 120 or 130 dB) below the transmitted carrier.

- The transmitter noise of concern in Doppler radar includes those noise components that
lie within the same frequency range as the Doppler frequencies. If complete elimination
of the direct leakage signal at the receiver could be achieved, it might not entirely solve
the isolation problem since echoes from nearby fixed targets (clutter) can also contain
the noise components of the transmitted signal.

- The receiver of a pulsed radar is isolated and protected from the damaging effects of
the transmitted pulse by the duplexer, which short-circuits the receiver input during the
transmission period. Turning off the receiver during transmission with a duplexer is not
possible in a CW radar since the transmitter is operated continuously.

- In CW Radars Isolation between transmitter and receiver might be obtained with a
single antenna by using a hybrid junction, circulator, turnstile junction, or with separate
polarizations. Separate antennas for transmitting and receiving might also be used.
  - The amount of isolation which can be readily achieved between the arms of
    practical hybrid junctions such as the magic-T, rat race, or short-slot coupler is of
    the order of 20 to 30 dB. In some instances, when extreme precision is exercised,
    an isolation of perhaps 60 dB or more might be achieved. But one limitation of the
    hybrid junction is the 6-dB loss in overall performance which results from the
    inherent waste of half the transmitted power and half the received signal power.
    Both the loss in performance and the difficulty in obtaining large isolations have
    limited the application of the hybrid junction to short-range radars.
  - Ferrite isolation devices such as the circulator do not suffer the 6-dB loss inherent
    in the hybrid junction. Practical devices have isolation of the order of 20 to 50 dB.
    Turnstile junctions achieve isolations as high as 40 to 60 dB.
  - The use of orthogonal polarizations for transmitting and receiving is limited to
    short range radars because of the relatively small amount of isolation that can be
    obtained.

- An important factor which limits the use of isolation devices with a common antenna is
the reflections produced in the transmission line by the antenna. The reflection
coefficient from a mismatched antenna with a voltage standing-wave ratio $\sigma$ is $1 = (\sigma - 1) / (\sigma + 1)$. Therefore, if an isolation of 20 dB is to be obtained, the VSWR must be less than 1.22. If 40 dB of isolation is required, the VSWR must be less than 1.02.

- The largest isolations are obtained with two antennas: one for transmission, the other for reception—physically separated from one another. Isolations of the order of 80 dB or more are possible with high-gain antennas. The more directive the antenna beam and the greater the spacing between antennas, the greater will be the isolation. A common radome enclosing the two antennas should be avoided since it limits the amount of isolation that can be achieved.

- Additional isolation can be obtained by properly introducing a controlled sample of the transmitted signal directly into the receiver. The phase and amplitude of this “buck-off” signal are adjusted to cancel the portion of the transmitter signal that leaks into the receiver. An additional 10 dB of isolation might be obtained.

- The transmitter signal is never a pure CW waveform. Minute variations in amplitude (AM) and phase (FM) can result in sideband components that fall within the Doppler frequency band. These can generate false targets or mask the desired signals. Therefore, both AM and FM modulations can result in undesired sidebands. AM sidebands are typically 120 dB below the carrier, as measured in a 1 kHz band, and are relatively constant across the usual Doppler spectrum of interest. The normal antenna isolation plus “feed through nulling” usually reduces the AM components below receiver noise in moderate power radars. FM sidebands are usually significantly greater than AM, but decrease with increasing offset from the carrier. These can be avoided by stabilizing the output frequency of the CW transmitter and by feeding back the extracted FM noise components so as to reduce the original frequency deviation.

**Intermediate-frequency receiver:**

**Limitation of Zero IF receiver:**

The receiver in the simple CW radar shown earlier is in some respects analogous to a super heterodyne receiver. Receivers of this type are called homodyne receivers, or super heterodyne receivers with zero IF. The function of the local oscillator is replaced by the leakage signal from the transmitter. Such a receiver is simpler than the one with a more conventional intermediate frequency since no IF amplifier or local oscillator is required. However, this simpler receiver is not very sensitive because of increased noise at the lower intermediate frequencies caused by flicker effect. Flicker-effect noise occurs in semiconductor devices such as diode detectors and cathodes of vacuum tubes. The noise power produced by the flicker effect varies as $1/f^\alpha$ where $\alpha$ is approximately unity. This is in contrast to shot noise or thermal noise, which is independent of frequency. Thus, at the lower range of frequencies (audio or video region), where the Doppler frequencies usually are found, and the detector of the CW receiver can introduce a considerable amount of flicker noise, resulting in reduced receiver sensitivity. For short-range, low-power, applications this decrease in sensitivity might be tolerated since it can be compensated by a modest increase in antenna aperture and/or additional transmitter power.
But for maximum efficiency with CW radar, the reduction insensitivity caused by the simple Doppler receiver with zero IF cannot be tolerated.

**Non zero IF Receiver:**

The effects of flicker noise are overcome in the normal super heterodyne receiver by using an intermediate frequency high enough to make the flicker noise small compared with the normal receiver noise. This results from the inverse frequency dependence of flicker noise. Figure below shows the block diagram of a CW radar whose receiver operates with a nonzero IF. Separate antennas are shown for transmission and reception. Instead of the usual local oscillator found in the conventional super heterodyne receiver, the local oscillator (or reference signal) is derived in the receiver from a portion of the transmitted signal mixed with a locally generated signal of frequency equal to that of the receiver IF. Since the output of the mixer consists of two sidebands on either side of the carrier plus higher harmonics, a narrowband filter selects one of the sidebands as the reference signal. The improvement in receiver sensitivity with an intermediate-frequency super heterodyne might be as much as 30 dB over the simple zero IF receiver discussed earlier.

![Block Diagram](image)

**Figure:** Block diagram of a CW Doppler radar with nonzero IF receiver, also called sideband super heterodyne Receiver.

**Receiver bandwidth requirements:**

One of the requirements of the Doppler-frequency amplifier in the simple CW radar (Zero IF) or the IF amplifier of the sideband super heterodyne (Non Zero IF) is that it has to be wide enough to pass the expected range of Doppler frequencies. In most cases of practical interest the expected range of Doppler frequencies will be much wider than the frequency spectrum.
occupied by the signal energy. Consequently, the use of a wideband amplifier covering the expected Doppler range will result in an increase in noise and a lowering of the receiver sensitivity. If the frequency of the Doppler-shifted echo signal were known beforehand, narrowband filter—that is just wide enough to reduce the excess noise without eliminating a significant amount of signal energy might be used. If the waveforms of the echo signal are known, as well as its carrier frequency, a matched filter could also be considered. Several factors tend to spread the CW signal energy over a finite frequency band. These must be known if an approximation to the bandwidth required for the narrowband Doppler filter is to be obtained.

If the received waveform were a sine wave of infinite duration, its frequency spectrum would be a delta function as shown in the figure (a) below and the receiver bandwidth would be infinitesimal. But a sine wave of infinite duration and an infinitesimal bandwidth cannot occur in nature. The more normal situation is an echo signal which is a sine wave of finite duration. The frequency spectrum of a finite-duration sine wave has a shape of the form \( \sin(n(t-f_0)\delta)/n(t-f_0) \) where \( f_0 \) and \( \delta \) are the frequency and duration of the sine wave, respectively, and \( f \) is the frequency variable over which the spectrum is plotted (Fig b).

![Spectrum](image.png)

**Figure:** Frequency spectrum of CW oscillation of (a) infinite duration and (b) finite duration

Note that this is the same as the spectrum of a pulse of sine wave, the only difference being the relative value of the duration \( \delta \). In many instances, the echo is not a pure sine wave of finite duration but is perturbed by fluctuations in cross section, target accelerations, scanning fluctuations, etc., which tend to broaden the bandwidth still further. Some of these spectrum broadening-effects are considered below.

**Causes for Spectrum broadening:**

- **Spread due to finite time on target:** Assume a CW radar with an antenna beam width of \( \theta_a \) deg. scanning at the rate of \( \theta_s \) deg/s. The time on target (duration of the received signal) is \( \delta = \theta_a/\theta_s \) sec. Thus, the signal is of finite duration and the bandwidth of the receiver must be of the order of the reciprocal of the time on target \( (\theta_s/\theta_a) \). Although this is not an exact relation, it is a good enough approximation for purposes of the
present discussion. If the antenna beam width is $2^0$ and the scanning rate is $36^0/s$ (6 rpm), the spread in the spectrum of the received signal due to the finite time on target would be equal to 18 Hz, independent of the transmitted frequency.

- In addition to the spread of the received signal spectrum caused by the finite time on target, the spectrum gets widened due to target cross section fluctuations. The fluctuations widen the spectrum by modulating the echo signal. The echo signal from a propeller-driven aircraft can also contain modulation components at a frequency proportional to the propeller rotation. The frequency range of propeller modulation depends upon the shaft-rotation speed and the number of propeller blades. It is usually in the vicinity of 50 to 60 Hz for World War 2 aircraft engines. This could be a potential source of difficulty in a CW radar since it might mask the target's Doppler signal or it might cause an erroneous measurement of Doppler frequency. In some instances, propeller modulation can be of advantage. It might permit the detection of propeller-driven aircraft passing on a tangential trajectory, even though the Doppler frequency shift is zero.

- The rotating blades of a helicopter and the compressor stages of a jet engine can also result in a modulation of the echo and a widening of the spectrum that can degrade the performance of a CW Doppler radar.

- If the target's relative velocity is not constant, a further widening of the received signal spectrum occurs. If $a_r$ is the acceleration of the target with respect to the radar, the signal will occupy a bandwidth

$$\Delta f_d = \left( \frac{2a_r}{\lambda} \right)^{1/2}$$

If, for example, $a_r$ is twice the acceleration due to gravity, the receiver bandwidth is approximately 20 Hz when the Radar wavelength is 10 cm.

When the Doppler-shifted echo signal is known to lie somewhere within a relatively wideband of frequencies, a bank of narrowband filters as shown below spaced throughout the frequency range permits a measurement of frequency and improves the signal-to-noise ratio.

- The bandwidth of each individual filter should be wide enough to accept the signal energy, but not so wide as to introduce more noise. The center frequencies of the filters are staggered to cover the entire range of Doppler frequencies.
Figure: (a) Block diagram of IF Doppler filter bank (b) frequency-response characteristic of Doppler filter bank.

- A bank of narrowband filters may be used after the detector in the video of the simple CW radar instead of in the IF. The improvement in signal-to-noise ratio with a video filter bank is not as good as can be obtained with an IF filter bank, but the ability to measure the magnitude of Doppler frequency is still preserved. Because of fold over, a frequency which lies to one side of the IF carrier appears, after detection, at the same video frequency as one which lies an equal amount on the other side of the IF. Therefore the sign of the Doppler shift is lost with a video filter bank, and it cannot be directly determined whether the Doppler frequency corresponds to an approaching or to a receding target. (The sign of the Doppler maybe determined in the video by other means.) One advantage of the fold over in the video is that only half the number of filters are required than in the IF filter bank.

- The equivalent of a bank of contiguous bandpass filters may also be obtained by converting the analog IF or video signal to a set of sampled, quantized signals which are processed with digital circuitry by using Fast Fourier Transform algorithm.

- A bank of overlapping Doppler filters, whether in the IF or video, increases the complexity of the receiver. When the system requirements permit a time sharing of the Doppler frequency range, the bank of Doppler filters may be replaced by a single
narrowband tunable filter which searches in frequency over the band of expected Doppler frequencies until a signal is found.

**Sign of the radial velocity.** In some applications of CW radar it is of interest to know whether the target is approaching or receding. This can be determined with separate filters located on either side of the intermediate frequency. If the echo-signal frequency lies below the carrier, the target is receding. If the echo frequency is greater than the carrier, the target is approaching.

\[ \text{Figure: Spectra of received signals. (a) No Doppler shift, no relative target motion; (b) approaching target; (c) receding target.} \]

Although the Doppler-frequency spectrum "folds over" in the video because of the action of the detector, it is possible to determine its sign from a technique available in single-sideband communications. If the transmitter signal is given by:

\[ E_t = E_0 \cos \omega_0 t \]

The echo signal from a moving target will be:

\[ E_r = k_2 E_0 \cos [(\omega_0 + \omega_d) t + \phi] \]

where

- \( E_0 \) = amplitude of transmitter signal
- \( k_2 \) = a constant determined from the radar equation
- \( \omega_0 \) = angular frequency of transmitter, rad/s
- \( \omega_d \) = Doppler angular frequency shift
- \( \phi \) = a constant phase shift, which depends upon range of initial detection

The sign of the Doppler frequency, and therefore the direction of target motion, may be found by splitting the received signal into two channels as shown in the figure below. In channel A the signal is processed as in the simple CW radar. The received signal and a portion of the transmitter are heterodyned in the detector (mixer) to yield a difference signal

\[ E_A = K_2 E_0 \cos (\pm \omega_d t + \phi) \]

The other channel is similar, except that a 90° phase delay is introduced in the reference signal. The output of the channel B mixer is

\[ E_B = K_2 E_0 \cos (\pm \omega_d t + \phi + \pi/2) \]
Figure: Measurement of Doppler direction using synchronous, two-phase motor.

If the target is approaching (positive Doppler) the outputs from the two channels are

\[ E_A(+) = K_2 E_0 \cos(\omega_d t + \phi) \quad \text{and} \quad E_B(+) = K_2 E_0 \cos(\omega_d t + \phi + \pi/2) \]

On the other hand, if the target is receding (negative Doppler)

\[ E_A(-) = K_2 E_0 \cos(\omega_d t - \phi) \quad \text{and} \quad E_B(-) = K_2 E_0 \cos(\omega_d t - \phi - \pi/2) \]

[Using the trigonometric relation \( \cos(-\omega_d) = \cos \omega_d \)]

The sign of \( \omega_d \) and the direction of the target's motion may be determined according to whether the output of channel B leads or lags the output of channel A. One method of determining the relative phase relationship between the two channels is to apply the outputs to a synchronous two-phase motor. The direction of motor rotation is an indication of the direction of the target motion. Electronic methods may be used instead of a synchronous motor to sense the relative phase of the two channels.

Applications of CW radar:

- Measurement of the relative velocity of a moving target, as in the police speed monitor or in the rate-of-climb meter for vertical-take-off aircraft.
- Control of traffic lights, regulation of tollbooths, vehicle counting.
- As a sensor in antilock braking systems, and for collision avoidance.
• In railways, as a speedometer to replace the conventional axle-driven tachometer. In such an application it would be unaffected by errors caused by wheel slip on accelerating or wheel slide when braking.

• Monitoring the docking speed of large ships.

• Measurement of the velocity of missiles, ammunition, and baseballs.

The principal advantage of CW Doppler radar over the other (non radar) methods of measuring speed is that there need not be any physical contact with the object whose speed is being measured. In industry this is used to measure turbine-blade vibration, the peripheral speed of grinding wheels, and the monitoring of vibrations in the cables of suspension bridges.

• Most of the above applications can be satisfied with a simple, solid-state CW source with powers in tens of milliwatts.

• High-power CW radars for the detection of aircraft and other targets have been developed and have been used in such systems as the Hawk missile systems. (Shown below)

• The difficulty of eliminating the leakage of the transmitter signals into the receiver has limited the utility of unmodulated CW radar for many long-range applications.

• The CW radar, when used for short or moderate ranges, is characterized by simpler equipment than a pulse radar. The amount of power that can be used with a CW radar is dependent on the isolation that can be achieved between the transmitter and receiver since the transmitter noise that finds its way into the receiver limits the receiver sensitivity. (The pulse radar has no similar limitation to its maximum range because the transmitter is not operative when the receiver is turned on.)

• Major disadvantage of the simple CW radar is its inability to obtain a measurement of range. This limitation can be overcome by modulating the CW carrier, as in the frequency-modulated radar.

• Some anti-air-warfare guided missile systems employ semi active homing guidance in which a receiver in the missile receives energy from the target, the energy having been transmitted from an "illuminator" external to the missile. The illuminator will be at the launch platform. CW illumination has been used in many successful systems. An example is the Hawk tracking illuminator shown in the figure below. It is tracking radar as well as an illuminator since it must be able to follow the target as it travels through space.

CW radar allows operation in the presence of clutter and has been well suited for low altitude missile defense systems. A block diagram of a CW tracking illuminator is shown in the figure above. Note that following the wide-band Doppler amplifier is a speed gate, which is a narrow-band tracking filter that acquires the targets Doppler and tracks its changing Doppler frequency shift.
FM-CW RADAR

Introduction:

The inability of the simple CW radar to measure range is mainly due to the lack of a Timing mark. The timing mark permits the time of transmission and the time of return to be recognized but it increases the spectrum of the transmitted waveform. The sharper or more distinct the mark, the more accurate the measurement of the transit time. But the more distinct the timing mark, the broader will be the transmitted spectrum. This follows from the properties of the Fourier transform. Therefore a finite spectrum of necessity must be transmitted if transit time or range is to be measured. The spectrum of a CW transmission can be broadened by the application of a modulation - amplitude, frequency, or phase. An example of an amplitude modulation is the pulse radar. A widely used technique to insert a timing mark is to frequency-modulate the carrier. The timing mark is the changing frequency. The transit time is proportional to the difference in frequency between the transmitter signal and the echo signal. The greater the transmitter frequency deviation in a given time interval, the more accurate is the measurement of the transit time but the transmitted spectrum also becomes larger.
Range and Doppler measurement:

In the frequency-modulated CW radar (abbreviated FM-CW), the transmitter frequency is changed as a function of time in a known manner. Assume that the transmitter frequency increases linearly with time, as shown by the solid line in the figure below.

![Diagram showing frequency-time relationships in FM-CW radar. Solid curve represents transmitted signal; dashed curve represents echo. (a) Linear frequency modulation (b) triangular frequency modulation (c) beat note of (b).]

If there is a reflecting object at a distance $R$, the echo signal will return after a time $T = 2R/c$. The dashed line in the figure represents the echo signal. When the echo signal is heterodyned with a portion of the transmitter signal in a nonlinear element such as a diode, a beat note $f_b$ will be produced. If there is no Doppler frequency shift, the beat note (difference frequency) is a measure of the target’s range and $f_b = f_r$, where $f_r$ is the beat frequency only due to the target’s range. If the rate of change of the carrier frequency is $\dot{f}_0$ (dot) then the beat frequency is given by:

$$f_r = \dot{f}_0 \cdot T = \frac{2R}{c} \cdot \dot{f}_0$$

In any practical CW radar, the frequency cannot be continually changed in one direction only. Periodicity in the modulation is necessary, as in the triangular frequency-modulation waveform shown in fig.b. The modulation need not necessarily be triangular. It can be saw tooth, sinusoidal, or some other shape. The resulting beat frequency as a function of time is shown in fig.c for triangular modulation. The beat note is of constant frequency except at the turnaround region. If a frequency change of $\Delta f$ is modulated at a rate $f_m$, then the beat frequency is

$$f_r = \frac{(2R/c) \cdot 2f_m \cdot \Delta f}{c} = 4Rf_m \cdot \Delta f / c$$
Or \( R = \frac{c f_r}{4f_{m,\Delta f}} \) ........[Eq.1]

Thus the measurement of the beat frequency determines the range \( R \).

A block diagram illustrating the principle of the FM-CW radar is shown in the figure below. A portion of the transmitter signal acts as the reference signal required to produce the beat frequency. It is introduced directly into the receiver via a cable or other direct connection.

![Block diagram of FM-CW radar]

**Figure: Block diagram of FM-CW radar**

Ideally the isolation between transmitting and receiving antennas is made sufficiently large so as to reduce to a negligible level the transmitter leakage signal which arrives at the receiver via the coupling between antennas. The beat frequency is amplified and limited to remove any amplitude fluctuations. The frequency of the amplitude-limited beat note is measured with a cycle-counting frequency meter calibrated in distance.

In the above, the target was assumed to be stationary. If this assumption is not applicable, a Doppler frequency shift will be superimposed on the FM range beat note and an erroneous range measurement results. The Doppler frequency shift causes the frequency-time plot of the echo signal to be shifted up or down as shown in the figure (a). On one portion of the frequency-modulation cycle the beat frequency (fig. b) is increased by the Doppler shift, while on the other portion, it is decreased.
Figure: Frequency-time relationships in FM-CW radar when the received signal is shifted in frequency by the Doppler effect (a) Transmitted (solid curve) and echo (dashed curve) (b) beat frequency

If for example, the target is approaching the radar, the beat frequency $f_b\text{(up)}$ produced during the increasing or up portion of the FM cycle will be the difference between the beat frequency due to the range $f_r$ and the Doppler frequency shift $f_d$. Similarly, on the decreasing portion, the beat frequency $f_b\text{(down)}$ is the sum of the two.

\[
\begin{align*}
    f_b\text{(up)} &= f_r - f_d \\
    f_b\text{(down)} &= f_r + f_d
\end{align*}
\]

The range frequency $f_r$ may be extracted by measuring the average beat frequency; that is, \(\frac{1}{2}(f_b\text{(up)} + f_b\text{(down)}) = f_r\). If $f_b\text{(up)}$ and $f_b\text{(down)}$ are measured separately, for example, by switching a frequency counter every half modulation cycle, one-half the difference between the frequencies will yield the Doppler frequency. This assumes $f_r > f_d$. If, on the other hand, $f_r < f_d$ such as might occur with a high-speed target at short range, the roles of the averaging and the difference-frequency measurements are reversed; the averaging meter will measure Doppler velocities, and the difference meter measures range.

If the FM-CW radar is used for single targets only, such as in the radio altimeter, it is not necessary to employ a linear modulation waveform. This is certainly advantageous since a sinusoidal or almost sinusoidal frequency modulation is easier to obtain with practical equipment than are linear modulations. The beat frequency obtained with sinusoidal modulation is not constant over the modulation cycle as it is with linear modulation. However,
It may be shown that the average beat frequency measured over a modulation cycle, when substituted into Eq. (1) yields the correct value of target range. If the target is in motion and the beat signal contains a component due to the Doppler frequency shift, the range frequency can be extracted, as before, if the average frequency is measured. To extract the Doppler frequency, the modulation waveform must have equal upsweep and down sweep time intervals.

**FM-CW Altimeter:**

The FM-CW radar principle is used in the aircraft radio altimeter to measure height above the surface of the earth. The large backscatter cross section and the relatively short ranges required of altimeters permit low transmitter power and low antenna gain. Since the relative motion between the aircraft and ground is small, the effect of the Doppler frequency shift also may usually be neglected. The band from 4.2 to 4.4GHz is reserved for radio altimeters, although they have in the past operated at UHF. The transmitter power is relatively low and can be obtained from a CW Magnetron, a backward-wave oscillator, or a reflex klystron, but now they have been replaced by the solid state transmitter. The altimeter can employ a simple homodyne receiver, but for better sensitivity and stability the super heterodyne is preferred whenever its more complex construction can be tolerated. The block diagram of the FM-CW radar with a sideband super heterodyne receiver is shown in the figure below.

**Figure: Block diagram of a FM-CW radar using sideband super heterodyne receiver**

A portion of the frequency-modulated transmitted signal is applied to a mixer along with the oscillator signal. The selection of the local-oscillator frequency is a bit different from that in the usual super heterodyne receiver. The local-oscillator frequency $f_o$ is the same as the intermediate frequency used in
the receiver, whereas in the conventional super heterodyne the LO frequency is of the same order of frequency as the RF signal.

The output of the mixer consists of the varying transmitter frequency \( f_t(t) \) plus two sideband frequencies, one on either side of \( f_t(t) \) and separated from \( f_t(t) \) by the local-oscillator frequency \( f_o \). The filter selects the lower sideband, \( f_o(t) - f_c \) and rejects the carrier and the upper sideband. The side band that is passed by the filter is modulated in the same fashion as the transmitted signal. The sideband filter must have sufficient bandwidth to pass the modulation, but not the carrier or other sideband. The filtered sideband serves the function of the local oscillator.

When an echo signal is present, the output of the receiver mixer is an IF signal of frequency \( (f_o+f_o) \)

where \( f_o \) is composed of the range frequency \( f_o \) and the Doppler velocity frequency \( f_o \). The IF signal is amplified and applied to the balanced detector along with the local-oscillator signal \( f_o \). The output of the detector contains the beat frequency (range frequency and the Doppler velocity frequency), which is amplified to a level where it can actuate the frequency-measuring circuits.

In the above figure, the output of the low-frequency amplifier is divided into two channels: one feeds an average-frequency counter to determine the range, and the other feeds a switched frequency counter to determine the Doppler velocity (assuming \( f_o \)). Only the averaging frequency counter need be used in an altimeter application, since the rate of change of altitude is usually small.

A target at short range will generally result in a strong signal at low frequency, while one at long range will result in a weak signal at high frequency. Therefore the frequency characteristic of the low frequency amplifier in the FM-CW radar may be shaped to provide attenuation at the low frequencies corresponding to short ranges and large echo signals. Less attenuation is applied to the higher frequencies, where the echo signals are weaker.

**Multiple-frequency CW Radar:**

Although it was indicated earlier that CW radar can not measure range, it is possible under some circumstances to do so by measuring the phase of the echo signal relative to the phase of the transmitted signal. Consider a CW radar radiating a single-frequency sine wave of the form \( \sin 2\pi f_o t \) (The amplitude of the signal is taken to be unity since it does not influence the result) the signal travels to the target at a range \( R \) and returns to the radar after a time \( T = 2R/c \) where \( c \) is the velocity of propagation. The echo signal received at the radar is \( \sin 2\pi f_o (t - T) \). If the transmitted and received signals are compared in a phase detector, the output is proportional to the phase difference between the two and is given by:

\[
\Delta \phi = 2\pi f_o T = 4\pi f_o R/c.
\]

The phase difference may therefore be used as a measure of the range, or

\[
R = \frac{c}{4\pi f_o} \frac{\Delta \phi}{4\pi} = \frac{\lambda}{4\pi} \Delta \phi
\]

[Eq. 2]

However, the measurement of the phase difference \( \Delta \phi \) is unambiguous only if \( \Delta \phi \) does not exceed \( 2\pi \) radians. Substituting \( \Delta \phi = 2\pi \) into the above equation (Eq.1) gives the maximum unambiguous range as \( \lambda/2 \). At radar frequencies this unambiguous range is much too small to be of any practical interest.

Unambiguous range may be extended considerably by utilizing two separate CW signals differing slightly in frequency. The unambiguous range in this case corresponds to half wavelength at the difference frequency.
The transmitted waveform is assumed to consist of two continuous sine waves of frequency $f_1$ and $f_2$, separated by an amount $\Delta f$. For convenience, the amplitudes of all signals are set equal to unity. The voltage waveforms of the two components of the transmitted signal $v_{1T}$ and $v_{2T}$ may be written as

$$v_{1T} = \sin \left(2\pi f_1 t + \phi_1\right)$$

$$v_{2T} = \sin \left(2\pi f_2 t + \phi_2\right)$$

where $\phi_1$ and $\phi_2$ are arbitrary (constant) phase angles. The echo signal is shifted in frequency by the Doppler Effect. The form of the Doppler-shifted signals corresponding to the two frequencies $f_1$ and $f_2$ are:

$$v_{1R} = \sin \left[2\pi f_1 t \pm \frac{4\pi f_1 R_0}{c} + \phi_1\right]$$

$$v_{2R} = \sin \left[2\pi f_2 t \pm \frac{4\pi f_2 R_0}{c} + \phi_2\right]$$

Where $R_o$ = range to target at a particular time $t = t_0$(range that would be measured if target were not moving)

$f_{dl}$ = Doppler frequency shift associated with frequency $f_1$

$f_{d2}$ = Doppler frequency shift associated with frequency $f_2$

Since the two RF frequencies $f_1$ and $f_2$ are approximately the same (that is $f_2 = f_1 + \Delta f$, where $\Delta f << f_1$) the Doppler frequency shifts $f_{dl}$ and $f_{d2}$ can be assumed to be equal to each other. Therefore we may write $f_{dl} = f_{d2} = f_d$

The receiver separates the two components of the echo signal and heterodynes each received signal component with the corresponding transmitted waveform and extracts the two Doppler-frequency components given below:

$$v_{1D} = \sin \left(\pm 2\pi f_d t - \frac{4\pi f_1 R_0}{c}\right)$$

$$v_{2D} = \sin \left(\pm 2\pi f_d t - \frac{4\pi f_2 R_0}{c}\right)$$

The phase difference between these two components is

$$\Delta \phi = \frac{4\pi (f_2 - f_1) R_0}{c} = \frac{4\pi \Delta f R_0}{c}$$

Hence

$$R_0 = \frac{c \Delta \phi}{4\pi \Delta f}$$

which is same as that of Eq..2, with $\Delta f$ substituted in place of $f_o$. 

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Important aspects of Multi Frequency Radar:

- The two frequencies of the two-frequency radar were described as being transmitted simultaneously. They may also be transmitted sequentially in some applications by rapidly switching a single RF source.

- A large difference in frequency between the two transmitted signals improves the accuracy of the range measurement since large Δf means a proportionately large change in Δφ for a given range. However, there is a limit to the value of Δf since Δφ cannot be greater than 2π radians if the range is to remain unambiguous. The maximum unambiguous range $R_{\text{unamb}}$ is

$$R_{\text{unamb}} = \frac{c}{2 \Delta f}$$

Therefore Δf must be less than $c/2R_{\text{unamb}}$. Note that when Δf is replaced by the pulse repetition rate, the above equation gives the maximum unambiguous range of a pulse radar.

- A qualitative explanation of the operation of the two-frequency radar may be had by considering both carrier frequencies to be in phase at zero range. As they progress outward from the radar, the relative phase between the two increases because of their difference in frequency. This phase difference may be used as a measure of the elapsed time. When the two signals slip in phase by one cycle, the measurement of phase, and hence range, becomes ambiguous.

- The two-frequency CW radar is essentially a single-target radar since only one phase difference can be measured at a time. If more than one target is present, the echo signal becomes complicated and the meaning of the phase measurement becomes doubtful.

- The theoretical rms range error with which range can be measured with the two-frequency CW radar was estimated to be

$$\delta R = \frac{c}{4\pi \Delta f (2E/N_0)^{1/2}}$$

Where $E$ = energy contained in received signal and $N_0$ = noise power per hertz of bandwidth.

The above Equation indicates that the greater the separation Δf between the two frequencies, the lesser will be the rms error.

- However if the frequency difference Δf increases unambiguous Range decreases. The selection of Δf represents a compromise between the requirements of accuracy and ambiguity. Both accurate and unambiguous range measurements can be made by transmitting three or more frequencies instead of just two.

For example, if the three frequencies $f_0, f_2$, and $f_4$ are such that $f_2 - f_4 = k(f_2 - f_0)$ where $k$ is a factor of the order of 10 or 20, the pair of frequencies $f_0, f_2$ (with greater Δf) gives an ambiguous but accurate range measurement while the pair of frequencies $f_2, f_4$ (with lesser Δf) resolve the ambiguities in the measurement of range. Likewise, further accuracy improvement with out reducing the ambiguous range can be obtained by adding more frequencies. As more frequencies are added the spectrum and target resolution approach that obtained with a pulse or an FM-CW waveform.
Important Formulae:

- Relation between Relative velocity \( V_r \) and Doppler frequency \( f_d \):
  \[ f_d = \frac{2V_r}{\lambda} = 2V_r f_0 / c \]

- Relation between reflection coefficient \( \sigma \) and VSWR \( \alpha \):
  \[ |\sigma| = \frac{\alpha-1}{\alpha+1} \]

- Change in Doppler frequency due to target’s acceleration:
  \[ \Delta f_d = \left( \frac{2\alpha_r}{\lambda} \right)^{1/2} \]

- In a FM CW Radar:
  - Target’s Range velocity \( f_r \) is given by (Assuming there is no Doppler shift):
    \[ f_r = 4RF_m \Delta f / c \]
    Where \( f_m \) = modulating frequency and \( \Delta f \) = frequency swing
  - Target’s Range velocity \( f_r \) and Doppler frequency \( f_d \) are given by (with Doppler shift for Approaching target):
    \[ f_r = \frac{1}{2}[f_b(up) + f_b(down)] \quad \text{and} \quad f_d = \frac{1}{2}[f_b(down) - f_b(up)] \]
    where:
    \[ f_b(up) = f_r - f_d \]
    \[ f_b(down) = f_r + f_d \]

Illustrative problems:

Example 1: Determine the Range and Doppler velocity of an approaching target using a triangular modulation FMCW Radar. Given: Beat frequency \( f_b(up) = 15 \text{KHz} \) and \( f_b(down) = 25 \text{KHz} \), modulating frequency \( 1 \text{MHz} \), \( \Delta f \) : 1KHz and Operating frequency : 3Ghz

Solution:
We know \[ f_r = \frac{1}{2}[f_b(up)+ f_b(down)] = \frac{1}{2}(15+25) = 20 \text{ KHz} \]

\[ f_d = \frac{1}{2}[f_b(down) - f_b(up)] = \frac{1}{2}(25-15) = 5 \text{ KHz} \]

The Range \( R \) in terms of \( f_r \), \( f_m \) and \( \Delta f \) is given by:
\[ R = \frac{c f_r}{4f_m \Delta f} \]

\[ = \frac{(3\times10^8)20	imes10^3}{4(10^8)(1\times10^8)} \text{ mtrs} = 1500 \text{ mtrs} = 1.5 \text{ Kms} \]

Example 2: What should be the VSWR of a mismatched antenna if an isolation of 20 dB is to be obtained between the receiver and the transmitter in a CW Radar using a common antenna.

Solution: Isolation 20 db corresponds to a reflection coefficient of 0.1
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[Since $20 \log_{10}(1/\lambda) = 20$, $\log_{10}(1/\lambda) = 1$, $1/\lambda = 10$ and $0.1]$

From the Relation between reflection coefficient and VSWR $\sigma : \lambda = (\sigma - 1)/(\sigma + 1)$ we can get

$\sigma = (1 + \lambda)/(1 - \lambda)$ and using the value of the reflection coefficient of $0.1$ in this relation we get

$VSWR = (1+0.1)/(1-0.1) = 1.1/0.9 = 1.22$

Questions from Previous Year Examinations:

1. The transmitter power is 1 KW and safe value of power which might be applied to a receiver is 10mW. Find the isolation between transmitter and receiver in dB. Suggest the appropriate isolator.

2. (a) What is the Doppler effect? What are some of the ways in which it manifests itself? What are its radar applications?
   (b) What is the relation between bandwidth and the acceleration of the target with respect to radar?

3. (a) How to find the target speed from Doppler frequency?
   (b) Write the applications of CW Radar.
   (c) What are the factors that limit the amount of isolation between Transmitter and Receiver of CW Radar?

4. (a) Explain the operation of the two frequency CW Radar.
   (b) How to select the difference between the two transmitted signals of CW radar?

5. (a) With the help of a suitable block diagram explain the operation of a CW Radar with non-zero IF amplifier
   (b) List down and explain the applications of CW radar

6. (a) Draw the block diagram of a FMCW Radar using side band super heterodyne receiver and explain it’s operation.
   (b) With a transmit (CW) frequency of 5GHz, calculate the Doppler frequency seen by a Stationary Radar when the target radial velocity is 100 km/h (62.5 mph)
UNIT- 3
MTI AND PULSE DOPPLER RADAR

• Introduction
• Principle of Operation
• MTI Radar with Power Amplifier Transmitter and Power Oscillator Transmitter
• Delay Line Cancellers- Filter Characteristics
• Blind Speeds
• Double Cancellations
• Staggered PRFs
• Range Gated Doppler Filters
• MTI Radar Parameters
• Limitations to MTI Performance
• MTI vs. Pulse Doppler Radar
  • Previous Years’ Examination Questions
MTI AND PULSE DOPPLER RADAR

Introduction:

The Doppler frequency shift \( f_d = \frac{2V}{\lambda} \) produced by a moving target may be used in a pulse radar just as in the CW radar, to determine the relative velocity of a target or to separate desired moving targets from undesired stationary objects (clutter). Although there are applications of pulse radar where a determination of the target’s relative velocity is made from the Doppler frequency shift, the use of Doppler to separate small moving targets in the presence of large clutter has been of greater interest. Such a pulse radar that utilizes the Doppler frequency shift as a means of discriminating moving targets from fixed targets is called a MTI (moving target indication) or a pulse Doppler radar. The two are based on the same physical principle, but in practice there are differences between MTI and Pulse Doppler radar:

- The MTI radar, usually operates with ambiguous Doppler measurement (so-called blind speeds) but with unambiguous range measurement (no second-time around echoes).
- A pulse Doppler radar operates with ambiguous range measurement but with unambiguous Doppler measurement. Its pulse repetition frequency is usually high enough to operate with unambiguous Doppler (no Blind speeds) but at the expense of range ambiguities.

The discussion in this chapter mostly is based on the MTI Radar, but much of what applies to MTI can be extended to Pulse Doppler Radar as well.

Salient Features of MTI:

- MTI is a necessity in high-quality air-surveillance radars that operate in the presence of clutter.
- Its design is more challenging than that of a simple pulse radar or a simple CW radar.
- A MTI capability adds to a radar's cost and complexity and often system designers must accept compromises they might not wish to.
- The basic MTI concepts were introduced during World War 2, and most of the signal processing theory on which MTI (and pulse Doppler) radar depends was formulated during the mid-1950s.
- However, the implementation of theory to practice was speeded up only subsequently after the availability of the necessary signal-processing technology.
- It took almost twenty years for the full capabilities offered by MTI signal-processing theory to be converted into practical and economical Radar equipment. The chief factor that made this possible was the development of reliable, small, and inexpensive digital processing hardware.

Principle of operation:

A simple CW radar studied earlier is shown in Fig.1 (a). In principle, the CW radar may be converted into a pulse radar as shown in Fig.1 (b) by providing a power amplifier and a modulator to turn the amplifier on and off for the purpose of generating pulses. The chief difference between the pulse radar of Fig. 1(b) and the one studied earlier is that a small portion of the CW oscillator power that generates the transmitted pulses is diverted to the receiver to take the place of the local oscillator. However, this CW signal does more than function as a replacement for the local oscillator. It acts as the coherent reference needed to detect the Doppler frequency shift. By coherent it means that the phase of the transmitted signal is preserved in the reference signal. The reference signal is the distinguishing feature of coherent MTI radar.
If the CW oscillator voltage is represented as $A_2 \sin 2\pi f_t t$ where $A_2$ = amplitude and $f_t$ the carrier frequency

- Then the reference signal is:
  $$V_{ref} = A_2 \sin 2\pi f_t t$$  \hspace{1cm} \text{(1)}

- And the Doppler-shifted echo-signal voltage is
  $$V_{echo} = A_3 \sin \left[ 2\pi (f_t \pm f_d) t - \frac{4\pi f_t R_0}{c} \right]$$  \hspace{1cm} \text{(2)}

Where $A_3$ = amplitude of reference signal

$A_3$ = amplitude of signal received from a target at a range $R_0$

$f_d$ = Doppler frequency shift

$t$ = time

$c$ = velocity of propagation

- The reference signal and the target echo signal are heterodyned in the mixer stage of the receiver. Only the low-frequency (difference-frequency) component from the mixer is of interest and is a voltage given by:
  $$V_{diff} = A_4 \sin \left( 2\pi f_d t - \frac{4\pi f_t R_0}{c} \right)$$  \hspace{1cm} \text{(3)}

---

**Figure 1:** (a) Simple CW Radar (b) Pulse Radar using Doppler Information

Note that the equations (1) to (3) above represent sine wave carriers upon which the pulse modulation is imposed. The difference frequency is equal to the Doppler frequency $f_d$. For stationary targets the Doppler frequency shift will be zero and hence $V_{diff}$ will not vary with time and may take on any constant...
value from $+A_t$ to $-A_t$ including zero. However, when the target is in motion relative to the radar, $f_d$ has a value other than zero and the voltage corresponding to the difference frequency from the mixer [Eq. (3)] will be a function of time.

- An example of the output from the mixer when the Doppler frequency $f_d$ is large compared with the reciprocal of the pulse width is shown in Fig. 2(b). The Doppler signal may be readily discerned from the information contained in a single pulse.
- If, on the other hand $f_d$ is small compared with the reciprocal of the pulse duration, the pulses will be modulated with an amplitude given by Eq. (4.3) [Fig. 2(c)] and many pulses will be needed to extract the Doppler information.
- The case illustrated in Fig. 2(c) is more typical of aircraft-detection radar, while the waveform of Fig. 2(b) might be more applicable to a radar used for the detection of extraterrestrial targets such as ballistic missiles or satellites.
- Ambiguities in the measurement of Doppler frequency can occur in the case of the discontinuous measurement of Fig. 2(c) but not when the measurement is made on the basis of a single pulse.
- The video signals shown in Fig. 2 are called bipolar, since they contain both positive and negative amplitudes.

\[ \text{(a)} \]
\[ \text{(b)} \]
\[ \text{(c)} \]

*Figure 2 (a) RF echo pulse train (b) video pulse train for Doppler frequency $f_d > 1/\tau$ (c) video pulse train for Doppler frequency $f_d < 1/\tau$.\]

Moving targets may be distinguished from stationary targets by observing the video output on an A-scope (amplitude vs. range). A single sweep on an A-scope might appear as in Fig. 3(a) shown below. This sweep shows several fixed targets and two moving targets indicated by the two arrows. On the basis of a single sweep, moving targets cannot be distinguished from fixed targets.
Figure 3 (a-e) Successive sweeps of a MTI radar A-scope display (echo amplitude as a function of time) (f) Superposition of many sweeps: arrows indicate position of moving targets.

Successive A-scope sweeps (pulse-repetition intervals) are shown in Fig. 3 (a) to (e). Echoes from fixed targets remain constant throughout, but echoes from moving targets vary in amplitude from sweep to sweep at a rate corresponding to the Doppler frequency. The superposition of the successive A-scope sweeps is shown in Fig. 3(f). The moving targets produce, with time, a "butterfly" effect on the A-scope.

Concept of delay-line canceller:

Although the butterfly effect is suitable for recognizing moving targets on an A-scope, it is not appropriate for display on the PPI. One method commonly employed to extract Doppler information in a form suitable for display on the PPI scope is with a delay-line canceller as shown in the Fig. 4 below.

![Figure 4: MTI Receiver with delay-line canceller](image)

The delay-line canceller acts as a filter to eliminate the d-c component of fixed targets and to pass the a-c components of moving targets. The video portion of the receiver is divided into two channels. One is a normal video channel. In the other, the video signal experiences a time delay equal to one pulse-repetition period (equal to the reciprocal of the pulse repetition frequency). The outputs from the two
channels are subtracted from one another. The fixed targets with unchanging amplitudes from pulse to pulse are canceled on subtraction. However, the amplitudes of the moving-target echoes are not constant from pulse to pulse and subtraction results in an uncanceled residue. The output of the subtraction circuit is a bipolar video just as was the input. Before bipolar video can intensity-modulate a PPI display it must be converted into unipotential voltages (unipolar video) by a full-wave rectifier.

**MTI Radar with Power Amplifier Transmitter:**

The simple MTI radar shown in Fig. 1(b) is not the most typical. The block diagram of a more common MTI radar employing a power amplifier is shown in the Fig. 5 below. The significant difference between this MTI configuration and that of Fig. 1(b) is the manner in which the reference signal is generated. In Fig. 5, the coherent reference is supplied by an oscillator called the coho, which stands for coherent oscillator. The coho is a stable oscillator whose frequency is the same as the intermediate frequency used in the receiver. In addition to providing the reference signal, the output of the coho is also mixed with the local-oscillator frequency \( f \). The local oscillator also must be a stable oscillator and is called stalo, for stable local oscillator. The RF echo signal is heterodyned with the stalo signal to produce the IF just as in the conventional super heterodyne receiver. The stalo, coho and the mixer in which they are mixed are called Receiver-Exciter because of the dual role they serve both the receiver and the transmitter.

![Figure 5](image)

**Figure 5:** Block diagram of MTI radar with power-amplifier transmitter.

The characteristic feature of coherent MTI radar is that the transmitted signal must be coherent (in phase) with the reference signal in the receiver. This is accomplished in the radar system shown in Fig. 5 by generating the transmitted signal from the coho reference signal. The function of the stalo is to provide the necessary frequency translation from the IF to the transmitted (RF) frequency. Although the phase of the stalo influences the phase of the transmitted signal, any stalo phase shift is canceled on
reception because the stalo that generates the transmitted signal also acts as the local oscillator in the receiver. The reference signal from the coho and the IF echo signal are both fed into a mixer called the Phase detector. The phase detector differs from the normal amplitude detector since its output is proportional to the phase difference between the two input signals.

Any one of a number of transmitting-tube types might be used as the power amplifier. These include the triode, tetrode, klystron, traveling-wave tube, and the crossed-field amplifier.

A transmitter which consists of a stable low-power oscillator followed by a power amplifier is sometimes called MOPA, which stands for Master-Oscillator Power Amplifier.

**MTI radar with power-oscillator transmitter:**

Before the development of the klystron amplifier, the only high-power transmitter available at microwave frequencies for radar application was the magnetron oscillator. In an oscillator, the phase of the RF bears no relationship from pulse to pulse. For this reason, the reference signal cannot be generated by a continuously running oscillator. However, a coherent reference signal may be readily obtained with the power oscillator by readjusting the phase of the coho at the beginning of each sweep according to the phase of the transmitted pulse. The phase of the coho is locked to the phase of the transmitted pulse each time a pulse is generated.

![Block diagram of MTI radar with power-oscillator transmitter](diagram)

**Delay Line Cancellers:**

The simple MTI delay-line canceller showed in Fig. 4 is an example of a time-domain filter. The capability of this device depends on the quality of the medium used in the delay line. The Pulse modulator delay line must introduce a time delay equal to the pulse repetition interval. For typical ground-based air surveillance radars this will be several milliseconds. Delay times of this magnitude cannot be achieved
with practical electromagnetic transmission lines. By converting the electromagnetic signal to an acoustic signal, it is possible to utilize delay lines of a reasonable physical length since the velocity of propagation of acoustic waves is about $10^6$ that of electromagnetic waves. After the necessary delay is introduced by the acoustic line, the signal is converted back to an electromagnetic signal for further processing. The early acoustic delay lines developed during World War 2 used liquid delay lines filled with either water or mercury. Liquid delay lines were large and inconvenient to use. They were replaced in the mid-1950s by the solid fused-quartz delay line that used multiple internal reflections to obtain a compact device. These analog acoustic delay lines were, in turn replaced in the early 1970s by storage devices based on digital computer technology. The use of digital delay lines requires that the output of the MTI receiver phase-detector be quantized into a sequence of digital words. The compactness and convenience of digital processing allows the implementation of more complex delay-line cancellers with filter characteristics not practical with analog methods. One of the advantages of a time-domain delay-line canceller as compared to the more conventional frequency-domain filter is that a single network operates at all ranges and does not require a separate filter for each range resolution cell. Frequency-domain Doppler filter banks are of interest in some forms of MTI and Pulse-Doppler radar.

**Filter Characteristics of the Delay Line Canceller:**

The delay-line canceller acts as a filter which rejects the d-c component of clutter. Because of its periodic nature, the filter also rejects energy in the vicinity of the pulse repetition frequency and its harmonics. The video signal of Eq. (3) received from a particular target at a range $R_o$ is

$$V_t = k \sin(2\pi f_d t - \phi_0)$$

where $\phi_0$ = phase shift and $k$ = amplitude of video signal. The signal from the previous transmission, which is delayed by a time $T$ = pulse repetition interval, is

$$V_2 = k \sin(2\pi f_d (t - T) - \phi_0)$$

Everything else is assumed to remain essentially constant over the interval $T$ so that $k$ is the same for both pulses. The output from the subtractor is

$$V = V_t - V_2 = 2k \sin(\pi f_d T) \cos[2\pi f_d (t - T/2) - \phi_0]$$

It is assumed that the gain through the delay-line canceller is unity. The output from the canceller Eq. (6) consists of a cosine wave at the Doppler frequency & with an amplitude $2k \sin(\pi f_d T)$: Thus, the amplitude of the canceled video output is a function of the Doppler frequency shift and the pulse-repetition interval, or prf. The magnitude of the relative frequency-response of the delay-line canceller [ratio of the amplitude of the output from the delay-line canceller, $2k \sin(\pi f_d T)$ to the amplitude of the normal radar video $k$] is shown in the Fig. 7 below.
Blind speeds:

The response of the single-delay-line canceller will be zero whenever the argument \((n f_c T)\) in the amplitude factor of Eq. (6) is 0, \(\pi\), \(2\pi\), \(\ldots\), etc., or when

\[
    f_d = \frac{n}{T} = nf_p
\]

where \(n = 0, 1, 2, \ldots\), and \(f_p\) is pulse repetition frequency. The delay-line canceller not only eliminates the d-c component caused by clutter \((n = 0)\), but unfortunately it also rejects any moving target whose Doppler frequency happens to be the same as the PRF or a multiple thereof. Those relative target velocities which result in zero MTI response are called blind speeds and are given by

\[
    v_n = \frac{n \lambda}{2T} = \frac{n \lambda f_p}{2} \quad n = 1, 2, 3, \ldots
\]

where \(v_n\) is the \(n^{th}\) blind speed.

The blind speeds are one of the limitations of pulse MTI radar which do not occur with CW radar. They are present in pulse radar because Doppler is measured by discrete samples (pulses) at the PRF rather than continuously. If the first blind speed is to be greater than the maximum radial velocity expected from the target, the product, \(\lambda f_p\) must be large. Thus, the MTI radar must operate at long wavelengths (low frequencies) or with high pulse repetition frequencies, or both. Unfortunately, there are usually constraints other than blind speeds which determine the wavelength and the pulse repetition frequency. Therefore, blind speeds might not be easy to avoid. Low radar frequencies have the disadvantage that antenna beam widths, for a given-size antenna, are wider than at the higher frequencies and would not be satisfactory in applications where angular accuracy or angular resolution is important. The pulse repetition frequency cannot always be varied over wide limits since it is primarily determined by the unambiguous range requirement. In Fig.8, the first blind speed \(v_1\) is plotted as a function of the maximum unambiguous range \(R_{\text{max}} = cT/2\), with radar frequency as the parameter. If
the first blind speed were 600 knots, the maximum unambiguous range would be 130 nautical miles at a frequency of 300 MHz (UHF), 13 nautical miles at 3000 MHz (S band), and 4 nautical miles at 10,000 MHz (X band). Since commercial jet aircraft have speeds of the order of 600 knots, and military aircraft even higher, blind speeds in the MTI radar can be a serious limitation.

![Diagram](image)

**Figure (8):** Plot of MTI radar first blind speed as a function of maximum unambiguous range.

**Double cancellation:**

The frequency response of a single-delay-line canceller (Fig. 7) does not always have as broad a clutter-rejection null as might be desired in the vicinity of d-c. The clutter-rejection notches may be widened by passing the output of the delay-line canceller through a second delay-line canceller as shown in Fig. 9 below. The output of the two single-delay line cancellers in cascade is the square of that from a single canceller. Thus the frequency response is $4 \sin^2 \pi f T$. The configuration of Fig. 9 is called a double-delay-line canceller, or simply a **double canceller**. The relative response of the double canceller compared with that of a single-delay-line canceller is shown in Fig. 10. The finite width of the clutter spectrum is also shown (hatched) in this figure so as to illustrate the additional cancellation of clutter offered by the double canceller.

The two-delay-line configuration of Fig. 9 (b) has the same frequency-response characteristic as the double-delay-line canceller. The operation of the device is as follows. A signal $f(t)$ is inserted into the adder along with the signal from the preceding pulse period, with its amplitude weighted by the factor $-2$, plus the signal from the previous two pulse periods. The output of the adder is therefore

$$f(t) - 2f(t + T) + f(t + 2T)$$

which is the same as the output from the double-delay-line canceller.

$$f(t) - f(t + T) - f(t + T) + f(t + 2T)$$
This configuration is commonly called the three-pulse canceller.

![Diagram of three-pulse canceller](image)

**Figure 9:** (a) Double-delay-line canceller (b) three-pulse canceller.

![Diagram of frequency response](image)

**Figure (10):** Relative frequency response of the single-delay-line canceller (solid curve) and the double delay-line canceller (dashed curve). Shaded area represents clutter spectrum.

**Multiple or staggered Pulse Repetition Frequencies:**
The use of more than one pulse repetition frequency offers additional flexibility in the design of MTI Doppler filters. It not only reduces the effect of the blind speeds of Eq. 8, but it also allows a sharper low-frequency cutoff in the frequency response than might be obtained with a cascade of single-delay-line cancellers with \( \sin^n \omega f_0 T \) response. The blind speeds of two independent radars operating at the same frequency will be different if their pulse repetition frequencies are different. Therefore, if one radar were "blind" to moving targets, it would be unlikely that the other radar would be "blind" also.
Instead of using two separate radars, the same result can be obtained with one radar which time-shares its pulse repetition frequency between two or more different values (multiple PRF's). The pulse repetition frequency might be switched every other scan or every time the antenna is scanned a half beam width, or the period might be alternated on every other pulse. When the switching is pulse to pulse, it is known as a staggered PRF. An example of the composite (average) response of a MTI radar operating with two separate pulse repetition frequencies on a time-shared basis is shown in Fig.11. The pulse repetition frequencies are in the ratio of 5:4. Note that the first blind speed of the composite response is increased several times over what it would be for a radar operating on only a single pulse repetition frequency. Zero response occurs only when the blind speeds of each PRF coincide. In the example of Fig.11, the blind speeds are coincident for $4/T_1 = 5/T_2$. Although the first blind speed may be extended by using more than one PRF, regions of low sensitivity might appear within the composite passband. The closer the ratio $T_1 : T_2$ approaches unity, the greater will be the value of the first blind speed. However, the first null in the vicinity of $f_b = 1/T_1$ becomes deeper. Thus, the choice of $T_1 / T_2$ is a compromise between the value of the first blind speed and the depth of the nulls within the filter pass band.

![Graphs showing frequency-response of single-delay-line canceller](image)

**Figure 11** (a) Frequency-response of a single-delay-line canceller for $f_b = 1/T_1$ (b) same for $f_b = 1/T_2$ (c) Composite response with $T_1 / T_2 = 4/5$. 

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The depth of the nulls can be reduced and the first blind speeds increased by operating with more than two interpulse periods. Figure 12 below shows the response of a five-pulse stagger (four periods) that might be used with a long-range air traffic control radar. In this example the periods are in the ratio 25 : 30 : 27 : 31 and the first blind speed is 28.25 times that of a constant prf waveform with the same average period. If the periods of the staggered waveforms have the relationship \( n_1 / T_1 = n_2 / T_2 = \ldots = n_N / T_N \) where \( n_1, n_2, \ldots, n_N \) are integers, and if \( v_B \) is equal to the first blind speed of a non-staggered waveform with a constant period equal to the average period \( T_0 = (T_1 + T_2 + \ldots + T_N) / N \) then the first blind speed \( v_1 \) is given by:

\[
\frac{v_1}{v_B} = \frac{n_1 + n_2 + \cdots + n_N}{N}
\]

![Graph showing frequency response of a five-pulse (four-period) stagger.](image)

**Figure 12**: Frequency response of a five-pulse (four-period) stagger.

**Range-gated Doppler filters:**

The delay-line canceller, which can be considered as a time-domain filter, is widely used in MTI radar to separate moving targets from stationary clutter. It is also possible to employ the more common
frequency-domain band pass filters of conventional design in MTI radar to separate the Doppler-frequency-shifted targets. The filter configuration however would be more complex, than the single, narrow-band pass filter. A narrowband filter with a pass band designed to pass the Doppler frequency components of moving targets will "ring" when excited by the usual short radar pulse. That is, its pass band is much narrower than the reciprocal of the input pulse width so that the output will be of much greater duration than the input. The narrowband filter "smears" the input pulse since the impulse response is approximately the reciprocal of the filter bandwidth. This smearing destroys the range resolution. If more than one target is present they cannot be resolved. Even if only one target is present, the noise from the other range cells that do not contain the target will interfere with the desired target signal. The result is a reduction in sensitivity due to a collapsing loss.

The loss of the range information and the collapsing loss may be eliminated by first quantizing the range (time) into small intervals. This process is called range gating. The width of the range gates depends upon the range accuracy desired and the complexity which can be tolerated, but they are usually of the order of the pulse width. Range resolution is established by gating. Once the radar return is quantized into range intervals, the output from each gate may be applied to a narrowband filter since the pulse shape need no longer be preserved for range resolution. A collapsing loss does not take place since noise from the other range intervals is excluded.

A block diagram of the video of an MTI radar with multiple range gates followed by clutter-rejection filters is shown in Fig. 13 below. The output of the phase detector is sampled sequentially by the range gates. Each range gate opens in sequence just long enough to sample the voltage of the video waveform corresponding to a different range interval in space. The range gate acts as a switch or a gate which opens and closes at the proper time. The range gates are activated once each pulse-repetition interval. The output for a stationary target is a series of pulses of constant amplitude. An echo from a moving target produces a series of pulses which vary in amplitude according to the Doppler frequency. The output of the range gates is stretched in a circuit called the boxcar generator, or sample-and-hold circuit, whose purpose is to aid in the filtering and detection process by emphasizing the fundamental of the modulation frequency and eliminating harmonics of the pulse repetition frequency. The clutter rejection filter is a band pass filter whose bandwidth depends upon the extent of the expected clutter spectrum.

![Figure 13: Block diagram of MTI radar using range gates and filters.](image-url)

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Following the Doppler filter is a full-wave linear detector and an integrator (a low-pass filter). The purpose of the detector is to convert the bipolar video to unipolar video. The output of the integrator is applied to a threshold-detection circuit. Only those signals which cross the threshold are reported as targets. Following the threshold detector, the outputs from each of the range channels must be properly combined for display on the PPI or A-scope or for any other appropriate indicating or data-processing device. The CRT display from this type of MTI radar appears "cleaner" than the display from a normal MTI radar, not only because of better clutter rejection, but also because the threshold device eliminates many of the unwanted false alarms due to noise. The frequency-response characteristic of the range-gated MTI appears as in Fig. 14. The shape of the rejection band is determined primarily by the shape of the band pass filter of Fig. 13. The band pass filter can be designed with a variable low-frequency cutoff that can be selected to conform to the prevailing clutter conditions. The selection of the lower cutoff might be at the option of the operator or it can be done adaptively. A variable lower cutoff might be advantageous when the width of the clutter spectrum changes with time as when the radar receives unwanted echoes from birds. A relatively wide notch at zero frequency is needed to remove moving birds. If the notch were set wide enough to remove the birds, it might be wider than necessary for ordinary clutter and desired targets might be removed. Since the appearance of birds varies with the time of day and the season, it is important that the width of the notch be controlled according to the local conditions.

![Frequency-response characteristic of an MTI using range gates and filters.](image)

MTI radar using range gates and filters is usually more complex than an MTI with a single-delay-line canceller. The additional complexity is justified in those applications where good MTI performance and the flexibility of the range gates and filter MTI are desired. The better MTI performance results from the better match between the clutter filter characteristic and the clutter spectrum.

**Limitations to MTI Performance:**

The improvement in signal-to-clutter ratio of an MTI is affected by factors other than the design of the Doppler signal processor such as:

- Instabilities of the transmitter and receiver
- Physical motions of the clutter
- Finite time on target (or scanning modulation)
- And limiting in the receiver

Before discussing these limitations, we shall study the related definitions.
Definitions related to MTI Performance:

**MTI improvement factor:** The signal-to-clutter ratio at the output of the MTI system divided by the signal-to-clutter ratio at the input, averaged uniformly over all target radial velocities of interest.

**Sub clutter visibility:** The ratio by which the target echo power may be weaker than the coincident clutter echo power and still be detected with specified detection and false alarm probabilities. All target radial velocities are assumed equally likely. A sub clutter visibility of, for example, 30 dB implies that a moving target can be detected in the presence of clutter even though the clutter echo power is 1000 times the target echo power. Two radars with the same sub clutter visibility might not have the same ability to detect targets in clutter if the resolution cell of one is greater than the other and accepts a greater clutter signal power. I.e., both radars might reduce the clutter power equally, but one starts with greater clutter power because its resolution cell is greater and "sees" more clutter targets.

**Clutter visibility factor:** The signal-to-clutter ratio, after cancellation or Doppler filtering that provides stated probabilities of detection and false alarm.

**Clutter attenuation.** The ratio of clutter power at the canceller input to the clutter residue at the output, normalized to the attenuation of a single pulse passing through the unprocessed channel of the canceller. (The clutter residue is the clutter power remaining at the output of a MTI system.)

**The improvement factor (I):** Is equal to the sub clutter visibility (SCV) times the clutter visibility factor (V_{cl}). In decibels, I(dB) = SCV(dB) + V_{cl}(dB). When the MTI is limited by noise like system instabilities, the clutter visibility factor should be chosen as is the signal to noise ratio as defined in Radar Equation.

Limitations:

**Equipment instabilities:** Pulse-to-pulse changes in the amplitude, frequency, or phase of the transmitter signal, changes in the Stalo or Coho oscillators in the receiver, jitter in the timing of the pulse transmission, variations in the time delay through the delay lines, and changes in the pulse width can cause the apparent frequency spectrum from perfectly stationary clutter to broaden and thereby lower the improvement factor of an MTI radar. The stability of the equipment in MTI radar must be considerably better than that of an ordinary radar. It can limit the performance of MTI radar if sufficient care is not taken in design, construction, and maintenance.

**Internal fluctuation of clutter:** Although clutter targets such as buildings, water towers, bare hills or mountains produce echo signals that are constant in both phase and amplitude as a function of time, there are many types of clutter that cannot be considered as absolutely stationary. Echoes from trees, vegetation, sea, rain, and chaff fluctuate with time, and these fluctuations can limit the performance of MTI radar. Because of its varied nature, it is difficult to describe precisely the clutter echo signal. However, for purposes of analysis, most fluctuating clutter targets may be represented by a model consisting of many independent scatterers located within the resolution cell of the radar. The echo at the radar receiver is the vector sum of the echo signals received from each of the individual scatterers i.e., the relative phase as well as the amplitude from each scatterer influences the resultant composite signal. If the individual scatters remain fixed from pulse to pulse, the resultant echo signal will also
remain fixed. But any motion of the scatterers relative to the radar will result in different phase relationships at the radar receiver. Hence the phase and amplitude of the new resultant echo signal will differ pulse to pulse.

Antenna scanning modulation: As the antenna scans by a target, it observes the target for a finite time equal to: 
\[ t_a = n_b f_p = \theta_b / \theta_s \]
where \( n_b \) = number of hits received, \( f_p \) = pulse repetition frequency, \( \theta_b \) = antenna beam width and \( \theta_s \) = antenna scanning rate. The received pulse train of finite duration \( t_a \) has a frequency spectrum (which can be found by taking the Fourier transform of the waveform) whose width is proportional to \( 1/t_a \). Therefore, even if the clutter were perfectly stationary, there will still be a finite width to the clutter spectrum because of the finite time on target. If the clutter spectrum is too wide because the observation time is too short, it will affect the improvement factor. This limitation has sometimes been called scanning fluctuations or scanning modulation.

Limiting in MTI Radar: A limiter is usually employed in the IF amplifier just before the MTI processor to prevent the residue from large clutter echoes from saturating the display. Ideally a MTI radar should reduce the clutter to a level comparable to receiver noise.
However, when the MTI improvement factor is not great enough to reduce the clutter sufficiently, the clutter residue will appear on the display and prevent the detection of aircraft targets whose cross sections are larger than the clutter residue. This condition may be prevented by setting the limit level \( L \), relative to the noise \( N \), equal to the MTI improvement factor \( I \); or \( L/N = I \). If the limit level relative to noise is set higher than the improvement factor. Clutter residue obscures part of the display. If it is set too low, there may be a “black hole” effect on the display. The limiter provides a constant false alarm rate (CFAR) and is essential to usable MTI Performance.
Unfortunately, nonlinear devices such as limiters have side-effects that can degrade performance. Limiters cause the spectrum of strong clutter to spread into the canceller pass-band, and result in the generation of additional residue that can significantly degrade MTI performance as compared with a perfect linear system.

Pulse Doppler Radar Vs MTI:

A Pulse radar that extracts the Doppler frequency shift for the purpose of detecting moving targets in the presence of clutter is either a MTI Radar or a Pulse Doppler Radar. The distinction between them is based on the fact that in a sampled measurement system like a pulse Radar, ambiguities arise in measuring both the Doppler frequency (relative velocity) and the Range (time delay). Range ambiguities are avoided with a low sampling rate (low pulse repetition frequency), and Doppler frequency ambiguities are avoided with a high sampling rate. However, in most radar applications the sampling rate, or pulse repetition frequency, cannot be selected to avoid both types of measurement ambiguities. Therefore, a compromise must be made and the nature of the compromise generally determines whether the radar is called an MTI or a Pulse Doppler Radar.

- MTI usually refers to a Radar in which the pulse repetition frequency is chosen low enough to avoid ambiguities in range (no multiple-time-around echoes) but with the consequence that the frequency measurement is ambiguous and results in blind speeds, Eq. (8).
- The pulse Doppler radar, on the other hand, has a high pulse repetition frequency that avoids blind speeds, but it experiences ambiguities in range. It performs Doppler filtering on a single spectral line of the pulse spectrum.
- A radar which employs multiple pulse repetition frequencies to avoid blind speeds is usually classed as an MTI if its average PRF would cause blind speeds. The justification for this
definition is that the technology and design philosophy of a multiple PRF radar are more like that of an MTI than a pulse Doppler radar.

- The pulse Doppler radar is more likely to use range-gated Doppler filter-banks than delay-line cancellers. Also, a power amplifier such as a klystron is more likely to be used than a power oscillator like the magnetron. A pulse Doppler radar operates at a higher duty cycle than does an MTI. Although it is difficult to generalize, the MTI radar seems to be the more widely used of the two, but pulse Doppler radar is usually more capable of reducing clutter.

**Previous Years’ Examination Questions:**

1. Explain the following limitations of MTI radar.
   (a) Equipment instabilities.
   (b) Scanning modulation.
   (c) Internal fluctuation of clutter.

2. (a) Explain the function of time domain filter in a MTI Radar with an example.
   (b) A MTI radar operates at 10GHz with a PRF of 300 pps. Calculate the lowest blind speed?

3. (a) An MTI radar is operated at 9GHz with a PRF of 3000 pps. Calculate the first two lowest blind speeds for this radar. Derive the formula used.
   (b) Discuss the limitations of non-coherent MTI Radar systems. [12+4]

4. (a) Write the description of Range gated Doppler filters.
   (b) Explain the operation of MTI radar with 2 pulse repetition frequencies. [8+8]

5. (a) Draw and explain the frequency response characteristics of a MTI using Range gates and Filters.
   (b) A MTI Radar operates at frequency of 6GHz with a PRF of 800 PPS. Calculate the lowest blind speeds of this Radar.

6. (a) Compare and contrast the situations with a Power amplifier and Power oscillator in the transmitter of a MTI system.
   (b) Calculate the blind speed for a Radar with the following specifications: Wave length: 0.1 Mtr. and PRF : 200 Hz

7. (a) Describe Range gated Doppler filters.
   (b) Differentiate blind phases from blind speeds.
   (c) Discuss the application of electrostatic storage tubes in MTI radar. [6+5+5]

8. (a) Briefly explain about range – gated Doppler filters.
   (b) Describe the importance of double cancellation.

9. (a) Compare MTI Radar with Pulse Doppler radar
   (b) Explain the function of a single delay line canceller and derive an expression for the frequency response function.

10. (a) What is an MTI Radar and how does it operate.
    (b) Define blind speed. A MTI radar operates at 5 Ghz with a PRF of 100PPS. Find the three lowest blind speeds of this Radar. Explain the importance of Staggered PRF. [8+7]
UNIT-4
TRACKING RADAR

• Tracking with Radar
• Sequential Lobing
• Conical Scan
• Monopulse Tracking Radar
• Amplitude Comparison Monopulse (One and two coordinate)
• Phase Comparison Monopulse
• Tracking in Range
• Acquisition and Scanning Patterns
• Comparison of Trackers
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TRACKING RADAR

Tracking with Radar:

Tracking is the process of continuously maintaining the antenna beam on the target and also the echo signal within the range gate (present Range value). The antenna beam in the continuous tracking radar is positioned in angle by a servomechanism actuated by an error signal. The various methods for generating the error signal are classified as sequential lobing, conical scan, and simultaneous lobing or monopulse. The range and Doppler frequency shift are continuously tracked, by a servo-control loop actuated by an error signal generated in the radar receiver. The data available from a tracking radar may be presented on a cathode-ray-tube (CRT) display for action by an operator, or in modern Radars supplied to an automatic computer which determines the target path and calculate it's probable future course.

The tracking radar must first find and acquire its target before it can track. Some radars operate in a search, or acquisition, mode in order to find the target before switching to a tracking mode. Although it is possible to use a single radar for both the search and the tracking functions, such a procedure usually results in certain operational limitations. Obviously, when the radar is used in its tracking mode, it has no knowledge of other potential targets. Also, if the antenna pattern is a narrow pencil beam and if the search volume is large, a relatively long time might be required to find the target. Therefore, many radar tracking systems employ a separate search radar to provide the information necessary to position the tracker on the target. A search radar when used for this purpose is called an acquisition Radar. The acquisition radar designates targets to the tracking radar by providing the coordinates where the targets are to be found. The tracking radar acquires a target by performing a limited search in the area of the designated target coordinates.

The availability of small, inexpensive minicomputers has made it practical to obtain target tracks, not just target detections, from surveillance radar. Such processing is usually called ADT (automatic detection and track).

It is also necessary to distinguish between a continuous tracking Radar and a track-while-scan (TWS) radar. The former supplies continuous tracking data on a particular target, while the track-while-scan supplies sampled data on one or more targets. In general, the continuous tracking radar and the TWS radar employ different types of equipment.

A surveillance radar that provides target tracks is sometimes called a track-while-scan radar. This terminology is also applied to radars that scan a limited angular sector to provide tracking information at a high data rate on one or more targets within its field of view. Landing radars used for GCA (ground control of approach) and some missile control radars are of this type. When the term tracking radar is used in this book, it generally refers to the continuous tracker, unless otherwise specified.

Sequential Lobing:

The antenna pattern commonly employed with tracking radars is the symmetrical pencil beam in which the, elevation and azimuth beam widths are approximately equal. However, a simple pencil-beam antenna is not suitable for tracking radars unless means are provided for determining the magnitude and direction of the target's angular position with respect to some reference direction, usually the axis of the antenna. The difference between the target position and the reference direction is the angular
error. The tracking radar attempts to position the antenna to make the angular error zero. When the angular error is zero, the target is located along the reference direction. One method of obtaining the direction and the magnitude of the angular error in one coordinate is by alternately switching the antenna beam between two positions (Fig 1). This is called lobe switching sequential switching, or sequential lobeing. Fig 1-a is a polar representation of the antenna beam (minus the side lobes) in the two switched positions. A plot in rectangular coordinates is shown in Fig.1-b, and the error signal obtained from a target not on the switching axis (reference direction) is shown in Fig.1-c. The difference in amplitude between the voltages obtained in the two switched positions is a measure of the angular displacement of the target from the switching axis. The sign of the difference determines the direction the antenna must be moved in order to align the switching axis with the direction of the target. When the voltages in the two switched positions are equal, the target is on axis and its position may be determined from the antenna direction.

Two additional switching positions are needed to obtain the angular error in the orthogonal coordinate. Thus a two-dimensional sequentially lobeing radar consists of a cluster of four feed horns illuminating a single antenna, arranged so that the right-left, up-down sectors are covered by successive antenna positions. Both transmission and reception are accomplished at each position. A cluster of five feeds might also be employed, with the central feed used for transmission while the outer four feeds are used for receiving. High-power RF switches are not needed since only the receiving beams, and not the transmitting beam, are stepped in this five-feed arrangement.

Figure 1: Lobe-switching antenna patterns and error signal (one dimension) (a) Polar representation of switched antenna patterns (b) Rectangular representation (c) error signal
One of the limitations of a simple unswitched non-scanning pencil-beam antenna is that the angle accuracy can be no better than the size of the antenna beam width. An important feature of sequential lobing (as well as the other tracking techniques to be discussed) is that the target-position accuracy can be far better than that given by the antenna beam width. The accuracy depends on how well equality of the signals in the switched positions can be determined. The fundamental limitation to accuracy is system noise caused either by mechanical or electrical fluctuations. Sequential lobing, or lobe switching, was one of the first tracking-radar techniques to be employed. It is not used as often in modern tracking-radar applications as some of the other techniques to be described.

Conical Scan:

A logical extension of the simultaneous lobing technique described in the previous section is to rotate continuously an offset antenna beam rather than discontinuously step the beam between four discrete positions. This is known as conical scanning (Fig.2). The angle between the axis of rotation (which is usually the axis of the antenna reflector) and the axis of the antenna beam is called the Squint Angle. Consider a target at position A. The echo signal amplitude will be modulated at a frequency equal to the rotation frequency of the beam. The amplitude of the echo signal modulation will depend upon the shape of the antenna pattern, the squint angle and the angle between the target line of sight & the rotation axis. The phase of the modulation depends on the angle between the target and the rotation axis. The conical scan modulation is extracted from the echo signal and applied to a servo-control system which continually positions the antenna on the target. [Note that two servos are required because the tracking is required in two-dimensions. When the antenna is on target, as in B of Fig. 2, the line of sight to the target and the rotation axis coincide, and the conical-scan modulation is zero.

A block diagram of the angle-tracking portion of a typical conical-scan tracking radar is shown in Fig.3. The antenna is mounted so that it can be positioned in both azimuth and elevation by separate motors. The antenna beam is offset by tilting either the feed or the reflector with respect to one another. One of the simplest conical-scan antennas is a parabola with an offset rear feed rotated about the axis of the reflector. If the feed maintains the plane of polarization fixed as it rotates, it is called a nutating feed. A rotating feed causes the polarization to rotate. The latter type of feed requires a rotary joint. The nutating feed requires a flexible joint. If the antenna is small, it may be easier to rotate the dish, which is offset, rather than the feed, thus avoiding the problem of a rotary or flexible RF joint in the feed. A typical conical-scan rotation speed is 30 r/s. The same motor that provides the conical-scan rotation of the antenna beam also drives a two-phase reference generator with two sinusoidal outputs 90° apart in phase. These two outputs serve as a reference to extract the elevation and azimuth errors. The received echo signal is fed to the receiver from the antenna via two rotary joints (not shown in the block diagram). One rotary joint permits motion in azimuth; the other, in elevation.

The receiver is conventional super heterodyne except for features peculiar to the conical-scan tracking radar. One feature not found in other radar receivers is a means of extracting the conical-scan modulation, or error signal. This is accomplished after the second detector in the video portion of the receiver. The error signal is compared with the elevation and azimuth reference signals in the angle-error detectors, which are phase-sensitive detectors.
Figure 2: Principle of Conical-scan tracking.

Figure 3: Block diagram of conical-scan tracking radar.

A phase sensitive detector is a nonlinear device in which the input signal (in this case the angle-error signal) is mixed with the reference signal. The input and reference signals are of the same frequency. The output d-c voltage reverses polarity as the phase of the input signal changes through 180°. The magnitude of the d-c output from the angle-error detector is proportional to the error, and the sign
(polarity) is an indication of the direction of the error. The angle-error detector outputs are amplified and drive the antenna elevation and azimuth servo motors. The angular position of the target is determined from the elevation and azimuth of the antenna axis. The position is read out by means of standard angle transducers such as synchros, potentiometers, or shaft encoders.

**Boxcar generator:**

When extracting the modulation imposed on a repetitive train of narrow pulses, it is usually convenient to stretch the pulses before low-pass filtering. This is called *boxcarking or sample and hold*. Here the device is called a *boxcar generator*. The boxcar generator was also mentioned in the discussion of the MTI receiver using range-gated filters. In essence, it clamps or stretches the video pulses of Fig.4-(a) in time so as to cover the entire pulse-repetition period (Fig.4-b). This is possible only in a range-gated receiver (Tracking radars are normally operated with range gates). The boxcar generator consists of an electric circuit that clamps the potential of a storage element, such as a capacitor, to the video-pulse amplitude each time the pulse is received. The capacitor maintains the potential of the pulse during the entire repetition period and is altered only when a new video pulse appears whose amplitude differs from the previous one. The boxcar generator eliminates the pulse repetition frequency and reduces its harmonics. It also has the practical advantage that the magnitude of the conical-scan modulation is amplified because pulse stretching puts more of the available energy at the modulation frequency. The pulse repetition frequency must be sufficiently large compared with the conical-scan frequency for proper boxcar filtering. If not it may be necessary to provide additional filtering to attenuate undesired cross modulation frequency components.

![Boxcar generator diagram](image)

**Fig 4:** (a) Pulse train with conical scan modulation (b) same pulse train after passing through boxcar generator.
Automatic Gain Control (AGC):

The echo-signal amplitude at the tracking-radar receiver will not be constant but will vary with time. The three major causes of variation in amplitude are:
1. The inverse-fourth-power relationship between the echo signal and range
2. The conical-scan modulation (angle-error signal) and
3. Amplitude fluctuations in the target cross section.

The function of the automatic gain control (AGC) is to maintain the d-c level of the receiver output constant and to smooth or eliminate as much of the noise like amplitude fluctuations as possible without disturbing the extraction of the desired error signal at the conical-scan frequency.

One of the purposes of AGC in any receiver is to prevent saturation by large signals. The scanning modulation and the error signal would be lost if the receiver were to saturate. In the conical-scanning tracking radar an AGC that maintains the d-c level constant results in an error signal that is a true indication of the angular pointing error. The d-c level of the receiver must be maintained constant if the angular error is to be linearly related to the angle-error signal voltage.

An example of the AGC portion of a tracking-radar receiver is shown in Fig. 5. A portion of the video-amplifier output is passed through a low-pass or smoothing filter and feedback to control the gain of the

![Diagram of AGC system]

**Figure 5:** Block diagram of the AGC portion of a tracking-radar receiver.

IF amplifier. The larger the video output, the larger will be the feedback signal and the greater will be the gain reduction. The filter in the AGC loop should pass all frequencies from direct current to just below the conical-scan-modulation frequency. The loop gain of the AGC filter measured at the conical-scan frequency should be low so that the error signal will not be affected by AGC action. (If the AGC responds to the conical-scan frequency, the error signal might be lost.) The phase shift of this filter must be small if its phase characteristic is not to influence the error signal. A phase change of the error signal is equivalent to a rotation of the reference axes and introduces cross coupling, or "cross talk," between the elevation and azimuth angle-tracking loops. Cross talk affects the stability of the tracking and might result in an unwanted nutating motion of the antenna. In conventional tracking radar applications, the phase change introduced by the feedback-loop filter should be less than $10^\circ$ and in some applications, it should be as little as $2^\circ$. For this reason, a filter with a sharp attenuation characteristic in the vicinity of the conical-scan frequency might not be desirable because of the relatively large amount of phase shift which it would introduce.
Other considerations:
- In both the sequential-lobing and conical-scan techniques, the measurement of the angle error in two orthogonal coordinates (azimuth and elevation) requires that a minimum of three pulses be processed.
- In practice, however, the minimum number of pulses in sequential lobing is usually four-one per quadrant. Although a conical scan radar can also be operated with only four pulses per revolution, it is more usual to have ten or more per revolution. This allows the modulation due to the angle error to be more that of a continuous sine wave.
- Thus, the PRF is usually at least an order of magnitude greater than the conical-scan frequency.
- The scan frequency also must be at least an order of magnitude greater than the tracking bandwidth.

A conical-scan-on-receive-only (COSRO) tracking radar radiates a non-scanning transmit beam, but receives with a conical scanning beam to extract the angle error. The analogous operation with sequential lobing is called lobe-on-receive-only (LORO).

Monopulse tracking radar:

The conical-scan and sequential-lobing tracking radars require a minimum number of pulses to extract the angle-error signal. In the time interval during which a measurement is made with either sequential lobing or conical scan, the train of echo pulses must not contain amplitude-modulation components other than the modulation produced by scanning. If the echo pulse-train contains additional modulation components, caused, for example, by a fluctuating target cross section, the tracking accuracy will be degraded, especially if the frequency of the fluctuations is at or near the conical-scan frequency or the sequential-lobing rate. The fluctuating echo can severely limit the accuracy of those tracking radars which require many pulses to be processed to extract the error signal.

Pulse-to-pulse amplitude fluctuations of the echo signal have no effect on tracking accuracy if the angular measurement is made on the basis of one pulse rather than many. There are several methods by which angle-error information might be obtained with only a single pulse. More than one antenna beam is used simultaneously in these methods, in contrast to the conical-scan or lobe-switching tracker, which utilizes one antenna beam on a time-shared basis. The angle of arrival of the echo signal may be determined in a single-pulse system by measuring the relative phase or the relative amplitude of the echo pulse received in each beam. The names simultaneous lobing and monopulse are used to describe those tracking techniques which derive angle-error information on the basis of a single pulse.

An example of a simultaneous-lobing technique is amplitude-comparison monopulse or more simply, monopulse. In this technique, the RF signals received from two offset antenna beams are combined so that both the sum and the difference signals are obtained simultaneously. The sum and difference signals are multiplied in a phase-sensitive detector to obtain both the magnitude and the direction of the error signal. All the information necessary to determine the angular error is obtained on the basis of a single pulse; hence the name monopulse.

Amplitude-comparison monopulse:

The amplitude-comparison monopulse employs two overlapping antenna patterns (Fig. 6-a) to obtain the angular error in one coordinate. The two overlapping antenna beams may be generated with a
single reflector or with a lens antenna illuminated by two adjacent feeds. (A cluster of four feeds may be used if both elevation- and azimuth-error signals are wanted.) The sum of the two antenna patterns of Fig. 6(a) is shown in Fig. 6(b), and the difference in Fig. 6(c). The sum pattern is used for transmission, while both the sum pattern and the difference pattern are used on reception. The signal received with the difference pattern provides the magnitude of the angle error. The sum signal provides the range measurement and is also used as a reference to extract the sign of the error signal. Signals received from the sum and the difference patterns are amplified separately and combined in a phase-sensitive detector to produce the error-signal characteristic shown in Fig. 6(d).

A block diagram of the amplitude-comparison-monopulse tracking radar for a single angular coordinate is shown in Fig. 7. The two adjacent antenna feeds are connected to the two arms of a hybrid junction such as a "magic T," a "rat race," or a "short-slot coupler." The sum and difference signals appear at the two other arms of the hybrid. On reception, the outputs of the sum arm and the difference arm are each heterodyned to an intermediate frequency and amplified as, in any super heterodyne receiver. The transmitter is connected to the sum arm. Range information is also extracted from the sum channel. A duplexer is included in the sum arm for the protection of the receiver. The output of the phase-sensitive detector is an error signal whose magnitude is proportional to the angular error and whose sign is proportional to the direction. The output of the monopulse radar is used to perform automatic tracking. The angular error signal actuates a servo-control system to position the antenna, and the range output from the sum channel feeds into an automatic-range-tracking unit.

The sign of the difference signal (and the direction of the angular error) is determined by comparing the phase of the difference signal with the phase of the sum signal. If the sum signal in the IF portion of the receiver were $A_s \cos (\omega_p t)$ the difference signal would be either $A_d \cos (\omega_p t)$ or $-A_d \cos (\omega_p t)(A_d > 0, A_d < 0)$, depending on which side of center is the target. Since $-A_d \cos (\omega_p t) = A_d \cos (\omega_p (t+\pi))$, the sign of the difference signal may be measured by determining whether the difference signal is in phase with the sum or $180^\circ$ out of phase.

Although a phase comparison is a part of the amplitude-comparison-monopulse radar, the angular-error signal is basically derived by comparing the echo amplitudes from simultaneous offset beams. The phase relationship between the signals in the offset beams is not used. The purpose of the phase-sensitive detector is only to conveniently furnish the sign of the error signal.
Figure 6: Monopulse antenna patterns and error signal. Left-hand diagrams in (a-c) are in polar coordinates. Right-hand diagrams are in rectangular coordinates. (a) Overlapping antenna patterns (b) sum pattern (c) difference pattern (d) product (error) signal.

Figure 7: Block diagram of amplitude-comparison Monopulse radar (one angular coordinate).
A block diagram of a monopulse radar with provision for extracting error signals in both elevation and azimuth is shown in Fig. 8. The cluster of four feeds generates four partial overlapping antenna beams. The feeds might be used with a parabolic reflector, Cassegrain antenna, or a lens. All four feeds generate the sum pattern. The difference pattern in one plane is formed by taking the sum of two adjacent feeds and subtracting this from the sum of the other two adjacent feeds. The difference pattern in the orthogonal plane is obtained by adding the differences of the orthogonal adjacent pairs. A total of four hybrid junctions generate the sum channel, the azimuth difference channel, and the elevation difference channel. Three separate mixers and IF amplifiers are shown, one for each channel. All three mixers operate from a single local oscillator in order to maintain the phase relationships between the three channels. Two phase-sensitive detectors extract the angle-error information, one for azimuth, the other for elevation. Range information is extracted from the output of the sum channel after amplitude detection.

Since a phase comparison is made between the output of the sum channel and each of the difference channels, it is important that the phase shifts introduced by each of the channels be almost identical and must be maintained to within $25^\circ$ or better for reasonably proper performance. The gains of the channels also must not differ by more than specified amounts.

An alternative approach to using three identical amplifiers in the monopulse receiver is to use but one IF channel which amplifies the sum signal and the two difference signals on a time-shared basis. The sum signal is passed through the single IF amplifier followed by the two difference signals delayed in time by

![Block diagram of two-coordinate (azimuth and elevation) amplitude-comparison Monopulse tracking radar.](image)

Figure 8: Block diagram of two-coordinate (azimuth and elevation) amplitude-comparison Monopulse tracking radar.
a suitable amount. Most of the gain and gain control take place in the single IF amplifier. Any variations affect all three signals simultaneously. After amplification, compensating delays are introduced to unscramble the time sequence and bring the sum signal and the two difference signals in time coincidence. Phase detection occurs as in the conventional monopulse.

Automatic gain control (AGC) is required in order to maintain a stable closed-loop servo system for angle tracking. The AGC in a monopulse radar is accomplished by employing a voltage proportional to the sum-channel IF output to control the gain of all three receiver channels. The AGC results in a constant angle sensitivity independent of target size and range. With AGC the output of the angle-error detector is proportional to the difference signal normalized (divided) by the sum signal. The output of the sum channel is constant.

**Important Requirements of Amplitude-comparison monopulse:**

- The monopulse antenna must generate a sum pattern with high efficiency (maximum bore sight gain), and a difference pattern with a large value of slope at the crossover of the offset beams.

  The greater the signal-to-noise ratio and the steeper the slope of the error signal in the vicinity of zero angular error, the more accurate is the measurement of angle.

- Furthermore, the side lobes of both the sum and the difference patterns must be low.
- The antenna must be capable of the desired bandwidth, and the patterns must have the desired polarization characteristics.

*It is not surprising that the achievement of all these properties cannot always be fully satisfied simultaneously. Antenna design is an important part of the successful realization of a good monopulse radar.*

**Phase comparison Monopulse:**

The tracking techniques discussed thus far in this chapter are based on the comparison of the amplitude of echo signals received from two or more antenna positions. The sequential-lobing and conical-scan techniques used a single, time-shared antenna beam while the monopulse technique used two or more simultaneous beams. The difference in amplitudes in the several antenna positions is proportional to the angular error. The angle of arrival (in one coordinate) may also be determined by comparing the phase difference between the signals from two separate antennas. Unlike the antennas of amplitude comparison trackers those used in phase-comparison systems are not offset from the axis. The individual bore sight axes of the antennas are parallel, causing the (far-field) radiation to illuminate the same volume in space. The amplitudes of the target echo signals are essentially the same from each antenna beam, but the phases are different.

The measurement of angle of arrival by comparison of the phase relationships in the Signals from the separated antennas of a radio interferometer has been widely used by the radio astronomers for precise measurements of the positions of radio stars. The interferometer as used by the radio astronomer is a passive instrument, the source of energy being radiated by the target itself. A tracking radar which operates with phase information is similar to an active interferometer and is also called *interferometer*.
radar. It has also been called *simultaneous phase comparison radar* or *phase-comparison monopulse*. The latter term is the one which will be used here.

In Fig.9 two antennas are shown separated by a distance $d$. The distance to the target is $R$ and is assumed large compared with the antenna separation $d$. The line of sight to the target makes an angle $\theta$ to the perpendicular bisector of the line joining the two antennas. The distance from antenna 1 to the target is: $R_1 = R + (d/2) \sin \theta$

and the distance from antenna 2 to the target is: $R_2 = R - (d/2) \sin \theta$

The phase difference between the echo signals in the two antennas is approximately

$$\Delta \phi = \frac{2\pi}{\lambda} \cdot d \cdot \sin \theta$$

For small angles where $\sin \theta \approx \theta$, the phase difference is a linear function of the angular error and may be used to position the antenna via a servo-control loop.

In the early versions of the phase-comparison monopulse radar, the angular error was determined by measuring the phase difference between the outputs of receivers connected to each antenna. The output from one of the antennas was used for transmission and for providing the range information. With such an arrangement, it was difficult to obtain the desired aperture illuminations and to maintain a stable bore sight. A more satisfactory method of operation is to form the sum and difference patterns in the RF and to process the signals as in a conventional amplitude-comparison monopulse radar.

In another phase-comparison principle as applied to missile guidance the phase difference between the signals in two fixed antennas is measured with a servo-controlled phase shifter located in one of the arms. The servo loop adjusts the phase shifter until the difference in phase between the two channels is a null. The amount of phase shift which has to be introduced to make a null signal is a measure of the angular error.
Tracking in range:

In most tracking-radar applications the target is continuously tracked in range as well as in angle. Range tracking might be accomplished by an operator who watches an A-scope and manually positions a hand wheel in order to maintain a marker over the desired target pip. The setting of the hand wheel is a measure of the target range and may be converted to a voltage that is supplied to a data processor. As target speeds increase, it is increasingly difficult for an operator to manually position the hand wheel at the required speed over a sustained period of time, and automatic tracking becomes a necessity. The technique for automatically tracking in range is based on the split range gate. Two range gates are generated as shown in Fig.10. One is the early gate, and the other is the late gate. The echo pulse is shown in Fig.10-a, the relative position of the gates at a particular instant in Fig.10-b, and the error signal in Fig.10-c. The portion of the signal energy contained in the early gate is less than that in the late gate. If the outputs of the two gates are subtracted, an error signal (Fig.10-c) will result which is used to reposition the center of the gates. The magnitude of the error signal is a measure of the difference between the center of the pulse and the center of the gates. The sign of the error signal determines the direction in which the gates must be repositioned by a feedback-control system. When the error signal is zero the range gates are centered on the pulse.
The range gating necessary to perform automatic tracking offers several advantages as by products. It isolates one target, excluding targets at other ranges. This permits the boxcar generator to be employed. Also, range gating improves the signal-to-noise ratio since it eliminates the noise from the other range intervals. Hence the width of the gate should be sufficiently narrow to minimize extraneous noise. On the other hand, it must not be so narrow that an appreciable fraction of the signal energy is excluded. A reasonable compromise is to make the gate width two to five times of the pulse width.

**Acquisition and Scanning patterns:**

A tracking radar must first find and acquire its target before it can operate as a tracker. Therefore it is usually necessary for the radar to scan an angular sector in which the presence of the target is expected. Most tracking radars employ a narrow pencil-beam antenna. Searching a volume in space for an aircraft target with a narrow pencil beam would be somewhat analogous to searching for a fly in a darkened auditorium with a flashlight. It must be done systematically in a fixed pattern if the entire volume is to be covered uniformly and quickly. Examples of the common types of scanning patterns employed with pencil-beam antennas are illustrated in Fig.11 below.

- In the **helical scan**, the antenna is continuously rotated in azimuth while it is simultaneously raised or lowered in elevation. It traces a helix in space.
- The **Palmer scan** consists of a rapid circular scan (conical scan) about the axis of the antenna, combined with a linear movement of the axis of rotation. When the axis of rotation is held stationary the Palmer scan reduces to the **conical scan**.
Figure 11: Examples of acquisition search patterns: (a) Trace of helical scanning beam (b) Palmer scan (c) spiral scan (d) raster, or TV, scan (e) nodding scan. The raster scan is sometimes called an n-bar scan, where n is the number of horizontal rows.

Because of this property, the Palmer scan is sometimes used with conical-scan tracking radars which must operate with a search as well as a track mode since the same mechanisms used to produce conical scanning can also be used for Palmer scanning. Some conical-scan tracking radars increase the squint angle during search in order to reduce the time required to scan a given volume. In general, conical scan is performed during the search mode of most tracking radars. The Palmer scan is suited to a search area which is larger in one dimension than another.

- The spiral scan covers an angular search volume with circular symmetry. Both the spiral scan and the Palmer scan suffer from the disadvantage that all parts of the scan volume do not receive the same energy unless the scanning speed is varied during the scan cycle. As a consequence, the number of hits returned from a target when searching with a constant scanning rate depends upon the position of the target within the search area.
- The raster or TV scan, unlike the Palmer or the spiral scan, scans the search area in a uniform manner. The raster scan is a simple and convenient means for searching a limited sector, rectangular in shape.
- Similar to the raster scan is the nodding scan produced by oscillating the antenna beam rapidly in elevation and slowly in azimuth. Although it may be employed to cover a limited sector as does the raster scan-nodding scan may also be used to obtain hemispherical coverage, that is, elevation angle extending to 90° and the azimuth scan angle to 360°.
Comparison of trackers:

Of the four continuous-tracking-radar techniques that have been discussed (sequential lobing, conical scan, amplitude-comparison monopulse, and phase-comparison monopulse), conical scan and amplitude-comparison monopulse have seen more application than the other two. The phase-comparison monopulse has not been too popular because of the relative awkwardness of its antenna (four separate antennas mounted to point their individual beams in the same direction), and because the side lobe level might be higher than desired. Although sequential lobing is similar to conical scan, the latter is preferred in most applications, since it suffers less loss and the antenna and feed systems are usually less complex. In this section, only the conical-scan radar and the amplitude-comparison monopulse will be compared. (The later will be referred to simply as monopulse).

- When the target is being tracked, the signal-to-noise ratio available from the monopulse radar is greater than that of a conical scan radar, all other things being equal, since the monopulse radar views the target at the peak of its sum pattern while the conical-scan radar views the target at an angle off the peak of the antenna beam. The difference in signal-to-noise ratio might be from 2 to 4 dB. For the same size aperture, the beam width of a conical-scan radar will be slightly greater than that of the monopulse because its feed is offset from the focus.

- The tracking accuracy of a monopulse radar is superior to that of the conical-scan radar because of the absence of target amplitude-fluctuations and because of its greater signal-to-noise ratio. It is the preferred technique for precision tracking. However, both monopulse and conical-scan radars are degraded equally by the wandering of the apparent position of the target (glint).

- The monopulse radar is the more complex of the two. Three separate receivers are necessary to derive the error signal in two orthogonal angular coordinates. Only one receiver is needed in the conical-scan radar. (There are certain monopulse implementations that can use either one or two receivers, but at some sacrifice in performance.) Since the monopulse radar compares the amplitudes of signals received in three separate channels, it is important that the gain and phase shift through these channels be identical. The RF circuitry that generates the sum and difference signals in a monopulse radar has been steadily improved, and can be realized without excessive physical bulk. A popular form of antenna for monopulse is the Cassegrain.

- With the monopulse tracker it is possible to obtain a measure of the angular error in two coordinates on the basis of a single pulse. A minimum of four pulses are usually necessary with the conical-scan radar. However, continuous-tracking radar seldom makes a measurement on a single pulse. (Phased array radars and some surveillance radars however might use the monopulse principle to extract an angle measurement on the basis of a single pulse.) In practice the two radars utilize essentially the same number of pulses to obtain an error signal if the servo tracking bandwidths and pulse repetition frequencies are the same. The monopulse radar first makes its angle measurement and then integrates a number of pulses to obtain the required signal-to-noise ratio and to smooth the error. The conical-scan radar, on the other hand, integrates a number of pulses first and then extracts the angle measurement.

- Because monopulse radar is not degraded by amplitude fluctuations, it is less susceptible to hostile electronic countermeasures than is conical scan.
• In brief, the monopulse radar is the better tracking technique; but in many applications where the ultimate in performance is not needed, the conical-scan radar is used because it is less costly and less complex.

**Previous year’s examination questions:**

1. Why is amplitude comparison monopulse more likely to be preferred over the Phase Comparison monopulse and conical scan tracker over sequential lobbing, or lobe switching Tracker? Explain. [16]

2. (a) Discuss in detail about the Amplitude fluctuations and how its effects are minimized.
   (b) Explain Mono pulse tracking in two angle coordinates. [8+8]

3. (a) Draw and explain block diagram of Conical-scan tracking radar.
   (b) Why does tracking radar have poor accuracy at low elevation angles? Explain. [8+8]

4. (a) Draw and explain the following with respect to Tracking in range:
   i. Echo pulse
   ii. Early-late range gates
   iii. Difference signal between early and late range gates.
   (b) Limitation of automatic detection and tracking. [8+8]

5. (a) Explain the operation of Monopulse tracking radar with a Block Diagram.
   (b) Write the differences between a Conical scanning Radar and a Monopulse Radar.

6. (a) With a neat diagram explain the operation of a conical scan Radar. Explain the various factors that need to be considered for optimum squint angle.
   (b) Explain with the help of a neat block diagram Amplitude comparison Monopulse radar for extracting error signals in both Azimuth and Elevation

7. (a) Compare the tracking techniques.
   (b) Explain in detail about limitations to tracking accuracy. [10+6]

8. (a) Draw and explain the wave front phase relationships in phase comparison monopulse Radar.
   (b) Write a brief note on acquisition and scanning patterns.
UNIT-5

DETECTION OF RADAR SIGNALS IN NOISE

- Introduction
- Matched filter receiver
- Response characteristics and derivation
- Correlation function and cross correlation receiver
- Efficiency of Non-matched filters
- Matched filter with non-white noise

RADAR RECEIVERS

- Noise figure and Noise Temperature
- Displays-Types
- Duplexers-Branch type and Balanced Type
- Circulators as duplexers
- Phased array Antennas
  - Introduction
  - Basic concepts
  - Radiation Pattern
  - Beam steering and Beam width changes
  - Applications
  - Advantages and Limitations
DETECTION OF RADAR SIGNALS IN NOISE

Introduction:
The two basic operations performed by radar are (1) detection of the presence of reflecting objects, and (2) extraction of information from the received waveform to obtain target data such as position, velocity, and perhaps size. Radar that is a good detection device is usually a good radar for extracting information, and vice versa. In this chapter, some aspects of the problem of detecting radar signals in the presence of noise will be studied. Noise ultimately limits the capability of any radar.

Matched filter receiver:

A network whose frequency-response function maximizes the output peak-signal-to-mean noise (Power) ratio is called a matched filter. This criterion, or its equivalent, is used for the design of almost all radar receivers.

The frequency-response function, denoted $H(f)$, expresses the relative amplitude and phase of the output of a network with respect to the input when the input is a pure sinusoid. The magnitude $|H(f)|$ of the frequency-response function is the receiver amplitude pass band characteristic. If the bandwidth of the receiver pass band is wide compared with that occupied by the signal energy, extraneous noise is introduced in the excess bandwidth which lowers the output signal-to-noise ratio. On the other hand, if the receiver bandwidth is narrower than the bandwidth occupied by the signal, the noise energy is reduced along with a considerable part of the signal energy. The net result is again a lower signal-to-noise ratio. Thus, there is an optimum bandwidth at which the signal-to-noise ratio is a maximum.

The rule of thumb quoted in pulse radar practice is that the Receiver bandwidth $B$ should be approximately equal to the reciprocal of the pulse width $T$.

The receiver frequency-response function, in our study, is assumed to apply from the antenna terminals to the output of the IF amplifier. (The second detector and video portion of a well-designed radar super heterodyne receiver will have negligible effect on the output signal-to-noise ratio if the receiver is designed as a matched filter.) Narrow banding is most conveniently accomplished in the IF. The bandwidths of the RF and mixer stages of the normal super heterodyne receiver are usually large compared to the IF bandwidth. Therefore, the frequency-response function of the portion of the receiver included between the antenna terminals to the output of the IF amplifier is taken to be that of the IF amplifier alone. Thus, we need only obtain the frequency-response function that maximizes the signal-to-noiseratio at the output of the IF. The IF amplifier is considered as a filter with gain and its frequency response is the property of interest to us.

For a received waveform $s(t)$ with a given ratio of signal energy $E$ to noise energy $N_0$ (or noise power per hertz of bandwidth), North showed that the frequency-response function of the linear, time-invariant filter which maximizes the output peak signal to noise (power) ratio for a fixed input signal to noise (energy) ratio is

$$H(f) = G_s S^*(f) \exp(-j2\pi f T)$$  \hspace{1cm} (1)

where $S(f) = \int_{-\infty}^{\infty} s(t) \exp(-j2\pi ft) \, dt$ = voltage spectrum (Fourier transform) of input signal.
\[ S^*(f) = \text{complex conjugate of } S(f) \]
\[ t_s = \text{fixed value of time at which signal is observed to be maximum} \]
\[ G_s = \text{constant equal to maximum filter gain (generally taken to be unity)} \]

The filter whose frequency-response function is given by Eq. (1) is called the matched filter. As can be seen, the frequency-response function of the matched filter is the conjugate of the spectrum of the received waveform except for the phase shift \( \exp(-j2\pi ft_s) \). This phase shift varies uniformly with frequency. Its effect is to cause a constant time delay.

The frequency spectrum of the received signal may be written as an amplitude spectrum \( |S(f)| \) and a phase spectrum \( \exp[-j\Phi(f)] \). The matched filter frequency response function may also be similarly written in terms of its amplitude spectrum \( |H(f)| \) and phase spectrum \( \exp[-j\Phi_m(f)] \). Ignoring the constant \( G_s \), then Eq. (1) for the matched filter may be written as:

\[ |H(f)| \exp[-j\Phi_m(f)] = |S(f)| \exp\left\{j[\Phi_s(f) - 2\pi ft_s]\right\} \quad \ldots (2) \]

Or

\[ |H(f)| = |S(f)| \quad \ldots (3.a) \]

And

\[ \Phi_m(f) = -\Phi_s(f) + 2\pi ft_s \quad \ldots (3.b) \]

Thus, the amplitude spectrum of the matched filter is the same as the amplitude spectrum of the signal, but the phase spectrum of the matched filter is the negative of the phase spectrum of the signal plus a phase shift proportional to frequency.

The matched filter may also be specified by its impulse response \( h(t) \) which is the inverse Fourier transform of the frequency-response function.

\[ h(t) = \int_{-\infty}^{\infty} H(f) \exp(j2\pi ft) \, df \quad \ldots (4) \]

Physically the impulse response is the output of the filter as a function of time when the input is an impulse (delta function). Substituting Eq. (1) into Eq. (4) gives

\[ h(t) = G_s \int_{-\infty}^{\infty} S^*(f) \exp[-j2\pi f(t - t_s)] \, df \quad \ldots (5) \]

Since \( S^*(f) = S(-f) \), we have

\[ h(t) = G_s \int_{-\infty}^{\infty} S(f) \exp[j2\pi f(t - t_s)] \, df = G_s s(t - t_s) \quad \ldots (6) \]

A rather interesting result is that the impulse response of the matched filter is the image of the received waveform, i.e., it is the same as the received signal run backward in time starting from the fixed time \( t_s \). Figure (1) below shows a received waveform \( s(t) \) and the impulse response \( h(t) \) of its matched filter.
Radar Systems Lecture Notes (K Subhas) Unit-5 : Detection of Radar signals in noise

\[ s(t) \]

(a)

\[ h(t) \]

(b)

Figure (1) (a) Received waveform \( s(t) \)(b) impulse response \( h(t) \) of the matched filter.

**Derivation of the matched-filter characteristic:**

We will derive here the frequency-response function of the linear, time-invariant filter which maximizes the output peak-signal-to-mean-noise (power) ratio as (using the Schwartz inequality):

\[ H(f) = G_m \cdot S^*(f) \cdot \exp(-j2\pi ft) \]

The ratio we wish to maximize is

\[ R_f = |S_d(t)|^2 \max / N \]

\[ \text{where } |S_d(t)|^2 \max = \text{maximum value of output signal voltage and } N = \text{mean noise power at receiver output.} \]

*The ratio \( R_f \) is not quite the same as the signal-to-noise ratio which has been considered previously in the radar equation. Note that the peak power as used here is actually the peak instantaneous power, whereas the peak power referred to in the discussion of the radar equation was the average value of the power over the duration of a pulse of sine wave. The ratio \( R_f \) is twice the average signal-to-noise power ratio when the input signal \( s(t) \) is a rectangular sine-wave pulse.*

The output voltage of a filter with frequency response function \( H(f) \) is
\[ |s_0(t)| = \left| \int_{-\infty}^{\infty} S(f)H(f) \exp(j2\pi ft) \, df \right| \] ...... (8)

where \( S(f) \) is the Fourier transform of the input (received) signal. The mean output noise power is

\[ N = \frac{N_0}{2} \int_{-\infty}^{\infty} |H(f)|^2 \, df \] ...... (9)

where \( N_0 \) is the input noise power per unit bandwidth. The factor \((1/2)\) appears before the integral because the limits extend from \(-\infty\) to \(+\infty\), whereas \( N_0 \) is defined as the noise power per cycle of bandwidth over positive values only.

Substituting Eqs. (8) and (9) into (7) and assuming that the maximum value of \(|s_0(t)|^2\) occurs at time \( t = t_1 \), the ratio \( R_f \) becomes

\[ R_f = \frac{\left| \int_{-\infty}^{\infty} S(f)H(f) \exp(j2\pi ft_1) \, df \right|^2}{\frac{N_0}{2} \int_{-\infty}^{\infty} |H(f)|^2 \, df} \] ...... (10)

Schwartz's inequality states that if \( P \) and \( Q \) are two complex functions, then

\[ \left| \int P^*P \, dx \right| \left| \int Q^*Q \, dx \right| \geq \left| \int P^*Q \, dx \right|^2 \] ...... (11)

The equality sign applies when \( P = kQ \), where \( k \) is a constant. Letting

\[ P^* = S(f) \exp(j2\pi ft_1) \quad \text{and} \quad Q = H(f) \]

and recalling that

\[ \int P^*P \, dx = \int |P|^2 \, dx \]

We get, on applying the Schwartz inequality to the numerator of Eq.(10),
\[
R_f \leq \frac{\int_{-\infty}^{\infty} |H(f)|^2 df \int_{-\infty}^{\infty} |S(f)|^2 df}{\frac{N_0}{2} \int_{-\infty}^{\infty} |H(f)|^2 df} = \frac{\int_{-\infty}^{\infty} |S(f)|^2 df}{\frac{N_0}{2}}
\]

\[\ldots (12)\]

From Parseval's theorem,

\[
\int_{-\infty}^{\infty} |S(f)|^2 df = \int_{-\infty}^{\infty} s^2(t) dt = \text{signal energy} = E
\]

\[\ldots (13)\]

Therefore we have

\[
R_f \leq \frac{2E}{N_0}
\]

\[\ldots (14)\]

The frequency-response function which maximizes the peak-signal-to-mean-noise ratio \( R_f \) may be obtained by noting that the equality sign in Eq. (10.11) applies when \( P = kQ \), or,

\[
H(f) = G_a S^*(f) \exp(-j2\pi ft_1)
\]

\[\ldots (15)\]

where the constant \( k \) has been set equal to \( 1/G_a \).

The interesting property of the matched filter is that no matter what the shape of the input-signal waveform, the maximum ratio of the peak signal power to the mean noise power is simply twice the energy \( E \) contained in the signal divided by the noise power per hertz of bandwidth \( N_0 \). The noise power per hertz of bandwidth, \( N_0 \), is equal to \( kT_0F \) where \( k \) is the Boltzmann constant, \( T_0 \) is the standard temperature (290 K), and \( F \) is the noise figure.

**The matched filter and the correlation function:**

The output of the matched filter is not a replica in shape of the input signal. However, from the point of view of detecting signals in noise, preserving the shape of the signal is not of importance. If it is necessary to preserve the shape of the input pulse rather than maximize the output signal-to-noise ratio, some other criterion must be employed.

The output of the matched filter may be shown to be proportional to the input signal cross-correlated with a replica of the transmitted signal, except for the time delay \( t_1 \). The cross-correlation function \( R(t) \) of two signals \( y(\lambda) \) and \( s(\lambda) \), each of finite duration, is defined as:
\[ R(t) = \int_{-\infty}^{\infty} y(\lambda)s(\lambda - t) \, d\lambda \quad .... (16) \]

The output \( y_0(t) \) of a filter with impulse response \( h(t) \) when the input is \( y_{in}(t) = s(t) + n(t) \) is

\[ y_0(t) = \int_{-\infty}^{\infty} y_{in}(\lambda)h(t - \lambda) \, d\lambda \quad .... (17) \]

If the filter is a matched filter, then \( h(\lambda) = s(t - \lambda) \) and Eq. (17) becomes

\[ y_0(t) = \int_{-\infty}^{\infty} y_{in}(\lambda)s(t_1 - t + \lambda) \, d\lambda = R(t - t_1) \quad .... (18) \]

Thus the matched filter forms the cross correlation between the received signal corrupted by noise and a replica of the transmitted signal. The replica of the transmitted signal is "built in" to the matched filter via the frequency-response function. If the input signal \( y_{in}(t) \) were the same as the signal \( s(t) \) for which the matched filter was designed (that is, the noise is assumed negligible), the output would be the auto correlation function. The auto correlation function of a rectangular pulse of width \( \tau \) is a triangle whose base is of width \( 2\tau \).

**Cross correlation receiver: (correlation detection)**

Equation (10.18) describes the output of the matched filter as the cross correlation between the input signal and a delayed replica of the transmitted signal. This implies that the matched-filter receiver can be replaced by a cross-correlation receiver that performs the same mathematical operation as shown in Fig. 2. The input signal \( y(t) \) is multiplied by a delayed replica of the transmitted signal \( s(t - T_r) \) and the product is passed through a low-pass filter to perform the integration.

![Figure 2: Block diagram of a cross-correlation receiver.](image-url)

The cross-correlation receiver of Fig. 2 tests for the presence of a target at only a single time delay \( T_r \). Targets at other time delays, or ranges, might be found by varying \( T_r \). However, this requires a longer
search time. The search time can be reduced by adding parallel channels, each containing a delay line corresponding to a particular value of $T$, as well as a multiplier and low-pass filter.

Since the cross-correlation receiver and the matched-filter receiver are equivalent mathematically, the choice as to which one to use in a particular radar application is determined by which is more practical to implement. The matched-filter receiver, or an approximation, has been generally preferred in the vast majority of applications.

**Efficiency of nonmatched filters:**

In practice the matched filter cannot always be obtained exactly. It is appropriate, therefore, to examine the efficiency of nonmatched filters compared with the ideal matched filter. The measure of efficiency is taken as the peak signal-to-noise ratio from the nonmatched filter divided by the peak signal-to-noise ratio ($2E/N_0$) from the matched filter.

The efficiency depends on the type of the input signal shape and the type of filter. It is optimum for a given $B\tau$ product.

Table 10.1 shown below lists the values of $B\tau$ which maximize the signal-to-noise ratio (SNR) for various combinations of filters and pulse shapes. It can be seen that the loss in SNR incurred by use of these nonmatched filters is small.

<table>
<thead>
<tr>
<th>Input signal</th>
<th>Filter</th>
<th>Optimum $B\tau$</th>
<th>Loss in SNR compared with matched filter, dB</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rectangular pulse</td>
<td>Rectangular</td>
<td>1.37</td>
<td>0.85</td>
</tr>
<tr>
<td>Rectangular pulse</td>
<td>Gaussian</td>
<td>0.72</td>
<td>0.49</td>
</tr>
<tr>
<td>Gaussian pulse</td>
<td>Rectangular</td>
<td>0.72</td>
<td>0.49</td>
</tr>
<tr>
<td>Gaussian pulse</td>
<td>Gaussian</td>
<td>0.44</td>
<td>0 (matched)</td>
</tr>
<tr>
<td>Rectangular pulse</td>
<td>One-stage, single-tuned circuit</td>
<td>0.4</td>
<td>0.88</td>
</tr>
<tr>
<td>Rectangular pulse</td>
<td>2 cascaded single-tuned stages</td>
<td>0.613</td>
<td>0.56</td>
</tr>
<tr>
<td>Rectangular pulse</td>
<td>5 cascaded single-tuned stages</td>
<td>0.672</td>
<td>0.5</td>
</tr>
</tbody>
</table>

**Table 1:** Efficiency of nonmatched filters compared with the matched filter

**Matched filter with nonwhite noise:**

In the derivation of the matched-filter characteristic [Eq. (10.15)], the spectrum of the noise accompanying the signal was assumed to be white, i.e., it is independent of frequency. If this
assumption were not true, the filter which maximizes the output signal-to-noise ratio would not be the same as the matched filter of Eq. (10.15). It has been shown that if the input power spectrum of the interfering noise is given by \[ (N(f))_0^2 \] the frequency-response function of the filter which maximizes the output signal-to-noise ratio is:

\[
H(f) = \frac{G_x S^*(f) \exp(-j2\pi f_1)}{(N(f))_0^2} \tag{19}
\]

When the noise is nonwhite, the filter which maximizes the output signal-to-noise ratio is called the NWN (nonwhite noise) matched filter. For white noise \[ (N(f))_0^2 = \text{constant} \] and the NWN matched-filter frequency-response function of Eq. (10.19) reduces to that of Eq. (10.15). Equation (10.19) can be written as:

\[
H(f) = \frac{1}{N(f)} \times G_x \left( \frac{S(f)}{N_0(f)} \right)^* \exp(-j2\pi f_1) \tag{20}
\]

This indicates that the NWN matched filter can be considered as the cascade of two filters. The first filter, with frequency-response function \(1/N_0(f)\), acts to make the noise spectrum uniform, or white. It is sometimes called the \textit{whitening filter}. The second is the matched filter described by Eq. (10.15) when the input is white noise and a signal whose spectrum is \(S(f)/N_0(f)\).

**RADAR RECEIVERS**

**Salient features of the radar receiver:**

- Detects desired echo signals in the presence of noise, interference, or clutter.
- Must separate desired signals from undesired signals, and amplify the desired signals to a level where target information can be properly displayed or further processed.
- The design of the radar receiver depends not only on the type of waveform to be detected, but on the nature of the noise, interference, and clutter echoes with which the desired echo signals must compete.
- Good receiver design is based on maximizing the output peak signal-to-average noise ratio. To achieve this, the receiver must be designed as a matched filter, or its equivalent. The matched filter specifies the frequency-response function of the IF part of the radar receiver. Obviously, the receiver should be designed to generate as little internal noise as possible, especially in the input stages where the desired signals are the weakest.
- Receiver design is also concerned with achieving sufficient gain, phase, and amplitude stability, dynamic range, tuning, ruggedness, and simplicity.
- Protection also must be provided against overloads or saturation, and burnouts from nearby interfering transmitters.
- Timing and reference signals must be provided to properly extract and synchronize target information.
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- Receivers that operate with a transmitter whose frequency can drift need some means of automatic frequency control (AFC).
- Radars that encounter hostile countermeasures need receivers that can minimize the effects of such interference.
- Although the super regenerative, crystal video, and tuned radio frequency (TRF) receivers have been employed in radar systems, the super heterodyne has seen almost exclusive application because of its good sensitivity, high gain, selectivity, and reliability. No other receiver type has been competitive to the super heterodyne receiver.

There are many factors that enter into the design of radar receivers. However, in this chapter, the receiver design is considered mainly as a problem of extracting desired signals from noise. Noise can enter the receiver via the antenna terminals along with the desired signals, or it might be generated within the receiver itself. At the microwave frequencies usually used for radar, the external noise which enters via the antenna is generally quite low so that the receiver sensitivity is usually decided by the internal noise generated within the receiver. The measure of receiver internal noise is the noise figure.

**Noise figure and Noise temperature:**

**Noise figure:**

Noise figure of a receiver is a measure of the noise produced by a practical receiver as compared with the noise of an ideal receiver. The noise figure $F_n$ may be defined as:

$$F_n = \frac{N_{out}}{S_{in}} = \frac{N_{out}}{S_{out}} \times \frac{S_{in}}{S_{out}} = \frac{N_{out}}{kT_0 B_n G}$$

(1)

Where $S_{in}$ = available input signal power

$N_{in}$ = available input noise power (equal to $kT_bB$)

$S_{out}$ = available output signal power

$N_{out}$ = available output noise power

"Available power" refers to the power which would be delivered to a matched load. The available gain $G$ is equal to $S_{out}/S_{in}$, $k = \text{Boltzmann's constant} = 1.38 \times 10^{-23} \text{J/deg} \cdot \text{K}$, $T_b = \text{standard temperature of 290 K (approximately room temperature)}$ and $B_n$ is the noisebandwidth (defined earlier). The product $kT_b \approx 4 \times 10^{-22} \text{W/Hz}$. The purpose of defining a standard temperature is to refer any measurements to a common basis of comparison.

**Equation (1)** effectively gives two different but equivalent interpretations of **noise figure**.

- It may be considered as the degradation of the signal-to-noise ratio caused by the receiver or
- It may be interpreted asthe ratio of the actual available output noise power to the noisepower which would be available if the network merely amplified the thermal noise.

Then the noise figure may also be written as:
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\[ F_n = \frac{kT_0 B_n G + \Delta N}{kT_0 B_n G} = 1 + \frac{\Delta N}{kT_0 B_n G} \]

Where \( \Delta N \) is the additional noise introduced by the receiver itself.
The noise figure is commonly expressed in decibels, that is, \( 10 \log F_n \). The term noise factor is also used at times instead of noise figure. The two terms are now synonymous.

Noise figure of networks in cascade:

Consider two networks in cascade, each with the same noise bandwidth \( B_n \) but with different noise figures and available gain as shown in Fig. 3 below.

\[ \begin{array}{c}
F_1, G_1, B_n \\
\rightarrow \quad \rightarrow \\
F_2, G_2, B_n \\
\rightarrow \quad \rightarrow \\
N_o
\end{array} \]

**Figure 3: Two networks in cascade**

Let \( F_1 \) and \( G_1 \) be the noise figure and available gain, respectively, of the first network, and \( F_2, G_2 \) be the same parameters for the second network. Let us now find out \( F_o \), the overall noise-figure of the two circuits in cascade. From the definition of noise figure [Eq. 1] the output noise \( N_o \) of the two circuits in cascade is given by:

\[ N_o = F_o G_1 G_2 kT_0 B_n \]

which is also equal to:

Noise from network 1 at output of network 2 + Noise \( \Delta N \) introduced by network 2

Hence

\[ N_o = F_o G_1 G_2 kT_0 B_n \]

Noise from network 1 at output of network 2 + Noise \( \Delta N \) introduced by network 2

\[ N_o = kT_0 B_n F_1 G_1 G_2 + \Delta N_2 = kT_0 B_n F_1 G_1 G_2 + (F_2 - 1) kT_0 B_n G_2 \]

Or

\[ F_o = F_1 + \frac{F_2 - 1}{G_1} \]

From this we see that the contribution of the second network to the overall noise-figure may be made negligible if the gain of the first network is large. This is of importance in the design of multistage receivers. It is not sufficient that only the first stage of a low-noise receiver have a small noise figure. The succeeding stage must also have a small noise figure, or else the gain of the first stage must be big enough to swamp the noise of the succeeding stages. If the first network is not an amplifier but is a network with loss (as in a crystal mixer), the gain \( G_1 \) should be interpreted as a number less than unity.
Extending the same method, the noise figure of \( N \) networks in cascade may be shown to be

\[
F_n = F_1 + \frac{F_2 - 1}{G_1} + \frac{F_3 - 1}{G_1 G_2} + \cdots + \frac{F_N - 1}{G_1 G_2 \cdots G_{N-1}}
\]

**Noise temperature:**

The noise introduced by a network may also be expressed as an **effective Noise temperature**, \( T_e \), defined as that (fictional) temperature at the input of the network which would account for the noise \( \Delta N \) at the output. Therefore \( \Delta N = kT_e B_0 G \) and

\[
F_n = 1 + \frac{T_e}{T_0}
\]

\[
T_e = (F_n - 1)T_0
\]

The **system noise temperature** \( T_s \) is defined as the effective noise temperature of the receiver system including the effects of antenna temperature \( T_0 \) (it is also sometimes called the system operating noise temperature) if the receiver effective noise temperature is \( T_0 \), then

\[
T_s = T_0 + T_e = T_0 F_s
\]

where \( F_s \) is the system noise-figure including the effect of antenna temperature.

The effective noise temperature of a receiver consisting of a number of networks in cascade is

\[
T_e = T_1 + \frac{T_2}{G_1} + \frac{T_3}{G_1 G_2} + \cdots + \frac{T_N}{G_1 G_2 \cdots G_{N-1}}
\]

where \( T_i \) and \( G_i \) are the effective noise temperature and gain of the \( i \) th network.

The effective noise temperature and the noise figure both describe the same characteristic of a network.

In general, the effective noise temperature has been preferred for describing low-noise devices, and the noise figure is preferred for conventional receivers. For radar receivers, the noise figure is the more widely used term, and is what is used in this text.

**Measurement of noise figure:**

The noise figure of a radar receiver can degrade during the period of operation of the Radar and reduce its sensitivity. Therefore must be monitored regularly while the Radar is in operation, so that a reduction of receiver sensitivity can be detected and corrected. The monitoring of the noise figure can be accomplished either automatically or manually by the operator.

The receiver noise-figure can be measured with a broadband noise source of known power level, such as a gas-discharge tube or a solid-state noise source. The noise figure is determined by measuring

1. The noise power output \( N_0 \) of the receiver when a matched impedance at temperature \( T_0 = 290K \) is connected to the receiver input and
2) The noise power output $N_2$ when a matched noise generator of temperature $T_2$ is connected to the receiver input.

The temperature $T_2$ is the equivalent noise temperature of the broadband noise generator. The noise figure can be shown to be

$$F_n = \frac{T_2/T_0 - 1}{Y - 1}$$

where $Y = N_2/N_1$.

The measurement of noise figure can be made during radar operation without degrading the receiver sensitivity by pulse-modulating the noise source in synchronism with the radar trigger and injecting the noise into the receiver during the "flyback" or "dead time" of the radar transmitter, just prior to the triggering of the next transmitter pulse. The measurement of the receiver output with the noise source ON($N_2$) and the noise source OFF($N_1$) can be made on alternate pulse periods.

Displays:

The purpose of the display is to visually present the information contained in the radar echo signal in a form suitable for operator interpretation and action. When the display is connected directly to the video output of the receiver, the information displayed is called raw video. This is the "traditional" type of radar presentation. When the receiver video output is first processed by an automatic detector or automatic detection and tracking processor (ADT), the output displayed is called synthetic video.

The cathode-ray tube (CRT) has been almost universally used as the radar display. There are two basic cathode-ray tube displays. One is the deflection-modulated CRT, such as the A-scope, in which a target is indicated by the deflection of the electron beam. The other is the intensity modulated CRT such as the PPI, in which a target is indicated by intensifying the electron beam and presenting a luminous spot on the face of the CRT.

In general, deflection-modulated displays have the advantage of simpler circuits than those of intensity-modulated displays, and targets may be more readily discerned in the presence of noise or interference. On the other hand, intensity-modulated displays have the advantage of presenting data in a convenient and easily interpreted form. The deflection of the beam or the appearance of an intensity-modulated spot on a radar display caused by the presence of a target is commonly referred to as a blip.

With the advent of technology in the display systems being used in other applications like computer monitors and TVs, the modern Radars now a days use the state of the art LCD and LED displays along with digital storage techniques overcoming many of the limitations of CRT displays used earlier.

Types of display presentations:

The various types of displays which were used for surveillance and tracking radars are defined as follows:
A-scope: A deflection-modulated display in which the vertical deflection is proportional to target echostrength and the horizontal coordinate is proportional to range.

B-scope: An intensity-modulated rectangular display with azimuth angle indicated by the horizontal coordinate and range by the vertical coordinate.

C-scope: An intensity-modulated rectangular display with azimuth angle indicated by the horizontal coordinate and elevation angle by the vertical coordinate.

D-scope: A C-scope in which the blips extend vertically to give a rough estimate of distance.

F-scope: A rectangular display in which a target appears as a centralized blip when the radar antenna is aimed at it. Horizontal and vertical aiming errors are respectively indicated by the horizontal and vertical displacement of the blip.

PPI, or Plan Position Indicator (also called P-scope): An intensity-modulated circular display on which echo signals produced from reflecting objects are shown in plan position with range and azimuth angle displayed in polar (rho-theta) coordinates, forming a map-like display. An offset, or off center PPI has the zero point of the time base at a position other than at the center of the display to provide the equivalent of a larger display for a selected portion of the service area.

R-scope: An A-scope with a segment of the time base expanded near the blip for greater accuracy in distance measurement.

RHI or Range-Height Indicator: An intensity modulated display with height (altitude) as the vertical axis and range as the horizontal axis.

The above definitions are taken from the IEEE Standard definition with some modifications. The terms A-scope and A-display, B-scope and B-display, etc., are used interchangeably. These letter descriptions of radar display date back to World War 2. All of them are not in current usage. However; the PPI, A-scope, B-scope, and RHI are among the more usual displays employed in radar.

Duplexers:

Introduction:

Duplexer is the device that allows a single antenna to serve both the transmitter and the receiver. During transmission it protects the receiver from burnout or damage, and on reception it channels the echo signal to the receiver. Duplexers, especially for high-power applications, sometimes employ a form of gas-discharge device. Solid-state devices are also utilized. In a typical duplexer application the transmitter peak power is typically a megawatt or more and the maximum safe power that can be tolerated at the receiver is typically less than a watt. Therefore, the duplexer must provide, in this example, more than 60 dB of isolation between the transmitter and receiver with only negligible loss of the desired signal. In addition, during the interpulse period or when the radar is shut down, the receiver must be protected from high-power radiation, such as from nearby radars, that might enter the radar antenna with less power than that needed to activate the duplexer, but with power greater than what the receiver can safely handle.

There are two basic methods employed that allow the use of a common antenna for both transmitting and receiving. The older method is represented by the branch-type duplexer and the balanced duplexer which utilize gas TR-tubes for accomplishing the necessary switching actions. The other method uses a
ferrite circulator to separate the transmitter and receiver, and a receiver protector consisting of a gas TR-tube and diode limiter.

**Branch-type duplexers:**

The branch-type duplexer, shown in Fig. 5 is one of the earliest duplexer configurations. It consists of a TR (transmit-receive) switch and an ATR (anti-transmit receive) switch, both of which are gas-discharge tubes.

When the transmitter is turned on, both the TR and the ATR tubes ionize i.e. they break down, or fire. The TR in the fired condition acts as a short circuit to prevent transmitter power from entering the receiver. Since the TR is located a quarter wavelength from the main transmission line, it appears as a short circuit at the receiver but as an open circuit at the transmission line so that it does not impede the flow of transmitter power. Since the ATR is displaced a quarter wavelength from the main transmission line, the short circuit it produces during the fired condition appears as an open circuit on the transmission line and thus has no effect on transmission.

During reception, the transmitter is OFF and neither the TR nor the ATR is fired. The open circuit of the ATR, being a quarter wave from the transmission line, appears as a short circuit across the line. Since this short circuit is located a quarter wave from the receiver branch-line, the transmitter is effectively disconnected from the line and the echo signal power is directed to the receiver. The diagram of Fig. 4 is a parallel configuration. Series or series-parallel configurations are possible.

The branch-type duplexer is of limited bandwidth and power-handling capability, and has generally been replaced by the balanced duplexer and other protection devices. It is used, in spite of these limitations, in some low-cost radars.

![Figure 4: Principle of branch-type duplexer.](image-url)
Balanced duplexers:

The balanced duplexer, Fig. 5, is based on the short-slot hybrid junction which consists of two sections of waveguides joined along one of their narrow walls with a slotted cut in the common narrow wall to provide coupling between the two. The short-slot hybrid may be considered as a broadband directional coupler with a coupling ratio of 3 dB.

![Diagram of balanced duplexer]

Figure 5: Balanced duplexer using dual TR tubes and two short-slot hybrid junctions. (a) Transmit Condition (b) Receive condition.

In the transmit condition (Figure 5a) power is divided equally into each waveguide by the first short slot hybrid junction. Both TR tubes break down and reflect the incident power out the antennaarm as shown. The short-slot hybrid has the property that each time the energy passes through the slot in either direction, its phase is advanced 90°. Therefore, the energy travels as indicated by the solid lines. Any energy which leaks through the TR tubes (shown by the dashed lines) is directed to the arm with the matched dummy load and not to the receiver. In addition to the attenuation provided by the TR tubes, the hybrid junctions provide an additional 20 to 30 dB of isolation.

On reception the TR tubes are unfired and the echo signals pass through the duplexer and into the receiver as shown in Fig.5b. The power splits equally at the first junction and because of the 90° phase advance on passing through the slot, the energy recombines in the receiving arm and not in the dummy load arm.

The power-handling capability of the balanced duplexer is inherently greater than that of the branch-type duplexer and it has wide bandwidth, over ten percent with proper design. A receiver protector, is usually inserted between the duplexer and the receiver for added protection.
Circulator and receiver protector: The ferrite circulator is a three- or four-port device that can in principle, offer separation of the transmitter and receiver without the need for the conventional duplexer configurations explained earlier. The circulator does not provide sufficient protection by itself and requires a receiver protector as in Fig. 7. The isolation between the transmitter and receiver ports of a circulator is seldom sufficient to protect the receiver from damage. However, it is not the isolation between transmitter and receiver port that usually determines the amount of transmitter power at the receiver, but the impedancemismatch at the antenna which reflects transmitter power back into the receiver. The VSWR is a measure of the amount of power reflected by the antenna. For example, a VSWR of 1.5 means that about 4 percent of the transmitter power will be reflected by the antenna mismatch in the direction of the receiver, which corresponds to an isolation of only 14 dB. About 11 percent of the power is reflected when the VSWR is 2.0, corresponding to less than 10 dB of isolation. Thus, a receiver protector is almost always required. It also reduces to safe level radiations from nearby transmitters. The receiver protector might use solid-state diodes for an all solid-state configurationor it might be a passive TR-limiter consisting of a radioactive primed TR-tube followed by a diode limiter. The ferrite circulator with receiver protector is attractive for radar applications because of its long life, wide bandwidth, and compact design.

Introduction to phased array antennas:

- The phased array is a directive antenna made up of individual radiating antennas, or elements, which generate a radiation pattern whose shape and direction is determined by the relative phases and amplitudes of the currents at the individual elements.
- By properly varying the relative phases it is possible to steer the direction of the radiation.
The radiating elements might be dipoles, open-ended waveguides, slots cut in waveguide, or any other type of antenna.

Became of interest to Radar due to the inherent flexibility it has offered in steering the beam by means of electronic control rather than by physical movement of the antenna.

It has been considered in those radar applications where it is necessary to shift the beam rapidly from one position in space to another, or where it is required to obtain information about many targets at a flexible, rapid data rate.

The full potential of a phased-array antenna requires the use of a computer that can determine in real time, on the basis of the actual operational situation, how best to use the capabilities offered by the array.

Initially in World War 2, the United States, Great Britain, and Germany used radar with fixed phased-array antennas in which the beam was scanned by mechanically actuated phase shifters.

A major advance in phased array technology was made in the early 1950s with the replacement of mechanically actuated phase shifters by electronic phase shifters.

The introduction of digitally switched phase shifters in the early 1960s made a significant improvement in the practicality of phased arrays that could be electronically steered in two orthogonal angular coordinates.

Basic concepts:

- An array antenna consists of a number of individual radiating elements suitably spaced with respect to one another. The relative amplitude and phase of the signals applied to each of the elements are controlled to obtain the desired radiation pattern from the combined action of all the elements.

- Two common geometrical forms of array antennas used in radar are the linear array and the planar array. A linear array consists of elements arranged in a straight line in one dimension. A planar array is a two-dimensional configuration of elements arranged to lie in a plane. The planar array may be thought of as a linear array of linear arrays.

- A broadside array is one in which the direction of maximum radiation is perpendicular, or almost perpendicular to the line (or plane) of the array.

- An end fire array has its maximum radiation parallel to the array. The end fire array is a special case of the linear or the planar array when the beam is directed along the array. End fire linear arrays have not been widely used in radar applications. They are usually limited to low or medium gains since an end fire linear antenna of high gain requires an excessively long array.

- The two-dimensional planar array is the most commonly used in radar applications since it is fundamentally the most versatile of all radar antennas. A rectangular aperture can produce a fan shaped beam. A square or a circular aperture produces a pencil beam. The array can be made to simultaneously generate many search and/or tracking beams with the same aperture.

- An array in which the relative phase shift between elements is controlled by electronic devices is called an electronically scanned array. In an electronically scanned array the antenna elements, the transmitters, the receivers, and the data-processing portions of the radar are often designed as a unit.
Radiation pattern:

Consider a linear array made up of \( N \) elements equally spaced a distance \( d \) apart shown in Fig. 8. The elements are assumed to be isotropic point sources radiating uniformly in all directions with equal amplitude and phase. Although isotropic elements are not realizable in practice, they are a useful concept in array theory, especially for the computation of radiation patterns. The array is shown as a receiving antenna for convenience, but because of the reciprocity principle, the results obtained apply equally well to a transmitting antenna. The outputs of all the elements are summed via lines of equal length to give a sum output voltage \( E_0 \). Element 1 will be taken as the reference signal with zero phase. The difference in the phase of the signals in adjacent elements is \( \psi = 2\pi \frac{d}{\lambda} \sin \theta \), where \( \theta \) is the direction of the incoming radiation. The amplitudes of the voltages in each element are assumed to be the same and, for convenience, will be taken to be unity. The sum of all the voltages from the individual elements, when the phase difference between adjacent elements is \( \psi \), can be written as

\[
E_a = \sin \omega t + \sin (\omega t + \psi) + \sin (\omega t + 2\psi) + \cdots + \sin (\omega t + (N - 1)\psi)
\]

where \( \omega \) is the angular frequency of the signal. The sum can be written

\[
E_a = \sin \left( \omega t + (N - 1)\frac{\psi}{2} \right) \sin \left( \frac{N\psi}{2} \right) \sin \left( \frac{\psi}{2} \right)
\]

Figure 7: N-element linear array.

The first factor is a sine wave of frequency \( \omega \) with a phase shift \( (N - 1) \frac{\psi}{2} \). The second term represents the amplitude factor of the form \( \sin (N\psi/2)/\sin (\psi/2) \). The field intensity pattern is the magnitude of the equation 2, or

\[
|E_a(\theta)| = \left| \frac{\sin \left[ N\pi\frac{d}{\lambda} \sin \theta \right]}{\sin \left[ \pi\frac{d}{\lambda} \sin \theta \right]} \right|
\]

\[\text{unit-5: Detection of Radar signals in noise}\]
The pattern has nulls when the numerator is zero. For discrete aperture antennas (such as phased arrays) in which the element spacing is greater than a half wavelength, the spatial aliasing effect causes some sidelobes to become substantially larger in amplitude, and approaching the level of the main lobe; these are called grating lobes, and they are identical, or nearly identical to the main beams.

The radiation pattern is equal to the normalized square of the amplitude, or

$$G_d(\theta) = \frac{|E_d|^2}{N^2} = \frac{\sin^2 \left[ N\pi(d/\lambda) \sin \theta \right]}{N^2 \sin^2 \left[ \pi(d/\lambda) \sin \theta \right]} \quad \ldots \ldots \ [4]$$

When directive elements are used, the resultant array antenna radiation pattern is

$$G(\theta) = G_e(\theta) \frac{\sin^2 \left[ N\pi(d/\lambda) \sin \theta \right]}{N^2 \sin^2 \left[ \pi(d/\lambda) \sin \theta \right]} = G_e(\theta)G_d(\theta) \quad \ldots \ldots \ [5]$$

where $G_e(\theta)$ is the radiation pattern of an individual element. The resultant radiation pattern is the product of the element factor $G_e(\theta)$ and the array factor $G_d(\theta)$, the latter being the pattern of an array composed of isotropic elements. The array factor has also been called the space factor. Grating lobes caused by a widely spaced array may therefore be eliminated with directive elements which radiate little or no energy in the directions of the undesired lobes. For example, when the element spacing $d = 2\lambda$, grating lobes occur at $\theta = \pm 30^\circ$ and $\pm 60^\circ$ in addition to the main beam at $\theta = 0^\circ$. If the individual elements have a beamwidth somewhat less than $60^\circ$, the grating lobes of the array factor will be suppressed.

In a two-dimensional, rectangular planar array, the radiation pattern may sometimes be written as the product of the radiation patterns in the two planes which contain the principal axes of the antenna. If the radiation patterns in the two principal planes are $G_1(\theta_x)$ and $G_2(\theta_y)$ the two-dimensional antenna pattern is

$$G(\theta_x, \theta_y) = G_1(\theta_x)G_2(\theta_y) \quad \ldots \ldots \ [6]$$

Thus, the normalized radiation pattern of a uniformly illuminated rectangular array is

$$G(\theta_x, \theta_y) = \frac{\sin^2 \left[ N\pi(d/\lambda) \sin \theta_x \right]}{N^2 \sin^2 \left[ \pi(d/\lambda) \sin \theta_x \right]} \cdot \frac{\sin^2 \left[ M\pi(d/\lambda) \sin \theta_y \right]}{M^2 \sin^2 \left[ \pi(d/\lambda) \sin \theta_y \right]} \quad \ldots \ldots \ [7]$$
Where \( N \) = number of radiating elements in \( \theta_a \) dimension with spacing \( d \) and \( M \) the number in \( \theta_e \) dimension.

**Beam steering and beam width changes:**

**Beam steering:**

The beam of an array antenna may be steered rapidly in space without physically moving large antennae by properly varying the phase of the signals applied to each element. Consider an array of equally spaced elements. The spacing between adjacent elements is \( d \), and the signals at each element are assumed to be of equal amplitude. If the same phase is applied to all elements, the relative phase difference between adjacent elements is zero and the position of the main beam will be broadside to the array at an angle \( \theta = 0 \). The main beam will point in a direction other than broadside if the relative phase difference between elements is other than zero. The direction of the main beam is at an angle \( \theta_0 \), when the phase difference is \( \phi = 2\pi \left( \frac{d}{\lambda} \right) \sin \theta_0 \). The phase at each element is therefore \( \phi + m \phi \) where \( m = 0, 1, 2, \ldots, (N - 1) \) and \( \phi \) is any constant phase applied to all elements. The normalized radiation pattern of the array when the phase difference between adjacent elements is \( \phi \) is given by:

\[
G(\theta) = \frac{\sin^2 \left[ N\pi(d/\lambda)(\sin \theta - \sin \theta_0) \right]}{N^2 \sin^2 \left[ \pi(d/\lambda)(\sin \theta - \sin \theta_0) \right]}
\]

(8)

The maximum of the radiation pattern occurs when \( \sin \theta = \sin \theta_0 \).

Equation (8) states that the main beam of the antenna pattern may be positioned to an angle \( \theta_0 \) by the insertion of the proper phase shift \( \phi \) at each element of the array. If variable, rather than fixed, phase shifters are used, the beam may be steered as the relative phase between elements is changed (Fig. 2) below.

![Figure 8: Steering of an antenna beam with variable phase shifters (parallel-fed array).](image)

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Change of beamwidth with steering angle:

The half-power beamwidth in the plane of scan increases as the beam is scanned off the broadside direction. The beamwidth is approximately inversely proportional to $\cos \theta_0$, where $\theta_0$ is the angle measured from the normal to the antenna.

This is proved by assuming that the sine in the denominator of Eq. (8) can be replaced by its argument, so that the radiation pattern is of the form $(\sin^2 u)/u^2$, where $u = N\pi(d/\lambda)(\sin \theta - \sin \theta_0)$. The $(\sin^2 u)/u^2$ antenna pattern is reduced to half its maximum value when $u = \pm 0.443$. Let us denote the angle corresponding to the half-power point when $\theta > \theta_0$ by $\theta_+$ and the angle corresponding to the half-power point when $\theta < \theta_0$ by $\theta_-$. That means $\theta_+$ corresponds to $u = +0.443\pi$ and $\theta_-$ corresponds to $u = -0.443\pi$.

By using these values and after further mathematical manipulations we can show that the half-power beamwidth $\theta_B$ is given by:

$$\theta_B = \theta_+ - \theta_- \approx \frac{0.886\lambda}{Nd \cos \theta_0}$$  \[13\]

Therefore, when the beam is positioned an angle $\theta_0$ off broadside, the beamwidth in the plane of scan increases as $(\cos \theta_0)^{-1}$.

Applications of the array in radar:

Though there are different types/forms of array antennas, the phased-array antenna alone has been of considerable interest to the radar systems engineer because its properties are different from those of the other microwave antennas.

The phased array antenna has seen application in radar for a wide variety of purposes:

- Aircraft surveillance from on board ship (AN/SPS-33)
- Satellite surveillance (AN/FPS-85)
- Ballistic missile defense (PAR, MSR)
- Air defense (AN/SPY-1 and Patriot)
- Aircraft landing systems (AN/TPN-19 and AN/TPS-32)
- Mortar (AN/TPQ-36) and artillery (AN/TPQ-37) location
- Tracking of ballistic missiles (Cobra Dane) and airborne bomber radar (EAR).

Many developmental array radars have been developed and built in USA. Although much effort and funds have been spent on this activity, except for limited-scan arrays there has not been any large serial production of such radars compared to the serial production of radars with mechanically rotating reflector antennas.
Advantages and limitations:

Advantages:

Inertia less rapid beam steering. The beam from an array can be scanned, or switched from one position to another, in a very short time limited only by the switching speed of the phase shifters. Typically, the beam can be switched in several microseconds, but it can be considerably shorter if desired.

Multiple, Independent beams. A single aperture can generate many simultaneous independent beams. Alternatively, the same effect can be obtained by rapidly switching a single beam through a sequence of positions.

Potential for large peak and for average power. If necessary, each element of the array can be fed by a separate high-power transmitter with the combining of the outputs made in “space” to obtain a total power greater than that can be obtained from a single transmitter.

Control of the radiation pattern. A particular radiation pattern may be more readily obtained with the array than with other microwave antennas since the amplitude and phase of each array element may be individually controlled. Thus, radiation patterns with extremely low sidelobes or with a shaped main beam may be achieved conveniently. Separate monopulse sum and difference patterns, each with its own optimum shape, can also be generated.

Graceful degradation. The distributed nature of the array means that it can fail only gradually and not at once (catastrophically).

Convenient aperture shape. The shape of the array permits flush mounting and it can be strengthened to resist blast.

Electronic beam stabilization. The ability to steer the beam electronically can be used to stabilize the beam direction when the radar is on an unstable platform, such as a ship or aircraft that is subject to roll, pitch, and yaw disturbances.

Limitations:

- The above advantages come for a price, and hence must be considered only when warranted. It is not obvious that they are always absolutely essential for the success of a particular application. For example, it is certainly true that a mechanically scanned reflector antenna cannot switch a beam from one direction to another as fast as a phased-array antenna. However, very rarely we may have such fast switching requirements and for our application a simple mechanically scanned antenna might be adequate in which case such a high cost electronically steered antenna might not be necessary.
• An N-element array can, in principle, generates N independent beams. However, in practice it is very rarely required that a radar generate more than a few simultaneous beams (perhaps no more than a dozen), since the complexity of the array radar increases with increasing number of beams.
• Although the array has the potential for radiating large power, it is rare that an array is required to radiate more power than
  ▪ That can be radiated by other antenna types or
  ▪ That can be generated by current high-power microwave tube technology that feeds a single transmission line.
• Conventional microwave antennas cannot generate radiation patterns with side lobes as low as can be obtained by an array antenna, especially a non-scanning array. However, when a planar array is electronically scanned, the change of mutual coupling that accompanies a change in beam position makes the maintenance of low side lobes more difficult.
• If an array has some margin in performance to permit graceful degradation, it is likely that this margin will be dispensed with during the procurement process if the cost of the radar escalates. Even if the radar is delivered with margin for graceful degradation, it is likely that after some time in operation it will always be at the degraded level because of a tendency to keep maintenance costs to a minimum. Another problem is to know when graceful degradation has gone too far beyond essential maintenance requirement.
• The full testing of an array radar system is often more complicated than with conventional radar systems.
• The major limitation that has limited the wide spread use of the conventional phased array in radar is its high cost, which is due to its complexity. The software for the computer system that is needed to utilize the inherent flexibility of the array radar also contributes significantly to the system cost and complexity.

Previous years’ Examination Questions:

1. (a) Explain the basic concept of phased array antennas.
(b) Explain characteristics of different radar displays. [8+8]
2. Discuss in detail about Matched filter Receiver with necessary expressions. [16]

3. (a) Explain the functioning and characteristics of PPI display and A-Scope. [8]
4. Derive the impulse response of a matched filter that is commonly used in a radar receiver. [16]
5. (a) Draw the structures of balanced duplexer during transmission and reception modes.
(b) List out the merits and demerits of phased array antennas. [8+8]
6. (a) Draw and explain the radiation pattern of phased array antennas.

(b) Write notes on various antenna parameters with reference to radar. [8+8]

7. Discuss the relation between the matched filter characteristics and correlation detection. [16]
8. (a) What is meant by correlation? Explain cross correlation with the help of a neat block diagram
(b) Derive the expression for the frequency response of a Matched filter receiver with non white noise input.

9.(a) Explain how the beam width of a Phased array antenna varies with the steering angle.
(b) What is a Duplexer and explain the principle of operation of typical Duplexer with a schematic diagram.

10.(a) Explain the principle and characteristics of a Matched filter. Hence derive the expression for it’s frequency response function
(b) Briefly explain about the efficiency of the nonmatched filters

11.(a) Briefly explain the concept of beam steering of Phased array antennas
(b) Derive an expression for the effective Noise figure of two cascaded networks.

12.(a) Establish the impulse response characteristic for a matched filter.

13. (a) Define noise figure. Derive the expression for the noise figure of two networks that are in cascade.

14. (a) Derive the frequency response function of the matched filter.
(b) Explain about the efficiency of nonmatched filters.

15. (a) Explain the principle of balanced duplexer.
(b) Write a detailed note on Matched filter Receiver

16. Explain about correlation function and cross correlation Receiver

17. Write short notes on the following:
   I. Phased array Radar
   II. Any two types of Radar Displays.