

# **Electrical Machines -1**

## **Lecture Notes**

**Prepared By**

**Dr.Karimulla Peerla Shaik**

**Associate Professor**

**Department of EEE**



**Department of Electrical & Electronics Engineering**

**Malla Reddy College of Engineering & Technology**

**Maisammaguda, Dhullapally, Secunderabad-500100**

**2022-23**

## **CONTENTS**

### ***Syllabus:***

**UNIT-I: D.C GENERATORS**

**UNIT-II: D.C. MOTORS**

**UNIT-III: TESTING OF D.C. MACHINES**

**UNIT-IV: SINGLE PHASE TRANSFORMERS**

**UNIT-V: TESTING OF TRANSFORMERS AND POLY-PHASE  
TRANSFORMERS**

(R20A0202) ELECTRICAL MACHINES - I

**COURSE OBJECTIVES:**

1. To understand the basic working principle, constructional details and operational features of DC Generators.
2. To study and understand the various characteristics DC Generators.
3. To learn the various methods of speed control of DC Motors.
4. To introduce the concept of regulation and its calculations.
5. To learn the concepts of single phase and three phase transformers circuits

UNIT – I

D.C GENERATORS: Principle of operation - constructional features - Action of commutator – armature windings – lap and wave windings – simplex and multiplex windings – use of laminated core – E. M.F Equation – Problems, Armature reaction – Cross magnetizing and de-magnetizing AT/pole – compensating winding - Commutation – reactance voltage – methods of improving commutation. Methods of Excitation – separately excited and self- excited generators – build-up of E.M.F - critical field resistance and critical speed - causes for failure to self-excite and remedial measures. Load characteristics of shunt, series and compound generators

UNIT – II

D.C. MOTORS: Principle of operation – Back E.M.F. - Torque equation – characteristics and application of shunt, series and compound motors – Armature reaction and commutation. Speed control of D.C. Motors - Armature voltage and field flux control methods. Motor starters (3 point and 4-point starters).

UNIT – III

TESTING OF D.C. MACHINES: Losses – Constant & Variable losses – calculation of efficiency condition for maximum efficiency. Methods of Testing – direct, indirect and regenerative testing – brake test – Swinburne’s test – Hopkinson’s test – Field’s test – Retardation test – separation of *stray losses in a DC Motor test*.

UNIT – IV

SINGLE PHASE TRANSFORMERS: Principle of operation - constructional features -Types - minimization of hysteresis and eddy current losses- EMF equation - operation on no load and on load - phasor diagrams. Equivalent circuit - losses and efficiency – regulation - All day efficiency - effect of variations of frequency & supply voltage on iron losses.

UNIT – V

TESTING OF TRANSFORMERS AND POLY-PHASE TRANSFORMERS: OC and SC tests - Sumpner’s test - predetermination of efficiency and regulation-separation of losses test- parallel operation with equal and unequal voltage ratios - auto transformers- equivalent

circuit - comparison with two winding transformers. Poly-phase transformers – Poly-phase connections -  $Y/Y$ ,  $Y/\Delta$ ,  $\Delta/Y$ ,  $\Delta/\Delta$  and open  $\Delta$

TEXT BOOKS:

1. Electric machinery – A.E. Fitzgerald, C.Kingsley and S.Umans, Mc Graw Hill Companies, 2004.
2. A. E. Clayton and N. N. Hancock, “Performance and design of DC machines”, CBS Publishers, 2004.
3. M. G. Say, “Performance and design of AC machines”, CBS Publishers, 2002.

REFERENCE BOOKS:

1. Electric Machinery Fundamentals, Stephen J. Chapman, Tata Mc Graw –Hill Publishers.
2. A. S. Langsdorf, “Alternating current machines”, McGraw Hill Education, 1984.
3. P. C. Sen, “Principles of Electric Machines and Power Electronics”, John Wiley & Sons, 2007.
4. Electrical Machines, P.S. Bimbra, Khanna Publishers.
5. Electric Machines by I.J. Nagrath & D.P. Kothari, Tata Mc Graw –Hill Publishers.

COURSE OUTCOMES:

At the end of this course the student would get

1. Explain the Constructional features of DC Generators, DC motors and transformers.
2. Understand different excitation and starting methods of DC machines.
3. Summarize Testing of different types of DC Generators and DC motors.
4. Carry out different testing methods and assess the performance of transformers.
5. Analyze single phase and three phase transformers.

## **UNIT – I**

### **D.C GENERATORS**

#### **CONTENTS:**

- **Principle of operation- Constructional features**
- **Action of Commutator**
- **Armature windings – lap and wave windings – simplex and multiplex windings**
- **Use of laminated core –**
- **E. M.F Equation – Problems,**
- **Armature Reaction – Cross magnetizing and demagnetizing AT/pole – compensating winding**
- **Commutation – reactance voltage – methods of improving commutation.**
- **Methods of Excitation – separately excited and self- excited generators – build-up of E.M.F - critical field resistance and critical speed - causes for failure to self-excite and remedial measures.**
- **Load characteristics of shunt, series and compound generators**
  - **Important concepts and Formulae:**
  - **Illustrative examples**

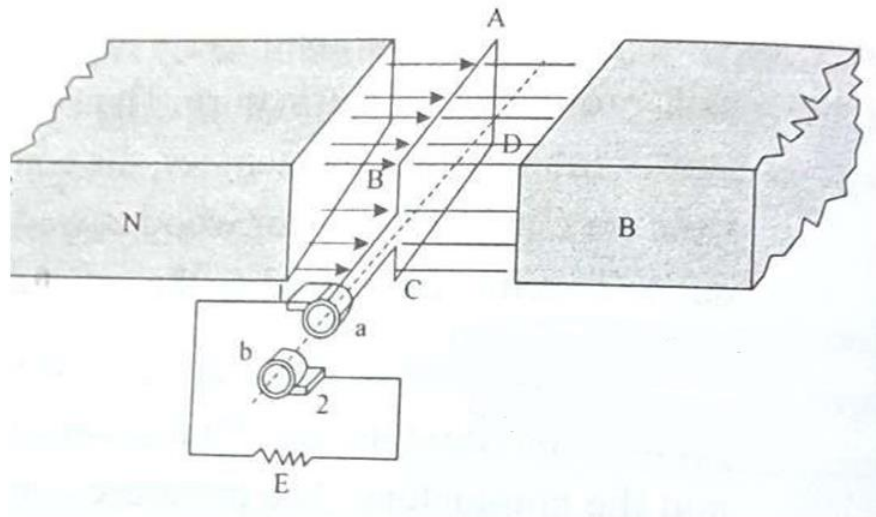
### Introduction:

A DC generator is a rotating machine which converts mechanical energy into DC electrical energy. It requires a prime mover like a Diesel engine, wind turbine or a steam turbine to rotate the DC generator. An EMF is induced in a DC Generator when there is a relative motion between a Magnetic field and a set of electrical conductors. The EMF induced is called a dynamically induced EMF or motional EMF . Normally the magnetic field is stationary and is obtained from stationary field coils placed on the Stator poles and the conductors are placed on a rotating shaft called Rotor. The basic constructional features of a DC generator and a DC Motor are same, and the same DC machine can work either as a DC generator or a DC motor.

The conversion of Mechanical energy into Electrical energy in DC generator is based on the principle of electromagnetic Induction. According to Faradays laws of Electromagnetic induction, whenever a conductor moves in a magnetic field a dynamically induced EMF is produced across the conductor. When the terminals of the conductor are connected to an electrical load the induced EMF enables a current flow through the load. Thus a mechanical energy in the form of a rotational motion given to a conductor is converted into Electrical energy. The EMF induced in a single conductor is very small. Hence a large set of conductors are used in practical generators and such a set of conductors placed on a rotating round shaft is called an armature.

### Principle of operation of DC Machines:

Let us consider a single turn of coil **ABCD** mounted on a cylindrical shaft and rotated in an anticlockwise direction at constant angular velocity of ' $\omega$ ' rad/sec within a uniform magnetic field of flux density **B webers/mtr<sup>2</sup>** as shown in the figure below .



Let  $l$  be the length and  $b$  be the breadth of the rectangular coil in meters. According to Faradays law the emf induced in a conductor is given by  $e = -N \cdot d\phi/dt$  where  $e$  is the induced emf ,  $N$  is the number of conductors ,  $\phi$  is the flux linkage and  $t$  is the time. The flux linkage  $\phi$  is given by :  $\phi = B \cdot \text{area of the coil} \cdot \cos \omega t = B \cdot l \cdot b \cdot \cos \omega t$

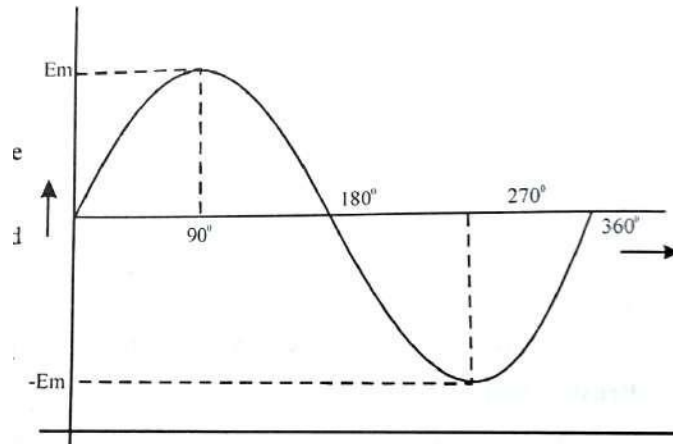
Since we are considering only one conductor the induced emf in the conductor is given by:

$$e = -d\phi/dt = -d(B \cdot l \cdot b \cdot \cos \omega t)/dt = B \cdot l \cdot b \cdot \omega \cdot \sin \omega t = E_m \sin \omega t \quad \text{where } E_m = B \cdot l \cdot b \cdot \omega$$

As can be seen from the above equation for induced emf the voltage in a given generator can be increased by either increasing the flux density ' $B$ ' or the rotational speed ' $\omega$ ' .

The induced emf ' $e$ ' at any position of the coil as a function of time ' $t$ ' as derived above is then given by :  $e = E_m \sin \omega t$  where  $E_m = B \cdot l \cdot b \cdot \omega$ . As can be seen  $d\phi/dt$  i.e rate of change of flux linkage is minimum ( $=0$ ) when the coil is at perpendicular position to the flux lines and hence the induced voltage  $e$  is also

minimum ( $=0$ ) . We will call this as position  $\Theta = 0^\circ$  at the instant of say  $t = 0$  sec. And  $d\phi/dt$  is maximum when the coil is at parallel position to the flux lines and hence the induced voltage  $e$  is also maximum(  $= E_m = B.l.b.\omega$ ) and this position will then be  $\Theta = 90^\circ$  .When  $\Theta = 180^\circ$  the induced emf is again zero and when  $\Theta = 270^\circ$  the emf induced is again maximum but now it would be negative. When  $\Theta = 360^\circ$  the coil is back to the original position and the induced emf is again equal to zero. For the two pole generator shown in the figure one complete cycle of change takes place in one rotation of the coil. A plot of the induced emf 'e' as function of coil position  $\Theta$  is an alternating voltage as shown in the figure below.

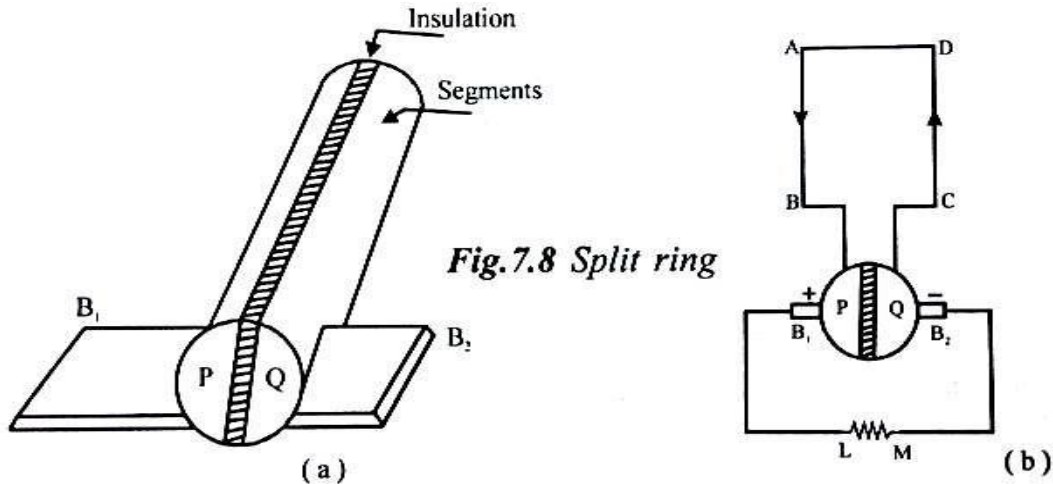


**Fig: emf induced in a single turn generator in one full revolution**

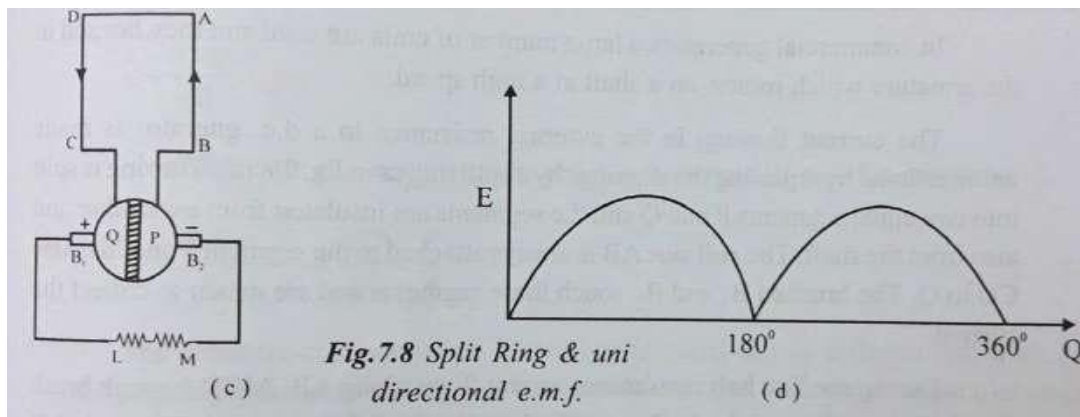
When the two terminals of the coil are connected to an external load (resistance in this case) through two separate rings (called slip rings) mounted on the armature current flows through the resistance and the current also would be sinusoidal.

**Action of commutator:** We have seen that the output from a simple single turn generator in one full revolution is a sinusoidal in nature (AC). Commutator is the most important part of a DC Generator which converts the AC to DC. The current flowing through the external load can be made unidirectional by replacing the two *slip rings* with two *split rings* as shown in the figure below which is the basis

for the operation of a commutator in a practical DC Machine with more number of Poles and multiple coils .

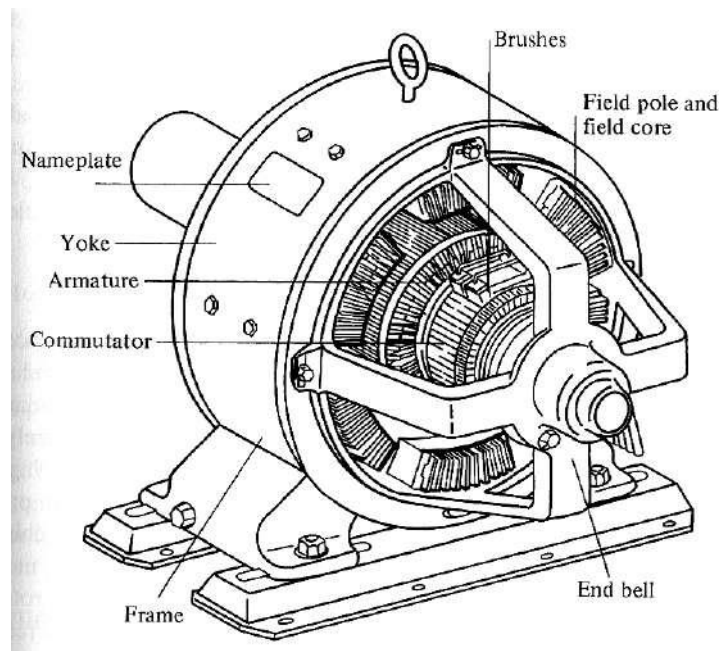


One slip ring is split into two equal segments **P** and **Q** which are insulated from each other and the armature shaft. The two coils **AB** and **CD** are connected to the two segments **P** and **Q** .Two fixed (stationary) brushes **B<sub>1</sub>** and **B<sub>2</sub>** sliding along these two split rings will be collecting the current from the generator. During the first half of the revolution segment **P** is positive and current flows along **ABPLMQCD** through brush **B<sub>1</sub>** which is positive and into brush **B<sub>2</sub>** into segment **Q** which is negative. Next during the other half cycle, the location of the segments **AB & CD** will reverse along with the respective segments **P** and **Q** . Now conductor **CD** and segment **Q** are positive and current flows along **DCQLMPBA** through the Brush **B<sub>1</sub>** which is again positive and into the brush **B<sub>2</sub>** which is again negative as shown in the figure below.



In each half revolution the positions of the conductors **AB & CD** and the segments **P & Q** reverse but the brushes **B<sub>1</sub>&B<sub>2</sub>** are stationary and continue to collect current from the Positive side and deliver current to the Negative side respectively. Hence the voltage across the load will be a unipolar voltage as shown in the waveform above. The changeover of brushes **B<sub>1</sub>&B<sub>2</sub>** between segments **P & Q** takes place when the voltage is minimum so as to avoid or minimize the arcing between the split segments. In practical generators there will be more number of conductors and also more number of Pole pairs and hence more number of split segments are required and such a set of more number of split segments is called *commutator*.

### Constructional features of a DC Generator:



**Fig: A simplified diagram of a dc machine:**

Major parts of a DC generator:

- Main frame or Yoke
- Poles
- Armature
- Commutator
- Brushes ,bearings and shaft

The physical structure of the machine consists of two parts: the stator and the rotor.

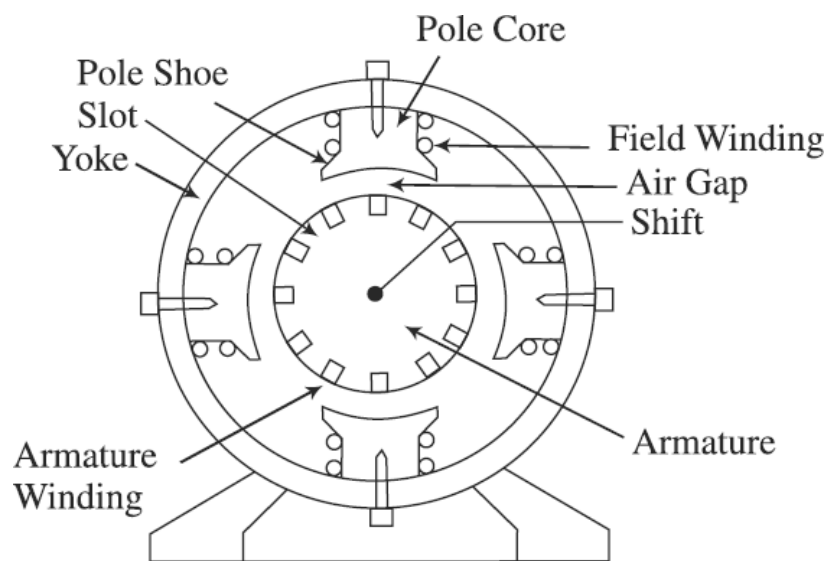
The stationary part consists of the main frame (yoke), and the pole pieces, which project inward and provide a path for the magnetic flux. The ends of the pole pieces that are near the rotor spread out over the rotor surface to distribute its flux evenly over the rotor surface. These ends are called the pole shoes. The exposed surface of a pole shoe is called a pole face, and the distance between the pole face and the rotor is the air gap.

There are two principal windings on a dc machine:

- The armature windings: the windings in which a voltage is induced (rotor)
- The field windings: the windings that produce the main magnetic flux (stator)

Because the armature winding is located on the rotor, a dc machine's rotor is mostly called an armature.

The terminal characteristic of a DC Machine is a plot of the output parameters of the Machine against each other. For a DC Generator the output quantities are the Terminal Voltage and the Line (Load) current.

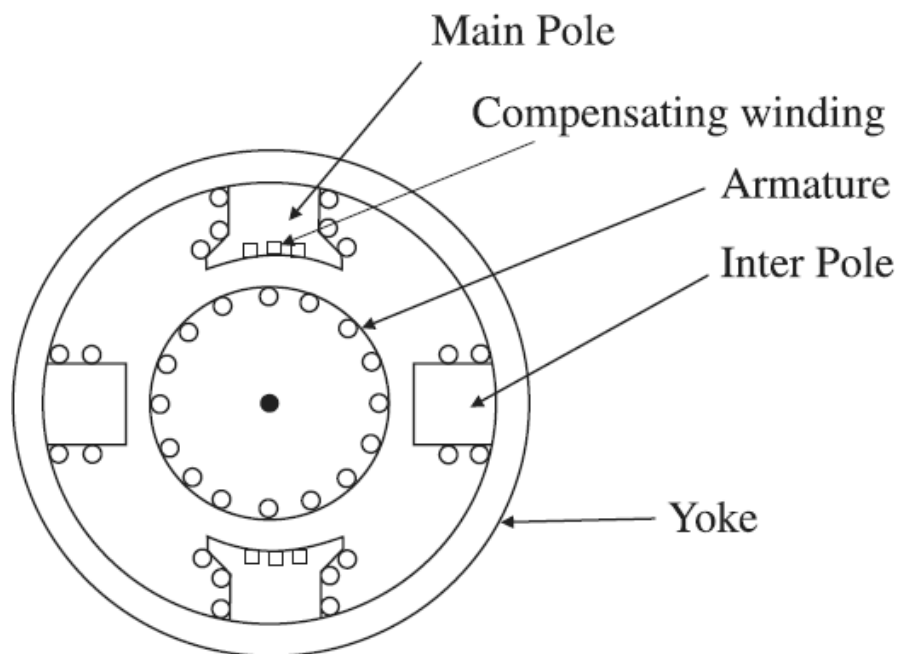


**Figure: Cross sectional view of a DC Machine**

The air-gap is kept very small to keep the reluctance of the magnetic circuit low. The armature is a laminated cylinder and is mounted on a shaft. The armature laminations are about 0.4–0.6 mm thick and are insulated from one another. The armature is laminated to reduce the eddy-current loss in the core. Slots are stamped on the periphery of the armature laminations. The armature slots house the armature windings. The stator core, the yoke and the poles may not be laminated as they encounter DC flux.

Due to the presence of slots on the armature surface, there is flux pulsation at the

Stator pole-face. The stator pole shoes, therefore, should be laminated to reduce the eddy-current loss. However, for mechanical reasons, in many cases the whole of the pole core is laminated. In DC Machines of high ratings slots are cut on the pole-faces to house a separate winding called the compensating winding. The compensating winding is connected in series with the armature winding and neutralises the effect of armature reaction. To neutralize the effect of armature reaction in the space in between two poles, smaller poles, called interpoles, are fixed on the yoke as shown in **Fig. 2.2**.



**Figure: Cross sectional view of a DC Machine showing Interpoles**

As mentioned earlier, the armature winding is placed inside the armature slots. The slots are lined with tough insulating material. This slot insulation is folded over the armature conductors. The conductors in the slots are secured in their places by hard wooden wedges or fiber glass wedges. The armature windings are first made on formers and then placed on slots.

Enamel insulated copper wires are used for the armature winding. Each armature coil end is connected with each segment of the commutator.

A commutator is a cylindrical body mounted on the shaft along with the armature. In fact, the armature core and the commutator form one single unit mounted on the shaft. Brushes are placed on the commutator surface to supply or collect current to the armature coils through the commutator segments. The commutator segments are insulated from each other.

The function of the commutator is to convert alternating currents induced in the armature conductors into direct currents in the external circuit in case of a DC Generator operation. In the case of a dc motor the function of the commutator is to produce a unidirectional torque. The commutator is of cylindrical structure and is built up of a wedge-shaped segment of hard-drawn copper. Mica insulation is provided between commutator segments. Brushes are made of carbon and are housed in brush-holders.

A spring in the brush-holder maintains the desirable pressure on the carbon brushes so that proper contact is maintained between the brushes and the commutator surface.

### **Armature windings:**

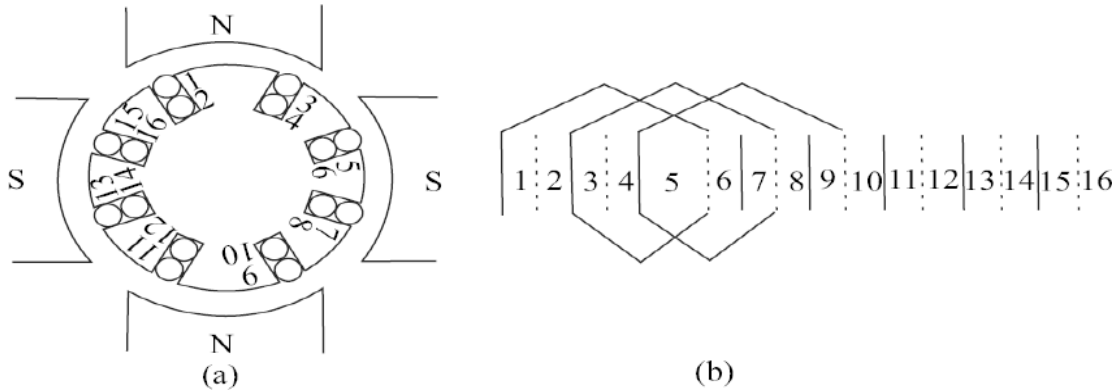
The armature windings are vital part of a DC Machine. This is where emf is induced in the case of a Generator and and force is developed that results in the turning of the rotor in the case of a Motor. The design of the armature winding is more critical than the design of other parts of a DC machine. The armature winding is housed in slots made on the armature surface. Formed coils are placed on slots. The ends of the coils are joined with commutator segments.

**Commutator:** The commutator is made up of a number of commutator segments. Coil-ends are connected to each commutator segment. The segments of the commutator are made of hard-drawn copper and are separated by thin sheets of mica or micanite ( insulator) .

The induced emf per conductor in a DC machine is small. The problem is how these conductors are to be connected together so as to form a complete winding. Figure below shows the cross-sectional view of the armature of a four-pole machine.

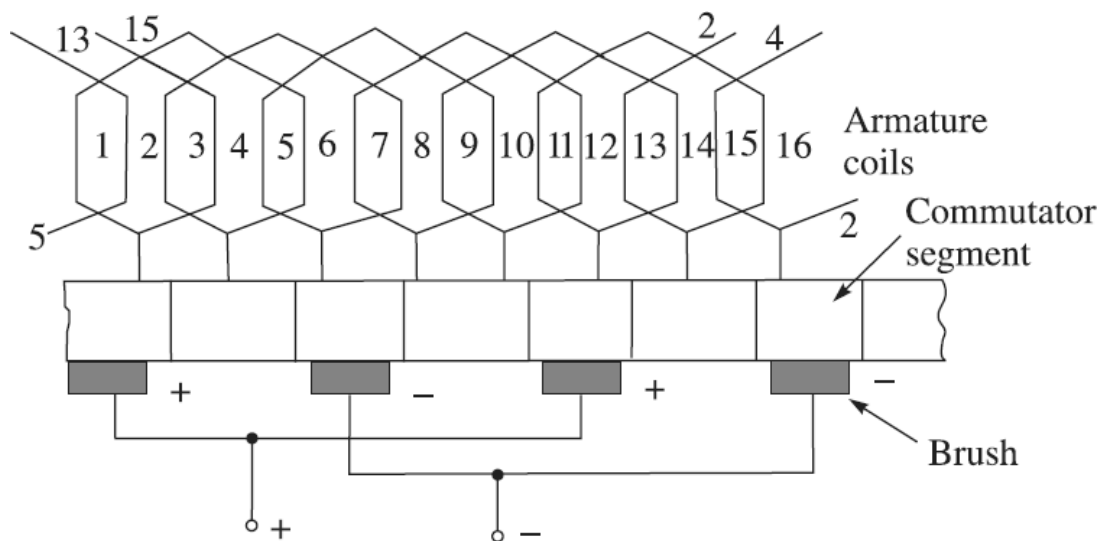
For ease of understanding, a developed diagram of armature of Fig. **(a)** below is drawn as shown in Fig. **(b)**. Conductors should be so connected that the total emf

is maximum. Therefore, conductor 1 should be connected to conductor 6 shown by dotted line as conductor 6 is placed below conductor 5 so that they occupy identical positions under two adjacent poles. Similarly conductor 3 should be connected with conductor 8 and so on.



**Figure : (a) Cross-sectional view of the armature of a 4-pole DC Machine (b) Incomplete developed diagram of the armature winding**

Figure **below** shows the developed winding diagram of the 16 armature conductors of Fig. (a) Shown earlier.



**Figure: Armature winding of a DC Machine**

The average pitch **Ya**, back pitch **Yb**, and the front pitch **Yf** are calculated as:

$$Y_a = 16/4 = 4$$

$$Y_a = (Y_b + Y_f) / 2$$

$$Y_b - Y_f = \pm 2$$

For progressive lap winding

$$Y_b - Y_f = 2$$

$$Y_b = 5, Y_f = 3$$

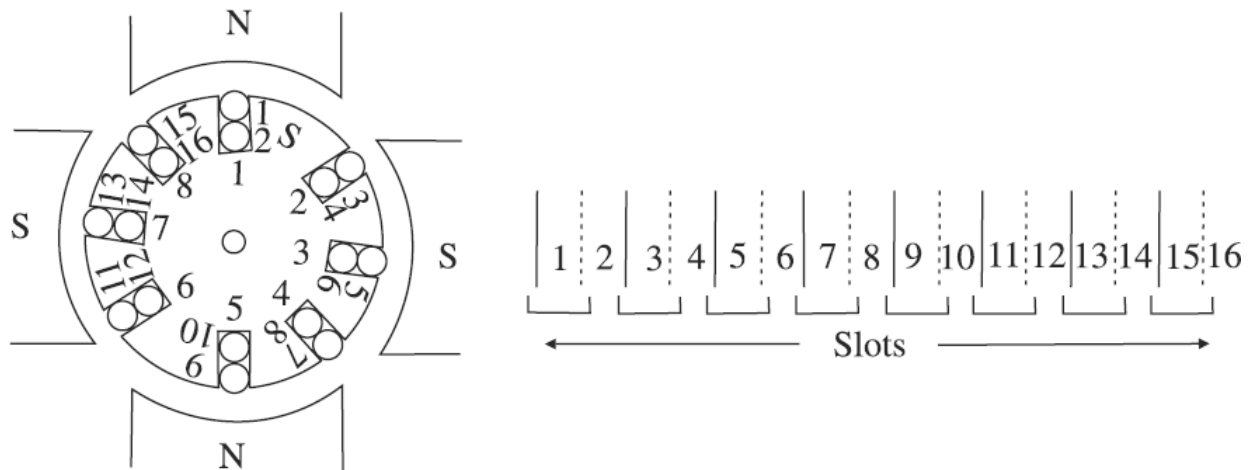
Figure gives the details of end connections of the conductors, connection of coils with commutator segments, and the position of brushes on the commutator surface with their polarities. This type of winding is called lap winding. In the winding shown in Fig. 2.11, single-turn conductors are used. As many as 16 conductors make eight coils. The coils are 1-6, 3-8, 5-10, 7-12, 9-14, 11-16, 13-2 and 15-4. The design of a lap winding of the type shown in Fig. 2.11 is described as follows.

### 2.3.2 Lap Winding

In a lap winding, the finishing end of one coil is connected via the commutator segment to the starting end of the adjacent coil situated under the same pole. In this way all the coils are connected. The winding is known as lap winding because the sides of successive coils overlap each other (see Fig. below). A coil may consist of any number of turns. The number of slots required on the armature is equal to the number of coil sides if two coil-sides are placed in each slot. With two coil-sides in each slot, a two layer winding is obtained. While making a winding diagram in a two-layer winding, all top coil-sides are numbered odd whereas the bottom coil-sides are numbered even (shown by dotted lines) as shown in Fig.. For an eight-coil armature, therefore, eight slots are required on the armature surface. The following terminologies are required to be understood for preparing an armature winding diagram.

**Pole Pitch:** It is equal to the number of coil-sides per pole. For a single turn, eight coil, four-pole armature pole pitch is calculated as:

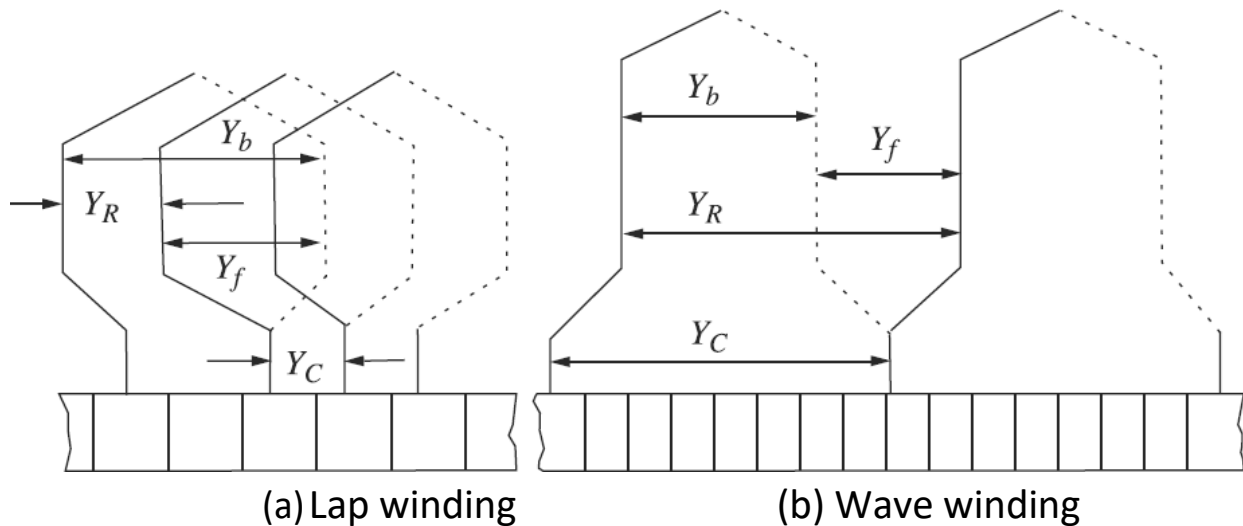
$$\text{Pole pitch ( } Y_a \text{ )} = (\text{No. of coils} \times 2) / \text{No. of poles} = (8 \times 2) / 4 = 4$$



**Figure: Position of coil-sides in slots of a two-layer armature winding**

**Coils and Coil-sides:** The DC armature windings are double-layer type having at least two coil-sides per slot. Each coil consists of an upper coil-side at the top of one slot and a lower coil-side situated at the bottom of another slot. The distance between the two coil-sides of a coil is approximately equal to the pole pitch. A coil may be of single turn or of many turns. If two coil-sides are placed in one slot, then the number of slots required on the armature for housing the coils is equal to the number of coils of the winding. For low-speed high-voltage winding, however, the number of coil-sides per slot is more than two. This is because the winding will have a large number of coils and it may not be possible to have an equal number of slots on the armature.

**Back Pitch:** The distance measured in terms of the number of armature conductors (coil sides) between the two coil-sides of a coil measured around the back of the armature, i.e., away from the commutator end of the armature is called the back pitch,  $Y_b$ .



**Figure:** Shows back pitch  $Y_b$  , front pitch  $y_f$  resultant pitch  $Y_r$ , and commutator pitch  $Y_c$  in (a) lap winding (b) wave winding

**Front Pitch:** The distance between two coil-sides connected to the same commutator segment is called the front pitch,  $Y_f$  .

**Resultant Pitch:** It is defined as the distance in terms of the number of coil-sides between the start of one coil and the start of the next coil to which it is connected.

**Commutator Pitch:** It is defined as the distance measured in terms of commutator segments between the segments to which the two ends of a coil are connected.

For calculating back pitch  $Y_b$  and front pitch  $Y_f$  for a lap winding, the following relations are used:

(i)  $Y_b - Y_f = \pm 2m$  Also,  $Y_b = (Z/P) \pm 1$

where  $m = 1$  for simplex winding

$= 2$  for duplex winding

When  $Y_b$  is greater than  $Y_f$  , the winding is a progressive one, i.e., it progresses from left to right. If  $Y_b$  is less than  $Y_f$  , the winding is called a retrogressive one, i.e., it progresses from right to left.

(ii) The back pitch and front pitch must be odd.

(iii) The average pitch,  $Y_a = (Y_b + Y_f)/2$  should be equal to the pole pitch, i.e., equal to  $Z/P$ , where  $Z$  is the number of coil sides.

- (iv) The commutator pitch is equal to  $m$ , i.e., equal to 1, 2, etc. for simplex, duplex etc. type of winding.
- (v) The number of parallel paths in the armature winding for a simplex lap winding is equal to the number of poles,  $P$ .
- (vi) The resultant pitch is always even, being the difference of two odd numbers.

**Example 1 :** Prepare a layout winding diagram for a simplex lap-type DC armature winding. The winding is for 4 poles. The armature has 16 slots and 16 commutator segments.

**Solution:** Number of armature coils = Number of commutator segments = 16

Number of coil-sides (conductors)  $Z = 16 \times 2 = 32$

Back pitch  $Y_b = (Z/P) \pm 1 = (32/4) \pm 1 = 9$  or  $7$

$Y_b - Y_f = 2$

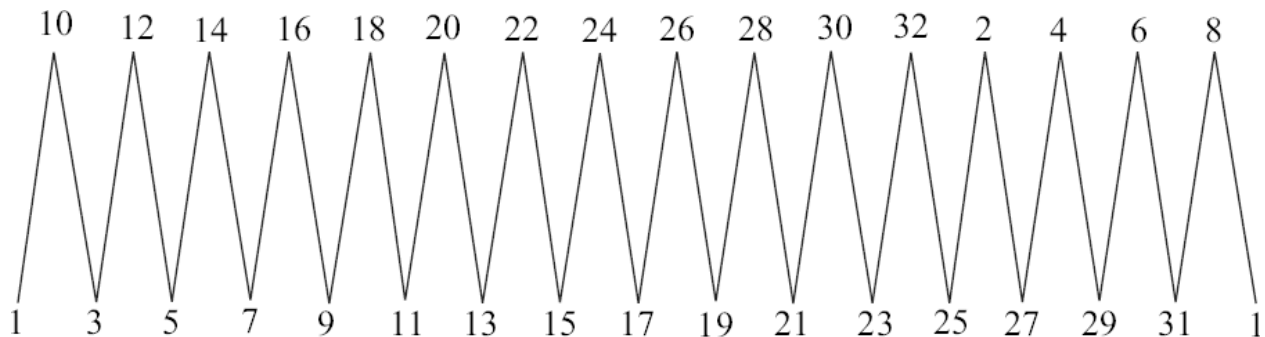
$Y_f = Y_b - 2 = 9 - 2$  (using  $Y_b = 9$ ) =  $7$

$Y_b = 9$

$Y_f = 7$

Since  $Y_b > Y_f$ , the winding is a progressive one.

As there are 32 coil-sides and 16 slots, the number of coil-sides per slot is 2. The connection scheme of the coil-sides is shown in Figure . below



**Figure: Scheme for connections of the coil-sides of a DC armature windings**

Coil-side 1 is connected to coil-side 10 on the other side of the commutator (since  $Y_b$  is 9, coil-side 1 is connected to coil-side  $1 + 9$ , i.e., 10). Coil-side 10 is connected

to coil-side 3 on the commutator end (Since  $Y_f$  is 7, coil-side 10 is connected to coil-side  $10-7$ , i.e., 3). The winding progresses according to the above scheme. It may be noted that each coil is used once and the winding is a closed one.

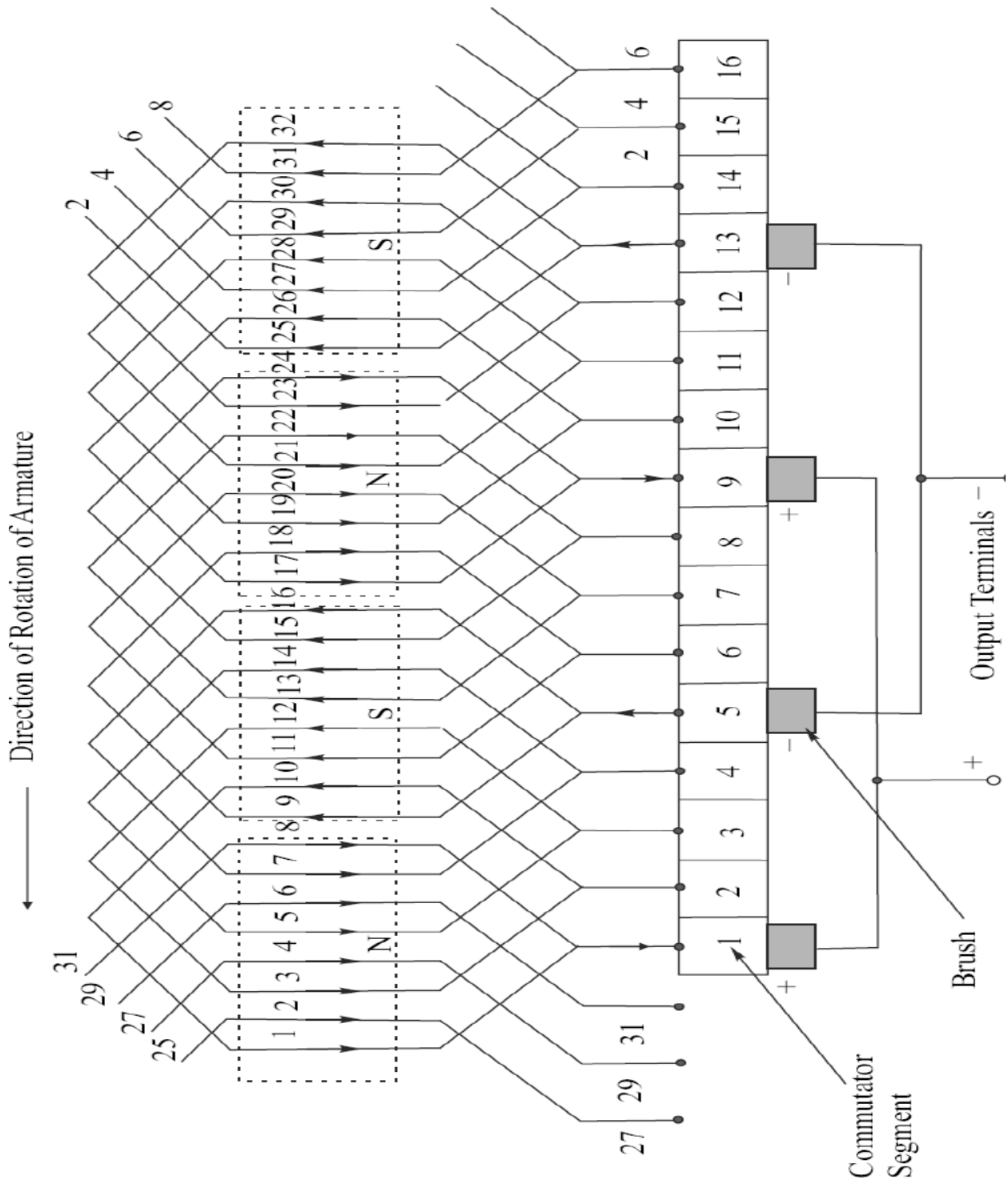
The layout diagram of the winding along with commutator connections and brush positions is shown below.

Connections of the coil-sides are made as follows: for connections at the back end of the armature, add the back pitch with the coil-side which is to be connected. Thus coil-side 1 is to be connected with coil-side  $1 + Y_b$ , i.e.,  $1 + 9 = 10$ . On the commutator end side, coil-side 10 is connected to coil-side 3. This is achieved by subtracting  $Y_f$ , i.e., 7 from coil-side number 10 ( $10 - 7 = 3$ ). Coil-side 3 is now connected to  $3 + Y_b = 3 + 9 = 12$ . In this way the winding is completed.

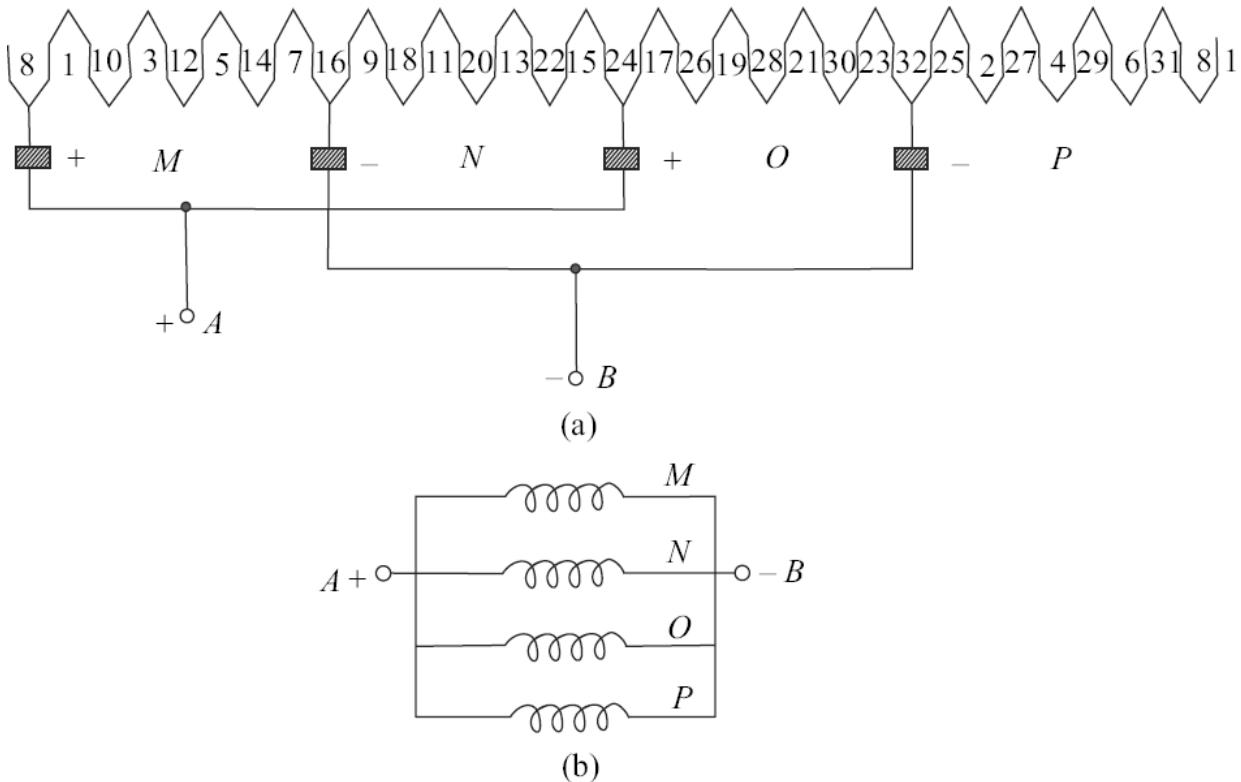
The positions of the four poles are also shown in Fig. 2.15. Eight coil-sides placed in four slots are under each pole. Assuming a direction of rotation of the armature, say anticlockwise in Fig. 2.15, the direction of the induced emf in the armature conductors is determined by applying Fleming's right-hand rule. The direction of the current in the coil-sides under north poles will be downward and under south poles upward as shown in Fig. 2.15.

The position of brushes can be determined by tracing the directions of current in various coil-sides. From Fig. 2.15, it can be observed that directions of current in coil-sides 1 and 8 are downward and they are connected to commutator segment 1. A brush placed on commutator segment 1 will have positive polarity. Similarly in coil-sides 9 and 16, the current is upwards. The two coil-sides are connected to commutator segment 5. The brush placed on commutator segment 5 will have negative polarity. Similarly the positions of the other two brushes are fixed. Two positive brushes and two negative brushes are joined together to output terminals A and B respectively.

The number of parallel paths of the armature winding across the output terminals is four (equal-to the number of poles) which can be examined as follows: Redraw the armature winding of Fig. 2.15 in a simplified manner as shown in Fig. 2.16. Between terminals A and B there are four parallel paths shown as M, N, O and P. The total emf generated in the machine is equal to the emf generated in one parallel path.



**Figure: Lay out diagram for a Lap winding given in example -1**



**Figure: (a) Armature winding of a dc machine shown in a simplified manner  
 (b) Shows the number of parallel paths in the armature**

**Equalizer Connections in Lap Winding:** As mentioned earlier, a simplex lap winding has as many number of parallel paths as there are poles. The emf induced in each parallel path may not be exactly equal due to a number of reasons, such as the difference in the lengths of the air-gap under each pole, the difference in the field strength due to some error in putting field windings, etc.

Unequal values of emf generated in the parallel paths will circulate a considerable amount of current in the armature circuit without doing any useful work. This circulating current will be large as the armature circuit resistance is generally very low. This circulating current will generate heat and while circulating through the brush contacts will cause commutation difficulties (like sparking on the commutator surface).

To overcome this problem arising from the circulating current, equalizer connections are made in lap wound armatures. These equaliser connections or equalisers are low-resistance copper conductors which connect those points in the winding which under ideal conditions should be at equal potential. The difference in potential between these points created due to reasons mentioned earlier will be equalised as a result of flow of current through these low resistance conductors which will bypass the current from flowing through the brushes.

### 2.3.3 Wave Winding

In a wave winding a coil-side under one pole is connected to a second coil-side which occupies approximately the same position under the next pole through back connection. The second coil-side is then connected forward to another coil-side under the next pole (in the case of lap winding the second coil is connected back through the commutator segment to a coil-side under the original pole). The difference in lap and wave winding connections has been illustrated in Fig. 2.13(a) and (b).

The characteristics of a wave winding are:

(i) Average pitch,  $Y_a = (Y_b \pm Y_f)/2 = (Z \pm 2)/P$

If  $Y_a$  is taken equal to  $Z/P$ , as is the case in a lap winding the winding after one round will close itself without including all the coils which is not desirable.

Hence the product of the average pitch and the number of pairs of poles must be two greater or less than the number of coil-sides.

Average pitch should be a whole number.

(ii) Both back pitch and front pitch should be odd numbers.

(iii) To make the average pitch a whole number, wave winding is not possible with any number of coil-sides. For example if  $Z = 32$  and  $P = 4$ ,

$$Y_a = (Z \pm 2)/P = (32 \pm 2)/4 = 8 \frac{1}{2} \text{ or } 7 \frac{1}{2}$$

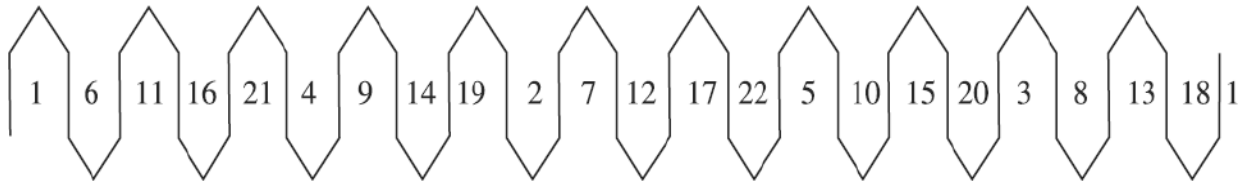
Thus wave winding is not possible with 32 coil-sides. In this case the number of effective coil-sides needs to be 30.

**Example-2 :** Prepare a winding diagram for a 4 pole wave-connected armature of a dc generator having 22 coil sides.

$$Y_a = (Z \pm 2)/P = (22 \pm 2)/4 = 6 \text{ or } 5$$

If  $Y_a$  is taken to be odd, i.e., 5, then the front pitch and back pitch will be equal. Thus,  $Y_a = Y_b = Y_f = 5$ .

Connections of the coil sides will be as shown in figure. The connection diagram is achieved by adding  $Y_b$  and  $Y_f$  with the coil numbers progressing in the forward direction. Coil-side 1 is connected at the back with coil side 6 ( $1 + Y_b = 6$ ). Coil side 6 is connected at the front with coil-side 11 ( $6 + Y_f = 11$ ) and so on.



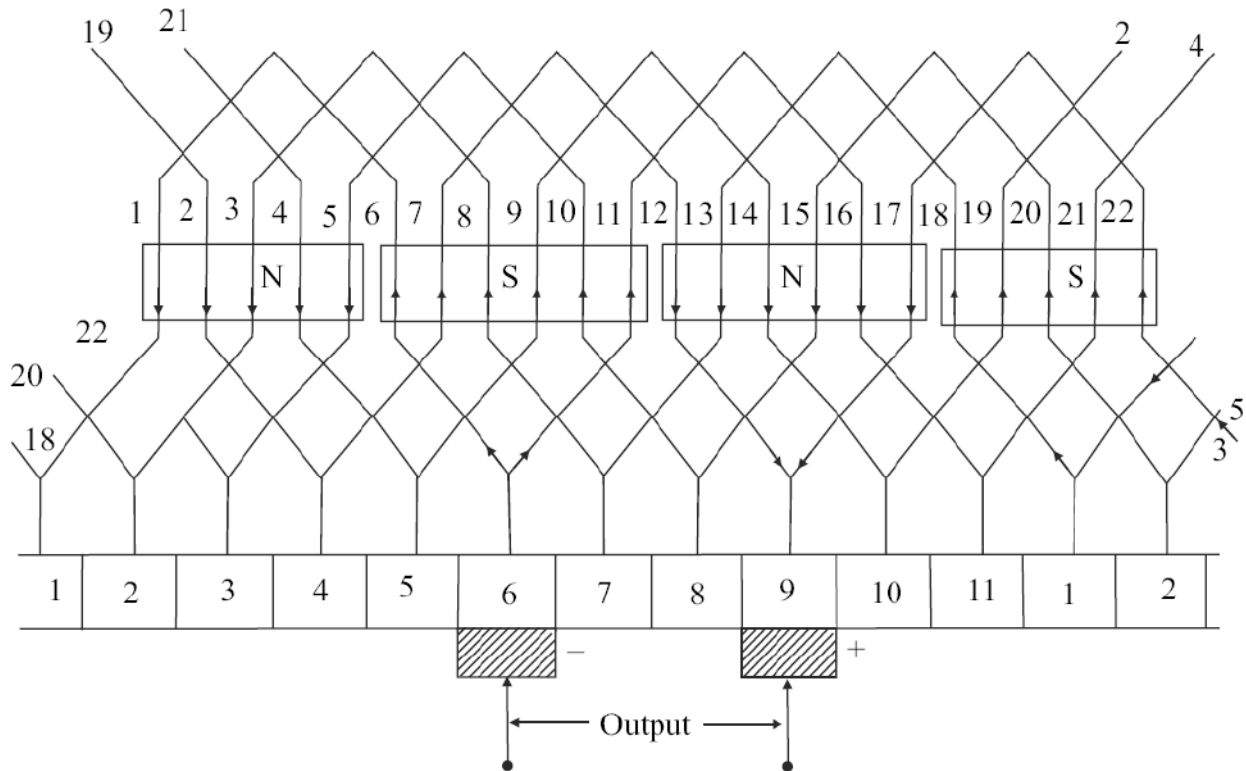
**Figure: Connection diagram of the coil-sides for a DC wave winding**

In Fig. 2.17 it is to be noted that coil-side 19 is connected with coil-side 2. This is obtained by adding  $Y_b$  to 19 which gives 24. Coil-side 24 does not exist as there are in all 22 coil-sides. Therefore after 22 count two more numbers starting from 1. This gives coil-side 2. Similarly it can be seen that coil-side 20 is connected in the front with coil-side 3. By adding  $Y_f (= 5)$  to 20, the number 25 is obtained. After 20 five numbers are counted as 21, 22, 1, 2, and 3. Thus coil-side 20 should be connected to coil-side 3. In this way, the whole winding is completed by connecting all the coil-sides with one another. The actual layout diagram of the winding along with the position of the poles and the direction of induced emf in the coil-sides for a particular direction of rotation of the armature are shown in Fig. 2.18. The positions of the four brushes are also shown in the figure.

The positions of brushes are fixed as follows: for ease of understanding, the connection diagram of Fig. 2.17 is reproduced in Fig. 2.19. The directions of current in the coil-sides are also shown by observing the directions from Fig. 2.18. By carefully examining the directions of current in the coil-sides it is seen that between points P and Q current gets divided in two parallel paths. From point P the current flows to Q via two paths, viz. through 11-16-21- ... 6-11-18-13-

The point P in Fig. 2.19 is the separating point of the emf in the two sections of the winding and therefore corresponds to the position of one of the brushes, viz. the negative brush. For placing of the positive brush, it is seen from Fig. 2.19 that

at point Q current is coming out from both the coil-sides. Therefore, point Q corresponds to the position of the positive brush.



**Figure: Layout diagram for the wave winding of example 2.2**

It may be noted from Fig. 2.18 that coil-sides 6 and 17 lie in the interpolar region. The direction of current in these coil-sides will depend upon the direction of current in the other coil side of the respective coils, viz. coils 1-6 and 17-22.

**Dummy Coils:** As mentioned earlier wave winding is possible with a particular number of coil-sides. But if standard stampings with a definite number of slots are to be used, the number of coil-sides needed to be placed in all the slots may be more than the required number. In such a case, the extra coils are left unconnected. These coils are called dummy coils. Dummy coils are used so as to make the armature dynamically balanced. They, otherwise, do not contribute to the induced emf or developed torque.

**Example-3 :** Calculate the winding pitches and draw developed and sequence diagrams of the winding for a four-pole wave connected armature winding of a dc generator having seven coils. In the diagram, show the position of poles and the position and polarity of brushes.

**Solution:**

Number of coil-sides =  $7 \times 2 = 14$

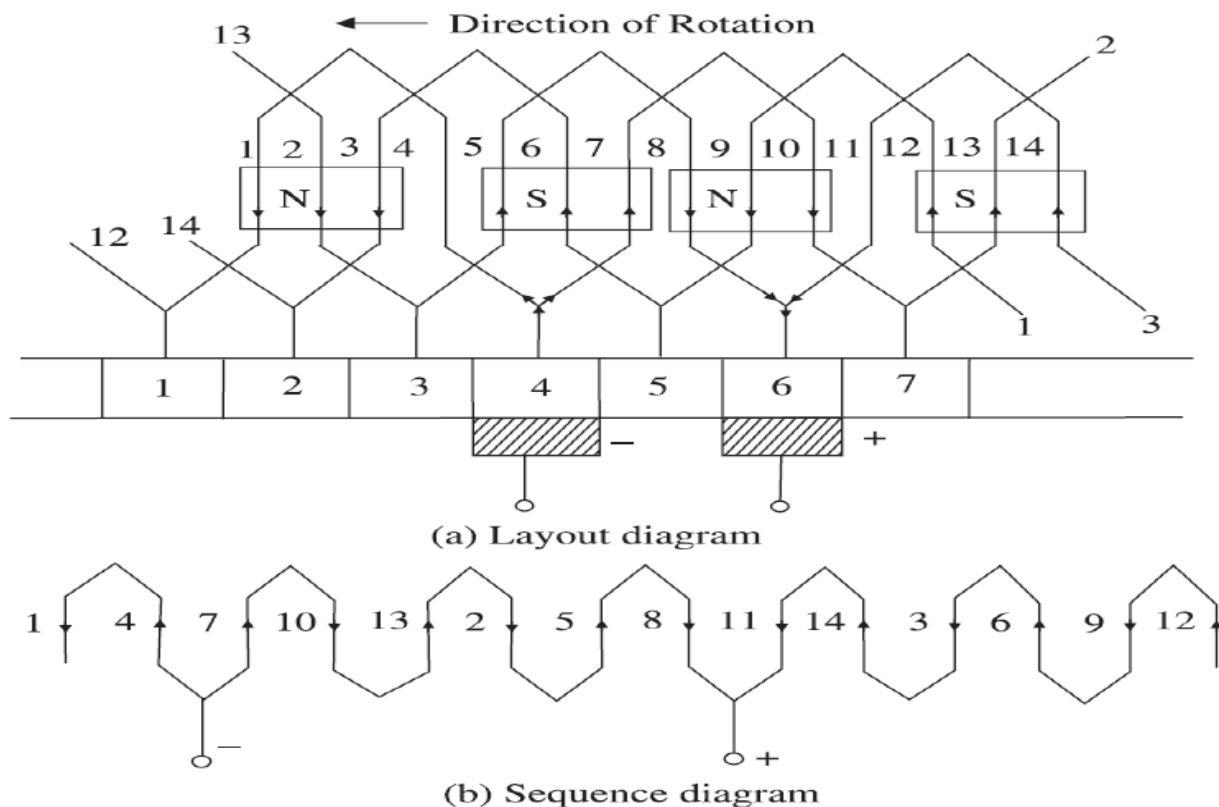
$$Y_a = (Z \pm 2)/P = (14 \pm 2)/4 = 3 \text{ or } 4$$

$Y_a$  should be an integer,  $Y_b$  and  $Y_f$  should be odd numbers.

Therefore, we choose

$$Y_a = Y_b = Y_f = 3$$

The sequence and layout diagrams of the winding are shown in Fig. 2.20.



**Figure: Layout and sequence diagram for a wave wound DC armature**

### **Simplex and Multiplex windings:**

Rotor (armature) windings are further classified according to the *plex* of their windings. A *simplex* rotor winding is a single, complete, closed winding wound on a rotor. A *duplex* rotor winding is a rotor with *two complete and independent sets* of rotor windings. If a rotor has a duplex winding, then each of the windings will be associated with every other commutator segment. One winding will be connected to segments 1, 3, 5, etc., and the other winding will be connected to segments 2, 4, 6, etc. Similarly, a *triplex* winding will have three complete and independent sets of windings, each winding connected to every third commutator segment on the rotor. Collectively, all armatures with more than one set of windings are said to have *multiplex windings*.

### **Use of laminated core :**

Faraday's law is the fundamental property of magnetic fields involved in transformer operation. The effect of Lenz's law in transformers is to predict the polarity of the voltages induced in transformer windings.

Faraday's law also explains the eddy current losses mentioned previously. A time-changing flux induces voltage *within* a ferromagnetic core in just the same manner as it would in a wire wrapped around that core. These voltages cause swirls of current to flow within the core, much like the eddies seen at the edges of a river. It is the shape of these currents that gives rise to the name *eddy currents*. These eddy currents are flowing in a resistive material (the iron of the core), so energy is dissipated by them. The lost energy goes into heating the iron core. The amount of energy lost to eddy currents is proportional to the size of the paths they follow within the core. For this reason, it is customary to break up any ferromagnetic core that may be subject to alternating fluxes into many small strips, or *laminations*, and to build the core up out of these strips. An insulating oxide or resin is used between the strips, so that the current paths for eddy currents are limited to very small areas. Because the insulating layers are extremely thin, this action reduces eddy current losses with very little effect on the core's magnetic properties. Actual eddy current losses are proportional to the

square of the lamination thickness, so there is a strong incentive to make the laminations as thin as economically possible.

### EMF Equation:

This being very important for understanding of a Generator performance we will derive a detailed expression for the exact induced emf in a generator in terms of all the following DC Machine parameters.

$\Phi$  The flux from a pole (webers)

$Z$  The total number of conductors on the armature

$a$  The number of parallel paths

- In a practical machine all the conductors are not connected in series. They are divided into groups of parallel conductors and then all the groups are connected in series to get higher voltage. In each group there are ' $a$ ' conductors in parallel and hence there are ' $a$ ' parallel current paths and each parallel path will have  $Z/a$  conductors in series.

$N$  The Speed of rotation (RPM)

$\omega$  The speed (Radians/sec)

$P$  The number of poles

Now consider one conductor on the armature. As this conductor makes one complete revolution it cuts  $P\Phi$  webers of flux.

Since the induced emf in a conductor is its rate of cutting of flux lines ( Rate of change of Flux linkage ) the emf ' $e$ ' induced in such a single conductor is equal to

$$e = P\Phi / \text{Time for one revolution in seconds} = P\Phi / (60/N) = NP\Phi / 60 \text{ volts}$$

There are  $Z/a$  conductors in series in each parallel path.

∴ the total induced emf  $'E' = (Z/a) NP\phi/60 = (NP\phi Z)/(a \cdot 60)$

$$E_A = (\phi ZN/60) \cdot (P/a)$$

The armature conductors are generally connected in two methods. Viz. Lap winding and Wave winding.

In Lap wound machines the number of parallel paths  $'a' = P$

$$\therefore 'E' = (\phi ZN/60)$$

In Wave wound machines the number of parallel paths  $'a' = 2$

$$\therefore 'E' = (\phi ZN/60) \cdot (P/2)$$

In general the emf induced in a DC machine can be represented as  $E_A = K_a \cdot \phi \cdot N$

Where

$$K_a = ZP/60 \cdot a$$

Sometimes it is convenient to express the emf induced in terms of the angular rotation  $\omega$  (Rad/sec) and then the expression for emf becomes:

$$E_A = (\phi ZN/60) \cdot (P/a) = (ZP/a) \cdot \phi \cdot N/60 = (ZP/a) \cdot \phi \cdot (\omega/2\pi) = (ZP/2\pi a) \cdot \phi \cdot \omega = K_a \cdot \phi \cdot \omega$$

(since  $N/60$  RPS =  $2\pi \cdot N/60$  Radians /sec =  $\omega$  Radians /sec and  $\therefore N/60 = \omega/2\pi$ )

Where  $K_a$  is the generalized constant for the DC machine's armature and is given by :

$$K_a = (ZP/2\pi a)$$

Where  $\phi$  is the flux/per pole in the machine (Webers),  $N$  is the speed of rotation (RPM)  $\omega$  is the angular speed (Radians/sec) and  $K_a$  is a constant depending on the machine parameters.

And thus finally  $E_A = K_a \cdot \Phi \cdot \omega$  and we can say in general, the induced voltage in any DC machine will depend on the following three factors:

1. The flux  $\Phi$  in the machine
2. The angular speed of rotation  $\omega$  and
3. A constant representing the construction of the machine.  $(ZP/2\pi a)$   
(i.e. the number of conductors 'Z', the number of poles 'P' and the number of parallel paths 'a' along with the other constant '2π')

### Armature Reaction:

If the magnetic field windings of a DC machine are connected to a power supply and the rotor of the machine is turned by an external source of mechanical power, then a voltage will be induced in the conductors of the rotor. This voltage will be rectified into a DC output by the action of the machine's commutator.

Now connect a load to the terminals of the machine, and a current will flow in its armature windings. This current flow will produce a magnetic field of its own, which will distort the original magnetic field from the machine's poles. This distortion of the flux in a machine as the load is increased is called *armature reaction*.

It causes two serious problems in real DC machines.

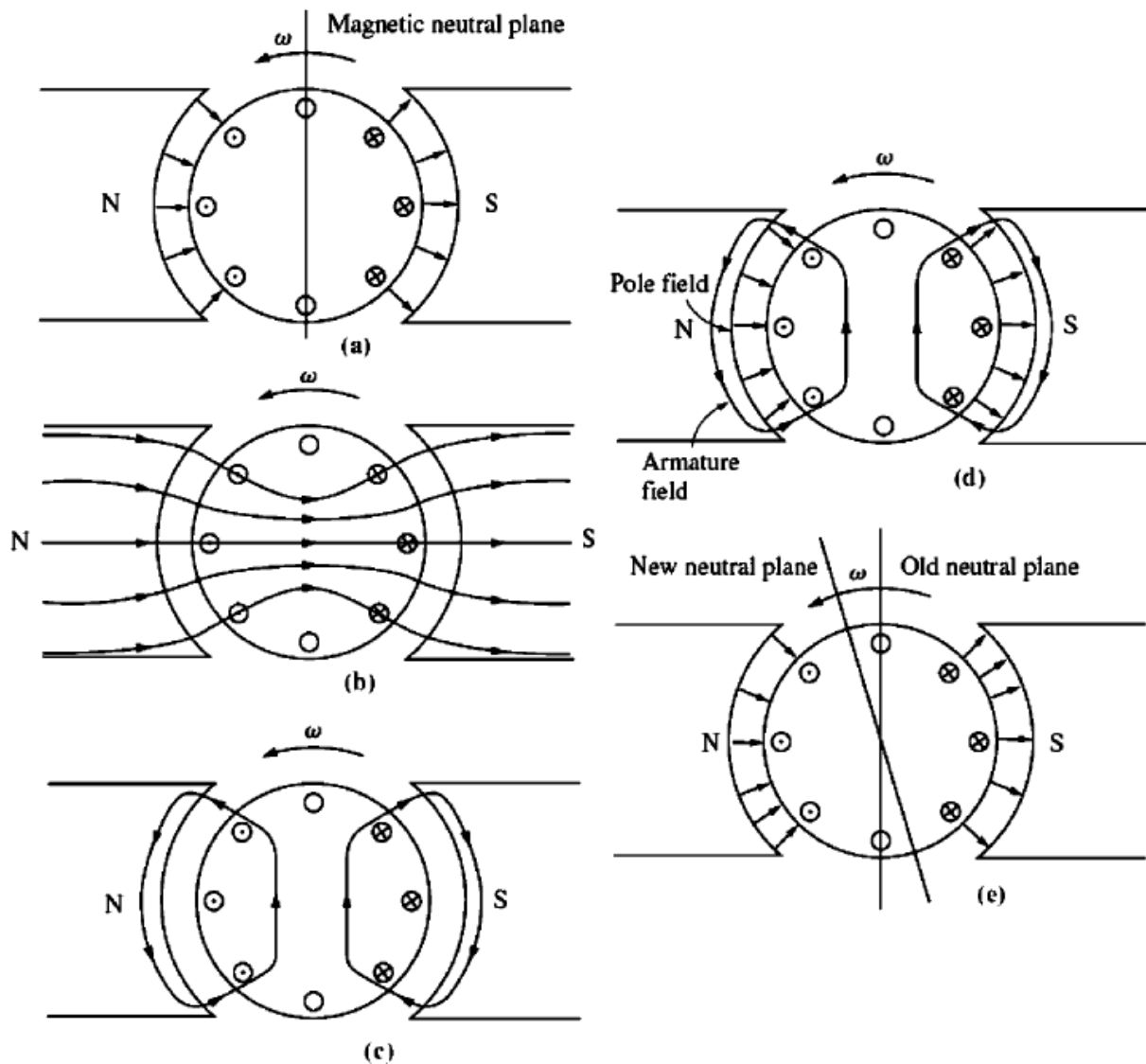
The first problem caused by armature reaction is *neutral-plane shift*. The *magnetic neutral plane* is defined as the plane within the machine where the velocity of the rotor wires is exactly parallel to the magnetic flux lines, so that  $e_{ind}$  in the conductors in the plane is exactly zero.

To understand the problem of neutral-plane shift, examine Figure 8- 23. Figure 8- 23a shows a two-pole dc machine. Notice that the flux is distributed uniformly under the pole faces. The rotor windings shown have voltages built up out of the page for wires under the North Pole face and into the page for wires under the South Pole face. The neutral plane in this machine is exactly vertical.

Now suppose a load is connected to this machine so that it acts as a generator. Current will flow out of the positive terminal of the generator, so current will be flowing out of the page for wires under the North Pole face and into the page for wires under the South Pole face. This current flow produces a magnetic field from

the rotor windings, as shown in Figure 8- 23c. This rotor magnetic field affects the original magnetic field from the poles that produced the generator's voltage in the first place. In some places under the pole surfaces, it subtracts from the pole flux, and in other places it adds to the pole flux. The overall result is that the magnetic flux in the air gap of the machine is skewed as shown in Figure 8- 23d and e. Notice that the place on the rotor where the induced voltage in a conductor would be zero (the neutral plane) has shifted.

For the generator shown in Figure 8- 23, the magnetic neutral plane shifted in the direction of rotation. If this machine had been a motor, the current in its rotor would be reversed and the flux would bunch up in the opposite corners from the bunches shown in the figure. As a result, the magnetic neutral plane would shift the other way.



**Figure 8-23: The development of armature reaction in a DC generator. (a) Initially the pole flux is uniformly distributed, and the magnetic neutral plane is vertical (b) the effect of the air gap on the pole flux distribution (c) the armature magnetic field resulting when a load is connected to the machine (d) both rotor and pole fluxes are shown indicating points where they add and subtract (e) the resulting flux under the poles. The neutral plane has shifted in the direction of motion.**

In general, the neutral-plane shifts in the direction of motion for a generator and opposite to the direction of motion for a motor. Furthermore, the amount of the shift depends on the amount of rotor current and hence on the load of the machine.

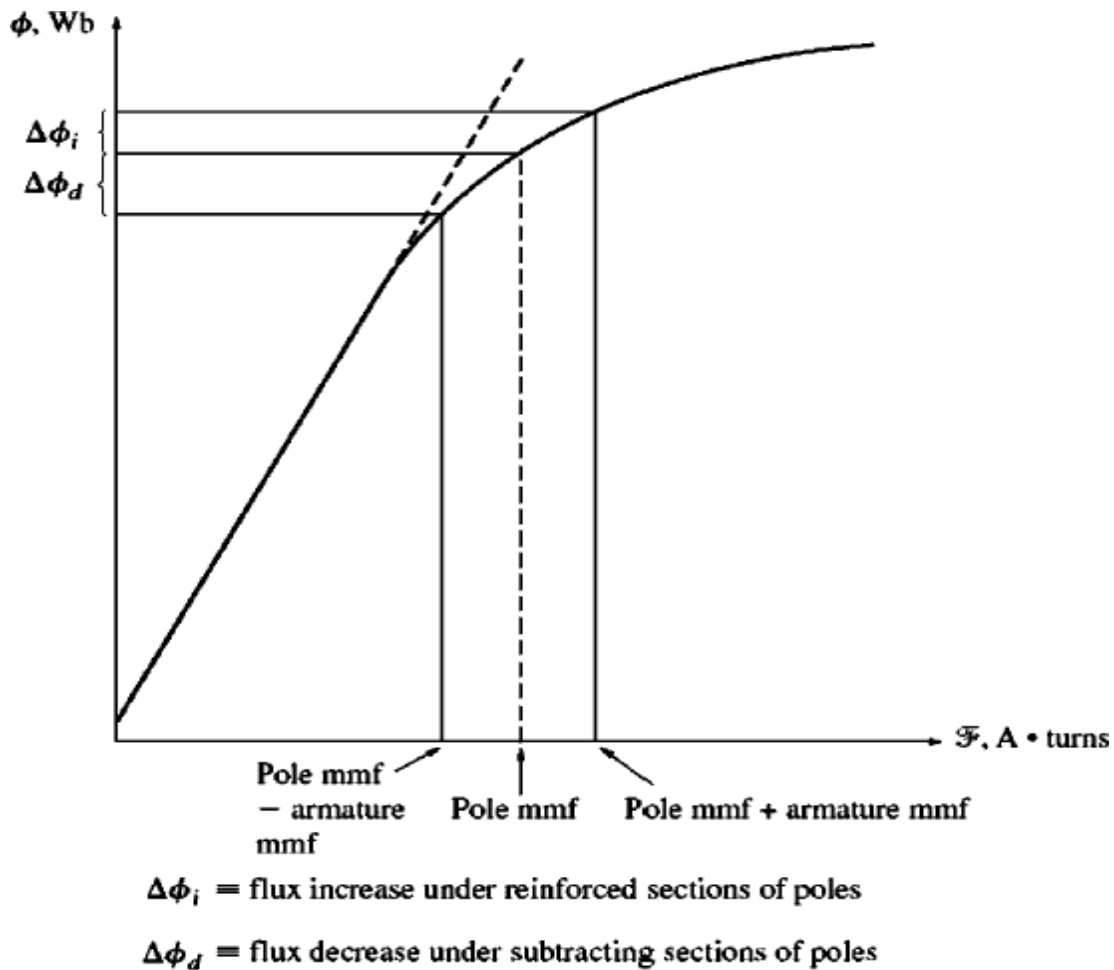
So what's the problem with neutral-plane shift? It 's just this: The commutator must short out commutator segments just at the moment when the voltage across them is equal to zero. If the brushes are set to short out conductors in the vertical plane, then the voltage between segments is indeed zero *until the machine is loaded*. When the machine is loaded, the neutral plane shifts, and the brushes short out commutator segments with a finite voltage across them. The result is a current now circulating between the shorted segments and large sparks at the brushes when the current path is interrupted as the brush leaves a segment. The end result is *arcing and sparking at the brushes*. This is a very serious problem, since it leads to drastically reduced brush life, pitting of the commutator segments, and greatly increased maintenance costs. Notice that this problem cannot be fixed even by placing the brushes over the full-load neutral plane, because then they would spark at no load.

In extreme cases, the neutral-plane shift can even lead to *flashover* in the commutator segments near the brushes. The air near the brushes in a machine is normally ionized as a result of the sparking on the brushes. Flashover occurs when the voltage of adjacent commutator segments gets large enough to sustain an arc in the ionized air above them. If flashover occurs, the resulting arc can even melt the commutator's surface.

The second major problem caused by armature reaction is called *flux weakening*. To understand flux weakening, refer to the magnetization curve shown in Figure 8-24. Most machines operate at flux densities near the saturation point. Therefore, at locations on the pole surfaces where the rotor magnetomotive force adds to the pole magnetomotive force, only a small increase in flux occurs. But at locations on the pole surfaces where the rotor magnetomotive force subtracts from the pole magnetomotive force, there is a larger decrease in flux. the net result is that *the total average flux under the entire pole face is decreased* (see Figure 8- 25).

Flux weakening causes problems in both generators and motors. In generators, the effect of flux weakening is simply to reduce the voltage supplied by the

generator for any given load. In motors, the effect can be more serious. As the early examples in this chapter showed, when the flux in a motor is decreased, its speed increases. But increasing the speed of a motor can increase its load, resulting in more flux weakening. It is possible for some shunt dc motors to reach a runaway condition as a result of flux weakening, where the speed of the motor just keeps increasing until the machine is disconnected from the power line or until it destroys itself.



**Figure 8-24: A typical magnetization curve shows the effects of pole saturation where armature and pole magnetomotive forces add.**

### Reactance voltage:

The voltage rise in the short circuited coil due to inductive property of the coil, which opposes the current reversal in it during the commutation period, is called the reactance voltage. It is given by :

Reactance voltage = Coefficient of self-inductance(L) x Rate of change of current (di/dt).

We know that the coil undergoes commutation when the two commutator segments get short-circuited by the brush. During this period the current say  $I$  changes from  $+I$  to  $-I$ . That means  $di = \text{change in current} = 2I$ .

The time taken for this change in current is given by  $dt = (W_b - W_c)/v$  where  
 $W_b$  = Width of the brush (cms)

$W_m$  = Width of the mica insulator between the commutator segments (cms)

$V$  = peripheral (linear) velocity of the commutator (armature) (cm/sec)

Then reactance voltage =  $L \cdot di/dt = L \cdot 2I \cdot v / (W_b - W_c)$

This reactance voltage also causes sparking at the brushes resulting in the same phenomenon as that produced by neutral phase shifting due to armature reaction.

We can produce reversing e.m.f in two ways. By brush shifting. By using inter-poles or commutating poles.

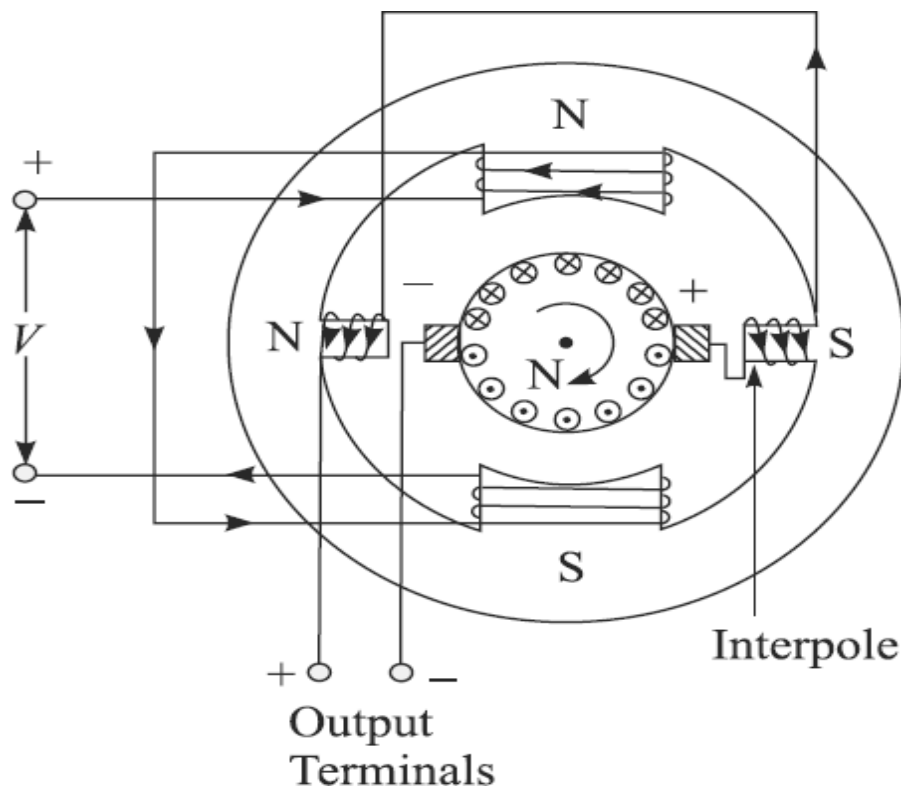
### Methods of improving commutation:

**Shifting of Brushes:** By shifting the brushes to the new MNA, sparking due to commutation can be avoided. The brushes are to be shifted by the same angle by which the MNA has shifted due to loading. They are to be shifted in the forward direction (in the direction of rotation) in a generator, and backward in a motor. The disadvantage with this method is that the angle of shift will depend upon the load on the machine and therefore is practically difficult to shift the brushes continuously with change in load.

**Commutating poles or Interpoles:** The basic idea behind this approach is that if the voltage in the conductors undergoing commutation can be made zero, then there will be no sparking at the brushes. To accomplish this, small poles, called

*commutating poles* or *interpoles*, are placed midway between the main poles. These commutating poles are located *directly over* the conductors being commutated. By providing a suitable amount of flux with proper polarity from the commutating poles, the voltage in the coils undergoing commutation can be exactly canceled. If the cancellation is exact, then there will be no sparking at the brushes. Exact cancellation of the voltage in the commutator segments is accomplished for all values of loads by connecting the interpole windings in *series* with the windings on the rotor, as shown in the figure below.

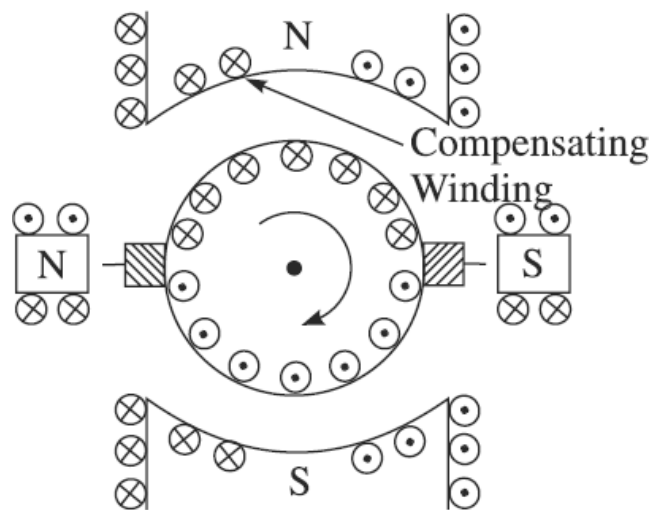
The commutating poles do not change the operation of the machine, because they are so small that they affect only the few conductors about to undergo commutation. The *armature reaction* under the main pole faces is unaffected, since the effects of the commutating poles do not extend that far. This means that the flux weakening problem in the machine is not solved by the commutating poles.



**Figure: Connection of commutating poles in a DC Generator**

As the load increases and the rotor current increases, the magnitude of the neutral-plane shift and the size of the  $L \frac{di}{dt}$  effects increase too. Both these effects increase the voltage in the conductors undergoing commutation. However, the interpole flux increases too, producing a larger voltage in the conductors that opposes the voltage due to the neutral-plane shift. The net result is that their effects cancel over a broad range of loads. Note that interpoles work for both motor and generator operation, since when the machine changes from motor to generator, the current both in its rotor and in its interpoles reverses direction. Therefore, the voltage effects from them still cancel.

The interpoles must induce a voltage in the conductors undergoing commutation with such a polarity that is *opposite* to the voltage caused by neutral-plane shift and  $L \frac{di}{dt}$  effects. In the case of a generator, the neutral plane shifts in the direction of rotation, meaning that the conductors undergoing commutation have the same polarity of voltage as the pole they just left (see Figure 8- 29).



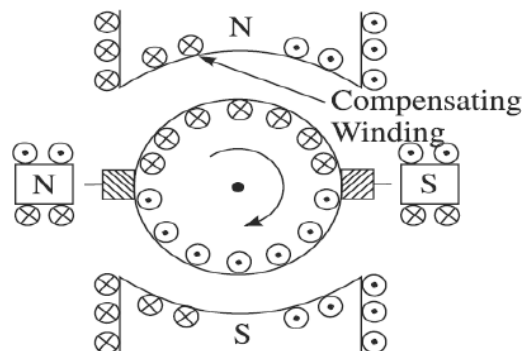
**Figure : Compensating windings used to neutralize the effect of armature reaction**

To oppose this voltage, the interpoles must have the opposite flux, which is the flux of the upcoming pole. In a motor, however, the neutral plane shifts opposite to the direction of rotation, and the conductors undergoing commutation have the same flux as the pole they are approaching. In order to oppose this voltage, the interpoles must have the same polarity as the previous main pole. Therefore,

1. The interpoles must be of the same polarity as the next upcoming main pole in a generator.
2. The interpoles must be of the same polarity as the previous main pole in a motor.

The use of commutating poles or interpoles is very common, because they correct the sparking problems of dc machines at a fairly low cost. They are almost always found in any dc machine of 1 hp or larger. It is important to realize, though, that they do *nothing* for the flux distribution under the pole faces, so the flux-weakening problem is still present. Most medium-size, general-purpose motors correct for sparking problems with interpoles and just live with the flux weakening effects.

**Compensating windings:** For very heavy, severe duty cycle motors, the flux-weakening problem can be very serious. To completely cancel armature reaction and thus eliminate both neutral-plane shift and flux weakening, a different technique was developed. This technique involves placing *compensating windings* in slots carved in the faces of the poles parallel to the rotor conductors, to cancel the distorting effect of armature reaction. These windings are connected in series with the rotor windings, so that whenever the load changes in the rotor, the current in the compensating windings also changes. Figure 8- 30 shows the basic



concept. In Figure 8-30(a), the pole flux is shown by itself. In Figure 8-30(b), the rotor flux and the compensating winding flux are shown. Figure 8-30(c) represents the sum of these three fluxes, which is just equal to the original pole flux by itself. The major disadvantage of compensating windings is that they are expensive, since they must be machined into the faces of the poles. Any motor that uses them must also have interpoles, since compensating windings do not cancel  $L di/dt$  effects. The interpoles do not have to be as strong, though, since they are canceling only  $L di/dt$  voltages in the windings, and not the voltages due to neutral-plane shifting. Because of the expense of having both compensating windings and interpoles on such a machine, these windings are used only where the extremely severe nature of a motor's duty demands them.

#### Important features of DC Generators:

- The terminal characteristic of a DC Machine is a plot of the output quantities of the Machine against each other. For a DC Generator the output quantities are the Terminal Voltage and the Line (Load) current.
- The various types of Generators differ in their terminal characteristics (Voltage-Current) and therefore to the application to which they are suited.
- The DC Generators are compared by their Voltages, Power ratings, their efficiencies and Voltage regulation. Voltage Regulation (**VR**) is defined by the equation:  $V_R = [(V_{nl} - V_{fl}) / V_{fl}] \cdot 100 \%$  Where  $V_{nl}$  is the No load terminal voltage and  $V_{fl}$  is the Full load terminal voltage. It is a rough measure of the Generator's Voltage- Current Characteristic. A positive voltage regulation means a drooping characteristic and a negative regulation means a rising characteristic.
- Since the speed of the prime movers affects the Generator voltage and prime movers can have varying speed characteristics, The voltage regulation and speed characteristics of the Generator are always compared assuming that the *Prime mover's speed is always constant*.

**Methods of excitation: (The method by which the field current is generated)**

The performance characteristics of a dc machine are greatly influenced by the way in which the field winding is excited with direct current. There are two basic ways of exciting a dc machine.

**1. Separate excitation:** The field is excited from a separate and independent DC source as shown in fig(a) below. It is flexible as full and independent control of both Field and Armature circuits is possible.

**2. Self- excitation:** The field is excited either from its own armature voltage (Shunt Excitation: fig-b) or own armature current (Series excitation : fig-c)

The dc machine excitation is also classified in three other ways:

**1. Shunt excitation :** Here the field winding is excited in parallel with armature circuit and hence the name shunt excitation. It is provided with a large number (hundreds or even thousands) of turns of thin wire and therefore, has a high resistance and carries a small current. Since the armature voltage of a dc machine remains substantially constant, the shunt field could be regulated by placing an external series resistance in its circuit.

**2. Series excitation :** Here the field winding has a few turns of thick wire and is excited with armature current by placing it in series with armature, and therefore it is known as series field winding. For a given field current, control of this field is achieved by means of a diverter, a low resistance connected in parallel to series winding. A more practical way of a series field control is changing the number of turns of the winding by suitable tappings which are brought out for control purpose.

**3. Compound Excitation:** In compound excitation both shunt and series fields are excited. If the two fields aid each other such that the resultant air gap flux per pole is increased (their ampere turns are additive), then the excitation is called

**cumulative compound excitation** as shown in Fig. (d). If the series field flux opposes the shunt field flux such that the resultant air gap flux per pole is decreased, then the excitation is called **differential compound excitation** as shown in Fig. (e). The series field is so designed that the increase or decrease in flux/pole is to a limited extent.

Further there are two types of compounding connections. **Long Shunt** and **Short shunt**. In long shunt compound of Fig. (f) the shunt field is connected across the output terminals. In short shunt compound, the shunt field is connected directly across the armature as shown in Fig. (g). There is no significant difference in machine performance for the two types of connections. The choice between them depends upon mechanical consideration or the reversing switches.

Figure below shows the physical arrangement of shunt and series field windings on one pole of a machine.

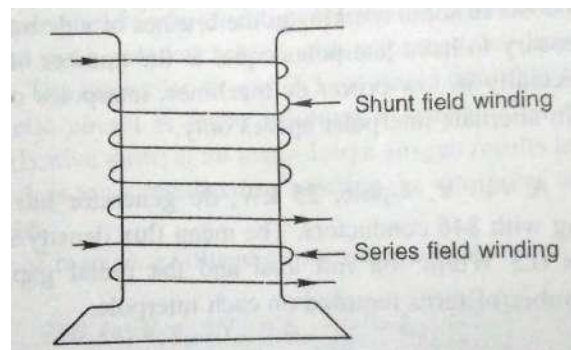


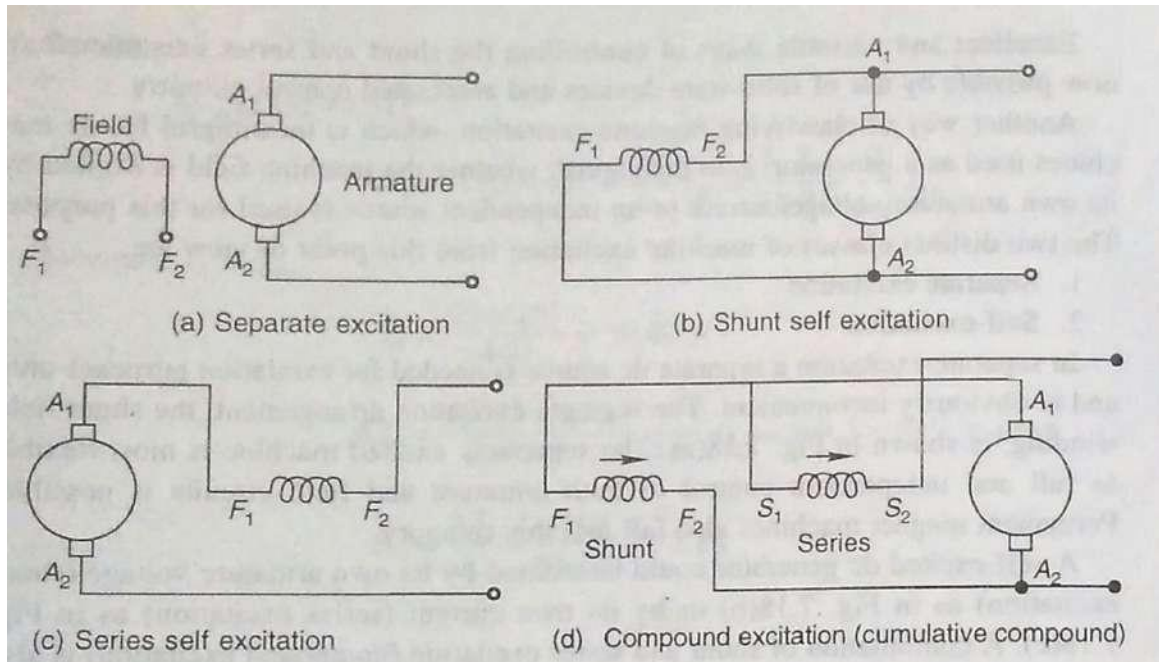
Fig: Arrangement of shunt and series field windings on one pole of a machine.

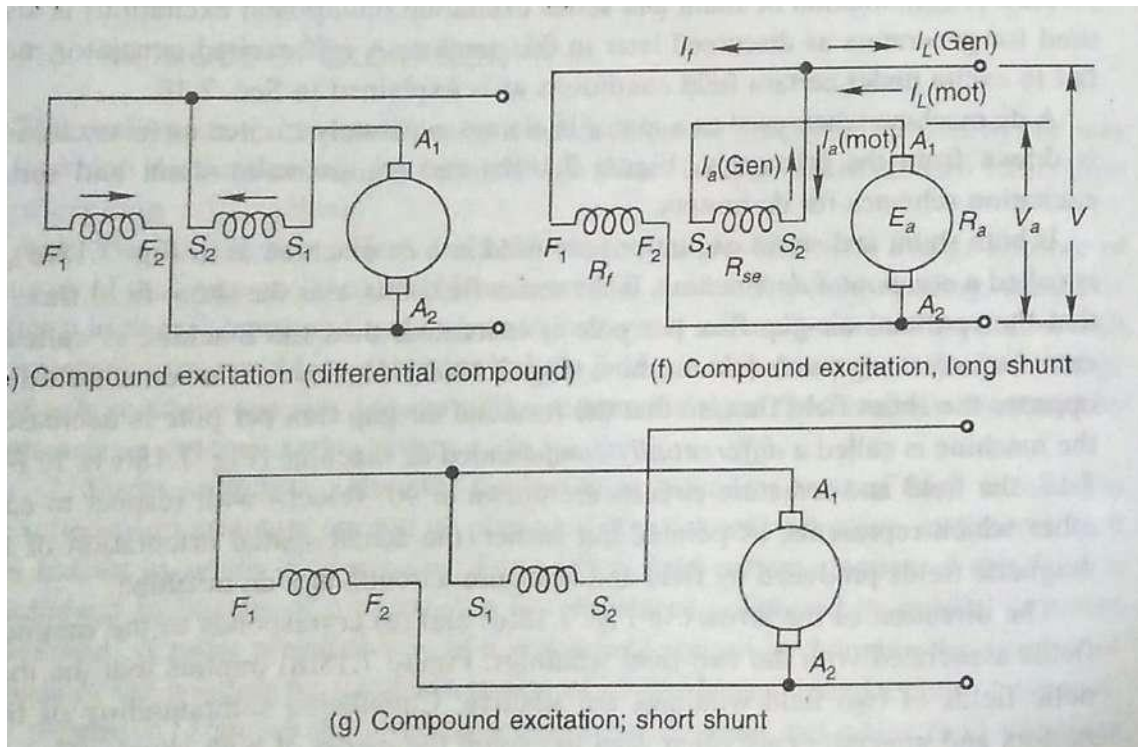
Excellent and versatile ways of controlling the shunt and series excitations are now possible by use of solid-state devices and associated control circuitry.

Electrical Machines -1 (EM-1): Lecture Notes: (Dr.Karimulla P Sk)  
Unit 1: D C Generators

---

In showing the excitation diagrams of a dc machine, the field winding is shown to be at 90° (electrical) with respect to the armature circuit which is the actual spatial orientation of the magnetic fields produced by the field and armature circuits in a DC machine.

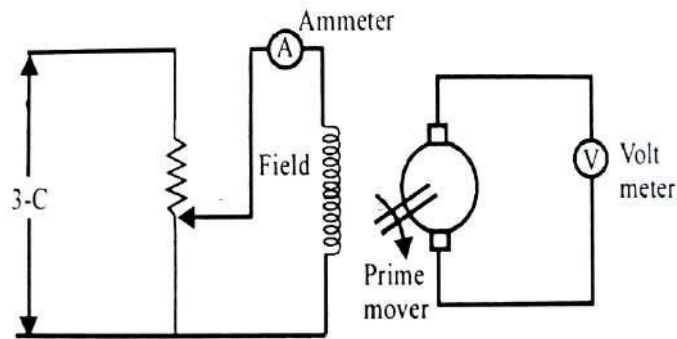




### Magnetization characteristics of DC Generators:

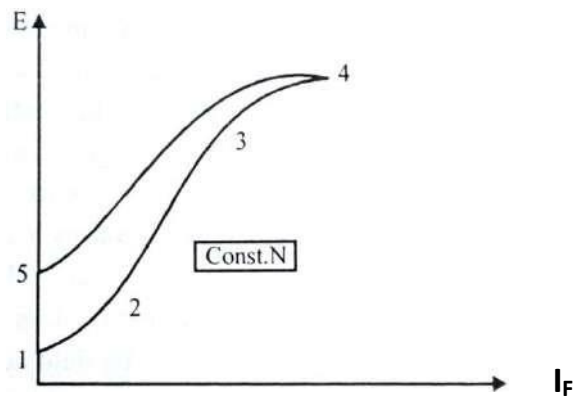
No load or Open circuit magnetization characteristic of any DC Machine is a plot of the Field flux versus the magnetizing current. Since measurement of field flux is difficult we use the relation for the emf induced in a DC machine  $E_A = K \cdot \Phi \cdot N$  from which we can see that the induced voltage is proportional to the Flux in the machine when the speed is maintained constant. Hence we conduct a test on the given DC machine to obtain data on the induced voltage as a function of the field current.

The diagram of the test setup required to obtain the above data is shown in the figure below.



**Fig: Test setup with a DC machine to obtain the No load magnetization Characteristic**

The prime mover gives the required mechanical energy to the DC Machine and it can be a small Diesel engine. The rheostat connected between the DC Input and the field winding is used to adjust and get the required field current. The field current is initially set to Zero and the Armature voltage is measured. Then the field current is gradually increased and the corresponding values of Armature voltage are measured until the output voltage saturates. Next the field current is brought back to zero gradually and the corresponding Armature voltages are measured at a few points. The corresponding data on Armature voltage is plotted against field current and is shown in the figure below.



**Fig: No load magnetization curve (or OCC) of a DC Machine ( Plot of Armature Voltage  $E_a$  Vs.Field current  $I_f$ )**

Though the field current is zero we get a small value of Armature voltage as seen at point 1 due to the residual magnetism present in the field coil. Subsequently armature voltage increases with field current upto some point 3 and then the rate of rise decreases. Finally at point 4 field flux gets saturated and hence the emf also gets saturated. The plot of armature voltage vs. field current is not same during the field current reduction as that during the field current increase and this is due to the property of magnetic hysteresis in the Ferro magnetic materials. In the return path the induced voltage at zero field current is higher than that during the field current increase. This is due to the combined effect of Hysteresis and the residual magnetism.

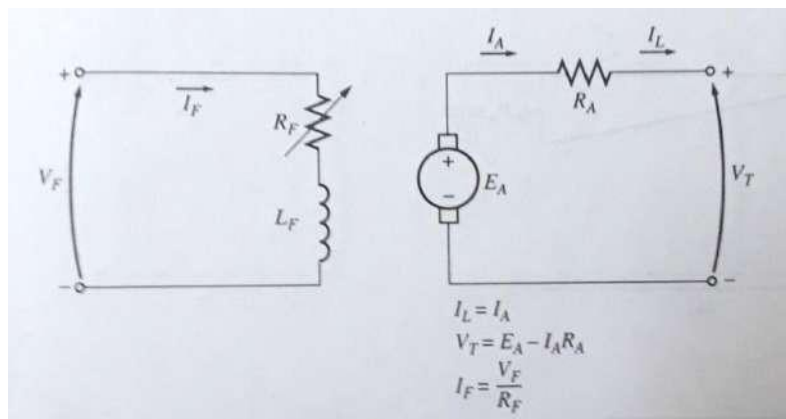
### **Different Types of DC Generators and their Terminal ( or Load ) Characteristics:**

The DC generators are classified according to the manner in which the field flux is produced. Let us consider the following important types of DC Generators and their characteristics along with their equivalent circuits.

The following notation is used uniformly in all the following circuits/characteristics:

- $V_T$  = Generator's Terminal Voltage
- $I_L$  = Load or line current
- $I_A$  = Armature current
- $E_A$  = Armature voltage
- $R_A$  = Armature Resistance
- $I_F$  = Field current
- $V_F$  = Field voltage
- $R_F$  = Field Resistance

**Separately Excited Generator:** In this type the field flux is derived from a separate power source which is independent of the Generator. The equivalent circuit of such a machine along with the governing equations is shown in the figure below.

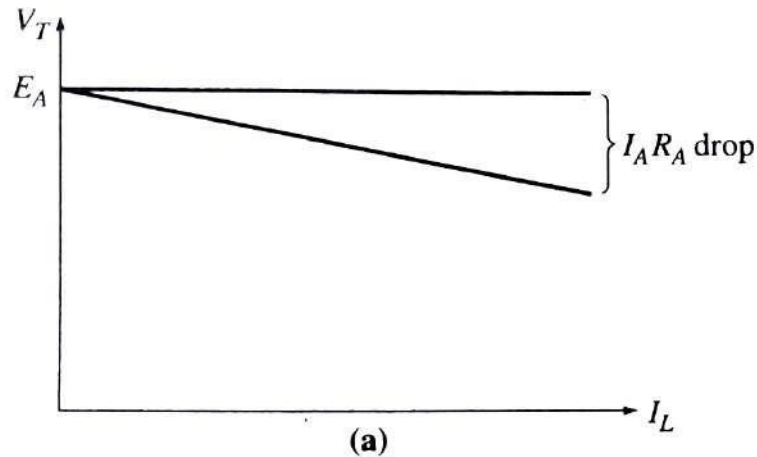


**Fig: Equivalent circuit of a separately excited DC Generator**

The terminal characteristic of this type of Generators is a plot of  $V_T$  vs.  $I_L$  for a constant speed  $\omega$  and the governing equations are :

- The Load or line current  $I_L$  = The armature current  $I_A$
- Generator's Terminal Voltage =  $V_T = (E_A - I_A R_A)$
- $I_F = V_F / R_F$

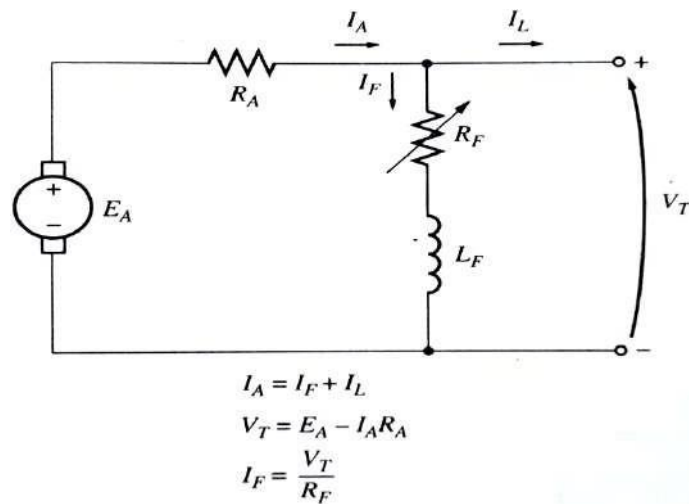
Since the internally generated voltage is independent of  $I_A$ , the terminal characteristic of a Separately Excited Generator is a straight line as shown in the figure below.



**Fig: The terminal Characteristics of a Separately Excited DC Generator**

When the load supplied by the generator increases, the load current  $I_L$  increases and hence the armature current  $I_A$  also increases. When the armature current increases, the  $I_A R_A$  drop increases, so the terminal voltage of the generator droops (falls). It is called a drooping characteristic.

**Shunt Generator:** In this the field flux is derived by connecting the Field directly across the Armature terminals. The equivalent circuit of such a generator is shown in the figure below along with the governing equations.



**Fig: The equivalent circuit of a DC Shunt generator along with the relevant governing equations**

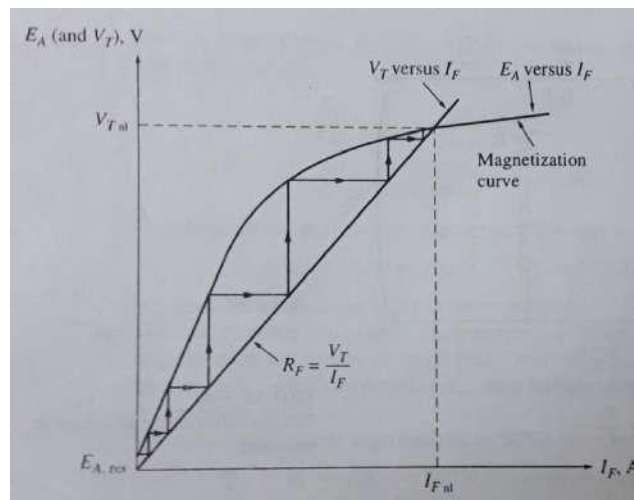
As could be seen, in this machine the armature current supplies both the load current and the field current. Using the Kirchhoff's voltage law the terminal voltage is seen to be same as that of a separately excited voltage i.e.  $V_T = (E_A - I_A R_A)$ . In this the advantage is that no external supply is required for the field circuit. *But this leaves an important question. If the generator supplies its own field current how does it get the initial field flux that is required to start the machine and generate voltage when it is first turned on? This is explained below.*

### **Build-up of E.M.F, Critical Field Resistance and Critical Speed :**

#### **Voltage build up in a Shunt Generator :**

The voltage build up in a shunt generator depends upon the presence of a **residual flux** in the poles of the generator. When a Shunt generator first starts to turn on an internal voltage is generated which is given by  $E_A = k \cdot \Phi_{res} \cdot \omega$ . This voltage ( which may be just one or two volts ) appears at the generator terminals. This causes a current to flow in the generator's field coil  $I_F = V_T / R_F$ . This produces

a m.m.f. in the poles which in turn increases the flux in them. The increase in the flux causes an increase in  $E_A = k \cdot \Phi \cdot \omega$  which in turn increases the terminal voltage  $V_T$ . When  $V_T$  rises,  $I_F$  increases further, increasing the flux more which increases  $E_A$  and so on. This voltage build up phenomenon is shown in the figure below.



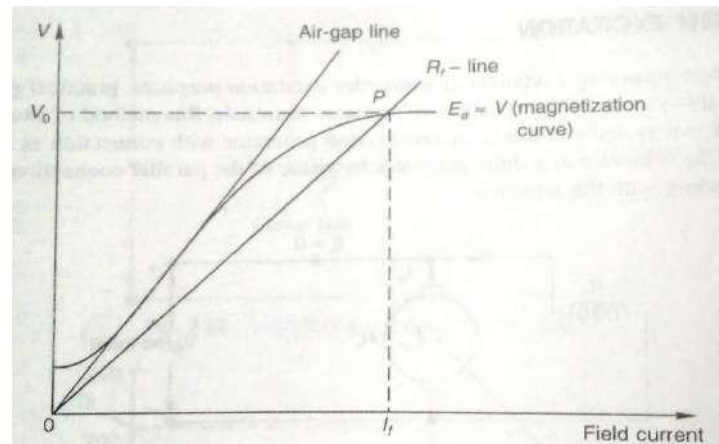
**Fig: Voltage build up on starting in a DC Shunt generator**

It is to be noted here that it is effect of **magnetic saturation** in the Pole faces which eventually limits the build of the terminal voltage.

The voltage build up in the figure above shows up as though it is building up in discrete steps. It is not so. These steps are shown just to make it clear the phenomenon of positive feedback between the Generator's internal voltage and the field current. In the DC Shunt generator both  $E_A$  and  $I_F$  increase simultaneously until the steady state conditions are reached.

**Critical Resistance:** For understanding the terms *critical Resistance* and *critical speed*, the open circuit characteristic (OCC) or the magnetization characteristic of a DC machine is shown again in the figure below along with *air gap* line and  $R_f$  line. The extension of the linear portion of the magnetization curve, shown dotted

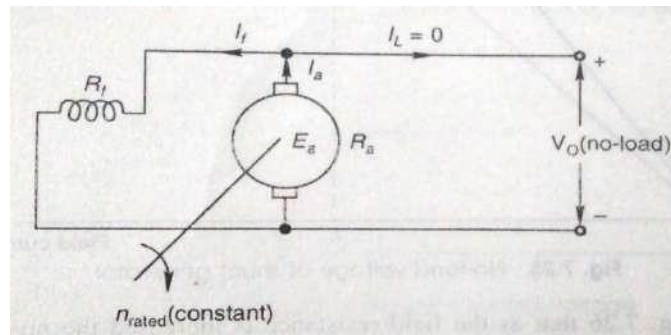
in the figure below is known as the *air-gap line* as it represents mainly the magnetic behavior of the machine's air-gap.



**Fig: Open Circuit Characteristic of DC machine along with Air gap and  $R_f$  lines**

As already explained in the topic *Build up of EMF in a DC shunt generator*: At the instant of switching on the field after the armature has been brought to rated speed, the armature voltage corresponds to a small residual value which causes a small field current to flow. If the field is connected such that this current increases the field mmf and therefore the induced emf, the machine voltage cumulatively builds up and settles at a final steady value because of the saturation characteristic of the machine's magnetic circuit.

Since the generator is assumed to be on no-load during the build-up process, the following circuit relationships apply with reference to the machines' equivalent circuit shown in the figure below.



**Fig: The equivalent circuit of DC shunt generator**

$$I_a = I_f$$

$$V = E_a - I_f R_a$$

The field current in a shunt generator being very small, the voltage drop  $I_f R_a$  can be neglected so that :

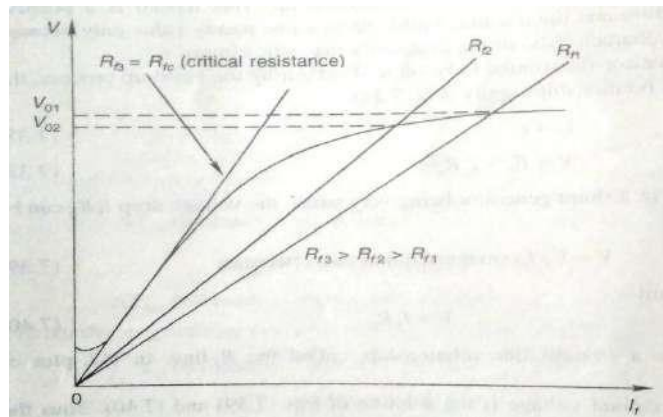
$$V_o = E_a(I_f) \quad (\text{magnetization characteristic})$$

And for the field circuit:

$$V_o = I_f R_f$$

which is a straight line relationship, called the  $R_f$ -line as shown in the OCC plot earlier. The no-load terminal voltage is the solution of the above two equations for  $V_o$ . Thus the intersection point  $P$  of the  $R_f$ -line with the magnetization characteristic as shown in the OCC gives the no-load terminal voltage ( $V_o$ ) and the corresponding field current. Further, it is easy to visualize from this figure that the no-load voltage can be adjusted to a desired value by changing the field resistance.

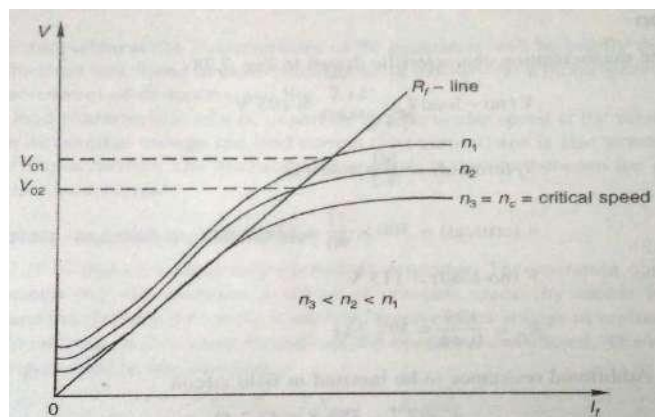
It can be seen in the figure below that as the field resistance is increased the no-load voltage decreases.



**Fig: Variation of No load voltage with field resistance**

The no-load voltage is undefined for a field resistance ( $R_{f3} = R_{fc}$ ) whose line coincides with the linear portion of the magnetization curve. With field resistance even slightly more than this value, the machine does not excite to any appreciable value and would give no-load voltage close to the residual value. The machine with this much resistance in the field fails to excite and the corresponding resistance is known as the *critical resistance* ( $R_{fc}$ ).

**Critical speed:** Consider now the operation with fixed  $R_f$  and variable armature speed as illustrated in the figure below. It can be observed that as the speed is reduced, the OCC proportionally slides downwards so that the no-load voltage *reduces*. At a particular speed, called the *critical speed*, the OCC becomes tangential to the  $R_f$  line and as a result the generator would fail to excite.



**Fig: Effect of speed on No load voltage**

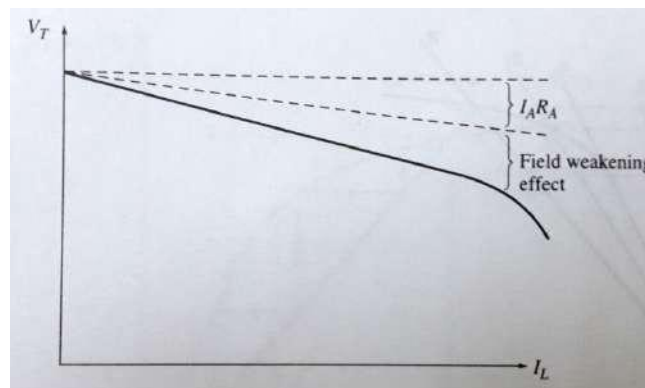
**Causes of failure to excite in a Shunt Generator:**

A shunt generator may not get excited in certain conditions. The causes of such failure to excite, the method of detection and the corresponding remedial measures are given in the table below.

<b>S.No</b>	<b>Cause</b>	<b>Method of Detection</b>	<b>Remedy</b>
1	Absence of residual magnetism due to ageing	Zero reading on Voltmeter after rotating the machine	Operate the Generator as separately excited machine first and then as separately excited
2	Wrong field winding connections. Due to this the flux gets produced in opposite direction to that of the residual flux and they cancel each other.	Voltmeter reading decreases rather than increasing as the field current is increased	Interchange the field connections
3	Field resistance is more than the Critical field resistance.	Voltmeter shows zero reading	Field resistance to be reduced using suitable field diverter
4	Generator is driven in opposite direction	This wipes out the residual flux and the machine fails to excite	Generator to be driven in the proper direction

**Terminal characteristics of a shunt generator :**

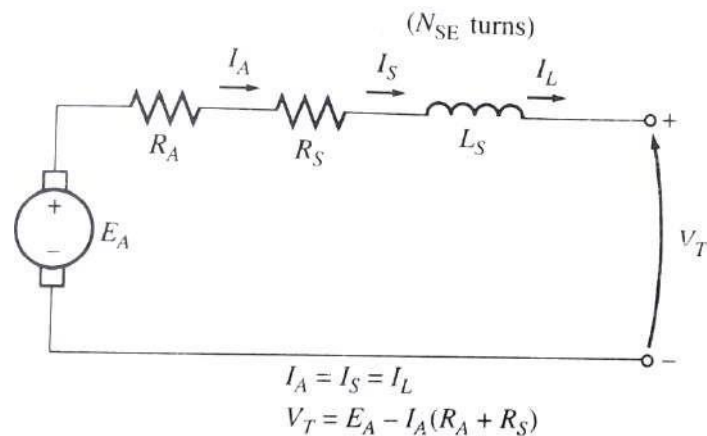
The terminal characteristics of the shunt generator differ from that of the separately excited generator because the amount of field current depends on its terminal voltage. As the generator load is increased, the load current  $I_L$  increases and so  $I_A = I_F + I_L \uparrow$  also increases. An increase in  $I_A$  increases the  $I_A R_A$  drop causing  $V_T = (E_A - I_A R_A)$  to decrease. This is precisely the same behavior we have seen in the case of separately excited generator. However, in the shunt generator when  $V_T$  decreases the field current decreases, hence the field flux decreases thus decreasing the generated Voltage  $E_A$ . Decreasing the  $E_A$  causes a further decrease in the terminal voltage  $V_T = (E_A \downarrow - I_A R_A)$ . The resulting characteristic is shown in the figure below.



**Fig: Terminal Characteristic of DC Shunt Generator**

It can be noticed that the drop with load is steeper than that of a separately excited motor due to the field weakening affect. This means that the regulation of a Shunt Generator is worse than that of a Separately Excited Generator.

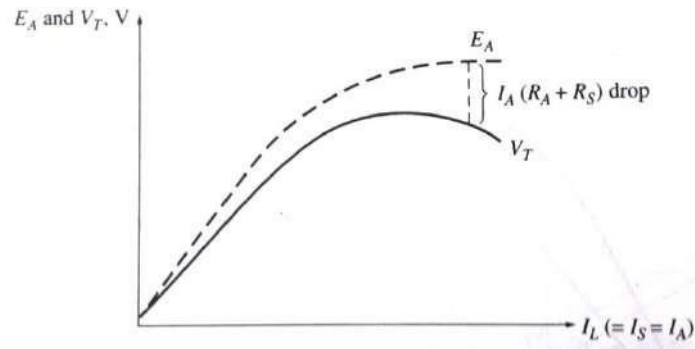
**DC Series Generator:** In this the field flux is derived by connecting the Field coil in series with the Armature of the Generator as shown in the figure below.



**Fig: Equivalent circuit of DC Series Generator along with the governing equations**

As shown, the armature current, load current and field current are same in a DC series generator. i.e  $I_A = I_F = I_L$ . Since the mmf produced by the fields is given by  $\mathcal{F}_s = NI$  and the field current is more in the DC series generator, the field winding is wound with lesser number of turns and also with a thicker gauge so as to offer less field resistance since full load current flows through the field winding.

The terminal characteristic of a DC Series Generator looks very much like the magnetization curve of any other type of generator and is shown in the figure below.

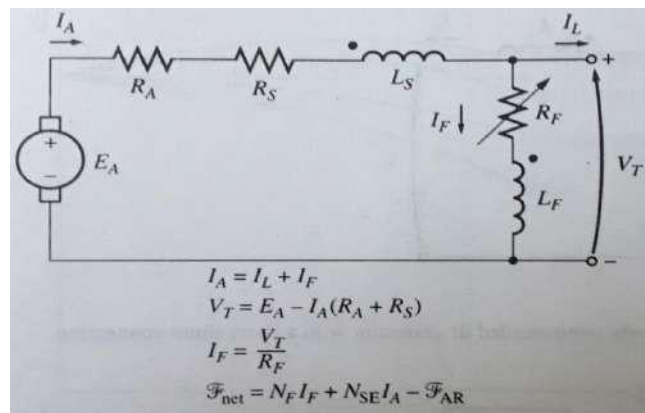


**Fig: Terminal Characteristic of DC Series Generator**

At no load however since there is no field current, armature voltage  $E_A$  and also the terminal voltage  $V_T$  are very small ( generated by the small amount of residual flux.) As the load increases, field current rises hence  $E_A$  also increases rapidly. The  $I_A (R_A + R_F)$  drop also goes up but this rise is less predominant compared to the rise in  $E_A$  initially and hence  $V_T$  also rises initially. After some time field flux gets saturated and hence the induced voltage  $E_A$  will be constant without any further rise. At this stage the resistive drop predominates and hence the **terminal voltage  $V_T$  starts drooping.**

#### **DC Compound generator:**

As we know in DC shunt Generator the terminal Voltage falls and in a DC series generator the terminal voltage increases on loading. A compound DC Generator is the one in which there will be both Series and shunt field coils. If they are wound such that they aid each other then it is called a Cumulative Compound DC Generator and if they are wound such that the two fields oppose each other, then it is called a differential Compound DC Generator. The equivalent circuit diagram of such Cumulative DC Generator along with relevant governing equations is shown in the figure below.



**Fig: Equivalent circuit of a Cumulative compound DC Generator**

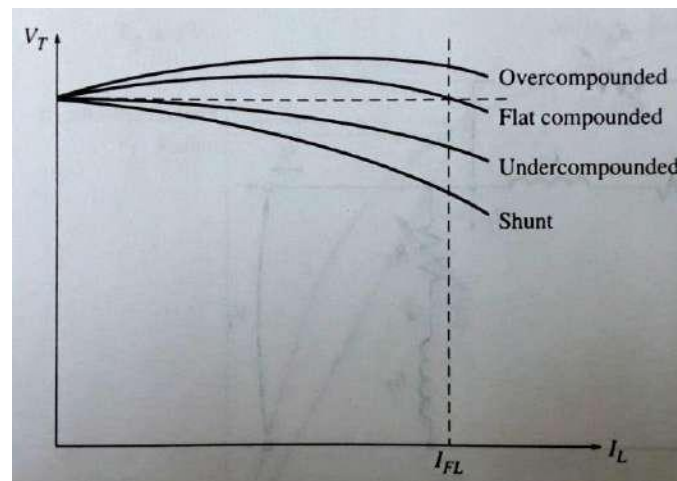
The circuit diagram is shown with standard **dot convention** on the field windings.

***i.e. The current flowing into the dot side of the winding produces a positive mmf***

.

And as can be seen that both  $I_F$  in the shunt winding and  $I_A$  in the series winding flow into the dot side and hence both produce magnetic fields which are positive and hence aid each other.

When the two fields are aiding each other we get a characteristic which will have the combined effect of **drooping** (due to the shunt coil) and **rising** (due to the field coil). Whichever coil current is more its effect will be more predominant. The terminal characteristics of a cumulative compound DC Generator are shown in the figure below for all the three cases.



**Fig: Terminal Characteristics of a DC Compound Generator**

1. If the Series field effect is more dominating than that of the Shunt field coil then we get the **Over compounded** characteristic where the full load terminal voltage is higher than the no load terminal voltage.
2. If the Series field effect is equal to that of the Shunt field coil then we get the **Flat compounded** characteristic where the full load terminal voltage is equal to the no load terminal voltage.
3. If the Shunt field effect is more dominating than that of the Series field coil then we get the **Under compounded** characteristic where the full load terminal voltage is lower than the no load terminal voltage.

The normal shunt characteristic is also shown in the figure for comparison.

#### Important concepts and Formulae:

- Voltage generated in a DC machine:  $E_A = (\Phi ZN/60) \cdot (P/a)$  and in terms of angular speed  $\omega$  :

$$E_A = K_a \Phi \omega \text{ where } K_a = ZP/2\pi a$$

### Illustrative Examples:

**Ex.1:** Calculate the e.m.f. generated by a 6 pole DC Generator having 480 conductors and driven at a speed of 1200 RPM. The flux per pole is 0.012 webers.  
(a) When the machine is lap wound (b) When the machine is wave wound

**Solution:** We know that the e.m.f. generated by a DC Generator is given by

$$E_A = (\Phi ZN/60)(P/a) \text{ where}$$

$$\Phi \text{ Flux per pole (webers)} = 0.012\text{Wb}$$

$$Z \text{ Total number of conductors on the armature} = 480$$

**a** The number of parallel paths = No of Poles  $P$  ( = 6 ) when Lap wound and = 2 when

wave wound

$$N \text{ Speed of rotation of the machine (RPM)} = 1200 \text{ RPM}$$

$$P \text{ The number of poles} = 6$$

(a) For Lap wound machine  $a = P = 6$

$$E_a = [ (0.012 \times 480 \times 1200) / 60 ] [ 6/6 ] = \mathbf{115.2 \text{ Volts}}$$

(b) For wave wound machine  $a = 2$

$$E_a = [(0.012 \times 480 \times 1200) / 60] [ 6/2 ] = \mathbf{345.6 \text{ Volts}}$$

**Ex.2 :** A 50 Kw ,250 V shunt generator operates at 1500 RPM .The armature has 6 poles and is lap wound with 200 turns. Find the induced e.m.f and the flux per pole at full load given that the armature and the field resistances are 0.01  $\Omega$  and 125  $\Omega$  respectively.

**Solution:**

$$\text{Output line current} = \text{Output power} / \text{Line voltage} = 50 \times 1000 / 250 = 200 \text{ A}$$

$$\text{Field current} = \text{Line Voltage} / \text{Field resistance} = 250 / 125 = 2 \text{ A}$$

$$\text{Armature current in a shunt generator:} = I_l + I_f = 200 + 2 = 202 \text{ A}$$

$$\begin{aligned} \text{Induced e.m.f } E_a : &= \text{Line Voltage} + \text{Armature drop } (I_a R_a \text{ drop}) \\ &= 250 + 202 \times 0.01 = \mathbf{252.02 \text{ V}} \end{aligned}$$

But we know that armature voltage in terms of the basic machine parameters is also given by

$$E_A = (\Phi ZN/60)(P/a) \text{ where}$$

$$\Phi : \text{ Flux per pole (webers)} = \text{To be determined}$$

$$Z : \text{ Total number of conductors on the armature} = \text{Number of turns} \times 2 \text{ (since each turn has two conductors)} = 200 \times 2 = 400$$

$$a : \text{ The number of parallel paths} = \text{No of Poles } P \text{ (} = 6 \text{ ) (since Lap wound)}$$

$$N : \text{ Speed of rotation of the machine (RPM)} = 1500 \text{ RPM}$$

$$P : \text{ The number of poles} = 6$$

$$\therefore \Phi = (E_A \times 60 \times a / ZNP) = 252.02 \times 60 \times 6 / 400 \times 1500 \times 6 = \mathbf{0.025202 \text{ Wb}}$$

**Ex.3:** A shunt generator connected in parallel to supply mains is delivering a power of 50 Kw at 250 V while running at 750 RPM. Suddenly its prime mover fails and the machine continues to run as a motor taking the same 50 Kw power from 250 V mains supply. Calculate the speed of the machine when running as a motor given that  $R_a = 0.01 \Omega$ ,  $R_f = 100 \Omega$  and brush drop is 1 V per brush.

**Solution:**

***First let us calculate the Voltage generated by the machine while running as a generator under the given conditions:***

$$\text{Output line current} = \text{Output power} / \text{Line voltage} = 50 \times 1000 / 250 = 200 \text{ A}$$

$$\text{Field current} = \text{Line Voltage} / \text{Field resistance} = 250 / 100 = 2.5 \text{ A}$$

$$\text{Armature current} : I_l + I_f = 200 + 2.5 = 202.5 \text{ A}$$

$$\text{Induced e.m.f } E_a : = \text{Line Voltage} + \text{Armature drop } (I_a R_a \text{ drop}) + \text{Brush drop (two brushes)}$$

$$= 250 + 202.5 \times 0.01 + 2 \times 1 = 254.025 \text{ V}$$

***Next let us calculate the Voltage generated by the machine while running as a motor under the given conditions :***

$$\text{Input line current} = \text{Input power} / \text{Line voltage} = 50 \times 1000 / 250 = 200 \text{ A}$$

$$\text{Field current} = \text{Line Voltage} / \text{Field resistance} = 250 / 100 = 2.5 \text{ A}$$

$$\text{Armature current} : I_l - I_f = 200 - 2.5 = 197.5 \text{ A}$$

$$\text{Induced e.m.f or back e.m.f } E_b : = \text{Line Voltage} - \text{Armature drop } (I_a R_a \text{ drop}) - \text{Brush drop (two brushes)}$$

$$= 250 - 197.5 \times 0.01 - 2 \times 1 = 246.025 \text{ V}$$

We know that the voltage induced in the machine is proportional to the speed  
i. e

Generator armature voltage is proportional to Generator speed :  $E_a \propto N_G$   
and similarly

Motor back e.m.f is proportional to Motor speed :  $E_b \propto N_M$

Hence  $E_a / N_G = E_b / N_M$  or  $N_M = (E_b / E_a) N_G = (246.025 / 254.025) \times 750 = \mathbf{726 \text{ RPM}}$

**Example 4 :** The following figures give the open-circuit characteristics of a dc shunt generator at 300 rpm:

$I_f$ (A)	0	0.2	0.3	0.4	0.5	0.6	0.7
$V_{oc}$ (V)	7.5	93	135	165	186	202	215

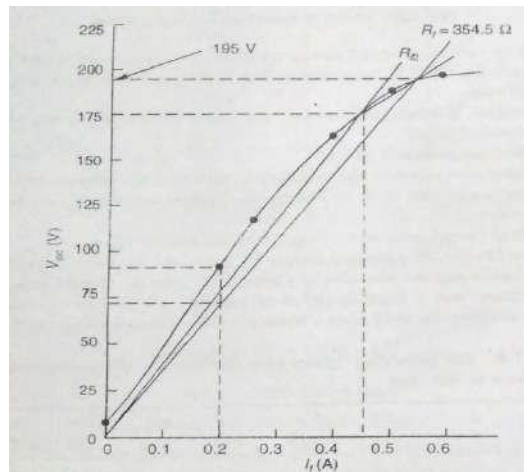
The field resistance of the machine is adjusted to 354.5  $\Omega$  and the speed is 300 rpm.

- (i) Determine graphically the no-load voltage.
- (ii) Determine the critical field resistance.
- (iii) Determine the critical speed for the given field resistance.
- (iv) What additional resistance must be inserted in the field circuit to reduce the no-load voltage to 175 V.

**Solution:**

Step-1 : Let us draw the Field resistance line corresponding to 354.5  $\Omega$  on the OCC ( magnetization characteristic). This can be done by identifying a point corresponding to a Voltage and current below the OCC corresponding to 354.5  $\Omega$  and extending the line joining this point with the origin.

1.  $V_{No \text{ load}}$  : This is the voltage corresponding to the point of intersection of the OCC and the  $R_f$  line corresponding to 354.5  $\Omega$  and is seen to be = 195 V
2.  $R_f \text{ critical}$  : To obtain this draw a line tangential to the OCC starting from the Voltage at  $I_f = 0$  A. Note down the voltage at which the tangential deviates from the OCC and the corresponding  $I_f$ . Dividing this voltage by the corresponding  $I_f$  we get the critical resistance viz.  $90/0.2 = 450 \Omega$



3. Critical Speed: We know that as speed reduces the armature voltage reduces. i.e. the OCC leans down wards with decrease in speed and becomes tangential to the existing  $R_f$  line itself. So to find out the critical speed we have to find out the new  $E_a$  from the OCC corresponding to the lesser speed which deviates from the existing  $R_f$  line. This is done by dropping a vertical perpendicular line from the point of deviation of the critical resistance line from the original OCC and identifying its intercept on the existing  $R_f$  line. Then by drawing a line parallel to the  $I_f$  axis from this point and locating its intercept with the Voltage axis, the new  $E_a$  is found out.

$$\text{Then Critical speed} = \text{Original RPM} \times \frac{\text{new } E_a}{\text{Original } E_a} = 300 \times \frac{71}{90} = 236.7 \text{ RPM}$$

4. To find out the additional resistance to be introduced into the field to get a new no load voltage of 175 V first we have to find out the value of  $I_f$  corresponding to the new no load voltage. This can be directly read from the OCC and then from these voltage and current values we can directly get the new value of  $R_f$  and thus the additional value of  $R_f$  to be introduced into the field circuit.

$$\text{Thus new } R_f = \frac{175}{0.44} = 397.7 \Omega \text{ and}$$

$$\text{The additional resistance to be introduced into the field} = 397.7 - 354.5 = 43.2 \Omega$$

## **UNIT – II**

### **D.C. MOTORS**

#### **CONTENTS:**

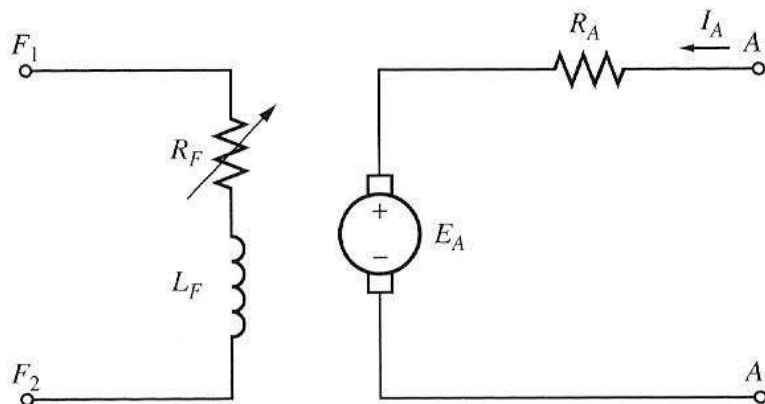
- **Principle of operation**
- **Back E.M.F - Torque equation**
- **Characteristics and application of shunt, series and compound motors**
- **Armature reaction and commutation.**
- **Speed control of D.C. Motors - Armature voltage and field flux control methods.**
- **Motor starters (3 point and 4-point starters)**
  - **Important concepts and Formulae**
  - **Illustrative examples**

**Principle of operation:** DC Motors are DC machines used as motors. A DC Motor converts the input DC power into output rotational mechanical power from the following principle. A current carrying conductor placed in a magnetic field experiences a mechanical force given by  $\mathbf{F} = i (\mathbf{I} \times \mathbf{B})$ .

When a group of such conductors is placed on a rotor and are connected properly the force experienced by the all the conductors together gets translated into a torque on the rotor (armature) and it starts rotating. We will derive an expression for such a Torque developed by a DC Motor from the first principles and its equivalent circuit by equating the Electrical power given to the motor (excluding the losses) to the mechanical power developed by the motor.

**Torque developed by a DC Motor:**

Consider the equivalent circuit of a DC motor as shown in the figure below.



**Fig: Equivalent circuit of a DC motor**

In this figure, the armature circuit is represented by an ideal voltage source  $E_A$  and the armature resistance  $R_A$ . The field coils, which produce the magnetic flux in the motor, are represented by inductor  $L_F$  and the field resistance  $R_F$ . The separate external variable resistor used to control the amount of current in the field circuit is also combined with the field resistance and is together shown as  $R_F$ .

We know from the earlier study of generators that the voltage generated in a DC Machine when It's armature is rotating in a magnetic flux of  $\Phi$  webers/pole is given by  $E_A = K_A \cdot \Phi \cdot \omega$  where  $K_A$  is given by:

$$K_A = (ZP/2\pi a)$$

Now in the DC Motor also, when it is rotating, from the same fundamental principle of Generator a Voltage is generated across the armature and it is now called back EMF and is normally shown as  $E_b$  to distinguish it from the voltage generated in the armature of a generator which was shown as  $E_A$  .

The governing equation of the DC Motor armature circuit now becomes:

$$V_T = E_b + I_a R_A \text{ or } E_b = V_T - I_a R_A$$

(as against  $V_T = E_A - I_a R_A$  in the case of a generator where  $I_a$  flows from armature towards the external terminals i.e external load )

Since now an external voltage  $V_T$  is applied to the motor terminals , direction of armature current changes i.e. now it flows from external terminals towards the armature.

The power delivered to the motor is given by :  $P_{in} = V_T \cdot I_a$  . From this, the loss of power in the armature is equal to  $I_a^2 R_A$  and hence the net power given to the motor armature is given by :

$$P_m = V_T \cdot I_a - I_a^2 R_A = I_a (V_T - I_a R_A) = I_a \cdot E_b$$
$$P_m = I_a \cdot E_b$$

This net electrical power is converted into mechanical power. We know that in mechanical rotational systems the power is equal to Torque times the speed. In the SI system of units which is the present Industry standard it is given by :

$$P_{mech} \text{ (watts) } = \tau \text{ (Nw.mtrs) } \cdot \omega \text{ (Radians/second) }$$

For simplification if we ignore the mechanical losses in the motor, then :

$$P_m = I_a \cdot E_b = P_{mech} = \tau \cdot \omega$$
$$\text{i.e. } \tau \cdot \omega = I_a \cdot E_b = E_b \cdot I_a$$

Substituting the value of  $E_A = K_A \cdot \Phi \cdot \omega$  we got in generators here for  $E_b$  since they are the same induced emfs we get  $\tau \cdot \omega = I_a \cdot K_A \cdot \Phi \cdot \omega$  or

$$\tau = K_A \cdot \Phi \cdot I_a$$

It is to be noted that this expression for the torque induced in a motor is similar to the voltage induced in a DC Generator except that the speed  $\omega$  in the DC Generator is replaced by the Armature current  $I_a$ . The constant  $K_A$  is same and is given by  $K_A = (ZP/2\pi a)$

***In general, the torque  $\tau$  in the DC motor will depend on the following 3 factors:***

- 1. The flux  $\Phi$  in the machine***
- 2. The armature current  $I_a$  in the machine***
- 3. The same constant  $K_A$  representing the construction of the machine***

### **Types of DC Motors and their output (or terminal) Characteristics:**

There are three important types DC Motors: DC separately excited, Shunt and Series motors. We will explain their important features and characteristics briefly.

The terminal characteristic of a machine is a plot of the machine's output quantities versus each other. For a motor, the output quantities are shaft **torque** and **speed**, so the terminal characteristic of a motor is a plot of its output **torque versus speed**. (Torque/Speed characteristics)

They can be obtained from the Motor's Induced voltage and torque equations we have derived earlier plus the Kirchhoff's voltage law around the armature circuit and are again given below for quick reference.

- The internal voltage generated in a DC motor is given by:  $E_b = K_a \cdot \Phi \cdot \omega$
- The internal Torque generated in a DC motor is given by:  $\tau = K_a \cdot \Phi \cdot I_a$
- KVL around the armature circuit is given by :  $V_T = E_b + I_a \cdot R_a$

Where  $\Phi =$  Flux per pole .... Webers  
 $I_a =$  Armature current .... Amperes

$V_T =$	Applied terminal Voltage	....	Volts
$R_a =$	Armature resistance	....	Ohms
$\omega =$	Motor speed	....	Radians/sec
$E_b =$	Armature Back EMF	....	Volts
$K_a =$	<b><math>(ZP/2\pi a)</math></b> : Motor Back EMF/Torque constant		

From the above three equations we get the relation between Torque and speed as:

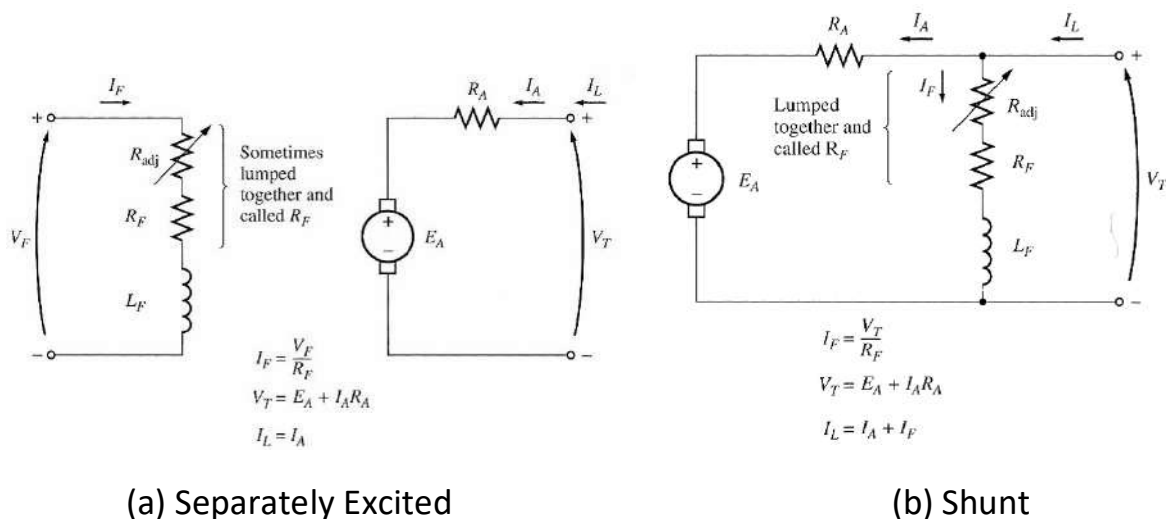
$$\omega = (V_T / K_a \cdot \Phi) - (R_a / K_a \cdot \Phi) \cdot I_a$$

$$\omega = (V_T / K_a \cdot \Phi) - [R_a / (K_a \cdot \Phi)^2] \cdot \tau$$

We will use this generalized equation in different types of motors and obtain their **Torque vs. Speed** characteristics.

### DC separately excited and Shunt Motors:

The Equivalent circuits of DC separately excited and Shunt Motors along with their governing equations are shown in the figure below.



**Fig: Equivalent circuit of DC separately excited and Shunt Motors**

In a separately excited DC motor the field and armature are connected to separate voltage sources and can be controlled independently. In a shunt motor

the field and the armature are connected to the same source and cannot be controlled independently. When the supply voltage to a motor is assumed constant and is same to the field and armature circuits, there is no practical difference in behavior between these two machines. Unless otherwise specified, whenever the behavior of a shunt motor is described, it would be same as that of a separately excited motor.

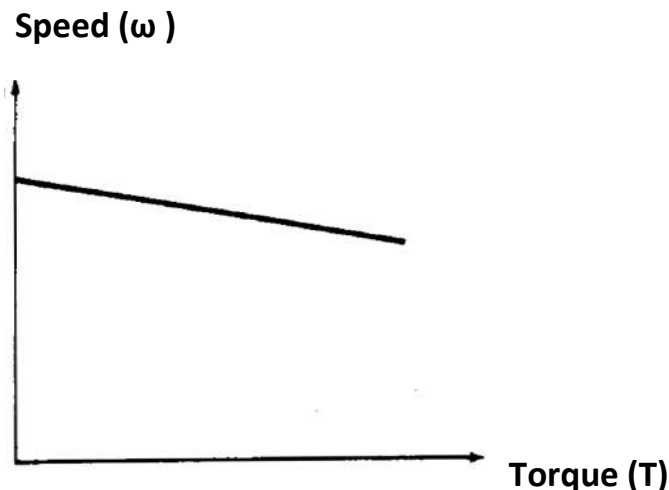
In both their cases, with a constant field current the field flux can be assumed to be constant and then  $(K_a \cdot \Phi)$  Would be another constant  $K$ . Then the above Generalized Torque speed relations would become:

$$\omega = V_T / K - (R_a / K) \cdot I_a$$

Substituting the value of  $I_a$  in terms of  $\tau$  ( $I_a = \tau / K_a \cdot \Phi = \tau / K$ ) we get

$$\omega = V_T / K - [R_a / (K)^2] \cdot \tau$$

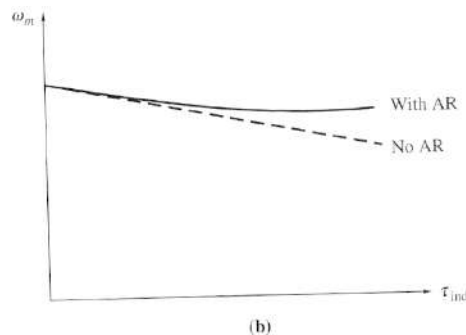
This equation is just a straight line with negative slope. The resulting Speed/Torque Characteristics of a DC Separately Excited /Shunt Motor for a rated terminal voltage and full field current are shown in the figure below. It is a drooping straight line.



**Fig: Speed/ Torque Characteristics of a DC Separately Excited/Shunt Motor**

The no load speed is given by the Applied armature terminal voltage and the field current. Speed falls with increasing load torque. The speed regulation depends on the Armature circuit resistance. The usual drop from no load to full load in the case of a medium sized motor will be around 5%. Separately excited motors are mostly used in applications where good speed regulation and adjustable speed are required.

If motor armature reaction is taken into account, then as its load increases, the flux-weakening effects reduce its flux. From the motor speed equation above, the effect of reduction in flux is to increase the motor's speed at any given load over the speed it would run at without armature reaction. Though at a first glance of the Speed torque equation it may appear that the effect of reduction in flux is to decrease the motor's speed at any given load (since  $\Phi^2$  is in the denominator) actually since the first positive term contains  $V_T$  which is much larger quantity compared to the second negative term,  $I_a R_a$  drop the net effect would only be to increase the motor's speed at any given load. The torque-speed characteristic of a shunt motor with armature reaction is shown below:

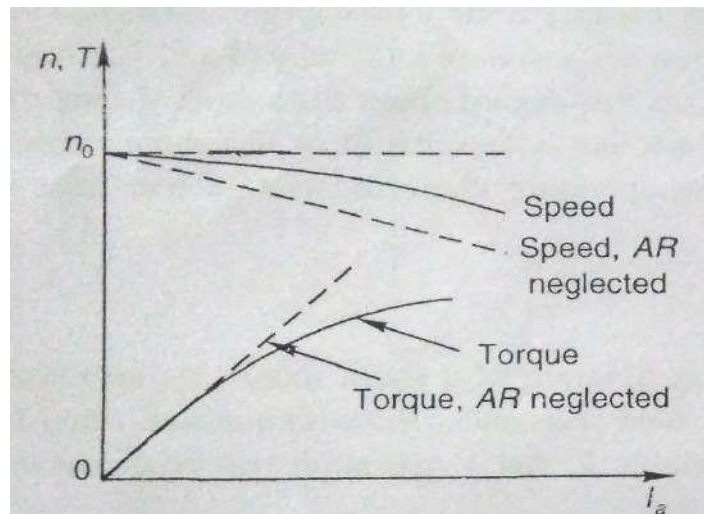


**Fig: Torque-speed characteristic of the motor with armature reaction considered**

**Motor's Other Characteristics:** Though the terminal characteristics (**Speed vs. Torque**) are only important for analysis of a DC motor performance, study and

understanding of the other characteristics like speed vs.  $I_a$  and Torque vs.  $I_a$  would also give additional insight into the performance of the motor and hence they are obtained from the basic equations and presented below:

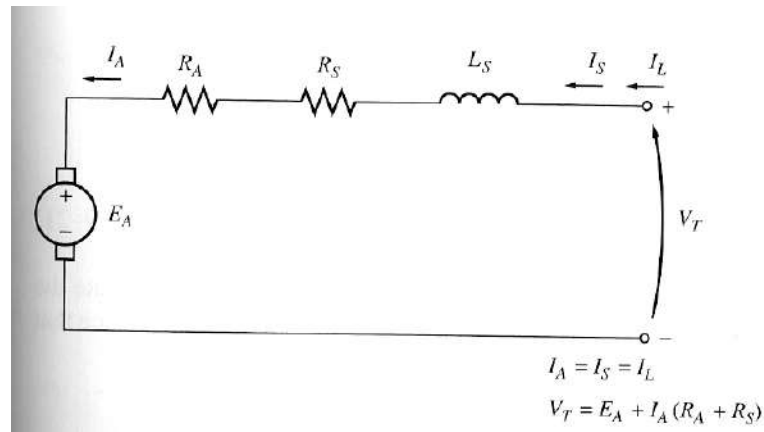
- **Speed vs.  $I_a$  :** 
$$E_b = K_a \cdot \Phi \cdot \omega = V_T - I_a \cdot R_a$$
$$\omega = (V_T - I_a \cdot R_a) / K_a \cdot \Phi$$
- **Torque vs.  $I_a$ :** 
$$\tau = K_a \cdot \Phi \cdot I_a$$



**Fig: Speed and torque vs. Armature current for a DC shunt motor**

### DC Series Motor:

The equivalent circuit of a DC Series motor is shown in the figure below.



**Fig: Equivalent Circuit of a DC Series Motor**

In a series motor the field current and armature current are same and hence the field flux is directly dependent on the armature current. Hence during the initial i.e unsaturated region of the magnetization characteristic the flux  $\Phi$  can be assumed to be proportional to the armature current.

$$\text{Then } \Phi = K_f \cdot I_a$$

And using this value in the first basic motor relation given earlier we get:

$$\tau = K_a \cdot \Phi \cdot I_a = K_a \cdot K_f \cdot I_a^2$$

$$\tau = K_{af} \cdot I_a^2 \quad (\text{where } K_{af} = K_a \cdot K_f)$$

Substituting the above two values of  $\Phi$  and  $\tau$  in the second basic motor equation

$$\omega = (V_T / K_a \cdot \Phi) - [R_a / (K_a \cdot \Phi)^2] \cdot \tau$$

We get

$$\omega = V_T / K_a \cdot K_f \cdot I_a - [R_a / (K_a \cdot K_f \cdot I_a)^2] \cdot K_{af} \cdot I_a^2$$

$$\omega = V_T / K_{af} \cdot I_a - [R_a / (K_{af} \cdot I_a)^2] \cdot K_{af} \cdot I_a^2$$

$$\omega = V_T / K_{af} \cdot I_a - [R_a / (K_{af})]$$

From the relation  $\tau = K_a \cdot \Phi \cdot I_a = K_a \cdot K_f \cdot I_a^2$  we get  $I_a = \sqrt{\tau/K_{af}}$  and substituting this in the above equation  $\omega = V_T / K_{af} \cdot I_a - [R_a / (K_{af})]$

We get  $\omega = *V_T / \sqrt{K_{af} \cdot \tau}] - [R_a / (K_{af})]$

Where  $R_a$  is now the sum of armature and field winding resistances and  $K_{af} = K_a \cdot K_f$  is the total motor constant. The Speed-Torque characteristics of a DC series motor as obtained from the above relation are shown in the figure below.

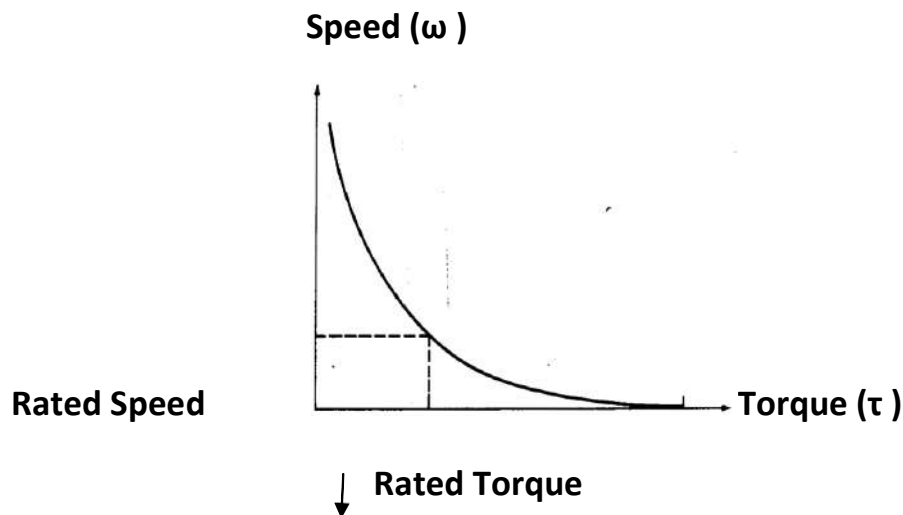


Fig: Speed-Torque characteristics of a DC series motor

**Motor's Other Characteristics:** Though the terminal characteristics (**Speed vs. Torque**) are only important for analysis of a DC motor performance, study and understanding of the other characteristics like speed vs.  $I_a$  and Torque vs.  $I_a$  would also give additional insight into the performance of the motor and hence they are obtained from the basic equations and presented below:

- **Speed vs.  $I_a$  :**

$$E_b = K_a \cdot K_f \cdot I_a \omega = V_T - I_a \cdot R_a$$

$$\text{i.e.} = K_{af} \cdot I_a \cdot \omega = V_T - I_a \cdot R_a \text{ and}$$

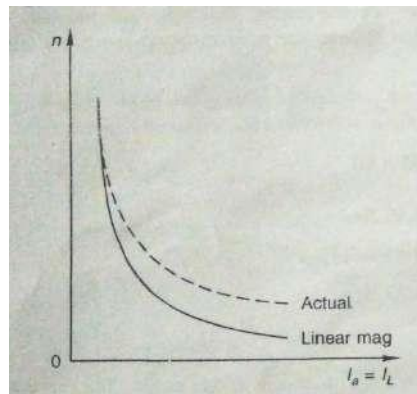
$$\omega = (V_T - I_a \cdot R_a) / K_{af} \cdot I_a = (V_T / K_{af} \cdot I_a) - (R_a / K_{af})$$

This is an inverse relationship and is shown plotted in the figure below.

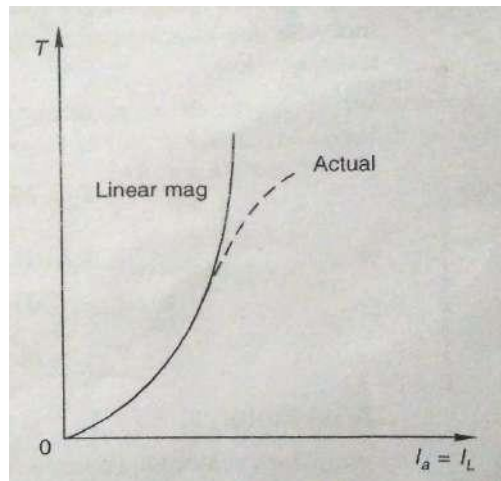
- **Torque vs.  $I_a$ :**  $\tau = K_a \cdot \Phi \cdot I_a = K_{af} I_a^2$

This is a direct relationship and is shown plotted in the figure below.

Saturation and armature reaction demagnetization both cause the flux per pole to increase (with respect to  $I_a$ ) at a rate slower than the assumed linear relationship. Actual characteristics are shown in dotted lines.



**Fig: Speed Vs. Armature current in a Series Motor**



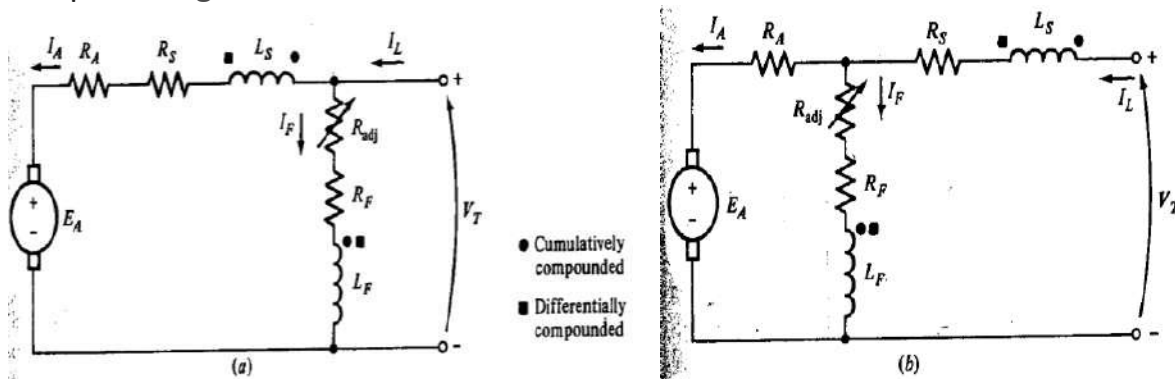
**Fig: Torque vs. Armature current in a DC Series Motor**

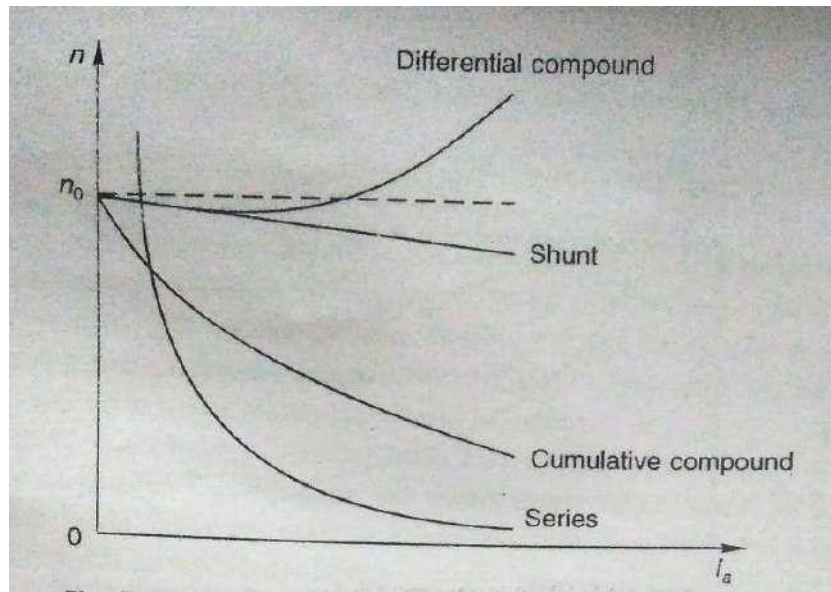
Series motors are suitable for applications requiring high starting torque and heavy overloads. Since Torque is proportional to square of the armature current, for a given increase in load torque the increase in armature current is less in case of series motor as compared to a separately excited motor where torque is proportional to only armature current. Thus during heavy overloads power overload on the source power and thermal overload on the motor are kept limited to reasonable small values. According to the above Speed torque equation, as speed varies inversely to the square root of the Load torque, the motor runs at a large speed at light load. Generally the electrical machine's mechanical strength permits their operation up to about twice their rated speed. Hence the series motors should not be used in such drives where there is a possibility for the torque to drop down to such an extent that the speed exceeds twice the rated speed.

#### **DC Compound Motor:**

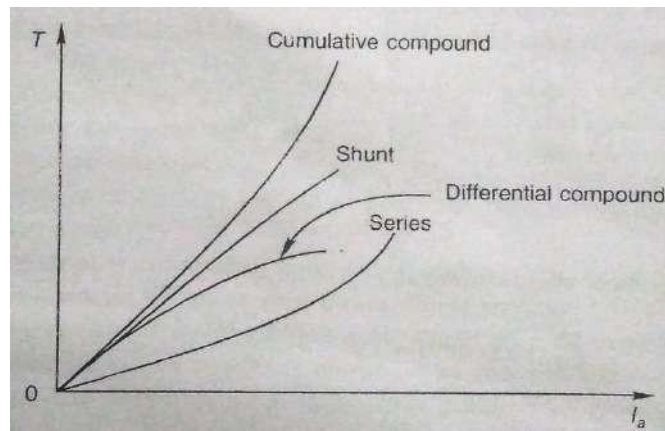
A compound motor is a motor with *both a shunt and a series field*. Such a motor is shown in the Figure below. The dots that appear on the two field coils have the same meaning as the dots on a transformer: *Current flowing into a dot produces a positive magneto motive force*. If current flows into the dots on both field coils, the resulting magneto motive forces add to produce a larger total magneto motive force.

This situation is known as *cumulative compounding*. If current flows into the dot on one field coil and out of the dot on the other field coil, the resulting magneto motive forces subtract. In the Figure below the round dots correspond to cumulative compounding of the motor, and the squares correspond to differential compounding.

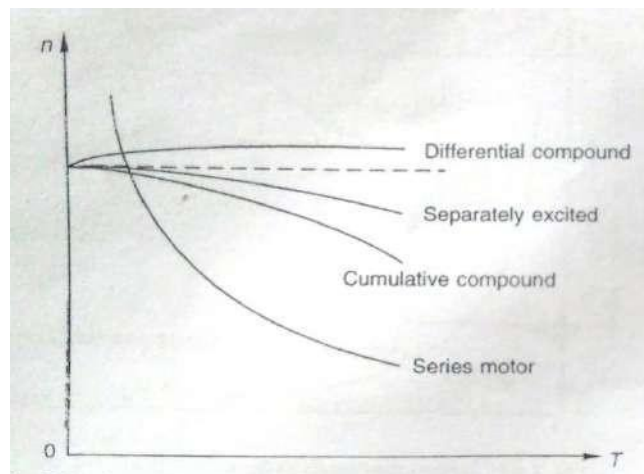




**Fig: Speed vs. Armature current in a DC Compound Motor Compared with other Motors**



**Fig: Torque vs. Armature current in a DC Compound Motor Compared with other Motors**



**Fig: Speed vs. Torque in a DC Compound Motor Compared with other Motors**

**The Torque-Speed Characteristic of a Cumulatively Compounded DC Motor :**

- In the cumulatively compounded DC motor, there is a component of flux which is constant and another component which is proportional to its armature current (and thus to its load) which aid each other. Hence the flux per pole increases with armature current and as consequence ( $n-I_a$ ) curve lies between that of a shunt motor ( $\Phi$  – constant ) and series motor ( $\Phi$  a  $I_a$  ).
- Therefore, the cumulatively compounded motor has a higher starting torque than a shunt motor (whose flux is constant) but a lower starting torque than a series motor (whose entire flux is proportional to armature current).
- In a sense, the cumulatively compounded DC motor combines the best features of both the shunt and the series motors. Like a series motor, it has extra torque for starting; like a shunt motor, it does not over speed at no load.
- At light loads, the series field has a very small effect, so the motor behaves approximately as a shunt DC motor. As the load gets very large, the series flux becomes quite important and the torque-speed curve begins to look like a series motor's characteristic. A comparison of the torque-speed characteristics of each of these types of machines is shown in Figures.

### The Torque-Speed Characteristic of a Differentially Compounded DC Motor:

- In a differentially compounded dc motor, *the shunt magneto motive force and series magneto motive force subtract from each other*. This means that as the load on the motor increases,  $I_a$  increases and *the flux in the motor decreases*. But as the flux decreases, the speed of the motor increases. This speed increase causes another increase in load, which further increases  $I_a$  further decreasing the flux, and increasing the speed again. The result is that a differentially compounded motor is unstable and tends to run away. This instability is *much* worse than that of a shunt motor with armature reaction. It is so bad that a differentially compounded motor is unsuitable for any application.
- Because of the stability problems of the differentially compounded DC motor, it is almost never *intentionally* used.
- However, a differentially compounded motor can result if the direction of power flow reverses in a cumulatively compounded generator. For that reason, if cumulatively compounded DC generators are used to supply power to a system, they will have a reverse-power trip circuit to disconnect them from the line if the power flow reverses. No motor-generator set in which power is expected to flow in both directions can use a differentially compounded motor, and therefore it cannot use a cumulatively compounded generator.

Typical terminal characteristics of differentially compounded dc motor are also included in the Figures.

### Speed control of DC Motors:

Speed control of DC Motors is easier as compared to the speed control of AC motors and much wider range of speeds is possible. That is one reason why even today they are preferred in modern industrial drives. From the two basic equations of DC machines

- $E_b = K_a \cdot \Phi \cdot \omega$
- $V_T = E_b + I_a \cdot R_a$

We have the expression for the speed  $\omega = (V_T - I_a \cdot R_a) / K_a \cdot \Phi$ . From this equation we can (Since  $K_a$  is a constant and  $I_a$  is load dependent) easily see that the speed can be controlled by two methods:

1. By varying the terminal voltage known as : **Armature Voltage Control (AVC)** and
2. By varying the field current and thus the flux per pole  $\Phi$  known as : **Flux control**

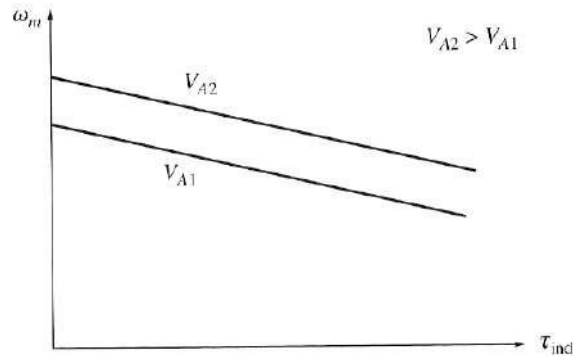
Let us study them one by one for all the three types of Motors

### Speed control of DC Shunt Motor:

#### Armature Voltage Control (AVC):

This method involves changing the voltage applied to the armature of the motor without changing the Voltage applied to the field. This is possible with a separately excited DC Motor only and not with DC Shunt Motor. So first we shall explain for a DC separately excited motor and extend the same logic to a shunt Motor. If the armature terminal Voltage  $V_T$  is increased, then the  $I_A$  will rise since  $[I_A = (V_T \uparrow - E_b) / R_A]$ . As  $I_A$  increases, the induced torque  $\tau = K_a \cdot \Phi \cdot I_a \uparrow$  increases, making  $\tau_{ind} > \tau_{load}$ , and the speed of the motor increases.

But, as the speed  $\omega$  increases,  $E_b = K_a \cdot \Phi \cdot \omega \uparrow$  increases, causing the armature current  $I_A$  to decrease since  $[I_A = (V_T - E_b \uparrow) / R_A]$ . This decrease in  $I_A$  decreases the induced torque, causing  $\tau_{ind}$  to become equal to  $\tau_{load}$  at a final higher steady state rotational speed  $\omega$ . Thus we can see that an increase in Armature voltage results in a higher speed and the resulting Speed Torque characteristics with **AVC** is shown in the figure below.



**Fig: The effect of armature voltage speed control**

Notice that the no-load speed of the motor is shifted by this method of speed control, but the slope of the curve remains constant

The cause-and-effect behavior in this method of speed control can be summarized as below:

1. An increase in  $V_T$  increases  $[ I_A = (V_T \uparrow - E_b)/R_A$
2. Increasing  $I_A$  increases  $\tau_{ind} = K_a \cdot \Phi \cdot I_a \uparrow$
3. Increasing  $\tau_{ind}$  makes  $\tau_{ind} > \tau_{load}$  increasing  $\omega$ .
4. Increasing  $\omega$  increases  $E_b = K_a \cdot \Phi \cdot \omega \uparrow$
5. Increasing  $E_b$  decreases  $I_A = (V_T - E_b \uparrow)/R_A$
6. Decreasing  $I_A$  decreases  $\tau_{ind}$  until  $\tau_{ind} = \tau_{load}$  corresponding to a higher  $\omega$ .

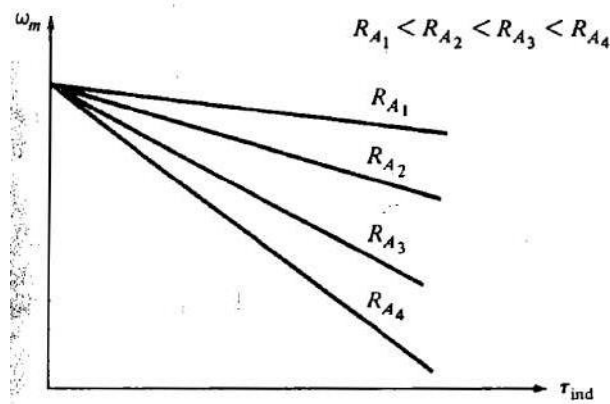
*In the case of a DC Shunt motor since changing the voltage applied to the armature of the motor without changing the voltage applied to the field is not possible, a variable resistance is introduced in series with the armature which results in a reduction in the armature current  $I_A$ . Effectively reduction of armature current is equivalent to reduction in armature voltage as seen in the above logic. Hence we get the same type of speed control as shown in the figure above except that the characteristic with  $V_{A2}$  represents the nominal rated speed and that with  $V_{A1}$  represents with additional resistance introduced in series with the armature. With this method, speed control is possible but speed can only be reduced from the rated or nominal speed. Even for a separately excited DC motor it can provide*

*speed control below Base speed only because armature voltage cannot exceed the rated value.*

**Inserting a resistor in series with the armature circuit :** If a resistor is inserted in series with the armature circuit, the effect is to drastically increase the slope of the motor's torque-speed characteristic, making it operate more slowly if loaded as shown in the figure below. This fact can easily be seen from the basic Equation:

$$\omega = (V_T / K_a \cdot \Phi) - [R_a / (K_a \cdot \Phi)^2] \cdot \tau$$

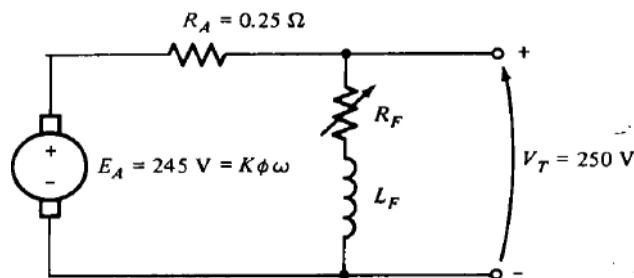
The insertion of a resistor is a very wasteful method of speed control, since the losses in the inserted resistor are very large. For this reason, it is rarely used.



**Figure: Effect of Armature Resistance on the Speed Torque characteristic of a DC shunt motor**

**Flux control:**

Another method of Shunt motor speed control is to change the flux in the field. In a shunt motor Field current and hence field flux cannot be changed without changing the armature voltage. Hence flux control in Shunt motor is achieved by changing the Field resistance. Field coil resistance being fixed we cannot reduce it but increase the field circuit resistance by adding a variable resistance in series with the field coil as shown in the figure below.



Accordingly, when the resistance increases, the field current decreases ( $I_F \downarrow = V_T/R_F \uparrow$ ), and as the field current decreases, the flux decreases. A decrease in flux causes an instantaneous decrease in the back emf ( $E_b \downarrow = K_a \cdot \Phi \downarrow \cdot \omega$ ) which causes an increase in the machine's armature current since,

$$I_A \uparrow = (V_T - E_B \downarrow)/R_A$$

The induced torque in a motor is given by  $\tau_{ind} = K_a \cdot \Phi \downarrow \cdot I_a \uparrow$  .

*Here since the flux in this machine decreases while the current  $I_A$  increases, which way does the induced torque change?*

From practical data it is seen that for an increase in field resistance the decrease in flux is much lesser than the increase in armature current i.e. the increase in current predominates over the decrease in flux.

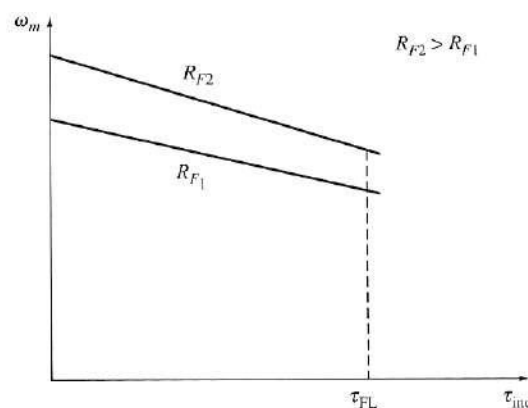
Hence,  $\tau_{ind}$  increases i.e.  $\tau_{ind} > \tau_{load}$ , and the motor speeds up.

However, as the motor speeds up,  $E_b$  rises, causing  $I_A$  to fall. Thus, induced torque  $\tau_{ind}$  too drops, and

Finally  $\tau_{ind}$  equals  $\tau_{load}$  at a higher steady-state speed than the original speed.

The cause-and-effect behavior involved in this method of speed control is summarized below :

1. Increasing  $R_F$  causes  $I_F \downarrow = V_T / R_F \uparrow$  to decrease
2. Decreasing  $I_F \downarrow$  decreases  $\Phi \downarrow$
3. Decreasing  $\Phi \downarrow$  lowers  $E_b \downarrow = K_a \cdot \Phi \downarrow \cdot \omega$
4. Decreasing  $E_b \downarrow$  increases  $I_A$  since  $I_A \uparrow = (V_T - E_b \downarrow) / R_A$
5. Increasing  $I_A$  increases  $\tau_{ind} = K_a \cdot \Phi \downarrow \cdot I_a \uparrow$ , with the change in  $I_A$  being dominant over the change in flux).
6. Increasing  $\tau_{ind}$  makes  $\tau_{ind} > \tau_{load}$ , and the speed  $\omega$  increases.
7. Increasing  $\omega$  increases  $E_b \uparrow = K_a \cdot \Phi \cdot \omega \uparrow$  again.
8. Increasing  $E_b$  decreases  $I_a$
9. Decreasing  $I_a$  decreases  $\tau_{ind}$  until  $\tau_{ind} = \tau_{load}$  at a higher speed  $\omega$ .



**Fig: Shunt Motor Speed control with Flux control (Change in field resistance) (over the normal operating Range )**

The Speed Torque characteristics with change in Field Resistance are shown in the figure below. Notice that with flux control i.e. with insertion of additional resistance in the field circuit, the flux in the machine decreases and hence:

- The no- load speed of the motor increases, while the slope of the torque-speed curve becomes steeper and also
- Speeds above base speed only can be achieved. (as against with Armature resistance insertion control , speeds below base speed only can be achieved) since to achieve speed below base speeds field current has to be increased beyond its rated value which is not permitted. In a normally designed motor the maximum speed can be twice the rated speed and in specially designed motors it can be up to six times the rated speed.

#### **Other important Limitation of field resistance speed control:**

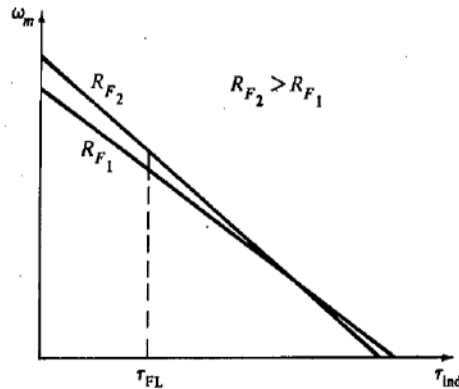
The effect of increasing the field resistance on the output characteristic of a DC shunt motor as seen and explained above is a consequence of the Equation

$$\omega = (V_T / K_a \cdot \Phi) - [R_a / (K_a \cdot \Phi)^2] \cdot \tau$$

which describes the technical characteristic of the motor. In this equation, the no-load speed is proportional to the reciprocal of the flux in the motor, while the slope of the curve is proportional to the reciprocal of the flux squared. Therefore, a decrease in flux causes the slope of the torque- speed curve to become steeper. The earlier figure shows the technical characteristic of the motor over the range from no-load to full-load conditions. Over this range, an increase in field resistance increases the motor's speed, as described above. Hence for motors operating between no- load and full-load conditions, an increase in  $R_f$  may reliably be expected to increase the operating speed.

Now let us examine the figure shown below. This figure shows the technical characteristic of the motor over the full range i.e. from no- load to stall conditions. It is apparent from the figure that at *very slow* speeds an increase in field resistance will actually *decrease* the speed of the motor. This effect occurs because , at very low speeds, the increase in armature current caused by the

decrease in  $E_b$  is no longer large enough to compensate for the decrease in flux in the induced torque equation. With the flux decrease being actually larger than the armature current increase, the induced torque decreases, and the motor slows down.



**Fig: Shunt Motor Speed control with Flux control (Change in field resistance)( over the complete operating Range i.e. from no load to stall condition)**

Some small DC motors used for control purposes actually operate at speeds close to stall conditions. For these motors, an increase in field resistance might have no effect, or it might even decrease the speed of the motor. Since the results are not predictable, field resistance speed control should not be used in these types of dc motors. Instead, the armature voltage method of speed control should be employed.

### Speed Control of Series DC Motors:

Unlike with the shunt dc motor, there is only one efficient way to change the speed of a series dc motor. That method is to change the terminal voltage of the motor. If the terminal voltage is increased, the first term in Equation

$$\omega = \frac{V_T}{K_{af} \tau} - \left[ \frac{R_a}{K_{af}} \right]$$

is increased, resulting in a *higher speed for any given torque*.

The speed of DC series motors can also be controlled by the insertion of a series or parallel (Diverter) resistor into the motor circuit as shown in the figures below along with the resulting effect on Speed torque characteristics. But in this technique large amount of power is dissipated as heat and thus wasted. Hence this method is used only for intermittent periods during the start-up of some motors.

Until the last 40 years or so, there was no convenient way to change  $V_T$ , so the only method of speed control available was the wasteful series resistance method. That has all changed today with the introduction of solid-state control circuits. We will study the techniques of obtaining variable terminal voltages subsequently in another subject '*Power Electronics*'

### **Speed Control of Cumulatively Compounded DC Motor:**

The techniques available for the control of speed in a cumulatively compounded DC motor are the same as those available for a shunt motor:

1. Change the field resistance  $R_F$
2. Change the armature voltage  $V_T$
3. Change the armature resistance  $R_A$ .

The analysis describing the methods and effects of changing  $R_F$  or  $V_T$  or  $R_A$  are similar to the analysis given earlier for the shunt motor.

### The ward Leonard system:

The speed of a separately excited, shunt, or compounded dc motor can be varied in one of three ways: by changing the field resistance, changing the armature voltage, or changing the armature resistance. Of these methods, perhaps the most useful is armature voltage control, since it permits wide speed variations without affecting the motor's maximum torque.

A number of motor-control systems have been developed over the years to take advantage of the high torques and variable speeds available from the armature voltage control of DC motors. In the days before solid-state electronic components became available, it was difficult to produce a varying DC voltage. In fact, the normal way to vary the armature voltage of a dc motor was to provide it with its own separate dc generator.

An armature voltage control system of this type known as ward Leonard Speed control system is shown in the Figure below .

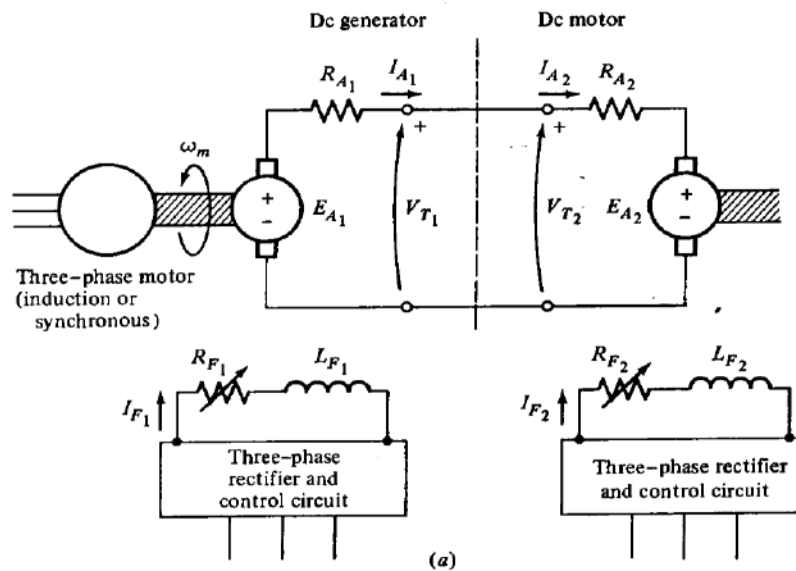
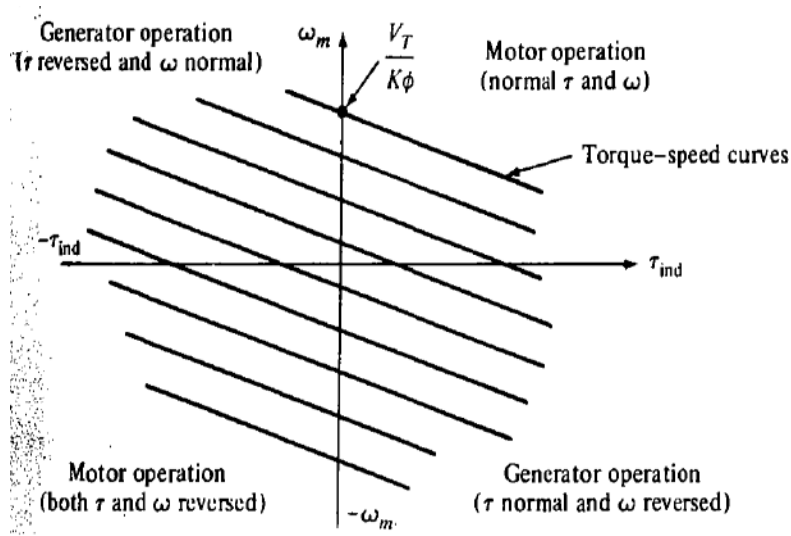


Figure: Ward Leonard DC Motor Speed control system



**Figure: The operating range of a Ward-Leonard motor-control system. The motor can operate as a motor in either the forward (quadrant - 1) or reverse (quadrant -3) direction and it can also regenerate in quadrants 2 and 4.**

In this an AC motor is serving as a prime mover for a DC generator, which allows the motor's speed to be smoothly varied between a very small value and the base speed. The speed of the motor can be adjusted above the base speed by reducing the motor's field current. With such a flexible arrangement, total motor speed control is possible.

Furthermore, if the field current of the generator is reversed, then the polarity of the generator's armature voltage will be reversed, too. This will reverse the motor's direction of rotation. Therefore, it is possible to get a very wide range of speed variations in *either direction of rotation* using a Ward-Leonard DC motor control system.

Another advantage of the Ward-Leonard system is that it can "regenerate," or return the machine's energy of motion to the supply lines. If a heavy load is first raised and then lowered by the DC motor of a Ward-Leonard system, when the load is being lowered, the DC motor acts as a generator and supplying power back

to the power system. In this fashion, much of the energy required to lift the load in the

first place can be recovered, reducing the machine's overall operating costs.

The possible modes of operation of the DC machine are shown in the torque-speed diagram shown in the above Figure. When this motor is rotating in its normal direction and supplying a torque in the direction of rotation, it is operating in the first quadrant of this figure. If the generator's field current is reversed, that will reverse the terminal voltage of the generator, in turn reversing the motor's armature voltage. When the armature voltage reverses with the motor field current remaining unchanged, both the torque and the speed of the motor are reversed, and the machine is operating as a motor in the third quadrant of the diagram. If the torque or the speed alone of the motor reverses while the other quantity does not, then the machine serves as a generator, returning power to the dc power system. Because a Ward-Leonard system permits rotation and regeneration in either direction, it is called a *four-quadrant control system*.

The disadvantages of a Ward-Leonard system should be obvious. One is that the user is forced to buy *three* full machines of essentially equal ratings, which is quite expensive. Another is that three machines will be much less efficient than one. Because of its expense and relatively low efficiency, the Ward-Leonard system has been replaced in new applications by SCR-based controller circuits.

### **Principle of 3 point and 4 point starters:**

Before studying the principle of operation of these starters let us understand the basic principles underlying the starters.

- DC motors are by themselves self starting type. Once the appropriate field and armature supply are given the motors start automatically. They do not need any additional device for the purpose of starting.
- But DC motor starters are required for safe starting of the motors. Initially just at the starting of the motor, the speed is zero and hence the back emf  $E_b$  is also zero. In this condition if the Rated terminal voltage  $V_t$  is applied to the motor we can see from the basic governing equation

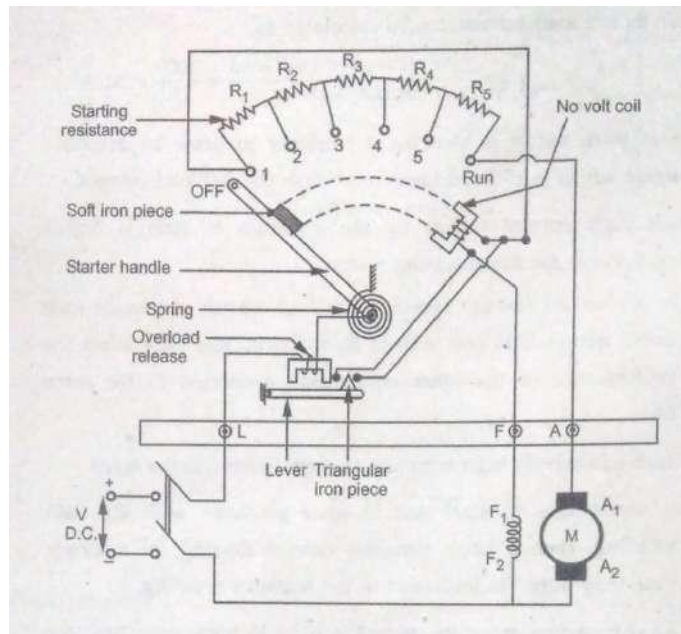
$$I_a = (V_t - E_b)/R_a$$

That the motor draws excessive current which would be easily 10 to 15 times that of the nominal rated current of the motor. This excessive current would flow till the motor develops the rated speed.

- During this transient period when the excessive current flows the torque developed also would be excessive. With the result the motor would get damaged both electrically and mechanically.
- To protect the motor from such damage, a resistance is introduced in series with the motor as shown in the figure below which would be withdrawn gradually in steps as the motor picks up speed.
- This is a basic arrangement of a DC motor starter and its operation is totally manual. But practical starters have been developed with additional protective and automatic starting features. They are called **3 point starters and 4 point starters**, the subject of our study.

### **3 Point starter:**

The circuit diagram and the arrangement of a three point starter are shown in the figure below.



**Figure: 3 Point starter**

**Basic features and working principles:**

- The basic component viz starter resistance comes in steps with contact points brought out as studs 1,2,3.. Run.
- The three points are:
  - L – The line terminal to be connected to the DC positive terminal through a two pole switch
  - A – The terminal to be connected to the terminal  $A_1$  of the armature.
  - F – The terminal to be connected to the terminal  $F_1$  of the Field winding
- The other ends  $A_2$  of the armature and  $F_2$  of the field are connected to the other contact of the two pole switch which gets connected to the negative terminal of the DC power supply when switched on.
- Point L in turn is connected to the pivot point of the handle through a protective device called OLR ( Over Load Relay )

- The handle which is spring loaded comes back to the OFF position under its own force until locked in the RUN position due to the electromagnetic pull of the other protective device known as NVC ( No Volt Coil)
- The field terminal F is connected to starting point 1 of the resistance in a parallel path through the NVC.

**Operation of the starter:** The starter is gradually moved from the initial position to the final RUN position manually against the spring force. When the handle comes in contact with stud -1 , the field supply gets extended to the field coil through the parallel path connected directly from stud -1 through NVC . In the starter initially entire resistance comes in series with the armature and as the handle is moved towards RUN, the portion of the series resistance that comes out of the armature circuit gets added to the field circuit. Finally when the handle is brought to the Run position, the entire resistance gets removed from the Armature circuit and the motor runs at the rated speed. The handle is held in RUN position due to the action of the NVC.

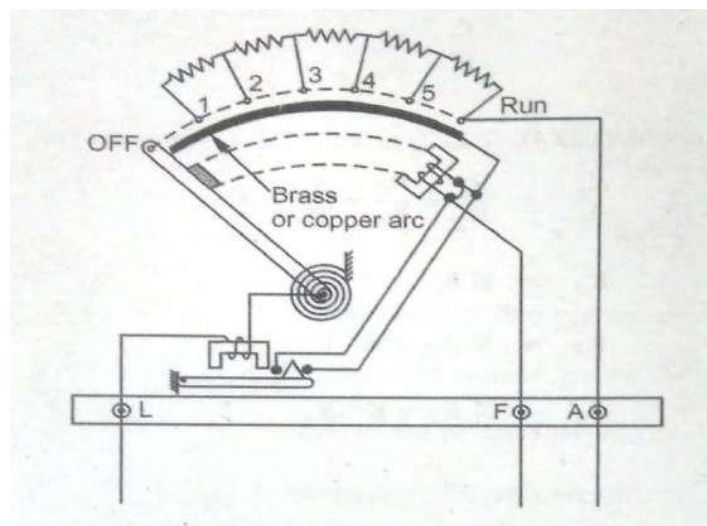
**Action of the NVC:** When the field current flows through the NVC it attracts the handle with the soft iron piece and keeps it in contact the NVC electromagnet. Hence NVC is also called as **Hold On Coil**. In addition to holding the handle in the final RUN position, the NVC works as a safety/protection device by releasing the handle back to the start position from the RUN position whenever there is a power failure or when the field circuit breaks. Thus the entire starting resistance comes into the armature circuit every time the motor is started from zero speed and prevents high inrush currents during every fresh starting attempt after a power failure.

**Action of the OLR:** As can be seen from the figure there is another protective relay called OLR (Over Load Relay) which is also an electromagnet which works in conjunction with an arm fixed on a fulcrum at one end and with a triangular iron piece fixed on the other end. Whenever there is an overload current beyond a set

safety value, the electromagnet activates and pulls the arm upwards and the triangular iron piece short circuits the two terminals which are connected to the two ends of the NVC coil. Thus with any overload due to a fault in the motor or associated circuit, the NVC gets deactivated and releases the handle back to the initial safe start condition. After the fault is rectified the motor can be started afresh with full resistance brought back into the armature circuit.

### 3 Point starter with a brass/copper arc:

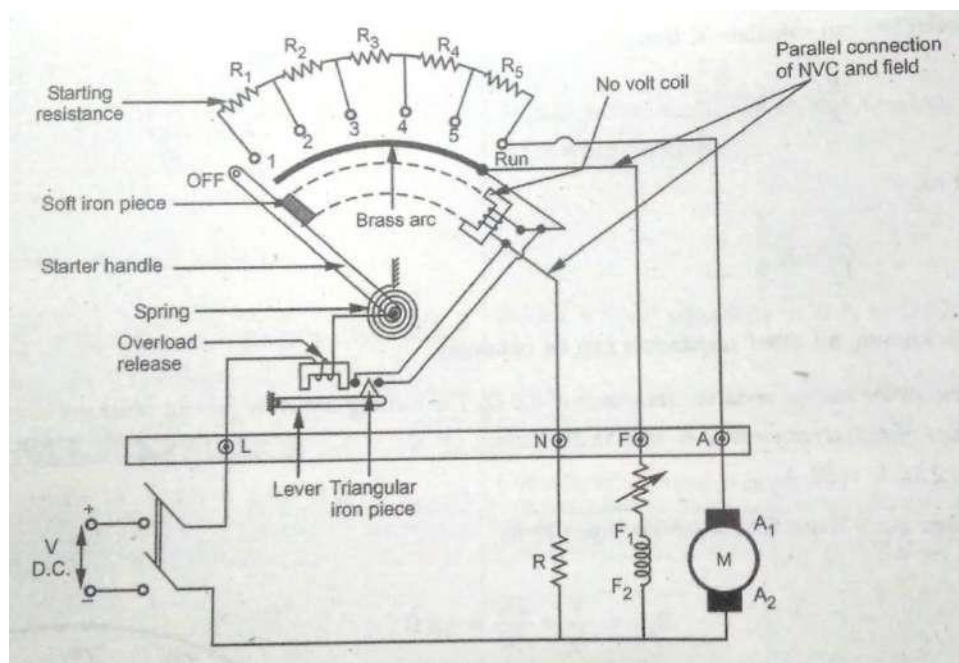
In the earlier version of the 3 Point starter as we have seen, as the handle is moved towards RUN position, the portion of the series resistance that comes out of the armature circuit gets added to the field circuit. Thus finally when the motor is running, the entire starter resistance gets added to the field circuit. But since the starting resistance value is very small compared to field winding resistance, this hardly reduces the field current and hence there is no any practical impact. However this addition of resistance in the field circuit can be avoided by providing a brass or copper arc with one end connected to the stud -1 and the other end connected to the NVC as shown in the figure below.



**Figure: 3 Point starter with Brass arc**

With such an arrangement when the handle moves on the arc the field current directly flows through the arc to the NVC thus avoiding the starting resistance. With such an arc in place, the earlier parallel connection from stud -1 to the NVC start terminal is no more required and hence is removed.

**4 Point starter:** The operation of the 4 point starter is explained along with the schematic diagram shown below.



**Figure: 4 Point starter**

- The basic difference between a 3 Point starter and a four point starter is : In a 3 point starter NVC was connected in series with the field coil while in a 4 point starter the NVC is connected independently to the supply through a fourth terminal termed as N in addition to L, F and A.
- With this arrangement any change in the field current due to the change field control resistance will not affect the performance of the NVC. This ensures

that NVC always produces a force enough to hold on the handle irrespective of the amount of field current. Adequate current required for the confirmed operation of the NVC is obtained by adjusting the resistor R connected in series with the NVC coil.

- However the 4 point starter has a separate disadvantage: Since now the NVC is connected separately excluding the field current, it cannot detect the field failure and hence the resulting **over speed** cannot be prevented.

### Important concepts and Formulae:

- Torque generated in a DC machine :  $\tau = K_a \cdot \Phi \cdot I_a$
- KVL around the armature circuit:  $V_T = E_b + I_a \cdot R_a$
- Generalized *Torque vs. Speed* equation in different types of motors:

$$\omega = (V_T / K_a \cdot \Phi) - [R_a / (K_a \cdot \Phi)] \cdot \tau$$

- Shunt Motor's Other Characteristics:

$$\text{Speed vs. } I_a : \quad \omega = (V_T - I_a \cdot R_a) / K_a \cdot \Phi$$

$$\text{Torque vs. } I_a : \quad \tau = K_a \cdot \Phi \cdot I_a$$

- Series Motor:

$$\omega = [V_T / \sqrt{K_{af} \cdot \tau}] - [R_a / (K_{af})]$$

- Speed vs.  $I_a$  :  $\omega = (V_T - I_a \cdot R_a) / K_{af} \cdot I_a = (V_T / K_{af} \cdot I_a) - (R_a / K_{af})$
- Torque vs.  $I_a$ :  $\tau = K_a \cdot \Phi \cdot I_a = K_{af} I_a^2$
- Speed control with armature voltage control is possible only below the rated or nominal speed (also known as base speed).
- Speed control with flux control is possible only above the base speed

### Illustrative examples:

**Ex.1:** A 500 V shunt motor with  $R_f = 250 \Omega$  and  $R_a = 0.2 \Omega$  runs at 2500 RPM taking a current of 25 A from the mains supply . Calculate the resistance to be added to the armature circuit to reduce the speed to 1500 RPM keeping the armature current constant.

**Solution:**

***First let us calculate the back e.m.f developed by the motor in the given first set of conditions:***

$$\text{Field current } I_f = \text{Rated terminal voltage} / R_f = 500 / 250 = 2 \text{ A}$$

$$\text{Armature current } I_a = I_l - I_f = (25 - 2) = 23 \text{ A}$$

$$\text{Back e.m.f } E_b = V_T - I_a R_a = 500 - 23 \times 0.2 = 495.4 \text{ V}$$

We know that the back e.m.f is proportional to the speed

$$\begin{aligned} \therefore E_{b1} / E_{b2} &= N_1 / N_2 \quad \text{i.e.} \quad 495.4 / E_{b2} = 2500 / 1500 \quad \therefore E_{b2} = 495.4 \times 1500 / 2500 \\ &= 297.24 \text{ V} \end{aligned}$$

But we also know that  $E_{b2} = V_T - I_a R_{a2}$  ( Since the terminal voltage and the armature current remain the same )

$$\therefore 297.24 = 500 - 23 \times R_{a2} \quad \text{from which we get} \quad R_{a2} = (500 - 297.24) / 23 = 8.82 \Omega$$

This is the total new resistance of the armature circuit (including the original armature resistance of  $0.2 \Omega$  to get a speed of 1500 RPM)

**Hence the new resistance to be added into the armature circuit =  $8.82 - 0.2 = 8.62 \Omega$**

**Ex.2:** A DC shunt motor takes 22 A from 250 V supply.  $R_a = 0.5 \Omega$  ,  $R_f = 125 \Omega$ . Calculate the resistance required to be connected in series with the armature to

halve the speed (a) when the load torque is constant (b) When the load torque is proportional to the square of the speed

**Solution :**

*First let us calculate the speed of the motor when the load current  $I_l$  is 22 A :*

$$\text{Field current } I_f = \text{Rated Terminal voltage} / \text{Field resistance} = 250/125 = 2 \text{ A}$$

$$\text{Armature current } I_a = I_l - I_f = 22 - 2 = 20 \text{ A}$$

$$\text{Back e.m.f } E_b = V_T - I_a R_a = 250 - 20 \times 0.5 = 240 \text{ V}$$

*(a) we have to find out the New  $R_a$  when the speed is halved with torque maintained constant :*

We know that Torque  $T = K_a \cdot \phi \cdot I_a$  . In this case since change is only in the armature resistance field current and hence flux  $\phi$  remains the same. Further since the torque is maintained constant the armature currents are also equal and hence  $I_{a1} = I_{a2} = 20 \text{ A}$

We also know that  $E_b = K_a \cdot \phi \cdot \omega$  . As already explained,  $K_a \cdot \phi$  remains same and hence when the speed is halved the back e.m.f also gets halved.

$$\text{Hence } E_{b2} = 120 \text{ V} = V_T - I_a R_{a2} \quad \text{i.e } 250 - 20 \times R_{a2} = 120 \text{ V} \quad \text{i.e } R_{a2} = (250 - 120)/20 = 6.5 \Omega$$

$$\text{Hence the } \mathbf{\text{Resistance to be added to halve the speed}} = R_{a2} - R_a = 6.5 - 0.5 = \mathbf{6.0 \Omega}$$

*(b) Next we have to find out the New  $R_a$  when the speed is halved when torque is proportional to square of speed.*

When the torque is proportional to the square of the speed  $\tau_1 = K \omega_1^2$  and  $\tau_2 = K \omega_2^2$

$$\therefore \tau_1 / \tau_2 = K \omega_1^2 / K \omega_2^2 = \omega_1^2 / \omega_2^2 = (1/0.5)^2 = 4$$

But Torque is also proportional to the product of flux (and hence field current) and Armature current. Here field circuit is not disturbed and hence the field current is same. Using this relation we can find out new armature current  $I_{a2}$

$$\therefore \tau_1 / \tau_2 = K \times I_f \times I_{a1} / K \times I_f \times I_{a2} = I_{a1} / I_{a2} = 4 \quad \text{i.e } I_{a2} = I_{a1} / 4 = 20/4 = 5 \text{ A}$$

Next using the relation between the speeds and the back emfs we can find out the armature resistance to be added.

$$\omega_1 / \omega_2 = 2 \text{ and also}$$

$$\omega_1 / \omega_2 = E_{b1} / E_{b2} = 240 / (250 - 5 \times R_{a2}) \quad \text{i.e } 250 - 5R_{a2} = 240/2 = 120 \quad \text{From which we get}$$

$$R_{a2} = (250-120)/5 = 26 \Omega \quad \therefore \quad \text{Finally } \mathbf{Resistance\ to\ b\ added\ is} = 26-0.5 = \mathbf{25.5 \Omega}$$

**Ex.3:** A 250 V DC series motor takes 40 A and runs at 1000 RPM. Find the speed at which it runs if its torque is halved. Assume that the motor is operating in the unsaturated region of its magnetization.  $R_f = 0.25 \Omega$   $R_a = 0.25 \Omega$

*First we will use the relation between torque and armature current and get the back e.mf when the torque is halved :*

In a DC motor we know that the torque is proportional to  $\phi \cdot I_a$ . In the case of a series DC motor flux is proportional to the armature current itself since  $I_f = I_a$ . Hence in a series motor  $\tau \propto I_a^2$

$$\text{Hence } \tau_1 / \tau_2 = I_{a1}^2 / I_{a2}^2 = 2 \quad (\text{Since torque is halved}) \quad I_{a1} / I_{a2} = \sqrt{2}$$

$$I_{a1} = 40 \text{ A} \quad \text{and } I_{a2} = 40 / \sqrt{2} = 28.28 \text{ A}$$

$$E_{b1} = 250 - 40(0.25 + 0.25) = 230 \text{ V} \quad \text{and } E_{b2} = 250 - 28.28(0.25 + 0.25) = 235.86 \text{ V}$$

*Next we will use the relation between back emf and speed and get the speed when the torque is halved:*

We know that  $E_{b1} = K_a \phi_1 N_1$  and  $E_{b2} = K_a \phi_2 N_2$ . But since the flux is proportional to  $I_a$  the relations become  $E_{b1} = K I_{a1} N_1$  and  $E_{b2} = K I_{a2} N_2$  where K is a new constant.

$$\text{Hence } E_{b1}/E_{b2} = K I_{a1}N_1 / K I_{a2}N_2 = I_{a1}N_1 / I_{a2}N_2 \quad \text{and } N_2 = ( I_{a1} / I_{a2} ) ( E_{b2}/ E_{b1} ) N_1$$

Substituting the above values we get  $N_2 = \sqrt{2} (235.86/230)1000 = \mathbf{1450 \text{ RPM}}$

**Ex.4:** A 500 V DC shunt motor runs at 1900 RPM taking an armature current of 150 A. The armature resistance is 0.16  $\Omega$ . Find the speed of the motor when a resistance is inserted in the field circuit which reduces the field current to 80 % and the armature current is 75 A.

**Solution:**

We know that the back e.m.f of a DC motor is proportional to the Flux and speed. And in the unsaturated region of the magnetization region the flux in turn is proportional to the field current. So Back e.m.f is proportional to field current and speed. We will find out the new speed by calculating the back e.m.fs [from the relation ( $E_b = V_T - I_a R_a$ )] and using the above proportionality relation in both the conditions as below.

$$E_{b1} = V_T - I_{a1} R_a = 500 - 150 \times 0.16 = 476 \text{ V} \quad \text{and is equal to } K_a \cdot \phi_1 \cdot N_1$$

$$E_{b2} = V_T - I_{a2} R_a = 500 - 75 \times 0.16 = 488 \text{ V} \quad \text{and is equal to } K_a \cdot 0.8\phi_1 \cdot N_2$$

$$\therefore 476/488 = K_a \cdot \phi_1 \cdot N_1 / K_a \cdot 0.8\phi_1 \cdot N_2$$

$$\text{And } N_2 = (488/476)(N_1/0.8) = (488/476)(1900/0.8) = \mathbf{2435 \text{ RPM}}$$

## UNIT – III

### TESTING OF D.C. MACHINES

#### CONTENTS:

- Losses – Constant & Variable losses
- Calculation of efficiency
- Condition for maximum efficiency.
- Methods of Testing
  - Direct, indirect and regenerative testing
  - Brake test
  - Swinburne's test
  - Hopkinson's test
  - Field's test
  - Retardation test
- Separation of stray losses in a DC motor test
  - **Important concepts and Formulae**
  - **Illustrative examples**

### Losses:

DC Generators convert Mechanical power into Electrical power and DC Motors convert Electric power to Mechanical power. In the process of conversion some power is lost. The difference between the input power and the output power of a machine is the **Power loss** that occurs inside the machine.

### Constant & Variable losses:

The losses are broadly classified as *constant losses* and *variable losses*. Constant losses are constant and are independent of the load where as the variable losses are dependent on the load. They are further classified in detail as below.

### Detailed Classification of Losses:

1. **Electrical or Copper Losses ( $I^2R$  Loss):** Current flow through the resistance of Armature and Field coils gives rise to  $I^2R$  losses and since the coils are normally made up of copper these losses are called Copper losses.

$$\text{Armature copper loss: } P_A = I_A^2 R_A$$
$$\text{Field copper loss: } P_F = I_F^2 R_F$$

2. **Brush losses:** The brush drop loss is the power lost across the contact potential at the brushes of the machine. It is given by the equation:

$$P_{BD} = V_{BD} \times I_A$$

where  $P_{BD}$  = brush drop loss  
 $V_{BD}$  = brush voltage drop  
 $I_A$  = armature current

The brush losses are calculated in this manner because the voltage drops across a set of brushes are

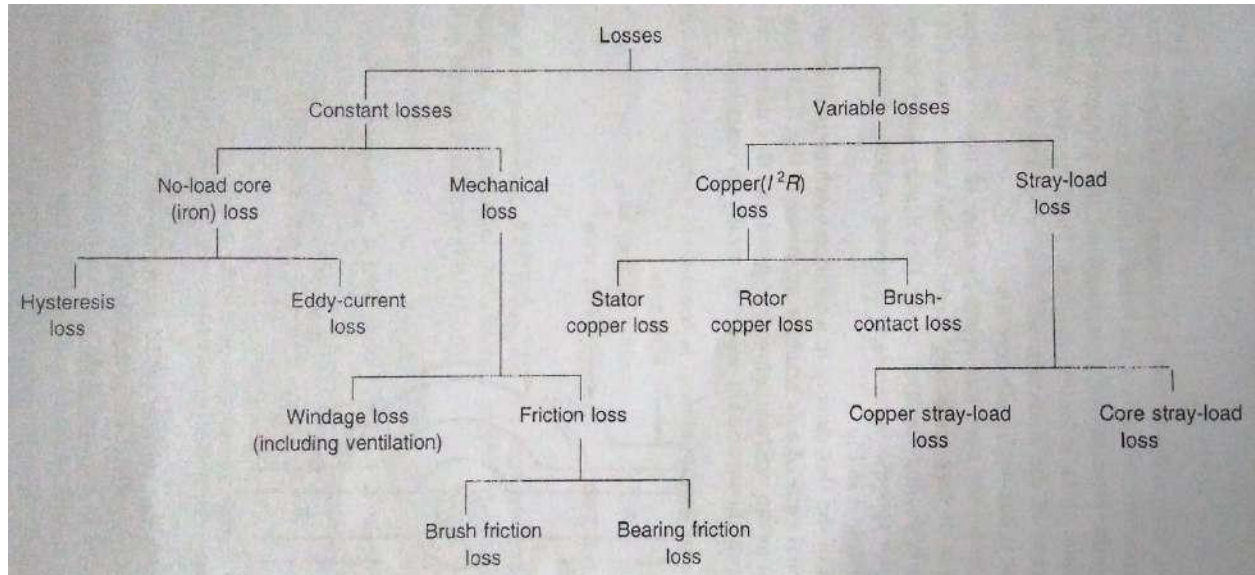
approximately constant over a large range of armature currents. Unless otherwise specified. The

brush voltage drop is usually assumed to be about 2 V.

3. **Core Losses:** *Hysteresis* and *eddy current* losses occurring in the Armature and Field cores together are called core losses.
- **Hysteresis loss:** in an iron core is the loss of power due to the hysteresis loop in the magnetization characteristic of the core in each cycle of the alternating current applied to the core. In the case of DC machines though there is no alternating current applied to the core, the change in the magnetic flux within the machine due to its constructional features result in a small *hysteresis loss*
  - **Eddy current losses:** A time-changing flux induces voltage within a ferromagnetic core in just the same manner as it induces voltage in the conductors around the core of the armature. These voltages cause swirls of current to flow within the core, much like the eddies seen at the edges of a river. It is the shape of these currents that gives rise to the name ***eddy currents***. These eddy currents flowing in a resistive material (the iron of the core) cause power loss thus heating the iron core and the resulting loss is called ***eddy current loss***. This loss is proportional to the thickness of the core material and hence to minimize this loss the core is made up of thin sheets called laminations instead of a single thick block. An insulating oxide or resin is used between the strips so that the current paths for eddy currents are limited to very small areas. Thus the eddy current losses have a very little effect on the core's magnetic properties.
4. **Mechanical Losses:** They are associated with the mechanical effects and they are mainly *Friction* and *windage* losses.
- ***Friction losses*** are losses caused by the friction in the bearings of the machine and
  - ***Windage losses*** are due to the friction between the moving parts of the machine and the air flow in the machine housing.
5. **Stray Losses:** They are other miscellaneous losses that cannot be grouped into any of the above categories.

*Out of the above, Core Losses and Mechanical Losses are grouped under Constant losses. Electrical or Copper Losses and Stray Losses are grouped under variable losses.*

The losses and their classification explained above is summarized in the form of a tree and is shown below.



**Figure: Classification of losses in DC Machine**

The power flow in DC machines showing the stages where the different losses occur is shown clearly in the figure below.

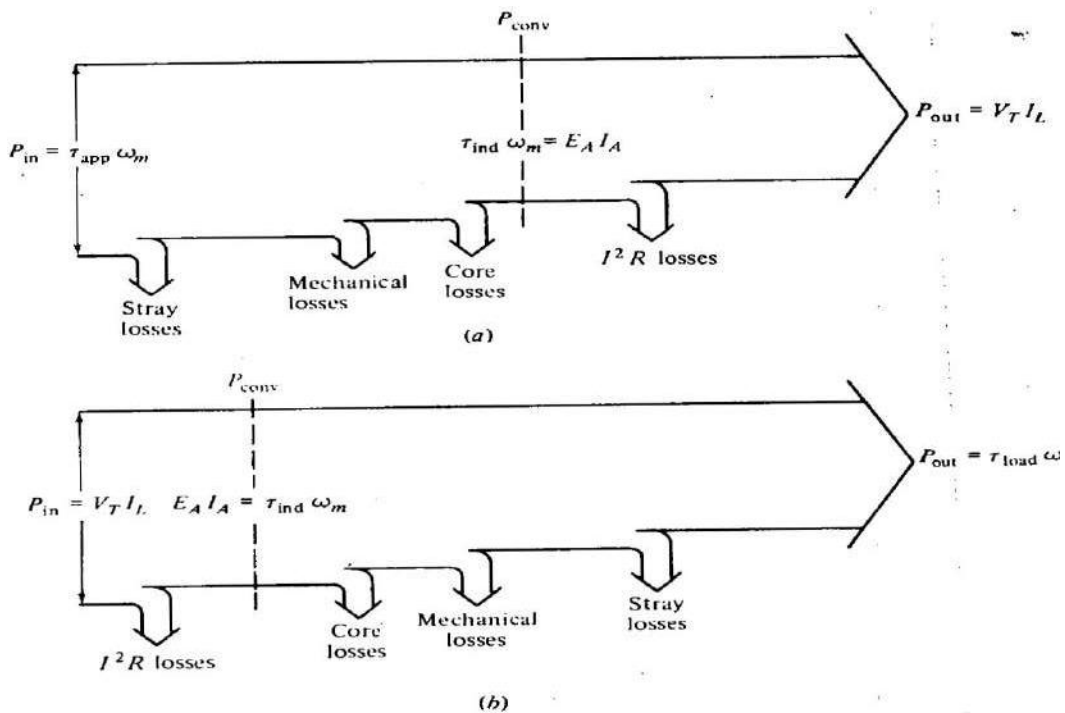
### Power flow diagram:

One of the most convenient techniques for accounting for power losses and showing them clearly in the order in which they occur in a machine is the *power-flow diagram*. A power-flow diagram for a DC generator is shown in the figure (a) below. In this figure, mechanical power is input into the machine, and then the stray losses, mechanical losses, and core losses are subtracted. After they have been subtracted, the remaining power is ideally converted from mechanical to electrical form at the point labeled  $P_{conv}$ . The mechanical power that is converted is given by:

$$P_{CONV} = \tau_{ind} \cdot \omega_m$$

and the resulting electric power produced is given by:  $P_{CONV} = E_A \cdot I_A$

However, this is not the power that appears at the machine's terminals. Before the terminals are reached, the electrical power losses like the copper losses and the brush losses must be subtracted.



**Figure: Power flow diagram of a DC Machine (a) Generator (b) Motor**

In the case of dc motors, this power-flow diagram is simply reversed. The Power flow diagram for a motor is shown in the above figure (b) above

**Efficiency:**

The efficiency of a DC Machine is defined as  $\eta = (P_{out}/P_{in}) \cdot 100 \% = [(P_{in} - P_{loss}) / P_{in}] \times 100 \%$

*Using this basic relation and from a clear understanding of the above Power flow the  $\eta$  calculations when the machine is working as a Generator and as a Motor are given below.*

**Efficiency calculations of Generator:**

- If  $I_L$  is the load current supplied by the Generator at a terminal voltage of  $V_T$  then the output power is given by  $P_{out} = V_T \cdot I_L$
- The armature current  $I_A = I_L + I_F$
- Armature copper loss  $P_A = I_A^2 R_A$
- Field copper loss  $P_F = I_F^2 R_F$
- Total losses  $= I_A^2 R_A + I_F^2 R_F + W_C$  where  $W_C$  is the sum of the core losses and stray losses. (also known as constant losses)
- Therefore Input  $= P_{out} + \text{Total losses} = P_{out} + I_A^2 R_A + I_F^2 R_F + W_C$

Hence  $\eta = (P_{out}/P_{in}) \cdot 100 \% = (V_T \cdot I_L) / (V_T \cdot I_L + I_A^2 R_A + I_F^2 R_F + W_C) \cdot 100\%$

**Efficiency calculations of Motor:**

- If  $I_L$  is the line current taken by the Motor at a terminal voltage of  $V_T$  then the input power is given by  $P_{in} = V_T \cdot I_L$

- The losses are same as in the Generator

- Therefore output  $P_{out} = P_{in} - \text{Total losses} = P_{in} - (I_A^2 R_A + I_F^2 R_F + W_C)$

Hence  $\eta = (P_{out}/P_{in}) \cdot 100\% = \{[P_{in} - (I_A^2 R_A + I_F^2 R_F + W_C)] / (V_T \cdot I_L)\} \cdot 100\%$

Of these losses  $(I_F^2 R_F + W_C)$  are called constant losses  $P_C$  since they are almost independent of load. The armature copper losses i.e.  $(I_A^2 R_A)$  is called the variable loss and is dependent on the load. The variable loss varies approximately as the square of load current. We say approximately since loss varies as the square of the armature current and not as the square of the load current. Hence if we know the loss at full load, the loss at half load, one fourth load etc can be calculated.

### Condition for Maximum efficiency:

The condition for maximum efficiency is developed by differentiating the expression for efficiency as a function of load current and equating it to zero since the variable losses are dependent on the load current.

#### Generator:

The efficiency is obtained as:  $\eta = (P_{out}/P_{in}) \cdot 100\% = (V_T \cdot I_L) / (V_T \cdot I_L + I_A^2 R_A + I_F^2 R_F + W_C) \cdot 100\%$

Neglecting the field current which is small compared to armature current we get

$$\eta = (V_T \cdot I_L) / (V_T \cdot I_L + I_L^2 R_A + W_C) \cdot 100\% = 1 / [1 + I_L R_A / (V_T \cdot I_L) + W_C / (V_T \cdot I_L)] \cdot 100$$

$$= 1 / [1 + I_L R_A / V_T + (W_C / (V_T I_L))] \cdot 100$$

The efficiency is maximum when the denominator is maximum. Hence the condition for maximum efficiency becomes:  $d/dI_L [1 + I_L R_A / V_T + (W_C / (V_T I_L))] = 0$   
i.e.  $R_A / V_T - (W_C / (V_T I_L^2)) = 0$

And finally the condition for maximum efficiency becomes:  $I_L^2 R_A = W_C$

Which means

**Variable losses = Constant**

**Losses**

And the current at maximum efficiency becomes:

$$I_L = \sqrt{W_C / R_A} = \sqrt{\text{Constant Losses} / \text{Armature resistance}}$$

**Motor:**

The efficiency is obtained as:  $\eta = (P_{out}/P_{in}) \cdot 100\% = \{[P_{in} - (I_A^2 R_A + I_F^2 R_F + W_C)] / (V_T \cdot I_L)\} \cdot 100\%$

$$= \{[V_T \cdot I_L - (I_A^2 R_A + I_F^2 R_F + W_C)] / (V_T \cdot I_L)\} \cdot 100\%$$

Neglecting the field current which is small compared to armature current we get

$$\eta = \{[V_T \cdot I_L - (I_L^2 R_A + W_C)] / (V_T \cdot I_L)\} \cdot 100\%$$

$$= 1 - [(I_L^2 R_A + W_C) / (V_T \cdot I_L)] \cdot 100$$

$\eta$  becomes maximum when the term in the square brackets becomes minimum

and thus the condition for maximum efficiency becomes  $d/d I_L [(I_L^2 R_A + W_C) /$

$(V_T \cdot I_L)] = 0$  which again finally becomes :

$$I_L^2 R_A = W_C$$

**Or Variable losses = Constant Losses**

And the current at maximum efficiency also becomes:

$$I_L = \sqrt{W_C / R_A} = \sqrt{\text{Constant Losses} / \text{Armature resistance}}$$

**Both same as that for the generator.**

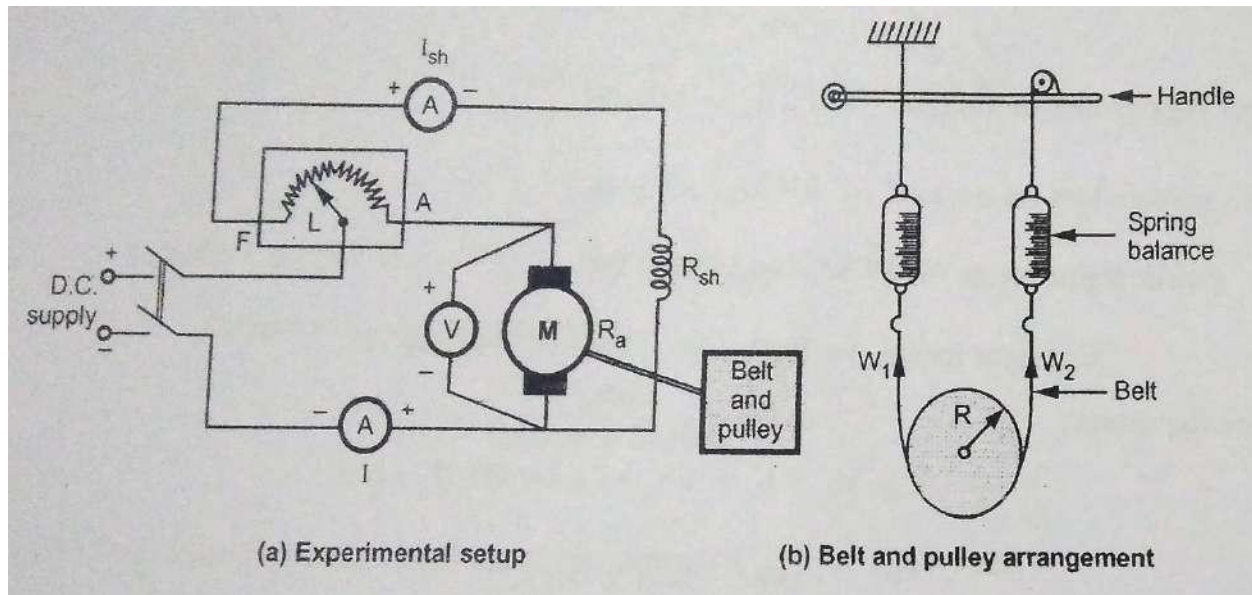
**Testing of DC machines:**

Involves the measurement of the various losses and then finding out the efficiency of the machine by various methods. The methods are broadly classified as:

1. Direct 2. Indirect and 3. Regenerative methods of testing

1. Direct method of testing : In this method the DC machine is actually loaded to the required extent, the Input and output are measured and then the  $\eta$  is calculated as  $\eta = \text{Output/Input}$ 
  1. This method is generally employed only for small motors. The motor is loaded by a friction pulley arrangement.(braking)
  2. The main drawback of this method is accuracy of output power measurement is limited.
  3. Difficult to provide braking load for a large capacity motor.
2. Indirect method of testing: In this method the machine is not subjected to full load. First on no load the constant losses are measured and then efficiency is estimated at various loads. Swinburne's test and Hopkinson's test come under this category. Only shunt motors can be tested using these methods. Series motors cannot be tested with this method since they cannot be run on no load.
3. Regenerative method of testing: In this method a motor generator is pair is used which are powered by each other. Thus only losses are drawn from the mains power supply. Hopkinson's test comes in this category.

**Brake Test:** This is a direct method of testing. In this method the motor is put on a direct friction load arrangement with a belt and a pulley as shown in the figure below.



**Figure: Brake Test setup**

By adjusting the tension in the pulley the motor can be subjected from no load to its full load capability. Since the load is applied by the physical braking action, the test is called the **Brake test**.

The tension in the belt is adjusted by using the handle. The tension (kgf) is obtained from the spring balance readings. The net force applied on the pulley by this braking arrangement is given by:

$$\text{Net force} = (W_1 - W_2) \text{ Kgf} = 9.81(W_1 - W_2) \text{ Nw}$$

Where  $R$  = Radius of the pulley in meters

$N$  = Speed in RPM

$W_1$  and  $W_2$  = Spring balance readings on the tight side and on the slack side of the

Pulley respectively.

With this force exerted on the pulley, the load torque applied on the motor shaft is given by:

$$\tau_{\text{load}} = \text{Net force} \times \text{Radius of the pulley} = 9.81(W_1 - W_2)R \text{ Nw.m}$$

With this applied load torque  $\tau_{\text{load}}$ , the output power (mechanical) of the motor is given by:

$$P_{\text{out}} = \tau_{\text{load}} \times \omega = \tau_{\text{load}} \times 2\pi N / 60 \text{ W}$$

The input power (electrical) to the motor is given by :  $P_{\text{in}} = VI$

Thus we have  $\eta = P_{\text{out}} / P_{\text{in}} = [\tau_{\text{load}} \times 2\pi N / 60 + VI$

Apart from the efficiency, we can also find out all the characteristics like **Torque vs Speed**, **Speed vs Armature current** and **Torque vs armature current** of the motor by noting down the currents and voltage along with the speed N at various load settings. The speed is measured by using physical contact type Tachometer.

*Advantages:*

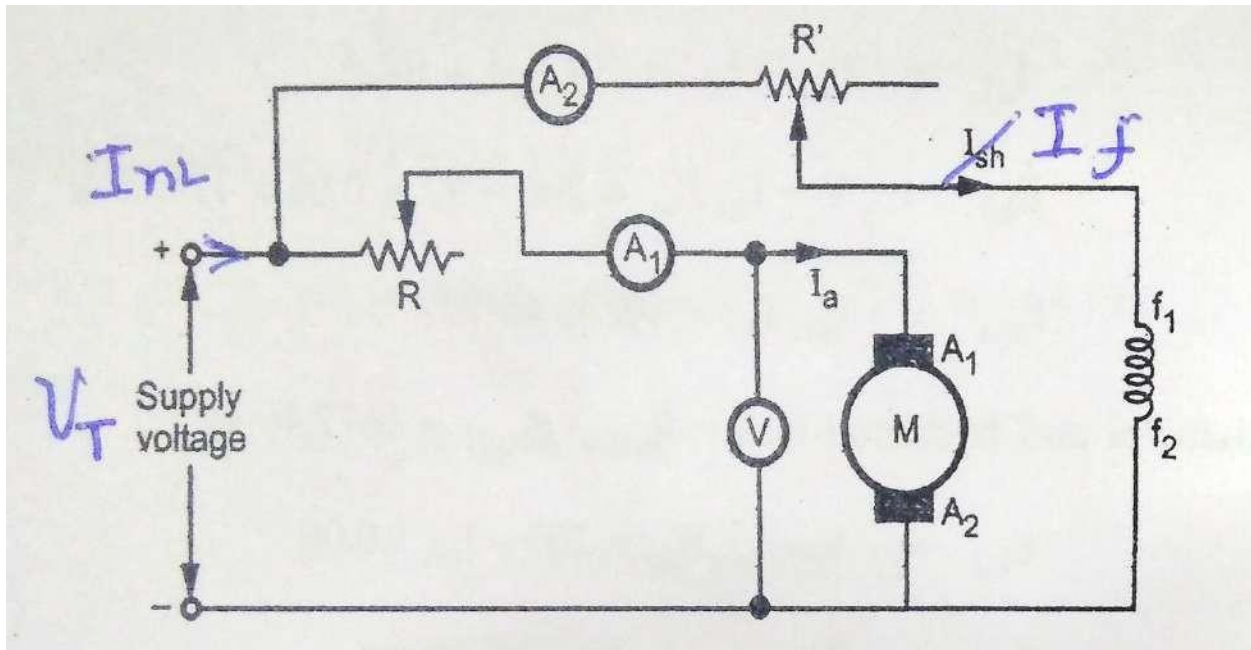
1. Efficiency can be found out in the actual working conditions.
2. The method is simple and easy to perform.
3. The test can be performed on any type of DC machine.

*Disadvantages:*

1. Due to the friction lot of energy is wasted in the form of heat. Hence the test is quite expensive and is suitable for only small machines.
2. Since heat energy is not accounted for, the efficiency observed would be inaccurate to that extent.

**Swinburne's test:**

This is a test to determine the efficiency of any DC Machine (Motor or Generator) without conducting the actual test at the required load. The test is conducted just at no load and the constant losses are found out when the machine is running as a motor. Then the efficiency is found out by calculating the variable losses at the required load. This method is formulated by Sir James Swinburne and hence it is called Swinburne's test. This comes under the category *indirect method of testing*. The test setup required to conduct this test is shown in the figure below.



**Figure: Swinburne's Test Setup**

**The machine is run as a motor** on no load at normal terminal voltage  $V_T$ , at normal speed and the armature current  $I_A$  & field current  $I_F$  ( $I_{sh}$  in figure) are measured.

- Then the no load armature current  $I_{NL} = I_A + I_F$
- Variable losses on no load  $= I_A^2 \cdot R_A$  (Machine's armature resistance can be measured directly and these losses can be calculated)

- Input to the motor =  $V_T \cdot I_{NL}$  = **Total losses** (Since the machine is on no load there is no output. i.e. the entire input power on no load goes as losses.)
- Therefore constant losses  $P_C = (\text{Total losses} - \text{Variable losses}) = (V_T \cdot I_{NL}) - (I_A^2 \cdot R_A)$

**Using these constant losses  $P_C$ , the efficiency of the machine can be estimated at any other load when working either as a Motor or as a Generator.**

*Working as a Generator delivering a load current of  $I_L$  amperes at a terminal voltage of  $V_T$  volts:*

$$\text{Power output} = V_T \cdot I_L$$

$$\text{Armature current } I_A = I_L + I_F \quad (I_F \text{ is same as obtained in the No load test})$$

$$\text{Variable loss} = I_A^2 R_A \quad (R_A \text{ is obtained from the no load test or from Machine data})$$

$$\text{Efficiency} = (\text{output/Input}) = [\text{output}/(\text{output} + \text{Total losses})] = (V_T \cdot I_L) / (V_T \cdot I_L + I_A^2 R_A + P_C)$$

( $P_C$  is obtained from the No load test and  $I_A^2 R_A$  is calculated using  $I_A$  corresponding to the required  $I_L$  at which the efficiency is to be calculated)

*Working as a Motor drawing a load current of  $I_L$  amperes from a supply terminal voltage of  $V_T$  volts:*

- Power in put =  $V_T \cdot I_L$

$$\text{Armature current } I_A = I_L - I_F \quad (I_F \text{ is same as obtained in the No load test})$$

$$\text{Variable loss} = I_A^2 R_A \quad (R_A \text{ is obtained from the no load test or from Machine data})$$

$$\text{Efficiency} = (\text{output/Input}) = [(\text{Input}-\text{Total losses})/\text{Input}] = [V_T \cdot I_L - (I_A^2 R_A + P_c)] / (V_T \cdot I_L)$$

( $P_c$  is calculated and obtained from the No load test and  $I_A^2 R_A$  is calculated using  $I_A$  corresponding to the required  $I_L$  at which the efficiency is to be calculated)

*Advantages of Swinburne's test:*

- This is a very simple to determine the efficiency of the machine at any load just by conducting the no load test.
- The power required is very less compared to the direct full load test.

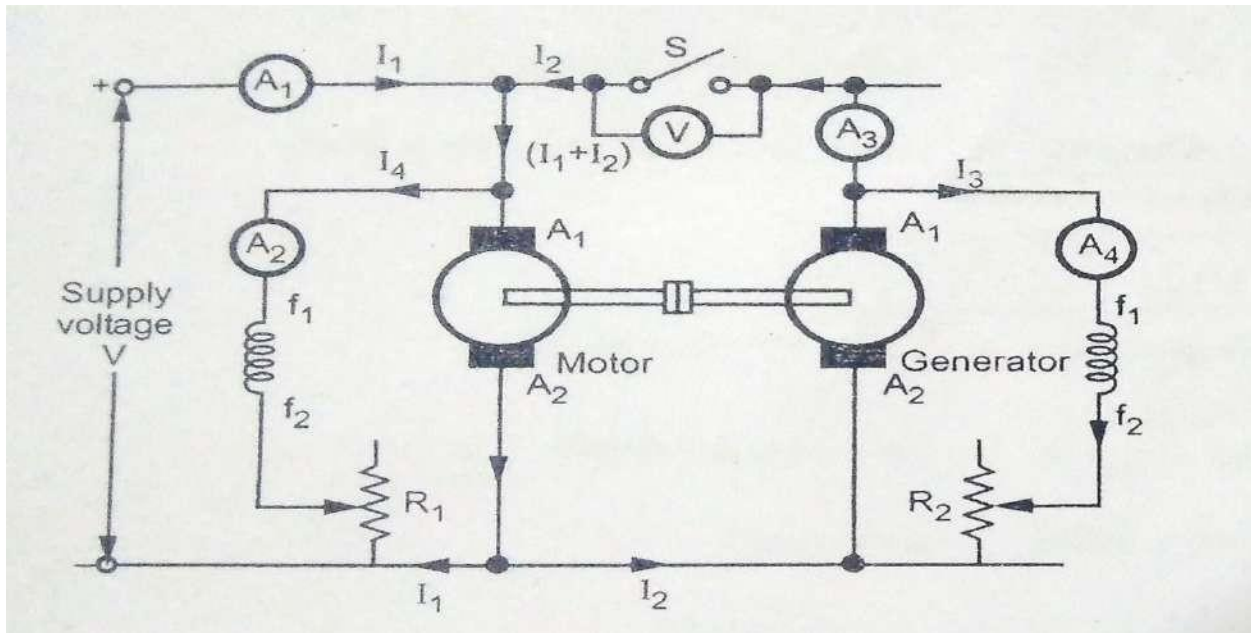
*Disadvantages of Swinburne's test:*

- This test can be done on Shunt machines only.
- The speed and flux are assumed constant. But the speed will fall with loading. Fall in speed results in lesser friction and windage losses. Change in flux will change the core losses.
- The temperature of the machine changes while running on load. Hence the assumption that  $R_A$  is same as that of the No load test is not correct.
- These reasons contribute to the difference in the efficiency obtained from the Swinburne's test and actual load test.

**Hopkinson's test:** In this set up two identical DC machines are coupled mechanically and tested together. One of the machines works as a motor and drives the other machine which works as generator and its electrical output in turn is connected back to the motor. Hence this comes under the regenerative category and is also called a Back to Back test. The motor is connected to the mains supply and it draws power from the mains only to compensate for the losses in the two machines since the major power required by each machine is

derived from the other machine. Since the power consumption is only to the extent of the losses they can be tested up to full load.

Figure below shows the Hopkinson's test up with all the measured voltage and current parameters marked clearly.



**Figure: Hopkinson's Test Setup**

**(Ammeters are to be redesignated as 'A2 to A4' , 'A3 to A2 ' and 'A4 to A3' so that the current designations will match )**

Initially the switch **S** is kept open and the Motor is run and brought to the rated speed by adjusting the field current using the field rheostat **R<sub>1</sub>** . The Generator Voltage is adjusted by adjusting it's field rheostat **R<sub>2</sub>** until the voltmeter reads zero volts. (This indicates that the Generator voltage is same as that of the Motor both in amplitude and polarity) This prevents flow of any high circulating currents when the switch **S** is closed and the two machines are connected back to back.

Now the switch is closed thus bringing the machines back to back and the load on both the machines can be increased gradually by increasing the Generator excitation or decreasing the Motor excitation. The readings from all the meters are taken at every load setting for further evaluation. Let us spell out clearly all the parameters for ease of further derivations.

**V = Terminal Voltage (Supply Voltage)**

**I<sub>1</sub> = Current drawn from mains supply**

**I<sub>2</sub> = Current supplied by the generator to the motor**

**I<sub>3</sub> = Excitation Current of Generator**

**I<sub>4</sub> = Excitation Current of Motor**

**R<sub>a</sub> = Armature resistance of each machine**

**1. Equal efficiency : Let us now first find out the Efficiency 'η' assuming it to be same for both the machines:**

**Input to the motor = V (I<sub>1</sub> + I<sub>2</sub>)**

**Output of the motor = η x Input to the motor = η x V (I<sub>1</sub> + I<sub>2</sub>)**

**This output of the motor is given as input to the generator. Hence**

**Input to the Generator = η x V (I<sub>1</sub> + I<sub>2</sub>)**

**Output of the generator = η x Input to the generator = η x η x V (I<sub>1</sub> + I<sub>2</sub>) = η<sup>2</sup> x V (I<sub>1</sub> + I<sub>2</sub>)**

**But the output of the generator can also be given as = V I<sub>2</sub> and equating these two we get**

**η<sup>2</sup> x V (I<sub>1</sub> + I<sub>2</sub>) = V I<sub>2</sub> from which we get**

$$\eta = \sqrt{[I_2 / (I_1 + I_2)]}$$

**2. Un equal efficiency: Let us now find out the Efficiency 'η' assuming it to be unequal for the two machines:**

In this analysis, the stray losses (Constant Losses) are assumed to be same for both the machines where as the field and armature copper losses are different (Since when the efficiencies are different it means that the currents are not the same and hence the copper losses also will not be same)

*So first let us determine the Copper losses of the two machines independently and then the constant losses of both machines together can taken as the difference between the input power from the main supply and the total copper losses.*

$$\text{Armature copper losses in Generator} = (I_2 + I_3)^2 \times R_a$$

$$\text{Armature copper losses in Motor} = (I_1 + I_2 - I_4)^2 \times R_a$$

$$\text{Field copper losses in Generator} = V I_3$$

$$\text{Field copper losses in Motor} = V I_4$$

We know that the total losses in both the machines put together are equal to the input power from the mains supply i.e  $V I_1$

Hence we can take that the total stray losses for both the machines put together are the difference between the input power and all the copper losses put together. Hence

$$\text{Stray losses of both machines together} = V I_1 - [(I_2 + I_3)^2 \times R_a + (I_1 + I_2 - I_4)^2 \times R_a + V I_3 + V I_4] = \text{Say } W_s \text{ and stray losses of each machine} = W_s/2$$

Now that we know the variable losses (Armature and Field Copper losses) and the constant (stray) losses for both the machines we can easily find out the efficiencies of both Generator and Motor using the above data as shown below.

**Efficiency of Generator:**

**Total losses = Generator's variable losses + Stray losses of one machine**

$$= [(I_2 + I_3)^2 \times R_a + V I_3 + W_s/2]$$

**Output of Generator =  $V I_2$**

**Efficiency of Generator  $\eta_G = \text{Output}/ \text{Input} = \text{Output}/ (\text{Output} + \text{Losses})$**

$$= V I_2 / V I_2 + [(I_2 + I_3)^2 \times R_a + V I_3 + W_s/2]$$

**Efficiency of Motor:**

**Total losses = Motor's variable losses + Stray losses of one machine**

$$= [(I_1 + I_2 - I_4)^2 \times R_a + V I_4 + W_s/2]$$

**Input to Motor =  $V ( I_1 + I_2 )$**

**Efficiency of Motor  $\eta_M = \text{Output}/ \text{Input} = (\text{Input} - \text{losses}) / \text{Input}$**

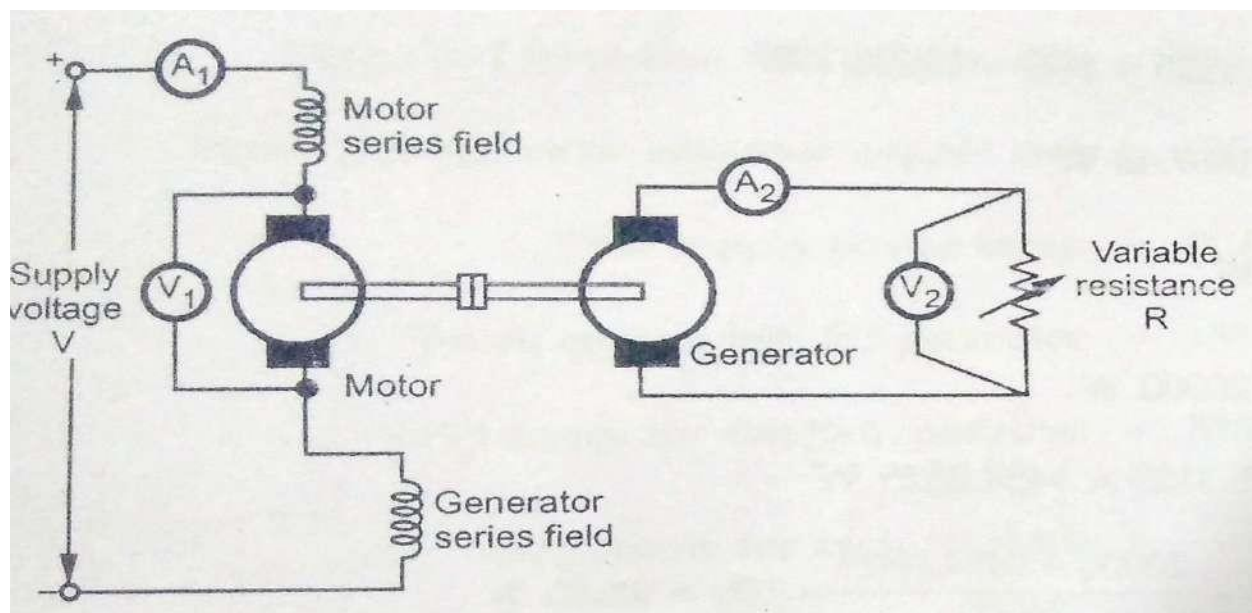
$$= [ V ( I_1 + I_2 ) - [(I_1 + I_2 - I_4)^2 \times R_a + V I_4 + W_s/2] ] / V ( I_1 + I_2 )$$

**Field's test:**

**Introduction and Methodology:**

This test is for finding out the losses and efficiency of DC Series Motors by direct testing, since series motors cannot be tested on no load. The test is named after the inventor of test method 'MB Field'. Series motors which are normally used for Traction are available as pairs and hence this test is devised on two motors

which are coupled mechanically. The test setup is shown in the figure below. One machine works as motor supplying power to the other one working as Generator. Their mechanical and iron (core) losses (put together called as *stray losses* or *constant losses*) are made equal by: (i) running them with equal speed and (ii) by connecting their both field windings in the motor armature circuit to the Motor input supply such that both the machines are equally excited . The load resistance is adjusted till the Motor draws the rated current as read by ammeter A1. In this condition all the other parameters are noted down from the respective meter readings as per the nomenclature given below.



**Figure: Field's Test Setup**

**Nomenclature:**

$V$  = Supply voltage

$V_1$  = Motor Supply voltage

$V_2$  = Generator output voltage connected across the variable load resistance  $R$

$I_1$  = Motor armature current and also the field current of both Motor and Generator

$I_2$  = Generator armature current

Also Let  $R_A$  = Motor armature resistance and  $R_F$  = Motor field resistance ( which can be measured independently or can be taken from the machine data )

***From this data obtained in this full load condition, the stray losses, copper losses and then efficiency of both the Motor and the Generator can be found out as below:***

**Stray Losses:**

Input to the total Motor Generator test setup:  $V I_1$

Output of Generator =  $V_2 I_2$

Total losses of both motor and generator =  $W_T = V I_1 - V_2 I_2$

But total losses of both motor and generator  $W_T$  are also equal to (Armature Copper losses + Field Copper losses + Stray losses ).Thus

$$W_T = V I_1 - V_2 I_2 = (I_1^2 + I_2^2) R_A + 2I_1^2 R_F + W_S$$

And total Stray Losses = Total Losses – Total Armature and Field copper losses

$$= V I_1 - V_2 I_2 - [(I_1^2 + I_2^2) R_A + 2I_1^2 R_F] \text{ and}$$

Stray losses per machine = Total Stray Losses/2 =  $W_S = [V I_1 - V_2 I_2 - [(I_1^2 + I_2^2) R_A + 2I_1^2 R_F]] / 2$

***Now using this value of stray losses of each machine, the efficiency of the machine as a Motor and as a Generator can be found out as below.***

**$\eta$  as a Motor:**

$$\text{Input to Motor} = V_1 I_1$$

$$\text{Total losses} = \text{Armature and Field Copper losses} + \text{Stray losses} = I_1^2(R_A + R_F) + W_s$$

$$\text{Output of Motor} = \text{Input} - \text{Losses} = V_1 I_1 - \{I_1^2(R_A + R_F) + W_s\}$$

$$\text{Efficiency of the motor} = \eta_M = \text{Output/Input} = \frac{[V_1 I_1 - \{I_1^2(R_A + R_F) + W_s\}]}{V_1 I_1}$$

**$\eta$  as a Generator:**

*This not very important because the machine is working in separately excited condition. However just for completion sake let us find it out.*

$$\text{Output of Generator} = V_2 I_2$$

$$\text{Total losses} = \text{Armature and Field Copper losses} + \text{Stray losses} = I_2^2 R_A + I_1^2 R_F + W_s$$

$$\text{Input to the Generator} = \text{Output} + \text{Losses} = V_2 I_2 + I_2^2 R_A + I_1^2 R_F + W_s$$

$$\text{Efficiency of the Generator} = \eta_G = \text{Output/Input} = \frac{V_2 I_2}{(V_2 I_2 + I_2^2 R_A + I_1^2 R_F + W_s)}$$

**Retardation test or Running down test:** This is an indirect test similar to the Swinburne's test where in the constant (Stray losses) losses are first determined and then the efficiency at any load when the machine is working both as a Generator and Motor are estimated on the same lines. However the constant losses in this test are determined by a different principle i.e. by finding out the

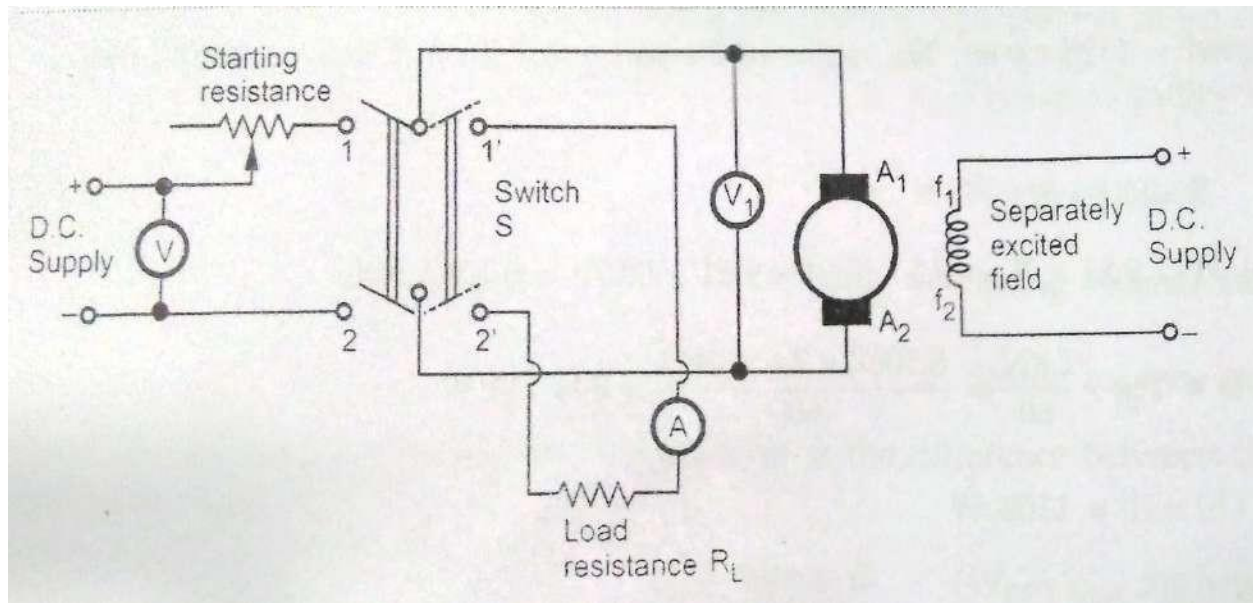
kinetic energy spent by a rotating mass during the process of retardation from the rated speed to zero speed and then calculating the rate of change of kinetic energy which is equal to the Power Loss.

The setup to conduct this test is shown in the figure below.

*Test Procedure:* The motor is started and taken to a speed higher than the rated speed of the machine. Then the supply to the motor is cutoff by moving the Two Pole Two way switch from the supply side to the open terminals. The armature then slows down with its' own inertia and its stored energy is used up to supply the constant rotational losses (stray losses) like iron, friction, windage etc. This power loss is found out from the following principle.

If ' $I$ ' is the moment of Inertia of the Armature, and ' $\omega$ ' is the angular velocity, then the kinetic energy of the armature is given by:

$$KE = \frac{1}{2} I\omega^2 \text{ and}$$



**Figure: Retardation test setup**

The power loss which is the rate of change of Energy is given by:

$$P_L = \frac{d}{dt} \left( \frac{1}{2} I \omega^2 \right) = I \cdot \omega \cdot \frac{d\omega}{dt}$$

Substituting the value of ' $\omega$ ' in terms of the speed in RPM ' $N$ ' ( $2\pi N/60$ ) we get

$$P_L = \left( \frac{2\pi}{60} \right)^2 I \cdot N \cdot \frac{dN}{dt}$$

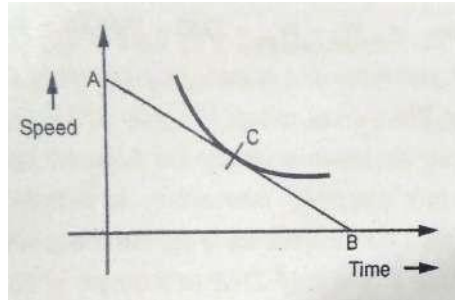
So, to find out the stray losses we must know ' $I$ ', the moment of inertia and  $\frac{dN}{dt}$ , the rate of change in speed. The method of finding out these quantities is given below.

*The method of finding out  $\frac{dN}{dt}$  :*

When the motor is cutoff from the input supply, the speed starts falling down. The motor back e.m.f. as read by the Voltmeter  $V_1$  connected across the motor is noted down as a function of time. Since we know that the back e.m.f is

proportional to the speed we can convert the  $V_1$  reading into speed in RPM and plot it as a function of time as shown in the figure below. In this figure, at the point **C** corresponding the rated speed a tangent AB is drawn whose slope gives the rate of change in speed  $dN/dt$ .

Thus:  **$dN/dt = OA(\text{RPM})/OB(\text{Time})$**



**Figure: Speed fall during the retardation test**

***Determination of Moment of Inertia (I):***

**Method -1: Using Flywheel**

The same test as earlier is repeated after adding a *flywheel* of known Moment of Inertia ' $I_1$ ' to the armature shaft of the motor and the resulting rate of change of speed is obtained. Let us call the earlier rate of change as  $dN/dt_1$  and the second rate of change with added inertia as  $dN/dt_2$ .

Then, since the losses can be assumed to be same with or without the new flywheel we have the following relations:

In the first case without flywheel:  **$P_L = (2\pi/60)^2 I . N . dN/dt_1$**

In the second case with added flywheel:  **$P_L = (2\pi/60)^2 (I+ I_1). N .$**

**$dN/dt$**

After equating the two equations and simplifying we get :  **$I = I_1 (t_1) / (t_2 - t_1)$**

### Method -2: Without using a Flywheel (Using Resistance Braking)

- First, the switch is taken from supply side to the open condition and then the time taken ( say  $t_1$ ) for the motor to slow down from a speed slightly higher than the rated speed to a speed slightly lower than the rated speed (say  $\delta N$ ) is noted down with just the Armature alone like in the earlier method step -1 (without any added external inertia)
- Then again the switch S is moved from the supply position to the Resistance position quickly and again the time taken(say  $t_2$ ) for the same change in speed (same  $\delta N$ )is noted down. By this effectively we are connecting an electrical load across the armature in which the stored electrical power is dissipated thus providing an additional retarding torque.
- This additional power loss due to the resistance is given by the product of the Average Voltage across the Armature (say  $V$  ) and the average current (say  $I_A$  ) that flows into the Braking Resistance R i.e.  $I_A^2 ( R_A+R ) \times V = \text{say } W'$
- Then the powers dissipated during the above two steps are given by

$$1. W = (2\pi/60)^2 I. N . dN/dt_1 \quad (\text{Just due to the armature Inertia})$$

$$2. W + W' = (2\pi/60)^2 I. N . dN/dt_2 \quad (\text{Due to the armature Inertia and the braking resistance )}$$

### Separation of Constant losses:

The theory required for the purpose of Separation of Constant losses is explained below.

At any given excitation:

- Friction losses and hysteresis losses are both proportional to speed .

- Windage losses and eddy current losses are both proportional to square of speed.
- Hence Friction losses =  $\mathbf{AN}$  Watts, Windage losses =  $\mathbf{BN}^2$  Watts, Hysteresis losses =  $\mathbf{CN}$  Watts, and Eddy current losses =  $\mathbf{DN}^2$  Watts where  $\mathbf{N}$  = speed and  $\mathbf{A,B,C}$  and  $\mathbf{D}$  are constant coefficients
- Further the coefficient  $\mathbf{C}$  of hysteresis losses is proportional to  $\mathbf{B}_{\max}^{1.6}$  and the coefficient  $\mathbf{D}$  of Eddy current losses is proportional to  $\mathbf{B}_{\max}^2$

*The other standard relation: For a motor on no load, power input to the armature is the sum of the armature copper losses and the above losses.*

Hence from a no load test we can get the constant losses as usual and then equate them to the constant losses with the above categorization as shown below:

Power input to the armature =  $\mathbf{V.I_a}$  watts.

Armature copper losses =  $\mathbf{I_a^2.R_a}$  watts

Constant Losses =  $\mathbf{W} = \mathbf{V.I_a} - \mathbf{I_a^2.R_a} = \mathbf{(A + C)N} + \mathbf{(B + D)N^2}$

$$\mathbf{W/N} = \mathbf{(A+C)} + \mathbf{(B+D)N}.$$

First the test is conducted with rated field current as per the following procedure:

1. The motor is started on no load with field current set to rated value by adjusting the field auto transformer.
2. The armature voltage is increased till the speed is about 200 rpm more than the rated value.
3. Now, the speed is gradually decreased by decreasing the armature voltage, the values of armature voltage, armature current and speed are noted down.

4. From this data **W** is calculated as explained above and the graph between **W/N & N** is plotted as shown in the figure below.

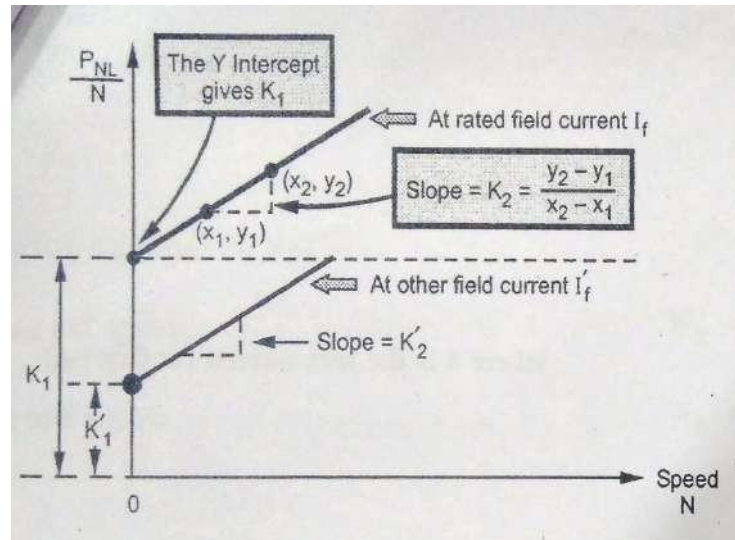


Figure : Plot of  $P_{NL} / N$  vs.  $N$

5. The graph between  $P_{NL} / N$  &  $N$  is a straight line from which **(A+C)** and **(B+D)** can be found. **(A+C)** is the **y** axis intercept and **(B+D)** is the slope of the straight line.
6. In order to separate out **A, B, C and D**, the test is repeated again with reduced field current and the graph between  $P_{NL} / N$  &  $N$  is drawn again as shown in the figure which is a straight line given by  $P_{NL} / N = (A+C') N + (B+D') N$  from which we can again find out **(A+C')** and **(B+D')**.

At the reduced excitation, friction and windage losses are still **AN** and **BN<sup>2</sup>**, but hysteresis losses become **C'N** and eddy current losses become **D' N<sup>2</sup>**. We can now obtain **(A+C')** and **(B+D')** as before from the second straight line .

As already indicated the coefficient of hysteresis loss  $C$  is proportional to  $B_{\max}^{1.6}$ , and the coefficient of eddy current loss  $D$  is proportional to  $B_{\max}^2$ . Since the core is common we can replace maximum flux density  $B_{\max}$  with maximum flux density  $\phi_{\max}$  in the core and then the ratio of fluxes corresponding to the normal and reduced excitation would become :

$$(C/C') = (\phi_{\max} / \phi'_{\max})^{1.6} \text{ also equal to } (E_b/E'_b)^{1.6} \text{ since Back e.m.f is proportional to } \phi_{\max}$$

$$(D/D') = (\phi_{\max} / \phi'_{\max})^2 \text{ also equal to } (E_b/E'_b)^2 \text{ since Back e.m.f is proportional to } \phi_{\max}$$

During the two tests we can get the values of back e.m.f s from the measured values of armature supply voltage  $V$ , armature current  $I_a$  and armature resistance  $R_a$  using the relation:  $E_b = V - I_a \cdot R_a$  and thus get the values of  $(C/C')$  and  $(D/D')$ . We can also get the values of  $(C'-C)$  and  $(D'-D)$  by subtracting  $(A+C)$  from  $(A+C')$  and  $(B+D)$  from  $(B+D')$

From these values we can get all the four coefficients  $A, B, C$  and  $D$  and thus separate the constant losses  $P_{NL}$  into **Friction, Windage, Hysteresis** and **Eddy current** losses

Since the change in speed  $\delta N$  is same in both the cases, dividing the expression 2 by expression 1 we get

$$(W + W')/W = (1/ dt_2) / (1/ dt_1) = dt_1/ dt_2 = t_1/ t_2 \text{ and after simplification we get}$$

$$W = W' * t_1 / (t_1 - t_2) \text{ which is equal to the stray losses ( Mechanical plus Core losses )}$$

### Important concepts and Formulae:

- The efficiency of a DC Machine is defined as  $\eta = (P_{out}/P_{in}). 100 \%$

#### *Efficiency of Generator:*

$$\eta = (P_{out}/P_{in}). 100 \% = (V_T \cdot I_L) / (V_T \cdot I_L + I_A^2 R_A + I_F^2 R_F + W_C). 100\%$$

#### *Efficiency of Motor:*

$$\eta = (P_{out}/P_{in}). 100 \% = [ \{ V_T \cdot I_L - ( I_A^2 R_A + I_F^2 R_F + W_C ) \} / ( V_T \cdot I_L ) ] . 100\%$$

- The condition for maximum efficiency :
- Constant losses  $P_C =$  Variable losses  $( I_L^2 R_A )$  or  $I_L = \sqrt{ ( P_C / R_A )}$

### Illustrative examples:

**Ex.1:** A DC shunt motor having a full load efficiency ( $\eta$ ) of 85 % takes a line current of 27A from 250 Volts mains on full load. If  $R_a = 0.5\Omega$  and  $R_f = 125 \Omega$ , find the constant losses, load current for maximum efficiency and the maximum efficiency.

#### **Solution:**

Input power at full load = Full load current x Rated voltage =  $250 \times 27 = 6750 \text{ W}$

Output power = Input power x  $\eta = 6750 \times 0.85$  ( $\eta = 85\%$ ) =  $5737.5 \text{ W}$

Hence Total losses = Input power – Output power =  $6750 - 5737.5 = 1012.5 \text{ W}$

We know that Total losses = Variable losses  $( I_a^2 R_a )$  + constant losses.

$I_f =$  Rated Terminal Voltage/ Field resistance =  $250 / 125 = 2 \text{ A}$

For the Shunt motor armature current  $I_a = I_l - I_f = 27 - 2 = 25 \text{ A}$

Variable losses =  $I_a^2 R_a = 25^2 \times 0.5 = 312.5$

Constant losses = Total losses – Variable losses = 1012.5 – 312.5 = 700 W

We know that the condition for maximum efficiency is: **Variable losses = Constant losses**

i.e.  $I_a^2 R_a$  at maximum efficiency = 700  $\therefore I_{a@max. \eta} = \sqrt{700 / 0.5} = \sqrt{1400} = 37.42 \text{ A}$

$\therefore$  The load current at maximum  $\eta$  :  $I_{l@max. \eta} = I_{a@max. \eta} + I_f = 37.42 + 2 = 39.42$

Input power at maximum  $\eta = I_{l@max. \eta} \times \text{Rated terminal voltage} = 39.42 \times 250 = 9855 \text{ W}$

O/P power at maximum  $\eta = \text{I/P power at maximum } \eta - \text{Total losses} = 9855 - (700 + 700) = 8455 \text{ W}$

(Since variable losses = constant losses = 700)

Maximum efficiency = Out power at maximum efficiency / Input power at maximum efficiency

$$= 8455 / 9855 = 0.858 \text{ or } 85.8 \%$$

**Ex.2:** A 100 Kw 500 V DC shunt machine when run as a motor on no load at rated speed and voltage takes a line current of 10 A and a shunt field current of 2.5 A . Resistance of the armature is 0.15  $\Omega$ . Estimate the efficiency of the DC machine when running as a generator (a) at full load (b) at half full load.

*First the constant losses of the Machine are obtained from the data we have when the machine is run as a motor on no load at rated speed and voltage:*

Input power on no load = Rated voltage x Input current on no load = 500 x 10 = 5000 W

Field current  $I_f = 2.5 \text{ A}$

No load Armature current  $I_a = I_{l \text{ no load}} - I_f = (10 - 2.5) = 7.5 \text{ A}$

Variable loss at no load =  $(I_{a \text{ on no load}})^2 \times R_a = 7.5^2 \times 0.15 = 8.4375 \text{ W}$

**Constant Losses** = ( Input power– Variable losses)(on no load) =  $5000 - 8.4375 = 4991.56 \text{ W}$

*Next we will calculate the efficiency in different conditions:*

*(a) Running as a generator at full load:*

Full load output (line) current =  $100 \times 1000 / 500 = 200 \text{ A}$

Full load armature current = Full load line current + Field current =  $200 + 2.5 = 202.5 \text{ A}$

Variable (Armature copper) losses on full load =  $I_a^2 R_a = 202.5^2 \times 0.15 = 6150.94 \text{ W}$

Total losses @ full load = Constant Losses + Variable losses on full load =  $4991.56 + 6150.94$

$= 11142.5 \text{ W}$

**Efficiency at full load (Working as Generator)** = Output / Input = Output / Output + Total losses @ full load =  $100000 / 100000 + 11142.5 = 0.8997$  or **89.97 %**

*(a) Running as a generator at half full load:*

Half Full load output (line) current =  $50 \times 1000 / 500 = 100 \text{ A}$

Half Full load armature current = Half Full load line current + Field current =  $100 + 2.5 = 102.5 \text{ A}$

Variable (Armature copper) losses on half full load =  $I_a^2 R_a = 102.5^2 \times 0.15 = 1575.94 \text{ W}$

Total losses @ half load = Constant Losses + Variable losses on half load =  $4991.56 + 1575.94$

$= 6567.5 \text{ W}$

**Efficiency at half full load (Working as Generator)** = Output / Input = Output / Output + Total losses @ full load =  $50000 / 50000 + 6567.5 = 0.8839$  or **88.39 %**

**Ex.3:** A 500 V DC shunt machine takes 5A when running light (on no load) at rated speed and rated voltage as a motor. Calculate the out output power and efficiency when the machine is run as a Motor and taking an Input current of 80 A. Calculate the line current at which the efficiency is maximum and the value of maximum efficiency.  $R_a = 0.2 \Omega$  and  $R_f = 250 \Omega$

***First the constant losses of the Machine are obtained from the data we have when the machine is run as a motor on no load at rated speed and voltage:***

Input power on no load = Rated voltage x Input current on no load =  $500 \times 5 = 2500 \text{ W}$

Field current  $I_f = \text{Rated voltage} / R_f = 500 / 250 = 2 \text{ A}$

No load Armature current  $I_a = I_{l \text{ no load}} - I_f = (5 - 2) = 3 \text{ A}$

Variable loss at no load =  $(I_{a \text{ on no load}})^2 \times R_a = 3^2 \times 0.2 = 1.8 \text{ W}$

**Constant Losses** = ( Input power– Variable losses)(on no load) =  $2500 - 1.8 = 2498.2 \text{ W}$

***Next we will calculate the output power and efficiency when the Machine is running as a motor and taking an input current of 80 A :***

Armature current = Line current (Input Current) - Field current =  $80 - 2 = 78 \text{ A}$

Variable (Armature Copper) losses (with armature current of 78A) =  $I_a^2 R_a = 78^2 \times 0.2 = 1216.8 \text{ W}$

Total losses = Constant Losses + Variable losses at 80 A line current =  $2498.2 + 1216.8 = 3715 \text{ W}$

Input Power =  $500 \times 80 = 40000 \text{ W}$

**Out Put Power at 80 A line current (Working as Motor)** = Input Power – Total losses at 80 A line current = 40000 -3715 = 36285 W

**Efficiency at 80 A line current (Working as Motor)** = Output Power /Input Power  
= 36285/40000 = 0.9071 or **90.71 %**

***Finally we will calculate the line current at which the efficiency is maximum and the value of maximum efficiency:***

We know that the condition for maximum efficiency is: **Variable losses = Constant losses**

i.e.  $I_a^2 R_a$  ( variable Losses ) at maximum efficiency = **2498.2 W**

$\therefore I_{a@max. \eta} = \sqrt{2498.2 / 0.2} = \sqrt{12491} = 111.76 \text{ A}$

**$\therefore$  The line current at maximum  $\eta$  :**  $I_{l@max. \eta} = I_{a@max. \eta} + I_f = 111.76 + 2 =$   
**113.76 A**

Input power at maximum  $\eta$  =  $I_{l@max. \eta} \times$  Rated terminal voltage = 113.76 x 500 = 56880 W

O/P power at maximum  $\eta$  = I/P power at maximum  $\eta$  – Total losses =  
56880 – (2498.2 + 2498.2) = 51883.6 W

(Since variable losses = constant losses = 2498.2)

**Maximum efficiency** = Out power at maximum efficiency/ Input power at maximum efficiency

= 51883.6 /56880 = 0.9122 or **91.22 %**

#### **Example: 4**

**The following readings are obtained when doing a load test on d.c. shunt motor using a brake drum. Spring balance readings =10kg, and 35 kg, Diameter of drum**

=40 cm, speed of the motor = 950 r.p.m, Applied voltage = 200 V Line current =30 A . Calculate the output power and efficiency.

**Solution :** We have  $W_1 = 10$  kg and  $W_2 = 35$  kg

Diameter of the drum  $D = 40$  cm,  $R = d/2 = 20$  cm = 0.2m

$I = 30$  A,  $V = 200$  volts,  $N = 950$  r.p.m

Applied Torque  $\tau$  (Newton meters ) = Net Force in (Newtons) x Diameter of the drum (mtrs)

$$\begin{aligned} &= (W_2 - W_1) \times 9.81 \times R \\ &= (35 - 10) \times 9.81 \times 0.2 = 49.05 \text{ N-m} \\ P_{\text{out}} &= \tau \text{ (Nw.mtrs.)} \times \omega \text{ (Rad/sec)} = \frac{2\pi N \tau}{60} = \frac{2\pi \times 950 \times 49.05}{60} \\ &= 4879.67 \text{ W} = \mathbf{4.87 \text{ kW}} \end{aligned}$$

$$P_{\text{in}} = V \cdot I = (200) (30) = 6000 \text{ W} = 6 \text{ kW}$$

$$\begin{aligned} \% \text{ Efficiency of motor} &= \frac{P_{\text{out}}}{P_{\text{in}}} \times 100 = \frac{4.87}{6} \times 100 \\ &= \mathbf{81.32\%} \end{aligned}$$

$\therefore$  Power output of DC shunt motor = **4.87 kW**

% Efficiency of DC shunt motor = **81.32**

**Example 5: A retardation test is carried out on a 1000 r.p.m. d.c. machine. The time taken for the speed to fall from 1030 r.p.m. to 970 r.p.m. is**

**i) 36 seconds with no excitation**

**ii) 15 seconds with full excitation**

**iii) 9 seconds with full excitation and the armature supporting an extra load of 10 A at 219 V.**

**Calculate : 1) The moment of inertia of the armature in kg.m<sup>2</sup>**

**2) Iron loss**

**3) Mechanical loss at the mean speed of 1000 r.p.m.**

**Solution:** It is to be remembered that,

1. When armature is slowing down and there is no excitation, then kinetic energy is used to overcome mechanical losses only. Iron losses are absent as excitation is absent.
2. When excitation is given, kinetic energy is used to overcome both mechanical as well as iron losses. Total called stray losses.
3. If moment of inertia is in kg-m<sup>2</sup>, then loss of energy is in watts.

The mechanical losses are,

$$W_m = \left\{ \frac{2\pi}{60} \right\}^2 I \cdot N \frac{dN}{dt 1}$$

The total stray losses are,

$$W_s = \left\{ \frac{2\pi}{60} \right\}^2 I \cdot N \frac{dN}{dt 3}$$

Now

$$dN = 1030 - 970 = 60 \text{ r.p.m.}$$

$$t_1 = 36 \text{ sec, } t_2 = 15 \text{ sec, } t_3 = 9 \text{ sec}$$

The additional loss  $W'$  due to additional load,

$$W' = VI = 219 \times 10 = 2190 \text{ W}$$

$$W_s = W' \times \frac{t_3}{t_2 - t_3} = 2190 \times \frac{9}{15 - 9} = 3285 \text{ W}$$

$$3285 = \left\{ \frac{2\pi}{60} \right\}^2 I \cdot N \frac{dN}{dt 2}$$

i.e. 
$$3285 = \left\{ \frac{2\pi}{60} \right\}^2 \times I \times 1000 \times \frac{60}{15}$$

$$I = \text{Moment of inertia} = \frac{(3285 \times 60^2 \times 15)}{(2\pi)^2 \times 1000 \times 60}$$

$$= \mathbf{74.88 \text{ kg-m}^2}$$

$$\frac{W_m}{W_s} = \frac{t_2}{t_1} = \frac{15}{36}$$

$$W_m = W_s \times \frac{15}{36} = 3285 \times \frac{15}{36}$$

$$= \mathbf{1368.75 \text{ W}}$$

$$\text{Iron losses} = W_s - W_m = 3285 - 168.75 = \mathbf{1916.85 \text{ W}}$$

**Example 6: A Retardation test is made on a separately excited d.c. machine as a motor. The induced voltage falls from 220 V to 190 V in 30 seconds on opening the armature circuits and 5 seconds on suddenly changing the armature connection from supply to a load resistance taking 12 A (average). Find the efficiency of the machine when running as a motor and taking a current of 22 A on a supply of 250 V. The resistance of its armature is 0.5  $\Omega$  and that of its field winding is 250 ohms.**

**Solution :** Average voltage across load =  $\frac{220+190}{2} = 205 \text{ V}$

$I_{av} = 12 \text{ A}$ , Armature resistance  $R_a = 0.5 \Omega$ , Field resistance  $R_f = 250 \Omega$

$\therefore$  Power absorbed by the load resistance ,

$$W' = (205) (12) = 2460 \text{ W}$$

$$t_1 = 30 \text{ sec}, t_2 = 5 \text{ sec}$$

Let  $W = \text{Stray losses}$

We have,  $\frac{W}{W'} = \frac{t_2}{t_1 - t_2} = \frac{5}{30 - 5} = \frac{5}{25} = 0.2$

$\therefore$   $\text{Stray losses } W = \frac{2460 \times 0.2}{1} = 492 \text{ W}$

Input current = 22 A,  $I_f = \frac{V}{R_f} = \frac{250}{250} = 1 \text{ A}$

Armature current  $I_a = \text{Input current} - I_f = 22 - 1 = 21 \text{ A}$

Armature copper loss =  $I_a^2 \times R_a = (21)^2 (0.5) = 220.5 \text{ W}$

Shunt field copper loss =  $V_s \times I_f = (250) (1) = 250 \text{ W}$

Total losses in the machine = *Stray losses + Armature copper loss + Shunt field copper loss*

$$= 492 + 220.5 + 250 = 962.5 \text{ W}$$

Motor input =  $V_s \times I_L = (250) (22) = 5500 \text{ W}$

$$\begin{aligned} \text{Motor output} &= \text{Motor input} - \text{Losses} = 5500 - 962.5 \\ &= 4537.5 \text{ W} \end{aligned}$$

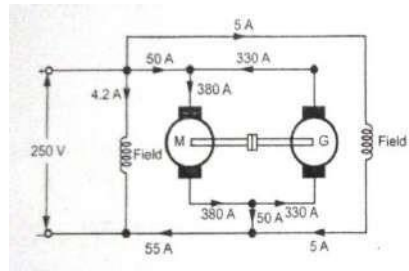
% Efficiency of the machine running as a motor ,

$$\eta_m = \frac{\text{Motor output}}{\text{Motor input}} \times 100$$

$$\% \eta_m (\text{Running as motor}) = \frac{4537.5}{5500} \times 100 = \mathbf{82.5\%}$$

**Example 7:** The Hopkinson's test on two shunt machines gave the following results for full load conditions. Line voltage 250 volts, line current excluding field current is 50 A, motor armature current 380 A, field currents for generator and motor are 5 A, 4.2 A respectively. Draw the circuit diagram with the above data and Calculate the efficiency of each machine. Armature resistance of each machine is  $0.02\Omega$

**Solution :** The connections and current distribution based on the given data are shown in the figure below.



$$\begin{aligned} \text{Armature copper loss for generator} &= I_a^2 R_a = (330)^2 \times 0.02 = \\ 2178 \text{ W} \end{aligned}$$

$$\begin{aligned} \text{Armature copper loss for motor} &= I_a^2 R_a = (380)^2 \times 0.02 = \\ 2888 \text{ W} \end{aligned}$$

$$\text{Power drawn loss from supply} = VI = 250 \times 50 = 12500 \text{ W}$$

$$\text{Stray losses for both machines} = VI - \text{Total armature}$$

copper loss

$$= 12500 - 2178 - 2888 =$$

7434 W

$$\text{Stray loss for each} = \frac{7434}{2} = 3717 \text{ W}$$

**For motor :**

$$\text{Field copper loss} = V I_f = 250 \times 4.2 = 1050 \text{ W}$$

$$\text{Total loss} = 2888 + 3717 + 1050 =$$

7655 W

$$\text{Motor input} = V \times I_a + V \times I_f = 250 \times 380 + 250 \times 4.2$$

= 96050 W

$$\text{Motor output} = \text{Motor input} - \text{Total loss}$$

= 88395 W

$$\% \eta_m = \frac{88395}{96050} \times 1000 = \mathbf{92.03 \%}$$

**For generator :**

$$\text{Field copper loss} = V I_f = 250 \times 5 = 1250 \text{ W}$$

$$\text{Total loss} = 2178 + 3717 + 1250 =$$

7145 W

$$\text{Generator input} = \text{Generator output} + \text{Total loss}$$

loss

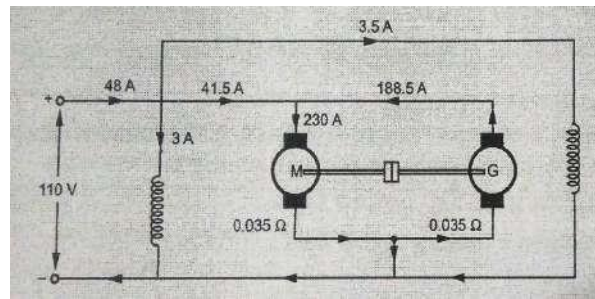
$$\text{Generator output} = V I_a = 250 \times 330 = 82500$$

W

$$\text{Generator input} = 82500 + 7145 = 89645 \text{ W}$$

$$\% \eta_m = \frac{82500}{89645} \times 100 = \mathbf{92.02 \%}$$

**Example 8:** The Hopkinson's test on two similar machines gave the following full load results: Line current = 48 A, Line voltage = 110 V, Motor armature current = 230 A the field currents of motor and generator are 3 A and 3.5 A respectively. Armature resistance of each machine is 0.035 ohms. Draw the circuit diagram with the above data and Calculate the efficiency of each machine assuming a brush contact drop of 1 V per brush.



**Solution:** The connections and current distribution based on the given data are shown in the figure below.

$$\text{Brush contact loss for motor} = 230 \times 2 = 460 \text{ W}$$

$$\text{Brush contact loss for generator} = 188.5 \times 2 = 377 \text{ W}$$

$$\text{Motor armature copper loss} = I_a^2 R_a = (230)^2 (0.035) = 1851.5 \text{ W}$$

$$\text{Generator armature copper loss} = I_a^2 R_a = (188.5)^2 (0.035) = 1243.62 \text{ W}$$

$$\text{Motor field copper loss} = V I_f = (110)(3) = 330 \text{ W}$$

$$\text{Generator field copper loss} = V I_f (110)(3.5) = 385 \text{ W}$$

$$\text{Motor and generator field copper loss} = 330 + 385 = 715 \text{ W}$$

$$\text{Motor and generator armature copper loss} = 1851.5 + 1243.62 = 3095.12 \text{ W}$$

$$\text{Total supply power taken} = (110)(48) = 5280 \text{ W}$$

Stray losses for both the machines =

$$5280 - 715 - 3095.12 - 460 - 377 = 632.88 \text{ W}$$

$$\text{Stray losses for each machine} = \frac{632.88}{2} = \mathbf{316.44 \text{ W}}$$

**Motor efficiency :**

$$\text{Motor armature copper loss} = 1851.5 \text{ W, Motor field copper loss} = 330 \text{ W}$$

$$\text{Stray loss} = 316.44 \text{ W; Brush loss} = 460$$

$$\text{Total motor losses} = 1851.5 + 330 + 316.44 + 460 = 2957.94 \text{ W}$$

$$\text{Motor input} = (110 \times 3) + (110 \times 230) = 25630$$

W

Motor output = Motor input – Total losses = 25630 – 2957.94  
= 22672.06 W

**Motor efficiency :**

$$\eta_m = \frac{\text{Machine output}}{\text{Machine input}} \times 100 = \frac{22672.06}{25630} \times$$

100 = **88.45 %**

Generator armature copper loss = 1243.62 W, Generator field  
copper loss = 385 W

Stray losses = 316.44 W, Brush contact loss for  
generator = 377 W

Total generator losses = 1243.62 + 385 + 316.44 + 377 =  
2322.06 W

Generator = Generator output + Total  
generator losses

$$= (110)(185) + 2322.06 = 22672.06$$

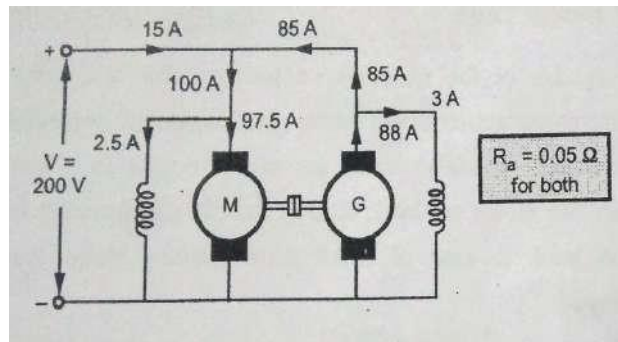
W

$$\text{Generator efficiency, } \eta_g = \frac{\text{Generator output}}{\text{Generator input}} \times 100 = \frac{(110)(185)}{22672.06} \times$$

100 = **89.75 %**

**Example 9: Two shunt machines loaded for the Hopkinson's test take 15 A at 200 V from the supply. The motor current is 100 A and the shunt currents are 3 A and 2.5 A for the generator and the motor respectively. The resistance of each armature is 0.05 Ω. Draw the circuit diagram with the above data and calculate the efficiency of each machine for its particular conditions of loading.**

**Solution:** The connections and current distribution are shown in the figure below.



**For generator :**

$$I_L = 15 \text{ A} , I_a = 88 \text{ A} , I_f = 3 \text{ A} , R_a = 0.05 \Omega , V_s =$$

200 V

$$\therefore \text{Armature copper loss} = I_a^2 R_a = 387.2 \text{ W}$$

$$\therefore \text{Field copper loss} = I_f V = 600 \text{ W}$$

**For motor :**  $I_a = 97.5 \text{ A} , I_f = 2.5 \text{ A} , R_a = 0.05 \Omega , V_s = 200 \text{ V}$

$$\therefore \text{Armature copper loss} = I_a^2 R_a = 475.3125 \text{ W}$$

$$\therefore \text{Field copper loss} = I_f V = 2.5 \times 200 = 500 \text{ W}$$

$$\text{Power drawn from supply} = VI = 200 \times 15 = 3000 \text{ W}$$

$$\text{Total copper losses for set} = 387.2 + 600 + 475.3125 + 500 = 1962.51$$

W

$$\therefore \text{Stray losses for both machine} = 3000 - 1962.51 = 1037.48 \text{ W}$$

$$\therefore \text{Stray losses for each machine} = \frac{1037.48}{2} = \mathbf{518.74 \text{ W}}$$

**Efficiency of motor :**  $P_{in} = VI = 200 \times 100 = 20000 \text{ W}$

$$\text{Total losses} = 518.74 + 500 + 475.3125 = 1494.0525$$

W

$$\therefore \% \eta = \frac{P_{in} - \text{Losses}}{P_{in}} \times 100 = \frac{20000 - 1494.0525}{20000} \times$$

$$100 = \mathbf{92.53 \%}$$

**Efficiency of generator :** Motor output = Generator input

$$P_{in} (\text{gen}) = 20000 - 1494.0525 = 18505.9475 \text{ W}$$

$$\text{Total losses} = 518.74 + 387.2 + 600 = 1505.94 \text{ W}$$

$$\therefore \% \eta = \frac{P_{in} - \text{Losses}}{P_{in}} \times 100 = \frac{18505.9475 - 1505.94}{18505.9475} \times 100$$

$$= 91.86 \%$$

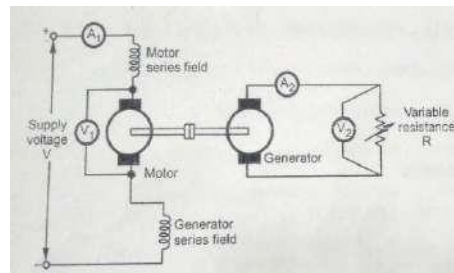
**Example 10: A Field's test on two mechanically coupled similar motors with their fields connected in series and with one machine running as motor and the other as a generator gave the following data :**

**Motor : Armature current 40 A, armature voltage 200 V the drop across its field winding 15 V.**

**Generator : Armature current 32 A, armature voltage 160 V, the drop across its field winding 15 V.**

**The resistance of each armature is 0.4 Ω. Draw the circuit diagram with the above data and calculate the efficiency of each machine at this load.**

**Solution: The circuit diagram with the above data is shown in the figure below.**



$I_1$  = Motor input current = 40 A

$V_1$  = Voltage across motor armature = 200 V

$I_2$  = Load current = 32 A

$V_2$  = voltage across generator = 160 V

$R_a, R_f$  = Armature and series field resistance of each machine

Total input voltage to Motor (excluding that of Generator field) =  $V = 200 + 15 = 215$  V

Total input power to motor =  $V \cdot I_1 = 215 \times 40 = 8600 \text{ W}$

Generator output =  $V_2 \cdot I_2 = 160 \times 32 = 5120 \text{ W}$

Total losses in the two machines = input – output =  $8600 - 5120 = 3480 \text{ W}$

Series field resistance for both machines,

$$R_f = (V/I_1) = \frac{15}{40} = 0.375 \Omega$$

$$\begin{aligned} \text{Total Cu loss} &= (R_a + 2 R_f) I_1^2 + I_2^2 R_a \\ &= (0.4 + 2 \times 0.375) (40)^2 + (32)^2 (0.4) \\ &= 1840 + 409.6 = 2249.6 \text{ W} \end{aligned}$$

Stray losses of the two machines = Total losses – Total Cu loss  
 $= 3480 - 2249.6 = 1230.4 \text{ W}$

Stray losses for each machine = Total stray Losses / 2 =  $\frac{1230.4}{2}$   
**= 615.2 W**

**For Motor :**

Motor input =  $V \cdot I_1 = 200 \times 40 = 8000 \text{ W}$

Armature and field Cu loss =  $(R_a + R_f) I_1^2 = (0.4 + 0.375) (40)^2$   
 $= 1240 \text{ W}$

Total losses = Armature and field Cu loss + Stray losses  
 $= 1240 + 615.2 = 1855.2 \text{ W}$

Motor output = Motor input – Total losses  
 $= 8000 - 1855.2 = 6144.8 \text{ W}$

Motor efficiency =  $\frac{\text{Motor output}}{\text{Motor input}} \times 100 = \frac{6144.8}{8000} \times 100$   
**=76.81%**

**For Generator :**

Armature Cu loss =  $I_2^2 R_a = (32)^2 (0.4) = 409.6 \text{ W}$

Series field Cu loss =  $V \cdot I_1 = 15 \times 40 = 600 \text{ W}$

Total losses = Armature Cu loss + Series field Cu loss + Stray loss  
 $= 409.6 + 600 + 615.2 = 1624.8 \text{ W}$

Output of generator = 5120 W

$$\begin{aligned}\text{Input to generator} &= \text{Output} + \text{Losses} = 5120 + 1624.8 \\ &= 6744.8 \text{ W}\end{aligned}$$

$$\text{Efficiency of generator} = \frac{\text{output}}{\text{input}} \times 100 = \frac{5120}{6744.8} \times 100 = \mathbf{75.91 \%}$$

**Example 11:** The total iron loss in a d.c. machine is 7 kW at the rated speed and excitation. With the same excitation and speed reduced by 30 % the total iron losses were found to be 4 kW. Calculate the hysteresis and eddy current losses at i) Full speed ii) Half of full speed

**Solution:** Hysteresis loss  $P_h = AN$ , Eddy current loss  $P_e = BN^2$

$$\text{And, } P_i = P_h + P_e = AN + BN^2$$

Let the rated speed  $N = 1$  then,

$$P_i = A + B = \mathbf{7 \text{ kW}} \text{ (given)}$$

$$\text{At new speed } N' = N - 30\% \text{ of } N = 1 - 0.3 = 0.7,$$

$$P_i = 0.7A + (0.7)^2 B = 4 \text{ kW}$$

$$\text{i.e. } 0.7A + 0.49B = \mathbf{4 \text{ kW}}$$

Solving equations (1) and (2),

$$\mathbf{A = 2.7143, B = 4.2857}$$

i) At full speed,

$$P_h = A = \mathbf{2.7143 \text{ kW}} \text{ and } P_e = B = \mathbf{4.2857 \text{ kW}}$$

ii) At half of full speed,

$$P_h = A \times (0.5)^1 = \mathbf{1.3515 \text{ kW}}$$

$$\text{and } P_e = B \times (0.5)^2 = \mathbf{1.07142 \text{ kW}}$$

**Example 12:** When running on no load, a 400 V shunt motor takes 5 A. Armature resistance is 0.5  $\Omega$  and field resistance 200  $\Omega$ . Find the output of the motor and efficiency when running on full load and taking a current of 50 A. Also, find the percentage change in speed from no load to full load.

**Solution :**  $V = 400 \text{ V}$ ,  $I_{NL} = 5 \text{ A}$ ,  $R_a = 0.5 \Omega$ ,  $R_f = 200 \Omega$ ,  $I_{FL} = 50 \text{ A}$

$$I_f = \frac{V}{R_{sh}} = \frac{400}{200} = 2 \text{ A}$$

$$I_{aNL} = I_{NL} - I_f = 5 - 2 = 3 \text{ A}$$

$$E_{bnl} = V - I_{aNL} R_a = 400 - 3 \times 0.5 = 398.5 \text{ V}$$

No load power input = Total losses

$$= V \times I_{NL} = 400 \times 5 = 2000 \text{ W}$$

$$\therefore \text{Constant losses} = 2000 - I_{aNL}^2 R_a = 2000 - (3)^2 \times 0.5 \\ = 1995.5 \text{ W}$$

On full load,  $I_{aFL} = I_{FL} - I_{sh} = 50 - 2 = 48 \text{ A}$

$\therefore$  Armature copper losses =  $I_{aFL}^2 R_a$

$$E_{bFL} = V - I_{aFL} R_a = 400 - 48 \times 0.5 = 376 \text{ V}$$

$$\text{Total losses} = \text{Constant losses} + \text{Armature copper losses} \\ = 1995.5 + 1152 = 3147.5 \text{ W}$$

$$P_{in} (FL) = V I_{FL} = 400 \times 50 = 20000 \text{ W}$$

$$\therefore P_{out} (FL) = P_{in} (FL) - \text{Total losses} = 20000 - 3147.5 \\ = 16852.5 \text{ W}$$

$$\therefore \% \eta_{FL} = \frac{P_{out} (FL)}{P_{in} (FL)} \times 100 = \frac{16852.5}{20000} \times 100 = 84.26 \%$$

$$N_a \frac{a E_b}{\phi} \quad (\text{Since } \phi \text{ is constant})$$

$$\therefore N_{nl} / N_{fl} = E_{bnl} / E_{bFL} = \frac{398.5}{376} = 1.05984$$

i.e.  $N_{nl} = 1.05984 N_{FL}$

$$\therefore \% \text{ Change in speed} = \frac{[(N_{nl} - N_{FL}) / N_{FL}] \times 100}{1.05984 N_{FL} - N_{FL}} \times 100 \\ = \frac{0.05984 N_{FL}}{N_{FL}} \times 100 \\ = 5.984 \%$$

## **UNIT – IV**

### **SINGLE PHASE TRANSFORMERS**

#### **CONTENTS:**

- **Principle of operation**
- **Constructional features -Types**
- **Minimization of hysteresis and eddy current losses**
- **EMF equation Operation on no load and on load**
- **Phasor diagrams**
- **Equivalent circuit Losses and efficiency**
- **Regulation**
- **All day efficiency**
- **Effect of variations of frequency & supply voltage on iron losses.**
  - **Illustrative Problems**

### **Introduction:**

A transformer is a device used to transfer Electric power from one circuit at a certain voltage level to another circuit at a different voltage level. The electrical power is transferred by Magnetic Induction between two coils in the magnetic circuit of the transformer. It is electrical equipment which has the highest efficiency since there are no moving parts. Transformers can carry only AC Electrical power. Transformers are available in single phase and three phases. We will study single phase transformers in this unit.

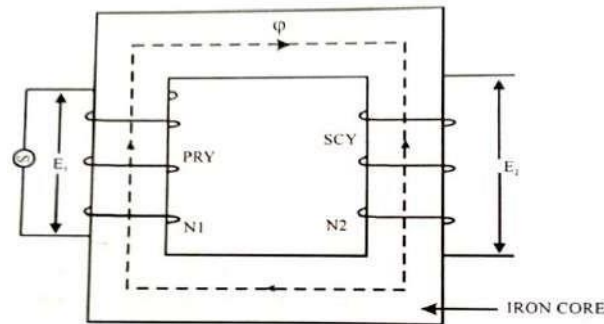
### **Types and applications of transformers:**

1. Step up/Unit transformers – Usually located at the output of a generator in Generating stations. Its function is to step up the voltage level to transmit power with minimum losses.
2. Step down/Substation transformers – Located at main distribution or secondary level transmission substations. Its function is to lower the voltage levels for first level distribution
3. Distribution Transformers – located at small distribution substation. It lowers the voltage levels  
for second level distribution purposes.
4. Special Purpose Transformers - E.g. Potential Transformer (PT) , Current Transformer (CT) etc which are used for metering applications.

### **Principle of operation of Transformers:**

In a generator a voltage is induced in a coil which moves past a stationary magnetic field emanating from field coils and the field flux is constant. But since the flux that links with the coil is changing, a voltage is induced in the coil which is proportional to the rate of change of flux linkage. In a transformer though both the coils and the magnetic circuit are stationary, a voltage is induced in the secondary since the current which flows in the primary is alternating and it produces a continuously changing flux. According to Faraday's laws of electromagnetic Induction when the magnetic flux linking with a conductor (or Coil) changes, an EMF is induced in that.

In a single phase transformer there are two coils one known as primary and the other known as secondary both wound on the same magnetic circuit (core) of high permeability (or low reluctance) as shown in the figure below.



**Fig: Single Phase Transformer**

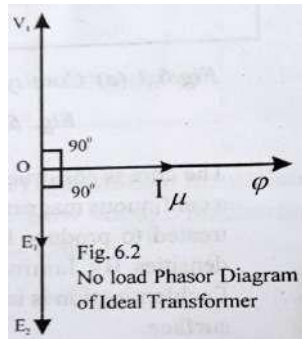
An alternating Voltage is applied to the primary coil. It sets up an alternating current in the primary coil and hence an alternating flux in the magnetic circuit (Core). This magnetic flux links with the secondary coil and induces in it an alternating voltage due to the changing flux linkage. The EMF induced in the secondary coil will be supplied to the load.

### **Ideal Transformer:**

An ideal transformer has no losses i.e. no  $I^2R$  losses (its windings have no resistance) and no core losses. In other words it consists of two pure inductive coils and a lossless core. But practically it is not possible to realize such an Ideal transformer.

In an Ideal transformer the no load current will be lagging the applied voltage by  $90^\circ$  since the current flows through a purely inductive primary coil. The flux  $\phi$  induced in the magnetic circuit also will be lagging the applied voltage  $V_1$  since the current in the primary is entirely a magnetizing current (the coil is assumed to be totally inductive) and will be in phase with the magnetic flux. The voltages induced in the primary and secondary coils  $E_1$  &  $E_2$  will be lagging the flux by  $90^\circ$

and in phase opposition to the applied Voltage  $V_1$ . All these relations are clearly shown in the No load phasor diagram given below.



**Fig: No load Phasor Diagram of an Ideal Single phase Transformer**

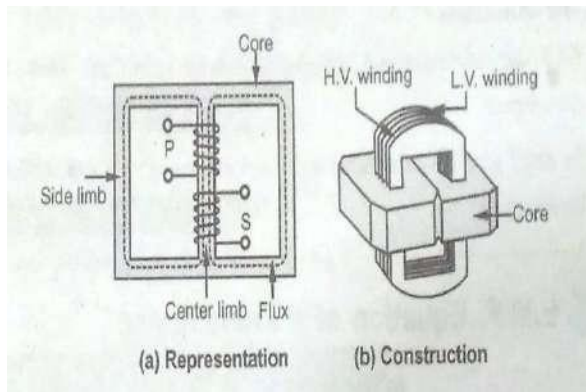
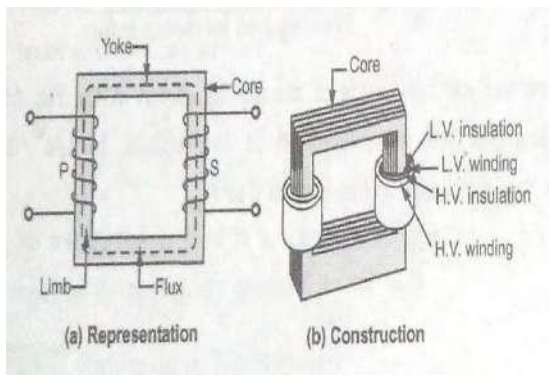
### **Types and constructional features:**

The essential parts of a Transformer are :

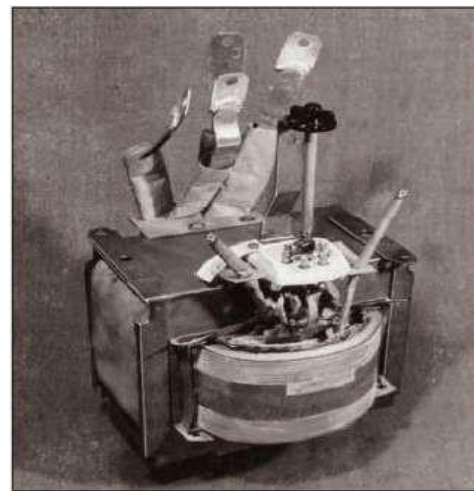
- Coils and the Laminated core
- Tank or container to house the coils & the core
- Suitable medium called the transformer oil to insulate the core and the windings from the tank
- Bushings (porcelain) for insulating and bringing out the coil terminals.

There are two types of core construction called **core** type and **shell** type and are shown in the figure below.

- **In Core Form** the transformer windings surround the two sides of a rectangular core.
- **In Shell Form** a three legged laminated core is used with the windings wrapped around the centre leg.

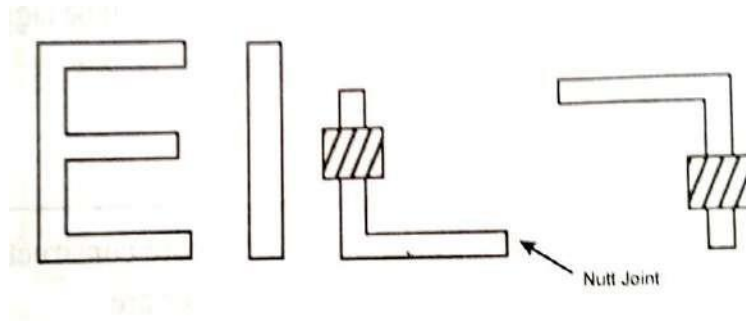


**Fig: Core type transformer**



**Fig: Shell type**

The core is constructed of laminations to minimize the **eddy current** losses. The laminations are made of thin sheet steel with high *silicon* content to produce *high permeability* and a *low hysteresis loss*. The laminations are insulated from each other by a coat of varnish. The laminations are cut in the form of strips of shape L, E & I called *core stampings* as shown in the figure below and then stacked together to get the *Core* and *shell* type cores.



**Fig: Types of core stampings**

**EMF Equation of a Transformer:**

- Number of turns in the primary and secondary windings :  $N_1$  and  $N_2$
- Maximum value of flux in the core :  $\Phi_m$
- Frequency :  $f$
- Time for change of Flux in one cycle :  $1/f$
- Time for change of flux from zero to  $\Phi_m$  :  $1/4f$
- Hence rate of change of flux :  $\Phi_m/1/4f$
- Average e.m.f. induced in each turn :  $4f \Phi_m$
- Form factor for a sine wave : **1.11**
- RMS value of voltage in a turn :  **$1.11 \times 4f$**
- **$\Phi_m = 4.44 f \Phi_m$**
- RMS value of voltage induced in the primary :  **$4.44 f \Phi_m$**   
 $N_1$
- RMS value of voltage induced in the secondary :  **$4.44 f \Phi_m$**   
 $N_2$

It is to be noted that when a sinusoidal voltage  $V_1$  is applied to the primary winding leading to the flow of current and thus generation of flux in the core, voltages are induced in both the primary and secondary windings as given by the equations above. Voltage  $E_1$  &  $E_2$  are opposite in polarity to  $V_1$  and the voltage  $E_2$  gives rise to the load current when the secondary is connected to a load.

We get the relation between the two voltages by dividing the expressions for  $E_1$  with that of  $E_2$  as

$$E_1/E_2 = N_1/N_2$$

And this ratio is called the **turns ratio 'a'** or **'k'** of the transformer.

Since the primary **VoltAmp (VA)** is nearly equal to that of the secondary due to the high efficiency of transformers we get the relation :  $E_1 I_1 = E_2 I_2$  i.e  $I_1/I_2 = E_2/E_1 = N_2/N_1 = 1/a$  or  $1/k$

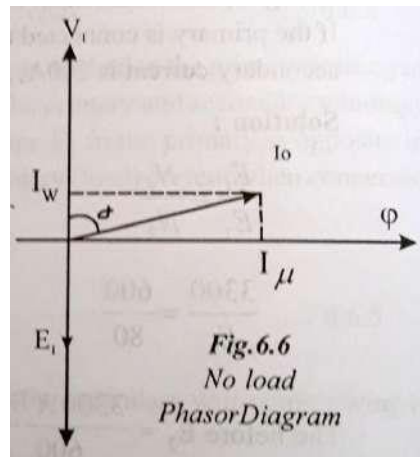
#### **Dot Convention:**

Given that the primary circuit's voltage is positive at a specific end of the coil, what would the *polarity* of the secondary circuit's voltage be? In real transformers, it would be possible to tell the secondary 's polarity only if the transformer were opened and its windings examined. To avoid this necessity, transformers utilize the *dot convention*. The dots appearing at one end of each winding in the figure tell the polarity of the voltage and current on the secondary side of the transformer. The relationship is as follows:

1. If the primary *voltage* is positive at the dotted end of the winding with respect to the undotted end, then the secondary voltage will be positive at the dotted end also. Voltage polarities are the same with respect to the dots on each side of the core.
2. If the primary *current* of the transformer flows *into* the dotted end of the primary winding, the secondary current will flow *out* of the dotted end of the secondary winding.

#### **A practical Transformer on no load:**

When the primary winding of a transformer is energized with an AC source of power and the secondary is left open without connecting to any load, the transformer is said to be on No load.



**Fig: Phasor diagram of a practical single phase transformer on No Load**

In a practical Transformer the primary winding cannot be a pure reactance and it would have some resistance also. Then the primary no-load current  $I_0$  would lag behind the applied voltage by an angle  $\alpha$  which is slightly less than  $90^\circ$  and it can be resolved into the following two components as shown in the Phasor diagram .

- $I_\mu = I_0 \sin \alpha$  along the direction of flux called the *magnetizing current* and is responsible for the generation of flux  $\phi$  in the core of the transformer
- $I_w = I_0 \cos \alpha$  along the direction of the primary voltage  $V_1$  called the *working (active) component* and is responsible to cover the **no load losses (Hysteresis, eddy current and small resistive)** or **core losses**

In this diagram the applied voltage  $V_1$  and the flux  $\phi$  will have a phase difference of  $90^\circ$  and are taken as reference.

The **no load losses** are given by:  $P_{NL} = V_1 \cdot I_w$

$E_1$  and  $E_2$  are the induced voltages in the primary and secondary and both lag behind the flux by  $90^\circ$  since the induced voltage is equal to the rate of change of flux linkages.

### **Equivalent circuit of a real transformer:**

The equivalent circuit of a transformer is a circuit which will take into account all the major imperfections in a practical transformer and are modeled appropriately as equivalent resistors and inductors as explained below.

***To develop the equivalent circuit of a real transformer, the following losses have to be taken into account in order to accurately model the transformer into its equivalent circuit:***

- ***Copper ( $I^2R$ ) Losses*** – Resistive heating losses in the primary and secondary windings of the transformer.

***They are modeled by placing a resistor  $R_P$  in the primary circuit and a resistor  $R_S$  in the secondary circuit.***

- ***Eddy current Losses*** – Resistive heating losses in the core of the transformer.
- ***Hysteresis Losses*** – These are associated with the rearrangement of the magnetic domains in the core during each half-cycle.

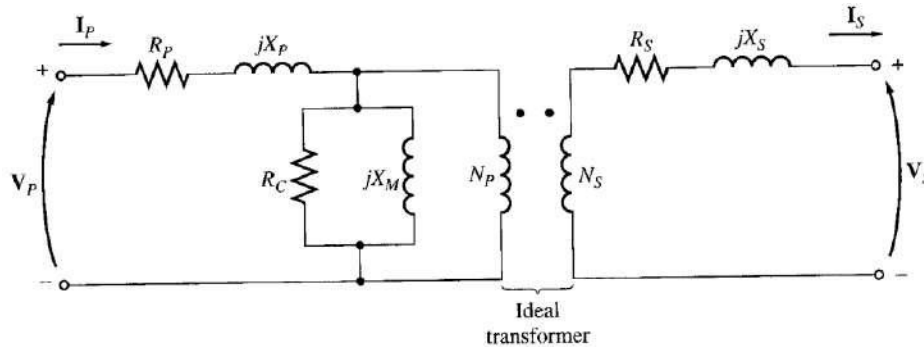
***Both these losses produce a core loss current  $I_{H+E}$  or  $I_{CL}$  ( $I_w$ ) which is a current proportional to the voltage applied to the core. Since this is in phase with the applied voltage this loss is modeled as a resistance  $R_C$  across the primary voltage source.***

- ***Leakage flux*** – The fluxes  $\Phi_{LP}$  and  $\Phi_{LS}$  which escape the core and pass through only one of the transformer windings are called leakage fluxes. They then produce self-inductances in the primary and secondary coils.

***They are modeled as equivalent Inductive reactances  $X_P$  and  $X_S$  in the primary and secondary circuits in series with the resistors  $R_P$  and  $R_S$ .***

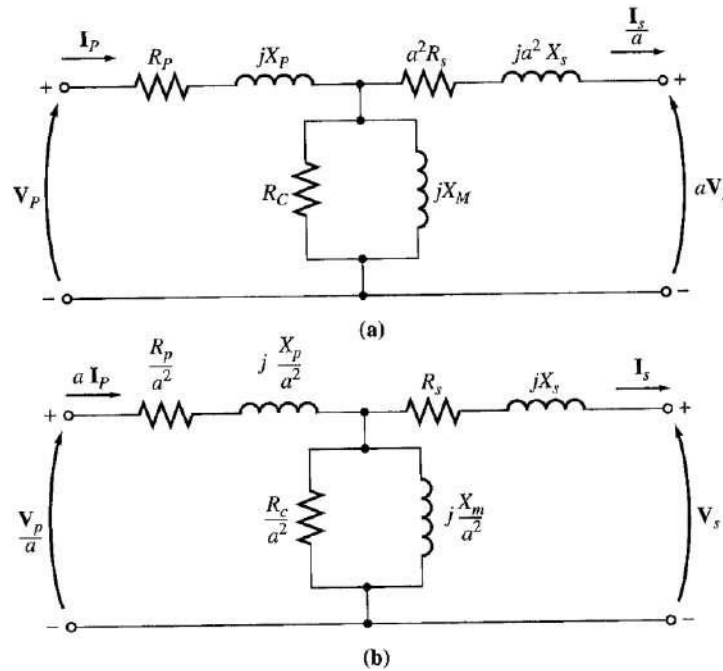
Apart from the above losses the transformer draws a magnetizing current  $I_\mu$  (or  $I_m$ ) and since this current lags behind the applied voltage by  $90^\circ$  it is modeled as a reactance  $X_M$

Complete Equivalent circuit of a Practical Transformer incorporating all the above aspects is shown in the figure below.



**Fig: Complete Equivalent circuit of a Practical Transformer**

For ease of circuit analysis and mathematical calculation this complete equivalent circuit is simplified by referring the impedances in the secondary to the primary and vice versa as shown below.



**Fig: Equivalent circuit of a Transformer (a) referring to the primary (b) referring to the secondary**

- When the **Secondary** is referred to the **Primary** the Secondary side parameters referred to the Primary will be come:  $a \cdot V_s$  ,  $I_s/a$  ,  $a^2 \cdot R_s$  and  $a^2 \cdot X_s$
- When the **Primary** is referred to the **Secondary** the Primary side parameters referred to the Secondary will be come:  $V_p/a$  ,  $a \cdot I_p$  ,  $R_p/a^2$  and  $X_p/a^2$

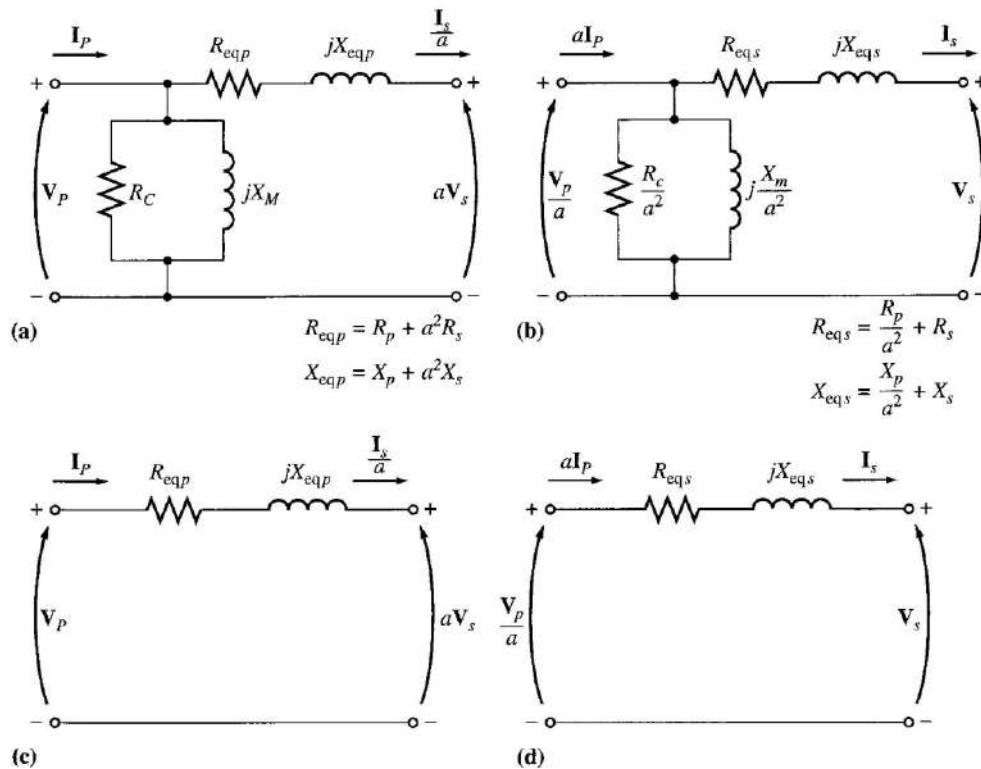
**Approximate Equivalent circuit of a Transformer:**

The derived equivalent circuit is detailed but it is complex for practical engineering applications. The main problem in analysis & calculations is

representation of the excitation and the eddy current & hysteresis losses which add an extra branch.

In practical situations, the excitation current  $I_\mu$  & the core loss current  $I_{CL}$  will be relatively small as compared to the load current, which makes the resultant voltage drop across  $R_p$  and  $X_p$  to be very small. Hence  $R_p$  and  $X_p$  may be lumped together with the secondary referred impedances to form an equivalent impedance. In some cases, the excitation current  $I_\mu$  & the core loss current  $I_{CL}$  are neglected entirely due to their small magnitude.

The equivalent circuits with these simplifications are shown in the figure below.



**Fig: Simplified equivalent circuits (a) Referred to the primary side (b) Referred to the secondary side**

(c) With no excitation branch, referred to the primary side (d) With no excitation branch, referred to the secondary side

**Voltage Regulation of a Transformer:**

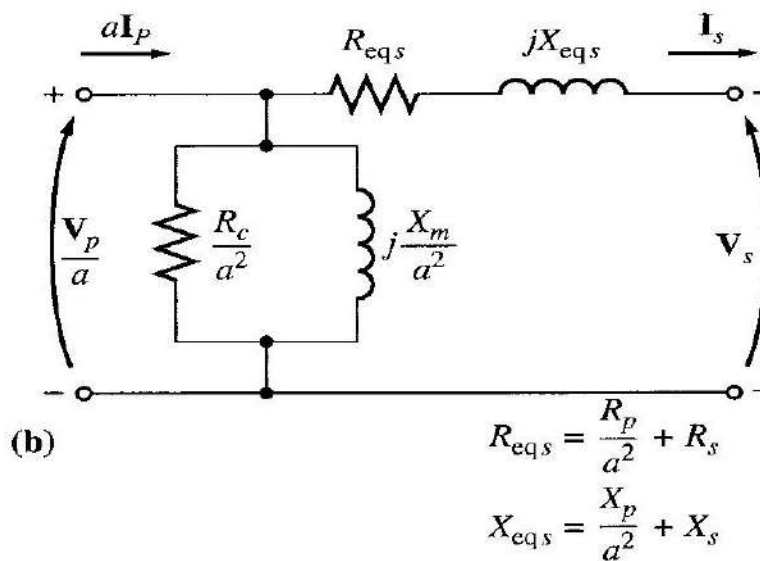
The output voltage of a transformer varies with the load even if the input voltage remains constant. This is because of the voltage drop in it's series impedance as seen in the equivalent circuit. It is normally defined for Full load as a percentage as given below:

$$\% \text{ Voltage Regulation } (\%VR) = [ (V_{S\text{NL}} - V_{S\text{FL}}) / V_{S\text{NL}} ] \cdot 100 \%$$

**The transformer phasor diagram:**

To determine the voltage regulation of a transformer, we must understand the voltage drops within the transformer and they can be seen easily with the help of the Phasor diagram **ON LOAD** .

To develop the Phasor diagram on load let us consider once again the simplified equivalent circuit referred to the secondary side:



**Fig: Simplified equivalent circuit Referred to the secondary side**

Ignoring the excitation branch (since the current flow through the branch is considered to be small), Series impedances ( $R_{eq} + jX_{eq}$ ) will be more predominant. Hence Voltage Regulation depends on the magnitude of the series impedance and the phase angle of the load current flowing through the secondary of the transformer with respect to the secondary voltage.

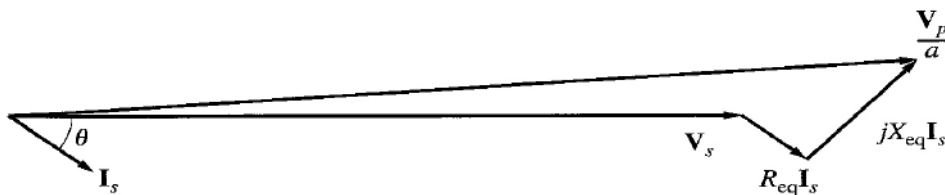
Phasor diagram will represent the effects of these factors on the voltage regulation. A phasor diagram consists of current and voltage vectors. In this Phasor diagram the secondary voltage  $V_s$  is assumed as the reference.

Applying Kirchoff Voltage Law to this simplified equivalent circuit we get the relation :

$$V_p/a = V_s + I_s \cdot R_{eqs} + I_s \cdot X_{eqs}$$

From this equation, the phasor diagram can be visualized and is shown below for a transformer operating at a lagging power factor. For lagging loads,  $V_p / a > V_s$  so the voltage regulation with lagging loads is  $> 0$ .

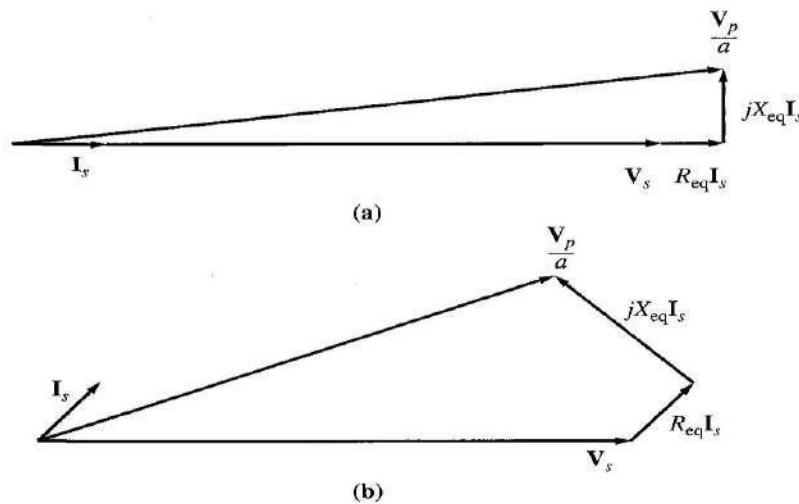
***From the Phasor diagram it can be seen that the addition is Vector addition and not an algebraic addition.***



**Fig: Phasor diagram of a Transformer on load with lagging power factor**

When the power factor is unity also  $V_s$  is lower than  $V_p$  and so  $VR > 0$ . But  $VR$  is smaller than before (during lagging PF) as shown in figure (a) below .

With a leading power factor ,  $V_s$  is higher than the referred  $V_p (V_p / a)$  so  $VR < 0$  as shown in figure(b) below .



**Fig: Phasor diagram of a Transformer on load with (a) unity power factor and (b) leading power factor**

### Losses:

As we know a transformer is a device used to transfer Electric power from one circuit at a certain voltage level to another circuit at a different voltage level. In this process of conversion some power is lost. The difference between the input power and the output power is termed as the **Power loss** or Losses .

### Constant & Variable losses:

The losses are broadly classified as *constant losses* and *variable losses*. Constant losses are constant and are independent of the load where as the variable losses are dependent on the load. They are further classified in detail as below.

### Detailed Classification of Losses:

1. **Electrical or Copper Losses (I<sup>2</sup>R Loss):** Current flow through the resistance of the transformer coils give rise to I<sup>2</sup>R losses and since the coils are normally made up of copper these losses are called Copper losses.

$$\text{Primary copper loss: } P_p = I_p^2 R_p$$

$$\text{Secondary copper loss: } P_s = I_s^2 R_s$$

2. **Core Losses:** *Hysteresis* and *eddy current* losses occurring in the transformer core are together called core losses.

3.

- **Hysteresis loss:** in an iron core is the loss of power due to the hysteresis loop in the magnetization characteristic of the core in each cycle of the alternating current applied to the core. It is proportional to the area of the Hysteresis loop and is given by :

$$\text{Hysteresis loss} = K_h B_m^{1.67} f v \text{ watts}$$

Where  $K_h$  = Hysteresis constant and it depends on the core material

$B_m$  = Maximum flux density in the core

$f$  = Frequency of operation

$v$  = Volume of the core.

- **Eddy current losses:** A time-changing flux induces voltage within a ferromagnetic core in just the same manner as it induces voltage in the coils around the core of the transformer. These voltages cause swirls of current to flow within the core, much like the eddies seen at the edges of a river. It is the shape of these currents that gives rise to the name **eddy currents**. These eddy currents flowing in a resistive material (the iron of the core) cause power loss thus heating the iron core and the resulting loss is

called **eddy current loss**. This loss is proportional to the thickness of the core material and hence to minimize this loss the core is made up of thin sheets called laminations instead of a single thick block. An insulating oxide or resin is used between the strips so that the current paths for eddy currents are limited to very small areas. Thus the eddy current losses have a very little effect on the core's magnetic properties.

The eddy current loss is given by:

$$\text{Eddy current loss} = K_e B_m^2 f^2 t^2 \text{ watts/ unit volume}$$

Where  $K_e$  = Eddy current constant

$t$  = Thickness of the core

*Out of the above, Core Losses are also called as Constant losses since the flux in the core is almost constant as supply voltage  $V_1$  at rated frequency  $f$  is always constant at all the loads and thus the flux density  $B_m$  is also constant on which both hysteresis and eddy current losses are dependent. Electrical or Copper Losses are also called as variable losses.*

*It is to be noted that there are no mechanical losses in transformers because there are no moving parts.*

#### **Minimization of hysteresis and eddy current losses:**

- Hysteresis losses are minimized by using high grade core material like silicon steel having very high permeability and low hysteresis loop.
- *Eddy current loss* is proportional to the thickness of the core material and hence this loss is minimized by making up the core with thin sheets called laminations instead of a single thick block. An insulating oxide or resin is used between the strips so that the current paths for eddy currents are limited to very small areas.

#### **Effect of variations of frequency and supply voltage on iron losses:**

---

We know that the iron losses in a transformer include two types of losses i.e. Hysteresis losses & Eddy current losses and are given by the relations:

$$\begin{aligned} \text{Hysteresis loss} &= K_h B_m^{1.67} f v \quad \text{watts and} && \text{-----} \\ (1) &&& \text{Eddy current loss} \\ &= K_e B_m^2 f^2 t^2 \text{ watts/ unit volume} && \text{-----} \quad (2) \end{aligned}$$

It is also known that for a transformer the voltage V is given by :

$$V = 4.44 f \phi_m N = 4.44 f B_m A N$$

From which we can see that for constant A and N

$$(3) \quad B_m \propto \frac{V}{f} \quad \text{-----}$$

From the above three equations we can see that:

- As voltage changes (with frequency remaining constant), the maximum flux density changes and thus both eddy current and hysteresis losses also change. As voltage increases, the maximum flux density in the core increases and total iron losses increase.
- As frequency increases, as per equation (3) the flux density in the core decreases but the iron loss increases as per equations (1) and (2). Hence the net effect depends on the relative effect of flux density  $B_m$  and frequency  $f$ . A closer look at equations (1) and (2) then reveals that :
  - In Hysteresis losses  $B_m$  is of higher degree compared to frequency and is more dominant. Hence the net Hysteresis losses **decrease with increase** in frequency .
  - In Eddy current losses  $B_m$  and  $f$  are of the same degree. Hence the net Eddy current losses remain the same with increase or decrease in frequency .

### **Volt-Ampere Rating of a Transformer:**

When electrical power is transferred from primary winding to secondary there are few power losses in between. These power losses appear in the form of heat which increases the temperature of the device. This temperature must be maintained below a certain limiting value as it causes deterioration of the insulation in the Transformer.

Hence the rating of the transformer is decided by the temperature rise which in turn is decided by the losses.

The copper losses ( $I^2R$ ) in the transformer depend on the current ' $I$ ' through the winding while the iron or core loss depends on the voltage ' $V$ ' as frequency of operation is constant.

As losses depend on  $V$  and  $I$  only and not on the load power factor, the rating of the transformer is specified as a product of these two parameters  $V \times I$  called **Volt-Ampere Rating or VA rating ( or kVA rating for bigger machines )**

On both sides of the transformer i.e. primary and secondary **VA** rating remains same. This rating is more generally expressed in **kVA** (kilo volt amperes rating) for large capacity machines.

### **Efficiency of a Transformer:**

Efficiency of a Transformer is defined as (Same as defined for motors and generators):

$$\text{Efficiency } \eta = (P_{\text{out}} / P_{\text{in}}) \cdot 100\% = (P_{\text{out}} / P_{\text{out}} + \text{Losses}) \cdot 100\%$$

Losses incurred in a transformer:

- Copper losses ( $I^2R$ )
- Core losses ( Hysteresis losses and Eddy current losses)

Therefore, for a transformer, efficiency may be calculated using the following relation :

From the phasor diagram we know that  $P_{out}$  in a transformer is given by  $P_{out} = V_s \cdot I_s \cdot \cos \theta$ . Hence

$$\eta = (V_s \cdot I_s \cdot \cos \theta / V_s \cdot I_s \cdot \cos \theta + P_{CU} + P_{CORE}) \cdot 100\%$$

#### All Day Efficiency of a Transformer:

The *ordinary or commercial efficiency* of a transformer is defined as the ratio of the output power to the input power.

$$\eta = \frac{\text{output power}}{\text{input power}} = \frac{\text{output power}}{\text{output power} + \text{losses}}$$

Transformers efficiency cannot be judged by ordinary or commercial efficiency as the load on certain transformers like distribution transformers (which serve residential and commercial loads) fluctuates throughout the day. Though the distribution transformers are energized for all 24 hours, they deliver very light loads for major portion of the day, rated or full load for minor portion of the day, and 50 to 75% load most of the time.

But the primary of such transformers is energized at its rated voltage for all the 24 hours to provide continuous supply to the consumer. As we know, there are various losses in the transformer such as iron and copper loss. The iron loss takes place in the core of the transformer and it occurs for the whole day in the distribution transformer. The second type of loss known as copper loss takes place in the windings of the transformer also known as the variable loss. It occurs only when the transformers are in the loaded condition and its value depends on the load. For no load the copper losses are very small and on full load it is at the rated value. Hence the normal power efficiency as defined earlier cannot give the efficiency of such transformers in its true sense.

Hence, the performance of such transformers cannot be judged by the commercial or ordinary efficiency, but the efficiency is calculated or judged by All

Day Efficiency also known as operational efficiency or energy efficiency which is computed by calculating the energy consumed during 24 hours.

**Definition:** All day efficiency means the power consumed by the transformer throughout the day. It is defined as the ratio of output power to the input power in kWh or wh of the transformer over 24 hours.

Mathematically, it is represented as:

$$\text{All day efficiency, } \eta_{\text{all day}} = \frac{\text{output in kWh}}{\text{input in kWh}} \quad (\text{for 24 hours})$$

All day efficiency of the transformer depends on their loading pattern.

### Condition for Maximum Efficiency :

The load current at which the efficiency attains maximum value is denoted as  $I_{sm}$  and maximum efficiency is denoted as  $\eta_{\max}$ . The efficiency is a function of load i.e. load current  $I_s$ , assuming the load power factor  $\cos \phi_s$  and the secondary terminal voltage  $V_s$  as constant. The condition for Maximum Efficiency can be obtained by differentiating the expression for the efficiency with respect to  $I_s$  and then equating the differential to zero. So for maximum efficiency,

$$\frac{d\eta}{dI_s} = 0 \quad \text{While } \eta = \frac{V_s I_s \cos \phi_s}{V_s I_s \cos \phi_s + P_i + I_s^2 R_{se}}$$

$$\therefore \frac{d\eta}{dI_s} = \frac{d}{dI_s} \left\{ \frac{V_s I_s \cos \phi_s}{V_s I_s \cos \phi_s + P_i + I_s^2 R_{se}} \right\} = 0$$

Effectively we can equate the numerator of  $\frac{d\eta}{dI_s}$  to zero.

$$\therefore (V_s I_s \cos \phi_s + P_i + I_s^2 R_{se}) (V_s \cos \phi_s) - (V_s I_s \cos \phi_s) (V_s \cos \phi_s + 2I_s R_{se}) = 0$$

$$\text{Cancelling } (V_s \cos \phi_s) \text{ from both the terms we get,}$$

$$V_s I_s \cos \phi_s + P_i + I_s^2 R_{se} - V_s I_s \cos \phi_s - 2 I_s^2 R_{se} = 0 \quad \text{i.e.} \quad P_i - I_s^2 R_{se} = 0$$

$$\therefore P_i = I_s^2 R_{se} = P_{Cu}$$

So the condition for maximum efficiency is :

$$\text{Copper losses} = \text{Iron losses i.e. } P_i = P_{Cu}$$

### Load current $I_s$ at maximum Efficiency:

$$\text{For } \eta_{\max}, \quad I_{s@ \max . \eta}^2 R_{se} = P_i$$

$$\therefore \text{The load current @max efficiency is given by: } I_{s@ \max . \eta} = \sqrt{\frac{P_i}{R_{se}}}$$

....

We can get Load current  $I_s$  at maximum Efficiency in terms of the full load current  $I_{s@FL}$  (or rated current) also:

$$\therefore \frac{I_{s@ \max . \eta}}{(I_s)@F.L} = \frac{1}{(I_s)@F.L} \sqrt{\frac{P_i}{R_{se}}} = \sqrt{\frac{P_i}{\{(I_s)@F.L\}^2 R_{se}}} = \sqrt{\frac{P_i}{(P_{Cu})@F.L}}$$

$$\therefore I_{s@ \max . \eta} = (I_s)@F.L \sqrt{\frac{P_i}{(P_{Cu})@F.L}}$$

### kVA supplied at Maximum Efficiency:

- For constant  $V_s$  the kVA supplied is a function of load current.

$$\therefore \text{kVA @ } \eta_{\max} = I_{s@ \max . \eta} V_s = V_s (I_s)@F.L \times \sqrt{\frac{P_i}{(P_{Cu})@F.L}} = (\text{kVA rating}) \times \sqrt{\frac{P_i}{(P_{Cu})@F.L}}$$

### Effect of Power Factor on Efficiency:

The efficiency of a transformer is given by,

$$\eta = \frac{\text{Output}}{\text{Input}} = \frac{\text{Input} - \text{Losses}}{\text{Input}} = 1 - \frac{\text{Losses}}{\text{Input}}$$

..... (1)

Now,  $\text{Input} = \text{output} + \text{Losses} = V_s I_s \cos \phi + \text{Losses}$

Using this in equation (1),  $\eta = 1 - \frac{\text{Losses}}{V_s I_s \cos \phi + \text{Losses}}$

$$\therefore \eta = 1 - \frac{\frac{\text{Losses}}{V_s I_s}}{\cos \phi + \frac{\text{Losses}}{V_s I_s}}$$

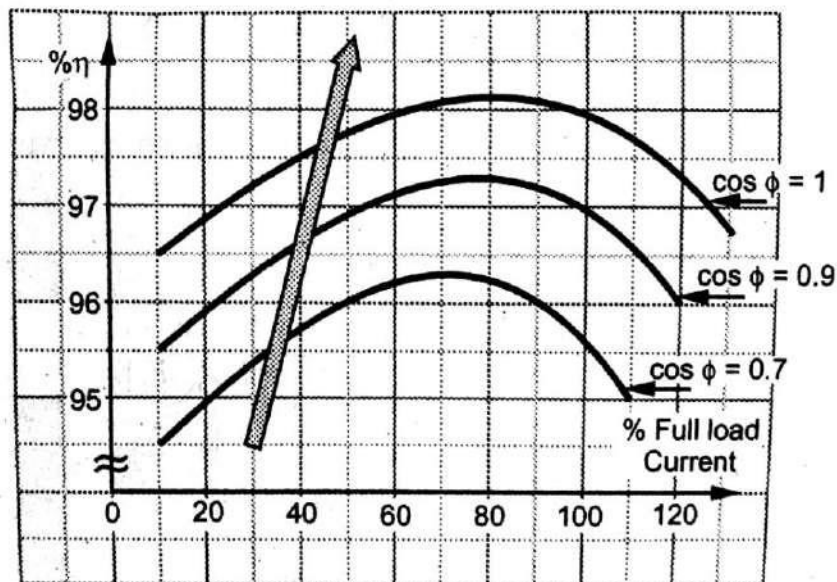
Let  $\frac{\text{Losses}}{V_s I_s} = x$  and using this in the above equation we get

$$\eta = 1 - \frac{x}{x + \cos \phi}$$

Thus as the power factor of the load i.e.  $\cos \phi$  becomes higher, the second term in the above equation becomes lesser and efficiency will become **more**.

The family of efficiency curves (as a function of load current) are obtained as power factor increases, are shown in the figure below. From these plots we can get the following two important observations.

- **Maximum efficiency comes at a value lower than the 100% full load current.**
- **As power factor increases, the efficiency increases.**



**Figure :** Family of efficiency curves (as a function of load current) as power factor increases

**Can D.C. Supply be used for Transformers? Explain.**

*The D.C. Supply cannot be used for the transformers for the following reasons.*

- The transformer works on the principle of mutual induction for which, current in one coil must change uniformly. If D.C. supply is given, the current will not change due to constant supply and transformer will not work.
- Practically a transformer is designed with a low winding resistance to reduce the copper losses and thus increase the efficiency. For D.C., the inductive reactance  $X_L$  is zero as D.C. frequency is zero. So total impedance of winding is very low for D.C. Thus winding will draw higher current if D.C. supply is given to it. This may cause the burning of windings due to extra heat generated and may cause permanent damage to the transformer.

### Illustrative Examples:

**Example 1 :** A single phase 50 Hz transformer has 100 turns on the primary and 400 turns on the secondary winding. The net cross-sectional area of core is 250 cm<sup>2</sup>. If the primary winding is connected to a 230 V, 50 Hz supply, determine,

- i) The e.m.f. induced in the secondary winding
- ii) The maximum value of flux density in the core.

**Solution:** We have the data:

$N_1 = 100, N_2 = 400, A = 250 \text{ cm}^2, E_1 = 230 \text{ V}.$

Hence turns ratio : 
$$K = \frac{N_2}{N_1} = \frac{400}{100} = 4$$

Now 
$$K = \frac{E_2}{E_1} \text{ i.e. } E_2 = KE_1$$

i)  $\therefore$  The e.m.f. induced in the secondary winding =  $E_2 = 4 \times 230 = 920 \text{ V}$

ii) We know that the e.m.f. induced in the secondary winding is given by  $E_1 = 4.44 \phi_m f N_1$  where  $\phi_m$  is the maximum value of flux in the core.

i.e.  $230 = 4.44 \phi_m \times 50 \times 100$

$\therefore \phi_m = 0.01045 \text{ Wb}$

$\therefore$  The maximum value of flux density in the core  $B_m = \frac{\phi_m}{A} = \frac{0.01045}{250 \times 10^{-4}} = \mathbf{0.4182 \text{ Wb/m}^2}$

**Example 2:** A single phase transformer is connected to a 230 V, 50 Hz supply. The net cross sectional area of the core is 50 cm<sup>2</sup>. The number of turns of the primary is 460 and the secondary is 80. Determine i) Transformation ratio ii) Peak value of the flux density in the core iii) e.m.f. in the secondary winding.

**Solution :** Refer example 1 above for the procedure and verify :

i)  $K = \frac{N_2}{N_1} = \mathbf{0.1739}$  ii)  $\mathbf{0.4504 \text{ Wb/m}^2}$  iii)  $\mathbf{40 \text{ V}}$

**Example 3: 2.4kV/115 V transformer has sinusoidal flux expressed by 0.113 sin188.5 t. Determine the primary and secondary turns.**

**Solution :** We have the data:

$$E_1 = 2.4 \text{ kV}, E_2 = 115 \text{ V}, \text{ and } \phi = 0.113 \sin 188.5 t \text{ Wb}$$

Comparing  $\phi = 0.113 \sin 188.5 t$  with the standard sinusoid expression  $\phi = \phi_m \sin \omega t$ , we get

$$\phi_m = 0.113 \text{ Wb and } \omega = 188.5 \text{ rad/sec.}$$

$$\therefore f = \frac{\omega}{2\pi} = \frac{188.5}{2\pi} = 30 \text{ Hz}$$

Induced e.m.f is given by  $E_1 = 4.44 \phi_m f N_1$ . Substituting the above values of  $E_1$ ,  $\phi_m$ , and  $f$  in this equation we get

$$N_1 = \frac{2.4 \times 10^3}{4.44 \times 0.113 \times 30} = 159.45 = 160$$

$$\text{We know that the turns ratio} = \frac{E_2}{E_1} = \frac{N_2}{N_1} \text{ and from which } N_2 = \frac{E_2}{E_1} \times N_1 = \frac{160 \times 115}{2.4 \times 10^3} = 7.666 \approx 8$$

**Example 4:** A 6300/210 V, 50 Hz, single phase transformer has per turn e.m.f of about 9 volts and maximum flux density of 1.2 T. Find the number of high voltage and low voltage turns and the net cross sectional area of the core.

**Solution:** We have the data:

$$E_1 = 6300 \text{ V}, E_2 = 210 \text{ V}, f = 50 \text{ Hz}, B_m = 1.2 \text{ T and } E/\text{turn} = 9\text{V}$$

$$\text{Hence } E_1 = (E/\text{turn}) \times N_1 \quad \text{i.e. } 6300 = 9 \times N_1$$

$$\text{Thus } N_1 = 6300/9 = \mathbf{700}$$

And similarly

$$E_2 = (E/\text{turn}) \times N_2 \quad \text{i.e. } 210 = 9 \times N_2$$

$$\text{Thus } N_2 = 210/9 = 23.33 \approx \mathbf{24}$$

But we also know that  $E_1$  is given by:  $E_1 = 4.44 \phi_m f N_1$

$$\text{i.e. } 6300 = 4.44 \phi_m \times 50 \times 700$$

$$\therefore \phi_m = 6300 / 4.44 \times 50 \times 700 = 0.04054 \text{ Wb}$$

And we have  $B_m = \frac{\Phi_m}{a}$

$$\therefore a = \frac{\Phi_m}{B_m} = \frac{0.04054}{1.2} = 0.03378 \text{ m}^2 = \mathbf{337.8 \text{ cm}^2}$$

**Example 5:** The e.m.f per turn for a single phase, 2310/220 volts, 50Hz transformer is 13 volts. Calculate.

- i) The number of primary and secondary turns.
- ii) The net cross sectional area of the core, for a maximum flux density of 1.4 T.

**Solution :** Refer example 4 for the procedure and verify :

$$N_1 = \mathbf{178}, N_2 = \mathbf{17},$$
$$a = \mathbf{417.5 \text{ cm}^2}$$

**Example 6:** A 1-phase transformer has 500 primary and 1200 secondary turns. The net cross sectional area of the core is 75 cm<sup>2</sup>. If the primary winding be connected to 400 V, 50 Hz supply, calculate the peak value of flux density in the core and voltage induced in the secondary winding.

**Solution :** Refer example 1 above for the procedure and verify the answers  
 **$B_m = 0.4804 \text{ Wb/m}^2$ ,  $E_2 = 960 \text{ V}$ .**

**Example 7:** A single-phase transformer has 400 primary and 1000 secondary turns. The net cross-sectional area of the core is 60 cm<sup>2</sup>. If the primary winding is connected to a 50Hz supply at 500 V. Calculate.

**Solution :** Refer example 1 for the procedure and verify the answer :  
**i)  $0.938 \text{ Wb/m}^2$ , ii) 1250 V**

**Example 8:** The no load current of a transformer is 4 A at 0.25 p.f when supplied at 250 V, 50 Hz. The number of turns on the primary winding is 200. Calculate

- i) The r.m.s value of the flux in the core
- ii) The core loss
- iii) The magnetizing current.

**Solution :**

We have the data :  $V_1 = 250$  V,  $f = 50$  Hz, No load current  $I_0 = 4$  A, P.F = 0.25 and  $N_1 = 200$

i) We know that  $V_1$  is given by :  $V_1 = 4.44 \Phi_m f N_1$  from which we get  $\Phi_m = \frac{250}{4.44 \times 50 \times 200}$

$\therefore \Phi_m = 5.63$  mWb,  $\Phi(\text{RMS}) = \frac{\Phi_m}{\sqrt{2}} = 3.98$  mWb

ii) Core loss =  $V_1 I_0 \text{ P.F} = 250 \times 4 \times 0.25 = \mathbf{250 \text{ W}}$

iii) P.F =  $\cos \theta = 0.25$  from which we get  $\sin \theta = 0.9682$

Then we can get magnetizing current as  $I_m = I_0 \sin \theta = 4 \times 0.9682 =$

**3.873 A**

**Example 9:** A 100 kVA, 2400/24 V, 50 Hz single phase transformer has an exciting current of 0.64 Amps and a core loss of 700 watts, when its high voltage side is energized at rated voltage and frequency. Calculate the two components of the exciting current.

**Solution :** We have the data

No load current  $I_0 = 0.64$  A, Core losses ( or iron losses or no load losses )  $P_i = 700$  W, In put rating  $V_1 = 2400$  V

We know that the no load or Iron losses are given by :  $P_i = V_1 I_0 (\text{p.f})$  which is = 700 Watts

From the given data then we get :  $700 = 2400 \times 0.64 \times (\text{p.f})$  from which we get  $\text{p.f} = 700 / (2400 \times 0.64) = 0.4557$

we also know that power factor =  $\cos \theta$  and hence  $\cos \theta = 0.4557$ , and thus  $\sin \theta = 0.8901$

The two components of exciting currents i.e core loss current  $I_c$  and magnetizing current  $I_m$  are then given by

$\therefore I_c = I_0 \cos \theta = 0.64 \times 0.4557$   
**= 0.2916 A**

$\therefore I_m = I_0 \sin \theta = 0.64 \times 0.8901$

$$= \mathbf{0.5696 \text{ A}}$$

**Example 10:** A 2000/200 V single phase transformer gives 0.5 A and 40 W as ammeter and wattmeter readings when supply is given to the low voltage winding and high voltage winding is kept open. Find : i) The magnetizing component, ii) The iron component, and iii) The power factor of no-load current.

**Solution :** From the given data we observe that the readings are on LV side (secondary) and h.v. winding is kept open.

$$\therefore \quad I_0 = 0.5 \text{ A}, W_0 = 40 \text{ W}, \\ V_0 = 200 \text{ V}$$

$$W_0 = V_0 I_0 \cos \theta$$

$$\text{i.e.} \quad \cos \theta = \frac{40}{0.5 \times 200} = 0.4$$

$$\sin \theta = 0.9165$$

$$\text{i)} \quad I_m = I_0 \sin \theta = 0.5 \times 0.9165 \\ = \mathbf{0.4582 \text{ A}}$$

$$\text{ii)} \quad I_c = I_0 \cos \theta = 0.5 \times 0.4 = 0.2 \text{ A}$$

**iii) No load power factor =  $\cos \theta = 0.4$  lagging**

**Example 11:** A single phase transformer with 10:1 turns ratio and rated at 50 kVA, 2400/240 volts, 50 Hz, is used to step down the voltage of a distribution system. The L.V. side voltage is kept constant at 240 V. Find the value of load impedance on L.V. side so that the transformer will be loaded fully. Find also the maximum flux in the core if L.V. side has 23 turns.

**Solution :**  $V_s = 240 \text{ V}, V_p = 2400 \text{ V}, N_s = 23, 50 \text{ kVA}$

$$(I_s)_{\text{F.L.}} = \frac{\text{VA rating}}{V_s} = \frac{50 \times 10^3}{240} = 208.333 \text{ A}$$

$$\therefore Z_s = \text{Load impedance on F.L} = \frac{V_s}{I_{s\text{FL}}} = \frac{240}{208.333} = \mathbf{1.152 \Omega}$$

$$V_s (\text{ Voltage on LV side }) = 4.44 \times \phi_m \times 50 \times 23$$

i.e.  $240 = 4.44 \times \phi_m \times 50 \times 23$

From which we get  $\phi_m$ (Maximum flux in core ) =  $240 / (4.44 \times 50 \times 23) = \mathbf{0.047 \text{ Wb}}$

**Example 12:** A 15 kVA, 2400/240 V, 60 Hz transformer has a magnetic core of 50 cm<sup>2</sup> cross section and a mean length of 66.7 cm. The application of 2400 V causes magnetic field intensity of 450 AT/m (RMS) and a maximum flux density of 1.5 T. Determine i) The turn's ratio, ii) The numbers of turns in each winding iii) The magnetizing current.

**Solution :** We have the data :  $V_p = 2400 \text{ V}$ ,  $V_s = 240 \text{ V}$ ,  $a = 50 \text{ cm}^2$ , mean length  $l = 66.7 \text{ cm}$ ,

maximum flux density  $B_m = 1.5 \text{ T}$ , magnetic field intensity  $H = 450 \text{ AT/m}$  (R.M.S) with

$V_p = 2400 \text{ V}$  is applied to primary,  $f = 60 \text{ Hz}$

i) Turns ratio  $K = V_p/V_s = N_p/N_s = 2400/240 = \mathbf{10}$

ii) maximum flux  $\phi_m = B_m \times a = 1.5 \times 50 \times 10^{-4} = 7.5 \times 10^{-3} \text{ Wb}$ , (Tesla = Wb/m<sup>2</sup>)

We know that  $V_p = 4.44 \phi_m f N_p$

From which we get  $N_p = \frac{2400}{4.44 \times 7.5 \times 10^{-3} \times 60} = 1201.2 = \text{say } \mathbf{1200}$

$K = N_p/N_s = \mathbf{10}$  i.e.  $N_s = N_p/10 = 1200/10 = \mathbf{120}$

iii) AT ( on 2400 V side ) =  $H \times l = 450 \times 66.7 \times 10^{-2} = 300.15 \text{ AT}$

(For calculating the magnetizing current we have to take that side where voltage is applied)

Hence magnetizing current  $I_0$  ( on 2400 V side ) =  $\frac{300.15}{1200}$   
 $= \mathbf{0.2501 \text{ A (RMS)}}$

**From here to be explained**

**Example 13:** A 6600/400 V single phase transformer has primary resistance of 2.5  $\Omega$  and secondary resistance of 0.01  $\Omega$ . Calculate total equipment resistance referred to primary and secondary.

**Solution :** The given values are,

$$R_1 = 2.5 \Omega, R_2 = 0.01 \Omega$$

$$K = 6600/400 = 16.5$$

For finding equivalent resistance referred to primary, transfer  $R_2$  to primary as

$R_{2eq}$ ,

$$R_{2eq} = K^2 R_2 = 0.01 \times 16.5^2 = 2.7225 \Omega$$

$$R_{1e} = R_1 + R_{2eq} = 2.5 + 2.7225 = \mathbf{5.2225 \Omega}$$

It can be observed that primary is high voltage side and hence high resistance side hence while transferring  $R_2$  from low voltage to high voltage side, its value increases.

Similarly to find total equivalent resistance referred to secondary, first calculate  $R_1$  referred to secondary as

$$R_{1eq} = R_1 / K^2 = 2.5 / 16.5^2$$

$$= 0.00918 \Omega$$

$$\therefore R_{2e} = R_2 + R_{1eq} = 0.01 + 0.00918 = \mathbf{0.01918 \Omega}$$

**Example 14:** A 20 kVA, 2000/200 V single phase transformer has the following parameters. H.V. Winding:  $R_1 = 3 \Omega$ ,  $X_1 = 5.3 \Omega$ , L.V. winding:  $R_2 = 0.05 \Omega$ ,  $X_2 = 0.1 \Omega$ . Find the voltage regulation at, i) Power factor of 0.8 lagging. ii) UPF iii) 0.707 power factor leading.

**Solution :** We have the data

$$V_1 = 2000 \text{ V}, V_2 = 200 \text{ V}, R_1 = 3 \Omega, R_2 = 0.05 \Omega, X_1 = 5.3 \Omega, X_2 = 0.1 \Omega.$$

$$K = V_1/V_2 = 2000/200 = 10$$

From this data let us find out equivalent resistances referred to primary.

$$R_{1eq} = R_1 + R_{2eq} = R_1 + K^2 R_2 = 3 + 0.05 \times 10^2 = \mathbf{8 \Omega}$$

$$X_{1eq} = X_1 + X_{2eq} = X_1 + K^2 X_2 = 5.3 + 0.1 \times 10^2 = \mathbf{15.3 \Omega}$$

$$(I_1)_{FL} = \text{VA rating} / \text{Primary voltage} = \frac{20 \times 10^3}{2000} = 10 \text{ A}$$

i)  $\cos \phi = 0.8$  lagging,  $\sin \phi = 0.6$

$$\begin{aligned} \therefore \% \text{ Regn.} &= (\text{Voltage drop with full load current} / \text{Input Voltage}) \\ &= [ I_1 \{ R_{1eq} \cos \phi + X_{1eq} \sin \phi \} / V_1 ] \times 100 \\ &= [ 10 \{ 8 \times 0.8 + 15.3 \times 0.6 \} / 2000 ] \times 100 = \mathbf{7.79 \%} \end{aligned}$$

ii)  $\cos \phi = 1$ ,  $\sin \phi = 0$

$$\begin{aligned} \therefore \% \text{ Regn} &= (\text{Voltage drop with full load current} / \text{Input Voltage}) \\ &= [ I_1 \{ R_{1eq} \cos \phi + X_{1eq} \sin \phi \} / V_1 ] \times 100 \\ &= [ 10 \{ 8 \times 1 + 15.3 \times 0 \} / 2000 ] \times 100 = \mathbf{4\%} \end{aligned}$$

iii)  $\cos \phi = 0.707$  leading,  $\sin \phi = 0.707$

$$\begin{aligned} \therefore \% \text{ Regn} &= (\text{Voltage drop with full load current} / \text{Input Voltage}) \\ &= [ I_1 \{ R_{1eq} \cos \phi - X_{1eq} \sin \phi \} / V_1 ] \times 100 \\ &= [ I_1 \{ 8 \times 0.8 - 15.3 \times 0.6 \} / 2000 ] \times 100 = \mathbf{-2.5805 \%} \end{aligned}$$

**Example 15:** A 230/460 volts transformer has a primary resistance of  $0.2 \Omega$  and reactance of  $0.5 \Omega$  and the corresponding values for secondary are  $0.75 \Omega$  and  $1.8 \Omega$  respectively. Find the secondary terminal voltage when supplying 10 amps at 0.8 power factor lagging.

**Solution :** We have the data:

$$V_1 = 230, V_2 = 460 \text{ V}, R_1 = 0.2 \Omega, X_1 = 0.5 \Omega, R_2 = 0.75 \Omega, X_2 = 1.8 \Omega$$

$$K = = 230/460 = 0.5$$

***We have to find out the secondary terminal voltage while supplying a load current of 10Amps. Since it is supplying current on the secondary side we have to find out the equivalent circuit parameters referred to secondary side.***

$$\therefore R_{2e} = R_2 + R_{1e} = R_2 + R_1 / K^2 = 0.75 + 0.2 / (0.5)^2 = 1.55 \Omega$$

$$\text{Similarly } X_{2e} = X_2 + X_{1e} = X_2 + X_1 / K^2 = 1.8 + 0.5 / (0.5)^2 = 3.8 \Omega$$

We have to find the secondary terminal voltage at  $\cos \phi = 0.8$  lagging and load current of  $I_2 = 10 \text{ A}$ .  $\sin \phi = 0.6$  ( from the value of  $\cos \phi = 0.8$ )

- $\therefore$  Voltage drop =  $I_2[R_{2e}\cos\phi + X_{2e}\sin\phi] = 10 [1.55 \times 0.8 + 3.8 \times 0.6] = 35.2 \text{ V}$   
 $\therefore$  Secondary terminal voltage  $V_2 = 460 - \text{Voltage drop} = 460 - 35.2 = \mathbf{424.8 \text{ V}}$

**Example 16:** A 100 kVA, 6.6 kV/415 V single phase transformer has an effective impedance of  $(3+j8) \Omega$  referred to HV side. Estimate the full load voltage regulation at

- i) 0.8 pf lagging    ii) 0.8 pf leading

**Solution :** Refer example 14 for the procedure with  $R_{1e} = 3 \Omega$ ,  
 $X_{1e} = 8 \Omega$  and verify the answers as,

- i) + 1.652 %    ii) -0.551 %**

**Example 17:** A 230/460 V transformer has a primary resistance of  $0.2 \Omega$  and the corresponding values for the secondary are  $0.75 \Omega$  and  $1.8 \Omega$  respectively. Find the secondary terminal voltages when supplying 10 A at 0.8 p.f lagging and its voltage regulation.

**Solution :** We have the data  $K = 230/460 = 0.5$ ,  $R_1 = 0.2 \Omega$ ,  $X_1 = 0.5 \Omega$   
 $R_2 = 0.75 \Omega$ ,  $X_2 = 1.8 \Omega$ ,  $I_2 = 10 \text{ A}$ ,  $\cos\phi = 0.8 \text{ lag}$

Since we have the current on the load side we will find out the equivalent resistance/reactance on secondary side.

$$R_{2e} = R_2 + R_1/K^2 = 0.75 + (2)^2 \times 0.2 = 1.55 \Omega$$

$$X_{2e} = X_2 + X_1/K^2 = 1.8 + (2)^2 \times 0.5 = 3.8 \Omega$$

$\therefore$  Voltage drop =  $I_2 [R_{2e} \cos\phi + X_{2e} \sin\phi]$   
 $= 10 [1.55 \times 0.8 + 3.8 \times 0.6] = 35.2 \text{ V}$

$\therefore$   $V_2$  (terminal) =  $V_2 - \text{drop} = 460 - 35.2 = \mathbf{424.8 \text{ V}}$

$\therefore$   $\% \text{ Reg} = \frac{\text{Voltage drop}}{V_2} \times 100 = \frac{35.2}{460} \times 100 = + \mathbf{7.652 \%}$

**Example 18 :** 1 kV/2kV transformer has 750 W hysteresis losses and 250 W eddy current losses. When the applied voltage is doubled and frequency is halved, find

the new losses.

**Solution :** The hysteresis loss is given by,

$$P_h \propto B_m^{1.6} f$$

The eddy current loss is given by,

$$P_e \propto B_m^2 f^2$$

But  $B_m \propto \frac{V}{f}$

$$\therefore P_h \propto \left(\frac{V}{f}\right)^{1.6} \times f \text{ and } P_e \propto \left(\frac{V}{f}\right)^2 \times f^2$$

From this the ratio of hysteresis losses is given by :

$$\frac{P_{h1}}{P_{h2}} = \left(\frac{V_1}{V_2}\right)^{1.6} \times \left(\frac{f_1}{f_2}\right)^{1.6} \times \frac{f_1}{f_2}$$

and the ratio of eddy current losses is given by:

$$\left(\frac{P_{e1}}{P_{e2}}\right) = \left(\frac{V_1}{V_2}\right)^2 \times \left(\frac{f_1}{f_2}\right)^2 \times \left(\frac{f_1}{f_2}\right)^2$$

Now voltage is doubled and frequency is halved and hence  $V_2 = 2V_1$  and  $f_2 = 0.5 f_1$

$\therefore$  The new hysteresis loss is given by:  $\frac{750}{P_{h2}} = (0.5)^{1.6} \times (0.5)^{1.6} \times 2$

and new eddy current loss is given by:  $\frac{250}{P_{e2}} = (0.5)^2 \times (0.5)^2 \times (2)^2$

From which we get  $P_{h2} = 3446.095 \text{ W}$  and  $P_{e2} = 1000 \text{ W}$

$\therefore$  Total new iron loss =  $P_{h2} + P_{e2} = 4446.095 \text{ W}$

**Example 19:** The flux in a magnetic core is alternating sinusoidally at a frequency of 600 Hz. The maximum flux density is 2 Tesla. The eddy current loss is 15 W. Find the eddy current loss in the core if the frequency is raised to 800 Hz and the maximum flux density is reduced to 1.5 Tesla.

**Solution :** The eddy current loss is given by:  $P_e = K_e B_m^2 f^2$

At  $f_1 = 600 \text{ Hz}$ ,  $B_{m1} = 2 \text{ T}$ ,  $P_{e1} = 15 \text{ W}$

$\therefore 15 = K_e \times (2)^2 \times (600)^2$

From which we get  $K_e = 15/(4 \times 360000) = 1.04166 \times 10^{-5}$

Then at  $f_2 = 800$  Hz and  $B_m = 1.5$  T we get

$$P_{e2} = 1.04166 \times 10^{-5} \times (1.5)^2 \times (800)^2 = \mathbf{15 \text{ W}}$$

The eddy current loss remains same.

**Example 20 :** A 2300/230 V, 50 Hz single phase transformer has the transformer parameters :  $R_1 = 0.3 \Omega$  ;  $R_{2e} = 0.295 \Omega$  ;  $X_1 = 0.375 \Omega$  ;  $X_{2e} = 0.685 \Omega$  ;  $R_{c1} = 4 \text{ k}\Omega$  ;  $X_{m1} = 1000 \Omega$ . The secondary load impedance  $Z_L = (0.4 + j0.3) \Omega$ . Find the voltage regulation, losses of the transformer using approximate equivalent circuit.

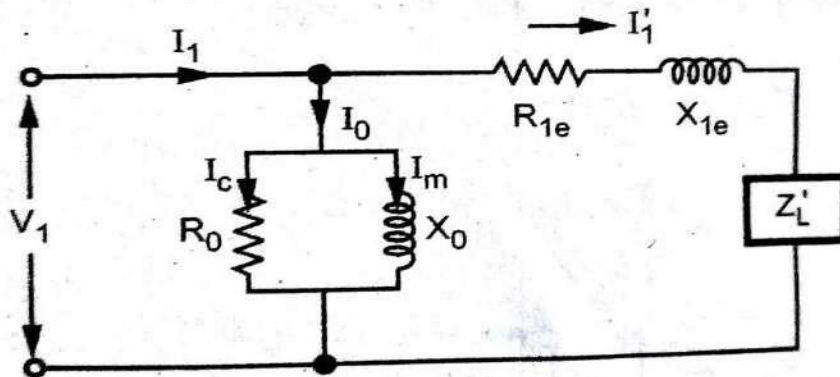
**Solution :**  $K = 2300/230 = 10$

$$R_{c1} = R_0 = 4 \text{ k}\Omega, X_{m1} = X_0 = 1000 \Omega$$

Let us find out equivalent circuit referred to primary because we are given the values of secondary referred to primary i.e.  $R_{2e}$  and  $X_{2e}$

$$R_{1e} = R_1 + R_{2e} = 0.595 \Omega,$$

$$X_{1e} = X_1 + X_{2e} = 1.06 \Omega$$



**Fig. 4.15.2**

Because we have taken equivalent circuit referred to primary. Let us find out the value of load impedance referred to primary

$$Z_{Le} (Z_L') = Z_L \times K^2 = Z_L \times 10^2 = (0.4 + j0.3) = 40 + j30 \Omega$$

Total equivalent impedance referred to primary  $Z_{1e} = (40+0.595)+j(30+1.06)= (40.595 + j31.06) \Omega$

Since we have taken primary as reference  $V_1 = 2300 \angle 0^\circ$  V will be the reference voltage.

If

$$\begin{aligned} I_1' &= \frac{V_1}{R_{1e} + jX_{1e} + Z_L} \\ &= 2300 \angle 0^\circ / (40.595 + j31.06) \\ &= 2300 \angle 0^\circ / 51.114 \angle 37.42^\circ = 45 \angle -37.42^\circ \text{ A} \\ &= 35.739 - j 27.344 \text{ A} \\ I_c &= \frac{V_1}{R_0} = \frac{2300}{4 \times 10^3} = 0.575 \text{ A}, I_m = \frac{V_1}{X_0} = 2.3 \text{ A} \end{aligned}$$

$\therefore I_0 = I_c - j I_m = 0.575 - j 2.3 \text{ A}$

$\therefore I_1 = I_0 + I_1' = 36.314 - j 29.644 \text{ A}$   
 $= 46.8772 \angle -39.22^\circ \text{ A}$

Core loss =  $P_i = I_c^2 R_c = 1322.5 \text{ W}$

Copper loss =  $(I_1')^2 R_{1e} = 45^2 \times 0.595 = 1204.875 \text{ W}$

Secondary voltage =  $I_2 Z_L = I_1' \times K \times Z_L$

$= 45 \times 10 \sqrt{(0.4)^2 + (0.3)^2} = 45 \times 10 = 225 \text{ V}$

$\therefore \% \text{ Reg} = \frac{230 - 225}{225} \times 100 = 2.22 \%$

**Example 21:** A single phase transformer has 500 turns on the primary and 40 turns on the secondary winding. The mean length of the magnetic path in the iron core is 150 cm and the joints are equivalent to an air gap of 0.1 mm. When a potential difference of 3000 V is applied to the primary, maximum flux density is  $1.2 \text{ Wb/m}^2$ . Calculate:

i) The cross-sectional area of the core ii) No-load secondary voltages.

iii) The no load current drawn by the primary iv) Power factor on no-load

Given that AT/cm for a flux density of  $1.2 \text{ Wb/m}^2$  in iron to be 5, the corresponding iron loss to be 2 watt/kg at 50 Hz, the density of iron as 7.8

gram/cm<sup>2</sup> and  $\mu_0 = 4\pi \times 10^{-7}$

**Solution :**  $E_1 = 3000$  V,  $B_m = 1.2$  Wb/m<sup>2</sup>,  $N_1 = 500$ ,  $f = 50$  Hz, length of the magnetic path in iron  $l_i = 150$ cm,

length of the magnetic path in air gap =  $l_g = 0.1$  mm =  $0.1 \times 10^{-3}$  mtrs

$$\therefore E_1 = 4.44 f \Phi_m N_1$$

$$\text{i.e. } \Phi_m = \frac{3000}{4.44 \times 50 \times 500} = 0.027 \text{ Wb}$$

$$\text{i) } B_m = \frac{\Phi_m}{a} \text{ i.e. } a = \frac{0.027}{1.2} = 0.0225 \text{ m}^2$$

$$= \mathbf{225 \text{ cm}^2}$$

$$\text{ii) } \frac{E_2}{E_1} = \frac{N_2}{N_1} \text{ i.e. } E_2 = \text{No load secondary voltage} = 3000 \times \frac{40}{500} = \mathbf{240}$$

V

iii) AT/cm = 5 for iron path

$$\text{AT} = 5 \times l_i = 5 \times 150 = 750$$

$$\text{AT for air gap} = H \times l_g = \frac{B}{\mu_0} \times l_g = \frac{1.2}{4\pi \times 10^{-7}} \times 0.1 \times 10^{-3} = 95.5$$

$$\therefore \text{Total AT for given } B_m = 750 + 95.5 = 845.5 = N_1 I_m$$

$$\therefore I_m = \frac{845.5}{500} = \mathbf{1.691 \text{ A}} \quad \text{..... Since } B_m \text{ (Maximum value) is used}$$

we get  $I_m$  (max)

$$\therefore I_m(\text{rms}) = \frac{I_m(\text{max})}{\sqrt{2}} = \frac{1.691}{\sqrt{2}} = 1.196 \text{ A} \quad \text{....Magnetizing current}$$

$$\text{Volume of iron} = l_i \times a = 150 \times 225 = 33750 \text{ cm}^3$$

$$\text{Density of iron} = 7.8 \text{ gm/cm}^3$$

$$\therefore \text{Mass of iron} = \text{Volume} \times \text{Density} = 33750 \times 7.8 = 263250 \text{ gm}$$

$$\text{Iron loss} = 2 \text{ W/kg i.e. Total iron loss} = 263250 \times 10^{-3} \times 2 = 526.5 \text{ W}$$

$$\therefore I_c = \text{Core loss component} = \frac{\text{Total iron loss}}{E_1} = \frac{526.5}{3000} = 0.176 \text{ A}$$

(Core loss component is working component which is in phase with the applied voltage.

Hence  $I_c = \text{Power/Voltage}$ )

$$\therefore I_0 = \sqrt{I_c^2 + I_m^2} = \sqrt{(0.176)^2 + (1.196)^2} = \mathbf{1.208 \text{ A}}$$

$$\text{iv) } \cos \phi_0 = \frac{I_c}{I_0} = \frac{0.176}{1.208} = \mathbf{0.1457 \text{ lagging.}}$$

**Additional Problems:**

**Example 1 :** A 4 kVA, 200/400 V, 50 Hz, single phase transformer has equivalent resistance referred to primary as 0.15 Ω. Calculate,

- i) The total copper losses on full load.
- ii) The efficiency while supplying full load at 0.9 p.f. lagging.
- iii) The efficiency while supplying half load at 0.8 p.f. leading.

Assume total iron losses equal to 60 W.

**Solution :** The given values are,

$$V_1 = 200 \text{ V}, V_2 = 400 \text{ V}, S = 4 \text{ kVA}, R_{1e} = 0.15 \text{ } \Omega, p_i = 60 \text{ W}$$

$$K = V_1/V_2 = 200/400 = 0.5$$

From the problem we can see that we have to find out the efficiency while **supplying** in different conditions. Hence we have to get the parameters reflected to secondary side. So we have to calculate

$$R_{2e} = R_{1e}/K^2 = 0.15/(0.5)^2 = 0.6 \text{ } \Omega$$

$$(I_2) \text{ F.L.} = \frac{kVA}{V_2} = \frac{4 \times 10^3}{400} = 10 \text{ A}$$

i) Total copper losses on full load,

$$(P_{cu}) \text{ F.L.} = [(I_2) \text{ F.L.}]^2 R_{2e} = (10)^2 \times 0.6 = \mathbf{60 \text{ W}}$$

ii)  $\eta$  @  $\cos \phi = 0.9$  lagging and full load

$$\therefore \% \eta_{FL} = \frac{V A \text{ rating } \cos \phi}{V A \text{ rating } \cos \phi + P_i + (P_{cu})_{F.L.}} \times 100$$

$$\therefore \% \eta_{FL} = \frac{4 \times 10^3 \times 0.9}{4 \times 10^3 \times 0.9 + 60 + 60} \times 100 = \mathbf{96.77\%}$$

iii)  $\cos \phi = 0.8$  leading half load i.e.  $n = 0.5$

$$(p_{cu}) \text{ H.L.} = n^2 \times (P_{cu})_{F.L.} = (0.5)^2 \times 60 = 15 \text{ W}$$

$$\therefore \% \eta_{HL} = \frac{n \times (V A \text{ rating}) \cos \phi}{n \times (V A \text{ rating}) \cos \phi + P_i + (P_{cu})_{H.L.}} \times 100$$

$$= \frac{0.5 \times 4 \times 10^3 \times 0.8}{0.5 \times 4 \times 10^3 \times 0.8 + 60 + 15} \times 100 = \mathbf{95.52\%}$$

**Example 2:** The maximum efficiency of a 500 kVA, 3300/500 Volts, 50Hz, single-phase transformer is 97 % and occurs at 3/4<sup>th</sup> full load with unity p.f. If the impedance is 10 %, calculate the regulation at full load and 0.8 p.f. lagging.

**Solution :**  $\eta_{\max} = 97\%$  occurs at  $n = \frac{3}{4} = 0.75$  and unity p.f

$$\% \eta_{\max} = \frac{nVA \cos \phi}{nVA \cos \phi + 2P_i} \times 100 \quad (\text{since at } \eta_{\max} P_i = P_{cu})$$

i.e. 
$$0.97 = \frac{0.75 \times 500 \times 10^3 \times 1}{0.75 \times 500 \times 10^3 \times 1 + 2P_i}$$

From which we get  $P_i = \text{Iron losses} = \mathbf{5798.969 \text{ W}}$

Also  $P_{cu @ n=0.75} = 5798.969$  as  $\eta_{\max}$  occurs at 3/4<sup>th</sup> load

We know that  $P_{cu \text{ at any fractional load } n} = n^2 (P_{cu})_{F.L.}$

i.e. 
$$5789.969 = (0.75)^2 \times (P_{cu})_{F.L.}$$

From which  $(P_{cu})_{@F.L.} = \text{Full load copper loss} = 5789.969 / (0.75)^2 =$

**10309.278 W**

$$(I_2)_{F.L.} = \frac{VA \text{ rating}}{V_1} = \frac{500 \times 10^3}{3300} = 151.515 \text{ A}$$

We are given the % impedance as 10% which can be taken on either primary side or secondary side for calculating the regulation. Accordingly the % resistance and %reactance also are to be calculated on either primary side or secondary side. Let us calculate on primary side.

Now  $(P_{cu})_{F.L.} = [(I_1)_{F.L.}]^2 R_{le}$

i.e. 
$$R_{le} = \frac{10309.278}{(151.515)^2} = \mathbf{0.4491 \Omega}$$

$\%R_{le}$  (in terms of primary side equivalent resistance ,voltage and current) = 
$$\frac{I_1 R_{le}}{V_1} \times 100$$

$$= \frac{151.515 \times 0.4491}{3300} \times 100$$

$$= \mathbf{2.062 \%}$$

$$\% Z_{le} = 10 \% \quad \text{(Given data)}$$

$$\% X_{le} = \sqrt{(Z_{le})^2 - (R_{le})^2} = \sqrt{(10)^2 - (2.062)^2}$$

$$= 9.785 \%$$

But  $\% X_{le}$  (again in terms of primary side equivalent resistance ,voltage and current) =  $\frac{I_1 X_{le}}{V_1} \times 100$

i.e.  $9.785 = \frac{151.515 X_{le}}{3300} \times 100$  from which  $X_{le} = 9.785 \times 3300 / (151.515 \times 100)$

$$\% \text{ Regulation} = \frac{I_1 [R_{le} \cos \phi + X_{le} \sin \phi]}{V_1} \times 100$$

$$= \frac{151.515 [0.4491 \times 0.8 + 2.1311 \times 0.6]}{3300} \times 100 \quad \text{(Given Cos } \phi = 0.8 \text{ lag we get Sin } \phi = 0.6)$$

$$= 7.52 \%$$

**Example 3:** The maximum efficiency of a 50 kVA transformer is 97.4 % and occurs at 90 % of the full load at unity power factor. Calculate the efficiency of transformer at,

- i) Full load 0.8 power factor lagging
- ii) Half load 0.9 power factor.

**Solution :** We have the data :

Rating: 50 kVA,  $\eta_{\max} = 97.4 \%$ ,  $n = 0.9$  for  $\eta_{\max}$  at  $\cos \phi = 1$ .

From this data using the relation  $\% \eta_{\max} = \frac{nVA \text{ rating} \times \cos \phi}{nVA \text{ rating} \times \cos \phi + 2P_i} \times 100$

(Since  $P_i = P_{cu}$  for  $\eta_{\max}$ )

we have  $97.4 = \frac{0.9 \times 50 \times 10^3 \times 1}{0.9 \times 50 \times 10^3 \times 1 + 2P_i} \times 100$

From which we get  $P_i = 600.616 \text{ W}$

We know that efficiency at Full load and any p.f  $\cos \phi$  is given by :

$$\% \eta_{FL} = \frac{VA \text{ rating} \times \cos \phi}{VA \text{ rating} \times \cos \phi + P_i + (P_{Cu})_{F.L.}} \times 100$$

We have the data of all the parameters in this equation except  $P_{Cu @ FL}$  which can be found from the relation

$$kVA \text{ for } \eta_{\max} = kVA \text{ rating} \times \sqrt{\frac{P_i}{(P_{Cu})_{F.L.}}}$$

$$\therefore 0.9 \times [50 \times 10^3] = [50 \times 10^3] \times \sqrt{\frac{600.616}{(P_{Cu})_{F.L.}}}$$

From which  $P_{Cu @ FL} = \text{Full load copper loss} = 741.5012 \text{ W}$

i) Efficiency at Full load and  $\cos \phi = 0.8 \text{ lag}$  :

$$\begin{aligned} \% \eta_{FL} &= \frac{VA \text{ rating} \times \cos \phi}{VA \text{ rating} \times \cos \phi + P_i + (P_{Cu})_{F.L.}} \times 100 \\ &= \frac{50 \times 10^3 \times 0.8}{50 \times 10^3 \times 0.8 + 600.616 + 741.5012} \times 100 \\ &= \mathbf{96.7536 \%} \end{aligned}$$

ii) Half load,  $n = 0.5, \cos \phi = 0.9$

$$\begin{aligned} \% \eta_{HL} &= \frac{\eta VA \text{ rating} \times \cos \phi}{\eta VA \text{ rating} \times \cos \phi + P_i + n^2 (P_{Cu})_{F.L.}} \times 100 \\ &= \frac{0.5 \times 50 \times 10^3 \times 0.8}{0.5 \times 50 \times 10^3 \times 0.8 + 600.616 + (0.5)^2 \times 741.5012} \times 100 \\ &= \mathbf{96.2186 \%} \end{aligned}$$

**Example 4:** The efficiency of a 1000 kVA, 110/200 V, 50 Hz single phase transformer is 98.5 % at half full load at 0.8 power factor leading and 98 % at full load, UPF. Determine

i) Iron loss ii) Copper loss iii) Maximum efficiency at UPF.

**Solution :** We have the data :

Rating : 1000 kVA,  $V_1 = 110 \text{ V}, V_2 = 200$  ,  $\eta_{@1/2FL} \text{ and } \cos \phi = 0.8 = 98.5 \%$ ,

Using this data in the basic efficiency relation

Electrical Machines -1 (EM-1): Lecture Notes: (Dr.Karimulla P Sk)  
Unit 4: Single Phase Transformers

---

$$\% \eta_{HL} = \frac{n \times VA \text{ rating} \times \cos \phi}{n \times VA \text{ rating} \times \cos \phi + P_i + n^2 (P_{cu})_{F.L.}} \times 100$$

we get :

$$\% \eta_{HL} = \frac{n \times VA \text{ rating} \times \cos \phi}{n \times VA \text{ rating} \times \cos \phi + P_i + n^2 (P_{cu})_{F.L.}} \times 100 \quad (\text{in this case } n =$$

0.5)

$$98.5 = \frac{0.5 \times 1000 \times 10^3 \times 0.8}{0.5 \times 1000 \times 10^3 \times 0.8 + P_i + (0.5)^2 \times (P_{cu})_{F.L.}} \times 100$$

From which we get  $P_i + 0.25 (P_{cu})_{F.L.} = 6091.3705$

.....(1)

Similarly in the second case:

$$\% \eta_{@FL} = \frac{\eta_{@FL \text{ and } \cos \phi = 1} \times n \times VA \text{ rating} \times \cos \phi}{n \times VA \text{ rating} \times \cos \phi + P_i + n^2 (P_{cu})_{F.L.}} \times 100 \quad (\text{in this case } n$$

= 1)

$$98 = \frac{1000 \times 10^3 \times 1}{1000 \times 10^3 \times 1 + P_i + (P_{cu})_{F.L.}} \times 100$$

From which we get:  $P_i + (P_{cu})_{F.L.} = 20408.1632$

.....(2)

Solving equations (1) and (2),

- i)  $P_i = 1319.1062 \text{ W}$  .....Iron loss
- ii)  $(P_{cu})_{F.L.} = 19089.0569 \text{ W}$  .....Full load copper loss
- iii) For  $\eta_{max}$  with  $\cos \phi = 1$ , we have to get the fractional kVA for maximum efficiency which is given by

$$\begin{aligned} \text{kVA for } \eta_{max} &= \text{kVA rating} \times \sqrt{\frac{P_i}{(P_{cu})_{F.L.}}} \\ &= 1000 \times \sqrt{\frac{1319.1062}{19089.0569}} = 262.874 \text{ kVA} \end{aligned}$$

At  $\eta_{max}$ ,  $P_i = P_{cu} = 1319.1062 \text{ W}$

$$\therefore \% \eta_{max} = \frac{[VA \text{ for } \eta_{max}] \times \cos \phi}{[VA \text{ for } \eta_{max}] \times \cos \phi + 2P_i} \times 100$$

$$= \frac{262.874 \times 10^3 \times 1}{262.874 \times 10^3 \times 1 + 2 \times 1319.1062} \times 100$$

$$= \mathbf{99 \%}$$

**Example 5:** A 100 Kva, 50 Hz, 440/11 Kv, single phase transformer has an efficiency of 98.5 % when supplying full load current at 0.8 p.f. lagging and an efficiency of 99 % when supplying half load current at unity power factor. Find the core losses and copper losses corresponding to full load current. At what value of load current will the maximum efficiency be attained?

**Solution :** Refer example 4 for the procedure and verify the answers :

$P_i = \mathbf{267.309 W}$ ,  $P_{cu}(FL) = \mathbf{950.964 W}$ .

$$I_2 (FL) = \frac{VA}{V_2} = \frac{100 \times 10^3}{11 \times 10^3} = 9.0909 \text{ A}$$

$$I_{2m} = I_2 (FL) \times \sqrt{\frac{P_i}{P_{cu}(FL)}}$$

$$= \mathbf{4.822 A}$$

**Example 6:** A 25 kVA, 2000/200 V, 50 Hz transformer has maximum efficiency at 75 % of full load. Its per unit resistance and impedance are 0.012 and 0.05 respectively. Determine its efficiency and voltage regulation at half of the full load and 0.8 p.f lagging.

**Solution :** kVA for  $\eta_{max} = 0.75 \times \text{kVA} = 0.75 \times 25 = 18.75 \text{ kVA}$

$$\text{Per unit R} = 0.012 = \frac{I_1 R_{1e}}{V_1}$$

Where  $I_1$  = full load current

$$I_1(FL) = \frac{VA}{V_1} = \frac{25 \times 10^3}{2000} = 12.5 \text{ A}$$

$$\therefore 0.012 = \frac{12.5 R_{1e}}{2000} \quad \text{i.e. } R_{1e} = \mathbf{1.92 \Omega}$$

$$\text{Per unit } X = 0.05 = \frac{I_1 X_{1e}}{V_1}$$

$$\text{i.e. } X_{1e} = \frac{0.05 \times 2000}{\frac{12.5}{1}} = \mathbf{8 \Omega}$$

$$\text{At half full load, } I_1(\text{HL}) = \frac{1}{2} \times 12.5 = 6.25 \text{ A}$$

$$\cos \phi = 0.8, \sin \phi = 0.6$$

$$\% \text{ Reg} = \frac{I_1(\text{HL})[R_{1e} \cos \phi + X_{1e} \sin \phi]}{V_1} \times 100 = 1.98 \%$$

$$\text{kVA for } \eta_{\max} = \text{kVA} \times \sqrt{\frac{P_i}{P_{cu}(\text{FL})}}$$

$$\therefore 18.75 = 25 \times \sqrt{\frac{P_i}{P_{cu}(\text{FL})}} \quad \text{i.e. } \frac{P_i}{P_{cu}(\text{FL})} = 0.5625$$

$$P_{cu}(\text{FL}) = [I_1(\text{FL})^2 R_{1e}] = 12.5^2 \times 1.92 = 300 \text{ W}$$

$$P_i = 0.5625 \times 300 = \mathbf{168.75 \text{ W}}$$

$$\begin{aligned} \therefore \% \eta_{\text{HL}} &= \frac{0.5 \times VA \cos \phi}{0.5 \times VA \cos \phi + P_i + [(0.5)^2 P_{cu}(\text{FL})]} \times 100 \quad (n = 0.5) \\ &= \frac{0.5 \times 25 \times 10^3 \times 0.8}{0.5 \times 25 \times 10^3 \times 0.8 + 168.75 + (0.5^2 \times 300)} \times 100 \\ &= \mathbf{97.62 \%} \end{aligned}$$

**Example 7:** A 250/500 V, 50 Hz single phase transformer has the following equivalent circuit parameters:  $R_{t1} = 0.715 \Omega$  ;  $X_{t1} = 0.375 \Omega$  ;  $R_{c1} = 780 \Omega$  ;  $X_{m1} = 265 \Omega$

Determine the voltage regulation, losses and efficiency of the transformer when it is **delivering** 6 kVA at 0.8 pf **leading** and at rated voltage.

**Solution :**

$$R_{t1} = R_{1e} = 0.175 \Omega, X_{t1} = X_{1e} = 0.375 \Omega$$

$$R_{c1} = R_o = 780 \Omega, X_{m1} = X_o = 265 \Omega$$

Because the kVA of 6 given is while **delivering** we have to calculate using the secondary equivalent parameters. So the given primary parameters are to be referred to secondary.

$$K = \frac{250}{500} = 0.5, R_{2e} = R_{1e} / K^2 = 0.175 / (0.5)^2 = 0.7 \Omega$$

$$X_{2e} = X_{1e} / K^2 = 0.375 / (0.5)^2 = 1.5 \Omega$$

$$I_2 = \frac{VA}{V_2} = \frac{6 \times 10^3}{500} = 12 \text{ A} \quad (\text{Since } 6 \text{ kVA given is on load side})$$

load side )

$$\therefore \% \text{ Reg} = \frac{I_2(R_{2e} \cos \phi - X_{2e} \sin \phi)}{V_2} \times 100 \quad (\text{negative sign is because } \cos \phi = 0.8 \text{ is leading})$$

Cos  $\phi = 0.8$  is leading)

$$= \frac{12(0.7 \times 0.8 - 1.5 \times 0.6)}{500} \times 100 = -0.816\%$$

$$R = \frac{V_1}{I} \quad \text{i.e. } I = \frac{500 V_1}{250} = 0.3205 \text{ A}$$

$$X = \frac{V_1}{I} \quad \text{i.e. } I = \frac{V_1}{X_0} = \frac{250}{265} = 0.9434 \text{ A}$$

$$\therefore \tan \phi = \frac{I_m}{I_c} = 2.9435, \phi = 71.2350$$

$$I_0 = \sqrt{I_c^2 + I_m^2} = 0.9963 \text{ A}$$

$$\therefore P_i = \text{Iron losses} = V_1 I_0 \cos \phi_0 = \mathbf{80.1242 \text{ W}}$$

$$P_{cu} = \text{Copper losses} = I_2^2 R_{2e} = 12^2 \times 0.7 = \mathbf{100.8 \text{ W}}$$

$$\therefore \% \eta = \frac{VA \cos \phi}{VA \cos \phi + P_i + P_{cu}} \times 100$$

$$= \frac{6 \times 10^3 \times 0.8}{6 \times 10^3 \times 0.8 + 80.1242 + 100.8} \times 100$$

$$= \mathbf{96.36 \%}$$

**Example 8:** A 25 kVA, 1-phase transformer, 2200/220 V, has a primary Resistance of 2.0  $\Omega$  and a secondary resistance of 0.05  $\Omega$ . Find the Equivalent secondary resistance and the full load efficiency at 0.8 p.f, if the iron loss of the transformer is 75 % of the full load copper loss.

**Solution :** We have the data :  $R_1 = 2 \Omega$   $R_2 = 0.05 \Omega$ ,  $K = 2200/220 = 10$

$\therefore$  Equivalent secondary resistance =  $R_{2e} = R_2 + R_{1e} = 0.05 + 2/(10)^2 = \mathbf{0.07 \Omega}$

$$I_2 (\text{FL}) = \frac{25 \times 10^3}{220} = 113.6363 \text{ A,}$$

$$P_{cu}(FL) = [I_2(FL)]^2 R_{2e}$$

$$\therefore P_{cu} (FL) = 903.925 \text{ W}, P_i = 0.75 \times 903.925 = 677.944 \text{ W}$$

$$\therefore \% \eta = \frac{25 \times 10^3 \times 0.8}{25 \times 10^3 \times 0.8 + 903.925 + 677.944} \times 100$$

$$= \mathbf{92.67 \%}$$

**Example 9:** The full load copper and iron losses of a 15 kVA single phase transformer are 320 W and 200 W respectively. Calculate the efficiency of the transformer at unity power factor at full load and half load.

**Solution :** ARting = 15 kVA,  $P_i = 200 \text{ W}$ ,  $P_{cu} (FL) = 320 \text{ W}$

$$\% \eta_{FL} = \frac{VA \cos \phi}{VA \cos \phi + P_i + P_{cu} (FL)} \times 100$$

We have to find out both FL and HFL efficiency with  $\cos \phi = 1$

$$\text{Hence } \% \eta_{FL} = \frac{15 \times 10^3 \times 1}{15 \times 10^3 \times 1 + 200 + 320} \times 100 = \mathbf{96.649 \%}$$

$$\% \eta_{HL} = \frac{nVA \cos \phi}{nVA \cos \phi + P_i + n^2 P_{cu} (FL)} \times 100 \quad (n = 0.5, \text{ and } \cos \phi = 1)$$

$$= \frac{0.5 \times 15 \times 10^3 \times 1}{0.5 \times 15 \times 10^3 \times 1 + 200 + [(0.5)^2 \times 320]} \times 100 = \mathbf{96.401 \%}$$

## **UNIT – V**

### **TESTING OF TRANSFORMERS AND THREE-PHASE TRANSFORMERS**

#### **CONTENTS:**

- **OC and SC tests**
- **Sumpner's test**
- **Predetermination of efficiency and regulation**
- **Separation of losses test**
- **Parallel operation with equal and unequal voltage ratios**
- **Auto transformers- equivalent circuit**
- **Comparison with two winding Transformers**
- **Poly phase Transformers**
- **Poly phase Connections**
- **Y/Y, Y/ $\Delta$ ,  $\Delta$ / $\Delta$ ,  $\Delta$ /Y open  $\Delta$  connections**

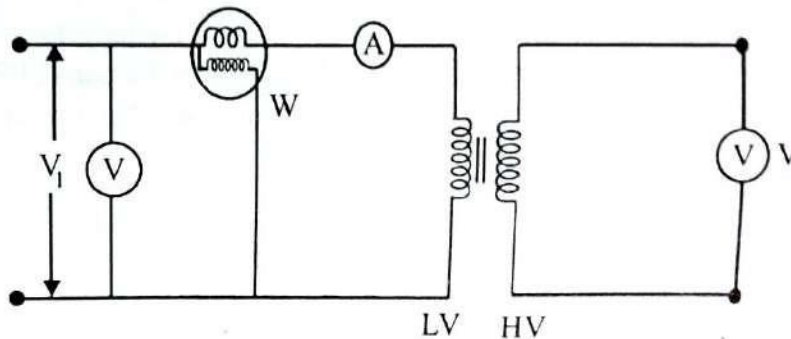
  - **Illustrative examples**

### Open Circuit (OC) and Short Circuit (SC) tests:

They are conducted on the transformer to find out the transformer losses and from them to determine the circuit constants that are used to represent the equivalent circuit. From these parameters the transformer efficiency and regulation can also be calculated. These tests are conducted without actually loading the transformer to its full load and hence the power consumed during the test is very small as compared to its full load (rated) power.

#### Open Circuit or No Load test:

The test setup to conduct the OC test is shown in the figure below.



**Fig: Test Setup to conduct the Open Circuit or No Load test**

Low voltage side is designated as Primary and High voltage side is designated as Secondary. Voltmeter  $V_1$ , Ammeter  $A$  and wattmeter  $W$  are connected in the primary as shown. Voltmeter  $V_2$  is connected in the open circuited Secondary. Since the secondary is open circuited a small value of no load current called  $I_0$  flows in the primary and this is measured by the ammeter  $A$ . The power loss in the transformer is due to core losses and a very small  $I^2R$  loss in the primary. There is no  $I^2R$  loss in the secondary since the secondary is open and there is no secondary current. Since  $I^2R$  loss in the primary also is very small the no load current is very small (usually 2 to 5 % of the full load current). The core loss is dependent on the flux which in turn depends on the applied voltage. Since full

rated voltage is applied to the primary in this test, full rated flux will be set up and the full Core losses will be present. And these core losses will be constant at all loads. Since the  $I^2R$  loss in the primary is very small compared to the core losses, they can be ignored and the full power consumed in the primary as read by wattmeter  $W$  can be regarded as the core losses. With this understanding the readings of the various meters in the OC test are as follows.

- Ammeter reading : No load current  $I_0$
- Volt meter reading : Applied rated Primary voltage  $V_1$
- Wattmeter reading : Input power totally consumed as Core losses  $P_{CL}$

From these measurements the parameters  $R_C$  and  $X_M$  shown in the equivalent circuit can be computed as shown below.

$$\text{No load power factor} : \cos \theta = P_{CL} / V_1 \cdot I_0 \text{ (Since Input power} = P_{CL} = V_1 \cdot I_0 \cdot \cos \theta \text{)}$$

We already know from the equivalent circuit description that  $I_{CL}$  is the core loss current corresponding to  $R_C$  and  $I_M$  is the magnetizing current corresponding to  $X_M$  and in terms of the present measurements are given by :

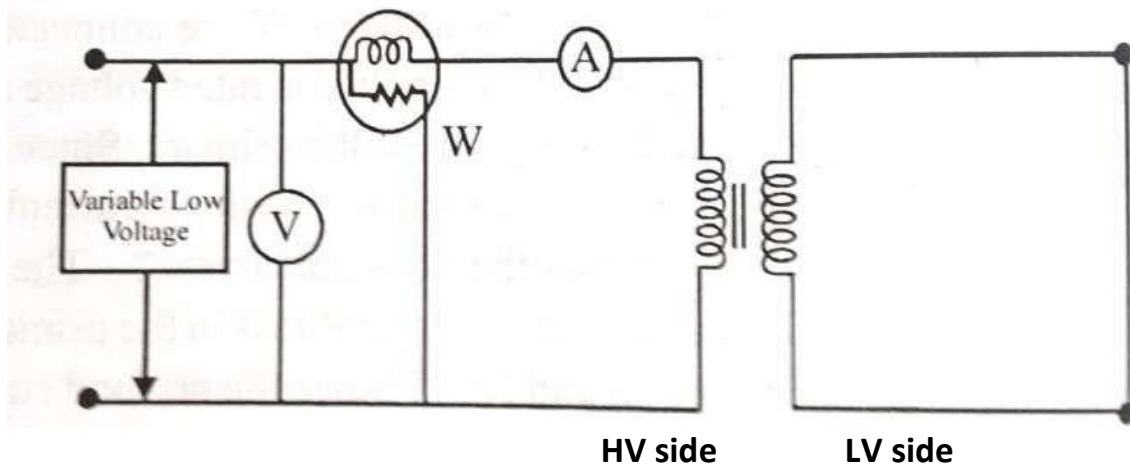
$$I_{CL} = I_0 \cdot \cos \theta \text{ and } I_M = I_0 \cdot \sin \theta$$

Therefore  $R_C$  and  $X_M$  are given by :

$$R_C = V_1 / I_{CL} \text{ and } X_M = V_1 / I_M$$

### Short Circuit test:

The test setup to conduct the SC test is shown in the figure below.



**Fig: Test Setup to conduct the Short Circuit test**

In this test high voltage side is designated as Primary (where an input voltage is applied) and the low voltage side is designated as Secondary which is usually short-circuited by a thick conductor (or sometimes through an Ammeter to read additionally the secondary load current). Voltmeter **V**, Ammeter **A** and wattmeter **W** are connected in the primary as shown.

A very low voltage through a Variac (Variable auto transformer) is applied to the primary gradually from zero Volts to about 5 to 10 % of the rated primary value till the primary current is just equal to the rated primary current. Since the secondary is short-circuited we will get rated primary current with a low value of voltage itself. Since the applied voltage is very low the flux produced is also very low. Hence the core losses also will be low and can be neglected. Now since the rated currents are flowing in both the Primary and the secondary the input power will be mostly consumed as copper losses. Since the secondary is short-circuited the secondary voltage is zero and the entire input voltage  $V_1$  drops in the total equivalent impedance  $Z_{EQP}$  of the transformer reflected to the primary.

$$\text{i.e. } V_1 = I_{PSC} \cdot Z_{EQP}$$

The readings in the short circuit test are as follows:

- Ammeter reading : Primary current  $I_{PSC}$  ( with secondary short circuited )
- Volt meter reading : Applied Primary voltage  $V_1$
- Wattmeter reading : Input power totally consumed as Copper losses  $P_{CUL}$

With this notation the power factor  $\cos \theta_{SC}$  in this test is given by:

$$P_{CUL} = V_1 \cdot I_{PSC} \cdot \cos \theta_{SC}$$

From the above readings and the governing equations we can calculate the Equivalent Resistance, Equivalent Impedance and Equivalent Reactance parameters of the transformer referred to the primary as below.

$$\text{Equivalent Resistance: } R_{EQP} = P_{CUL} / I_{PSC}^2$$

$$\text{Equivalent Impedance: } Z_{EQP} = V_1 / I_{PSC}$$

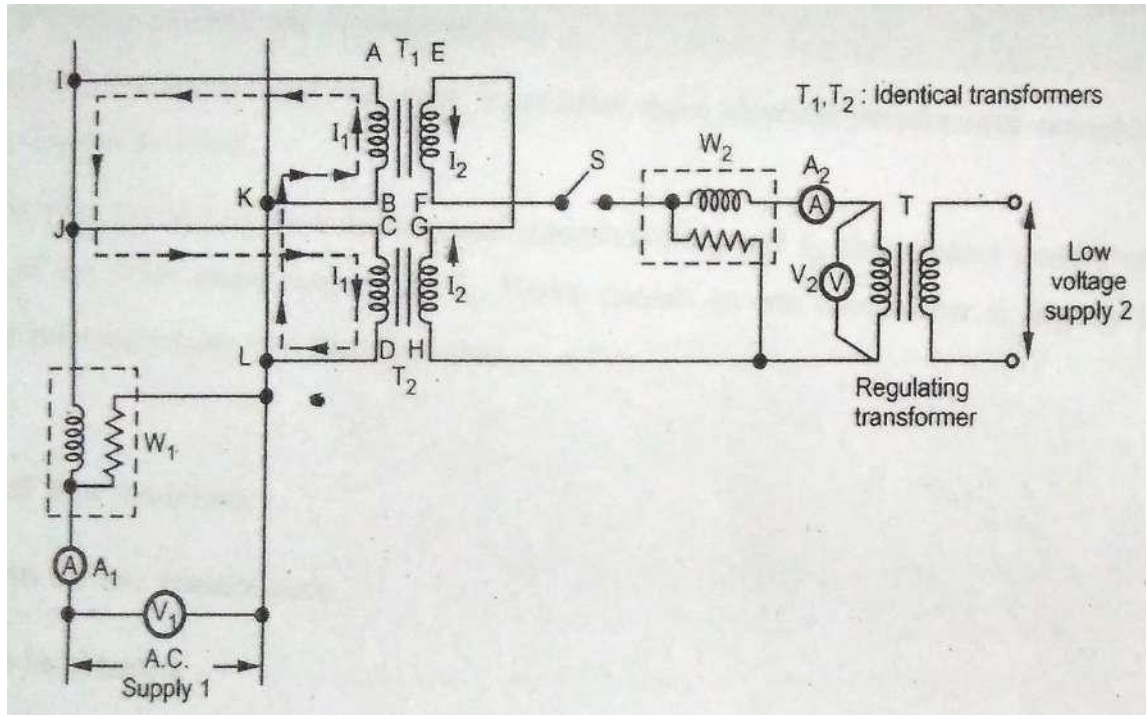
$$\text{Equivalent Reactance : } X_{EQP} = \sqrt{Z_{EQP}^2 - R_{EQP}^2}$$

### Sumpner's Test :

Sumpner's test or back to back test on transformer is another method for determining [transformer efficiency](#), voltage regulation and heating under loaded conditions. [Short circuit and open circuit tests on transformer](#) can give us parameters of [equivalent circuit of transformer](#), but they cannot help us in finding the heating information. Unlike O.C. and S.C. tests, actual loading is simulated in Sumpner's test. Thus the Sumpner's test gives more accurate results of regulation and efficiency than O.C. and S.C. tests.

Sumpner's test or back to back test can be employed only when two identical [transformers](#) are available. Both transformers are connected to supply such that one transformer is loaded on another. Primaries of the two identical

transformers are connected in parallel across a supply. Secondaries are connected in series opposition such that their e.m.f's are opposite to each other. Another low voltage supply is connected in series with secondaries to get the readings, as shown in the circuit diagram shown below.



**Figure : Sumpner's Test setup**

In the above diagram,  $T_1$  and  $T_2$  are identical transformers. Their secondaries are connected in voltage opposition, i.e. both the emf's  $E_{EF}$  and  $E_{GH}$  cancel each other, as transformers are identical. In this case, as per superposition theorem, no current flows through secondary. And thus the no load test is simulated. The current drawn from  $V_1$  is  $2I_0$ , where  $I_0$  is equal to no load current of each transformer. Thus, input power measured by wattmeter  $W_1$  is equal to iron losses of both the transformers.

i.e. Iron loss per transformer  $P_i = W_1/2$ .

Now, a small voltage  $V_2$  is injected into secondary with the help of a low voltage transformer. The voltage  $V_2$  is adjusted such that, the rated current  $I_2$  flows through the secondary. In this case, both primaries and secondaries carry rated current. Thus short circuit test is simulated and wattmeter  $W_2$  shows total full load copper losses of both transformers. i.e. Copper loss per transformer  $P_{Cu} = W_2/2$ .

From above test results, the **full load efficiency of each transformer** can be given as:

$$\% \text{ full load efficiency of each transformer} = \frac{\text{output}}{\text{output} + \frac{W_1}{2} + \frac{W_2}{2}} \times 100$$

And Output = kVA rating x Cos  $\theta$  (power factor)

### **Predetermination of efficiency and regulation:**

#### **Calculation of efficiency:**

We have obtained Iron losses  $P_i$  as the input power  $W_0$  from the OC test and then Full load ( or at any load ) copper losses  $P_{Cu}$  as the input power  $P_{sc}$  from the SC test.

Using this data we can predetermine efficiency and regulation at any load and power factor as shown below.

$$\% \eta \text{ at full (rated load)} = \left[ \frac{\text{Rated KVA} \times \cos \theta}{\text{Rated KVA} \times \cos \theta + P_i + P_{sc}} \right] \times 100$$



Then

$$\% \text{ Regulation (referred to primary)@FL} = \left[ \frac{I_P (R_{EQP} \times \cos \theta + X_{EQP} \times \sin \theta)}{V_P} \right] \times 100$$

$$\% \text{ Regulation (referred to secondary)@FL} = \left[ \frac{I_S (R_{EQS} \times \cos \theta + X_{EQS} \times \sin \theta)}{V_P/a} \right] \times 100$$

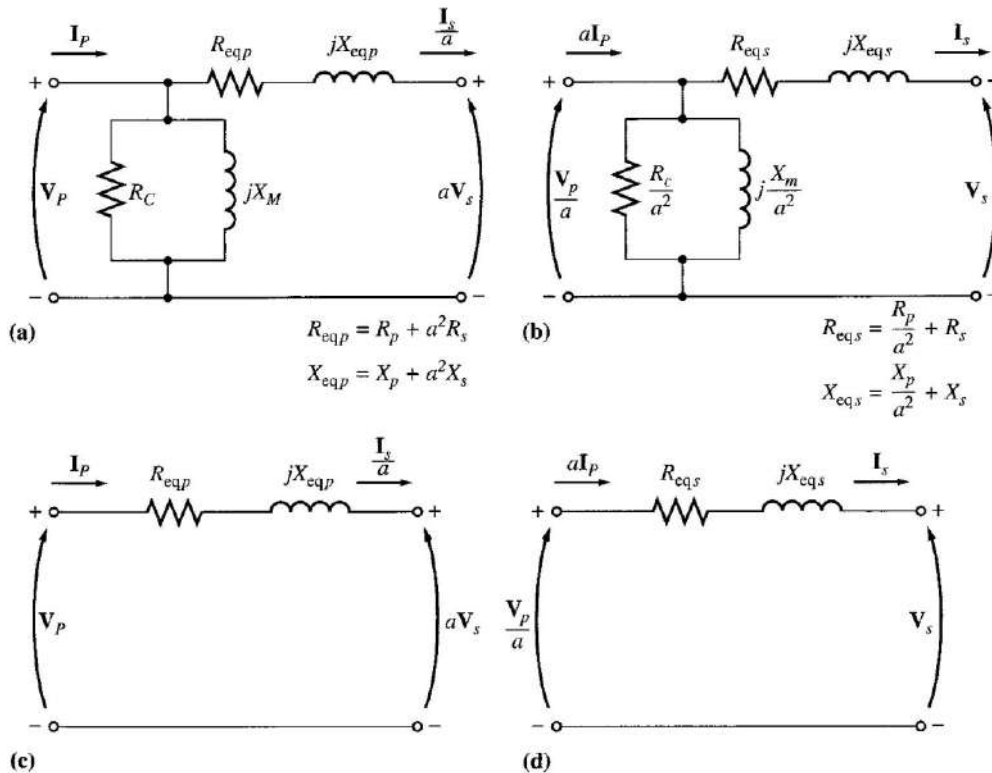
Where 'a' is the transformer turns ratio

Regulation at any other load can be calculated using the same relation as above except that the currents **I<sub>p</sub>** and **I<sub>s</sub>** are to be multiplied by the corresponding load fractions 'n' and the relations will become

$$\% \text{ Regulation (referred to primary)@any fractional FL} = \left[ \frac{n I_P (R_{EQP} \times \cos \theta + X_{EQP} \times \sin \theta)}{V_P} \right] \times 100$$

$$\% \text{ Regulation (referred to secondary)@any fractional FL} = \left[ \frac{n I_S (R_{EQS} \times \cos \theta + X_{EQS} \times \sin \theta)}{V_P/a} \right] \times 100$$

***Transformer equivalent circuits (Both Complete and simplified) referred to both Primary and secondary are appended below again for a quick reference :***



**Fig: Simplified equivalent circuits (a) Referred to the primary side (b) Referred to the secondary side**

**(c) With no excitation branch, referred to the primary side (d) With no excitation branch, referred to the secondary side**

**Parallel operation of transformers:**

**Need for Parallel operation of transformers:**

When the load outgrows the capacity of an existing transformer, it may be economical to install another one in parallel with it rather than replacing it with a single larger unit. Also, sometimes in a new installation, two units in parallel, though more expensive, may be preferred over a single unit for reasons of reliability—half the load can be supplied with one unit out. Further, the cost of maintaining a spare is less with two units in parallel. However, when spare units are maintained at a central location to serve transformer installations in a certain region, single-unit installations would be preferred. It is, therefore, seen that

parallel operation of the transformer is quite important and desirable under certain circumstances.

**Conditions for successful parallel operation of transformers:**

The satisfactory and successful operation of transformers connected in parallel on both sides requires that they fulfill the following conditions:

(i) The transformers must be connected properly as far as their polarities are concerned so that the net voltage around the local loop is zero. A wrong polarity connection results in a dead short circuit.

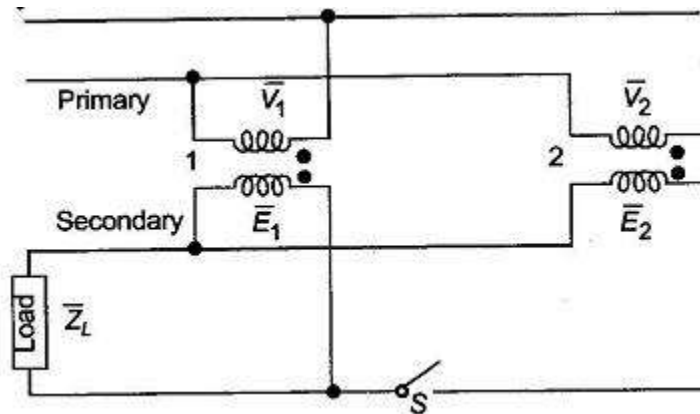
(ii) Three-phase transformers must have zero relative phase displacement on the secondary sides and must be connected in a proper phase sequence. Only the transformers of the same phase group can be paralleled. For example, Y/Y and Y/D transformers cannot be paralleled as their secondary voltages will have a phase difference of  $30^\circ$ . Transformers with  $+30^\circ$  and  $-30^\circ$  phase shift can, however, be paralleled by reversing the phase-sequence of one of them.

(iii) The transformers must have the same voltage-ratio to avoid no-load circulating current when transformers are in parallel on both primary and secondary sides. Since the leakage impedance is low, even a small voltage difference can give rise to considerable no-load circulating current and extra  $I^2R$  loss.

(iv) There should exist only a limited disparity in the per-unit impedances (on their own bases) of the transformers. The currents carried by two transformers (also their kVA loadings) are proportional to their ratings if their ohmic impedances (or their pu impedances on a common base) are inversely proportional to their ratings or their per unit impedances on their own ratings are equal. The ratio of equivalent leakage reactance to equivalent resistance should be the same for all the transformers. A difference in this ratio results in a divergence of the phase angle of the two currents, so that one transformer will be operating with a higher, and the other with a lower power factor than that of the total output; as a result, the given active load is not proportionally shared by them.

**Parallel Transformers on No-load:**

The parallel operation of transformers can be easily conceived on a per phase basis. Figure below shows two transformers paralleled on both sides with proper polarities but on no-load.

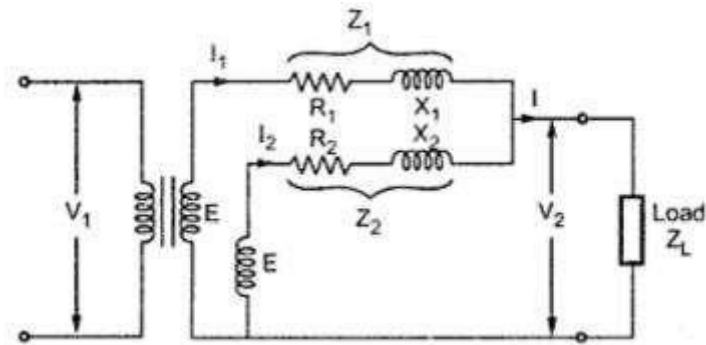


**Fig: Two transformers connected in parallel**

The primary voltages  $V_1$  and  $V_2$  are obviously equal. If the voltage-ratio of the two transformers are not identical, the secondary induced emf's,  $E_1$  and  $E_2$  though in phase will not be equal in magnitude and the difference  $(E_1 - E_2)$  will appear across the switch  $S$ . When secondaries are paralleled by closing the switch, a circulating current appears even though the secondaries are not supplying any load. The circulating current will depend upon the total leakage impedance of the two transformers and the difference in their voltage ratios. Only a small difference in the voltage-ratios can be tolerated.

### **Parallel Operation of Two Ideal Transformers:**

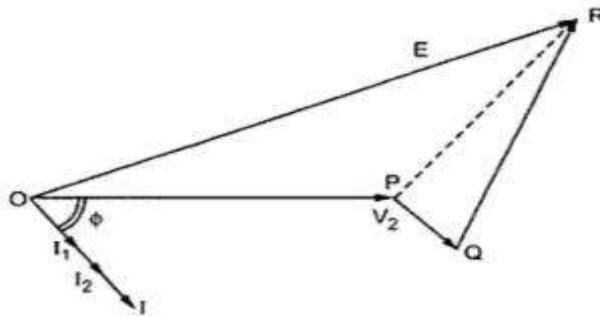
Now we will consider ideal case of two transformers having the same voltage ratio and their voltage triangles are equal in size and shape. i.e. apart from their impedances being same, their  $X/R$  ratios are also same. The circuit shown in the figure below consists of two transformers in parallel.



**Fig: Two ideal transformers connected in parallel**

The corresponding phasor diagram is shown in the figure below.

As seen from this figure the impedance voltage triangles of both the transformers are same.  $I_1$  and  $I_2$  are the currents flowing through transformers 1 and 2 which are in parallel. These currents are in phase with the load current and are inversely proportional to the respective impedances.



**Fig: Phasor diagram of two ideal transformers connected in parallel**

**Current sharing :** Let us now find out the distribution of the given load current for this condition between the two transformers. Let  $I$  be the total current and  $I_1$  and  $I_2$  be the currents shared by the two transformers.

Then applying KCL to the transformer secondaries ,

$$I = I_1 + I_2$$

Secondary voltage,

$$V_2 = E - I_1 Z_A = E - I_2 Z_2$$

Also  $I_1 Z_1 = I_2 Z_2$

$$I_1 / I_2 = Z_2 / Z_1$$

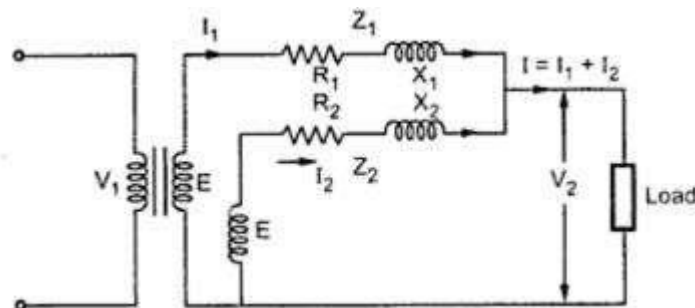
Applying current divider formulae we get

$$I_1 = I Z_2 / (Z_1 + Z_2)$$

and  $I_2 = I Z_1 / (Z_1 + Z_2)$

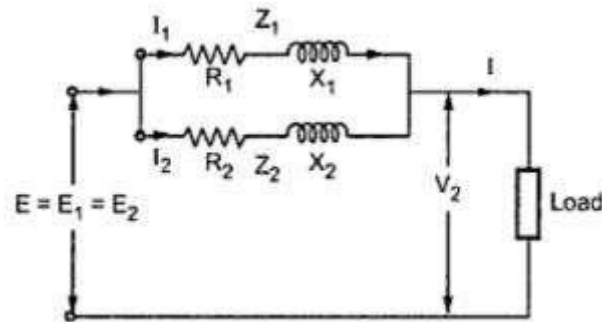
### Parallel Operation of Transformers with Equal Voltage Ratios:

Let us now consider the case of two transformers connected in parallel having equal voltage ratios. The two transformers are having no load secondary voltage same. i.e.  $E_1 = E_2 = E$ . These voltages are in phase with each other. This is possible if the magnetizing currents of the two transformers are not much different. With this case the primaries and secondaries of the two transformers can be connected in parallel and no current will circulate under no load condition. This is represented in the figure below.



**Fig: Two transformers with equal voltage ratios connected in parallel**

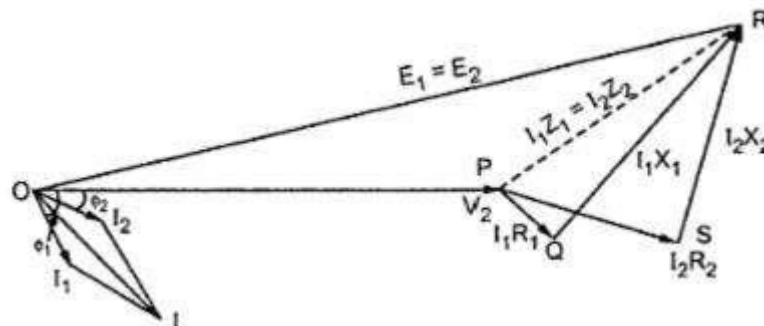
If we neglect magnetizing components, the two transformers are represented as shown in the figure below .



**Fig: Simplified equivalent circuit neglecting the magnetizing components**

The phasor diagram under this case is shown in the figure below .

The two impedances  $Z_1$  and  $Z_2$  are in parallel. The values of  $Z_1$  and  $Z_2$  are with respect to secondary. The impedances  $Z_1$  and  $Z_2$  are equal in magnitude but not in phase. i.e. the ratios  $X_1/R_1$  and  $X_2/R_2$  are not equal.



**Fig: Phasor diagram of the two transformers with equal voltage ratios connected in parallel**

**Current distribution:**

$Z_1$  and  $Z_2$  are in parallel therefore the equivalent impedance is given by,

$$1/Z_{eq} = 1/Z_1 + 1/Z_2 \quad Z_{eq} = Z_1 Z_2 / (Z_1 + Z_2)$$

As seen from the phasor diagram

$$I_1 Z_1 = I_2 Z_2 = I Z_{eq}$$

$$I_1 = I Z_{eq} / Z_1 = I Z_2 / (Z_1 + Z_2)$$

$$I_2 = I_{Zeq} = I Z_1 / (Z_1 + Z_2)$$

Multiplying both terms of above equation by voltage  $V_2$ ,

$$V_2 I_1 = V_2 I Z_2 / (Z_1 + Z_2)$$

$$V_2 I_2 = V_2 I Z_1 / (Z_1 + Z_2)$$

But  $V_2 I \times 10^{-3}$  is  $Q$  i.e. the combined load in KVA

From this, the KVA carried by each transformer is calculated as,

and

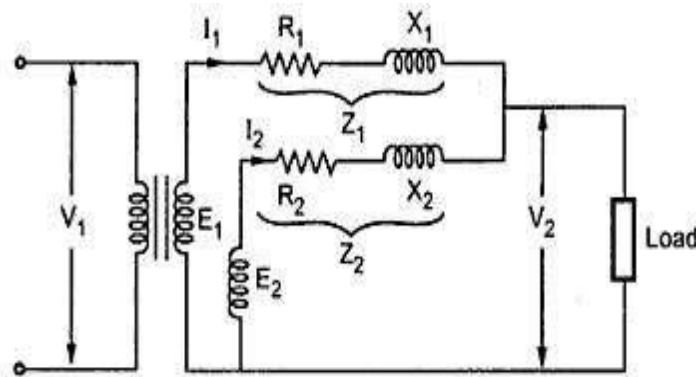
$$Q_1 = Q \cdot \frac{Z_2}{Z_1 + Z_2} = Q \cdot \frac{1}{1 + \frac{Z_1}{Z_2}}$$
$$Q_2 = Q \cdot \frac{Z_1}{Z_1 + Z_2} = Q \cdot \frac{1}{1 + \frac{Z_2}{Z_1}}$$

The above expressions are useful in determining the values of  $Q_1$  and  $Q_2$  in magnitude and in phase.

The two transformers work at different power factor. One operates at high p.f. while the other at low p.f. If the impedances  $Z_1$  and  $Z_2$  are equal both in magnitude and quality i.e. ( $X_1/R_1 = X_2/R_2$ ), then both transformers operate at the same p.f. which is the p.f. of the load.

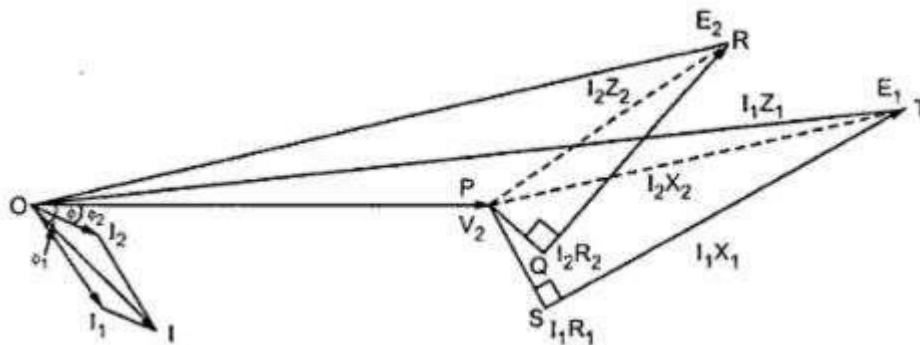
### **Parallel Operation of Transformers with Unequal Voltage Ratios:**

Now we will consider the case of two transformers working in parallel and having unequal voltage ratio. This is shown in the figure below. The voltage ratios of the two transformers are not equal. The parallel operation under this case is still possible. But as seen previously there would be a circulating current under no load condition.



**Fig: Two transformers with unequal voltage ratios connected in parallel**

The phasor diagram of this transformer is shown below:



**Fig: Phasor diagram of two transformers with unequal voltage ratios connected in parallel**

As can be seen the two transformers apart from having unequal output voltages  $E_1$  and  $E_2$ , their impedances  $Z_1$  and  $Z_2$  and their  $X/R$  ratios are also unequal.  $\Phi_1$  and  $\Phi_2$  are the power factor angles of these two transformers and  $\Phi$  is the combined p.f. angle.

**Current distribution (Load sharing):**

Let us consider that the voltage ratio of transformer-1 is slightly more than that of transformer-2. So the induced e.m.f  $E_1$  is greater than  $E_2$ . Thus the resultant

terminal voltage will be  $(E_1 - E_2)$  which will cause a circulating current  $I_c$  under no load condition.

$$I_c = (E_1 - E_2)/(Z_1 + Z_2)$$

From the circuit diagram we have,

$$E_1 = V_2 + I_1 Z_1$$

$$E_2 = V_2 + I_2 Z_2$$

Also,

$$I_L = I_1 + I_2$$

$$V_2 = I_L Z_L = (I_1 + I_2) Z_L$$

$$E_1 = (I_1 + I_2) Z_L + I_1 Z_1 \dots\dots\dots (1)$$

$$E_2 = (I_1 + I_2) Z_L + I_2 Z_2 \dots\dots\dots (2)$$

Subtracting equation (b) from (a) we have,

$$E_1 - E_2 = I_1 Z_1 - I_2 Z_2 \text{ from which we get}$$

$$I_1 = [(E_1 - E_2) + I_2 Z_2] / Z_1 \dots\dots\dots (3)$$

and

$$I_2 = [I_1 Z_1 - (E_1 - E_2)] / Z_2 \dots\dots\dots (4)$$

Substituting the value of  $I_1$  from (3) in equation (2) we get :

$$E_2 = I_2 Z_2 + \left[ \left\{ \frac{(E_1 - E_2) + I_2 Z_2}{Z_1} \right\} + I_2 \right] Z_L$$

From which we get:  $I_2 = [E_2 Z_1 - (E_1 - E_2)Z_L] / (Z_1 Z_2 + Z_L (Z_1 + Z_2)) + \dots\dots\dots (5)$

Similarly by Substituting the value of  $I_2$  from equation (4) in equation (1) we get :

$$E_1 = I_1 Z_1 + [I_1 + \{I_1 Z_1 - (E_1 - E_2)\} / Z_2] Z_L$$

From which we get:  $I_1 = [E_1 Z_2 + (E_1 - E_2)Z_L] / (Z_1 Z_2 + Z_L (Z_1 + Z_2)) + \dots\dots\dots (6)$

If impedances  $Z_1$  and  $Z_2$  are small in comparison with load impedance  $Z_L$  then the product  $Z_1 Z_2$  in the above equations (6) and (5) for currents  $I_1$  and  $I_2$  may be neglected so that finally we get the current distribution between the two transformers as,

$$I_1 = \frac{E_1 Z_2}{Z_1 (Z_1 + Z_2)} + \frac{E_1 - E_2}{Z_1 + Z_2}$$
$$I_2 = \frac{E_2 Z_1}{Z_1 (Z_1 + Z_2)} - \frac{E_1 - E_2}{Z_1 + Z_2}$$

Where the second term  $(E_1 - E_2) / (Z_1 + Z_2)$  is the circulating current  $I_c$  which adds to the load shared by transformer-1 whose output voltage  $E_1$  is higher and subtracts from the load shared by transformer-2 whose output  $E_1$  is lower. **Hence transformer-1 gets overloaded and the transformers will not share the load according to their ratings.**

### Introduction:

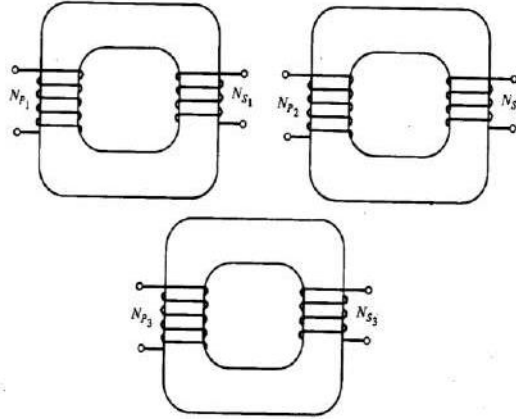
Almost all the major power generation and distribution systems in the world today are three-phase ac systems. Since three-phase systems play such an important role in modern life, it is necessary to understand how transformers are used in them.

Transformers for three-phase circuits can be constructed in two ways. One approach is simply to take three single-phase transformers and connect them in a three-phase bank. An alternative approach is to make a three-phase transformer consisting of three sets of windings wrapped on a common core.

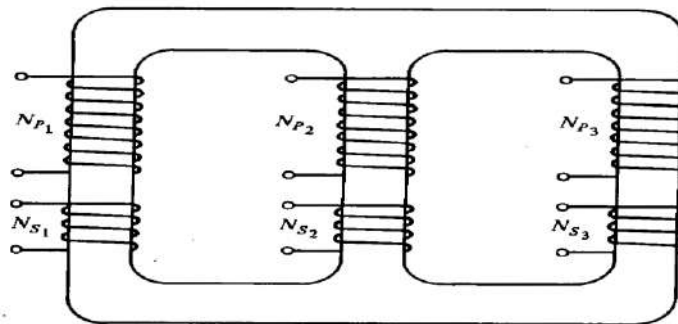
These two possible types of transformer construction are shown in the figures below.

The construction of a single three-phase transformer is the preferred practice today, since it is lighter, smaller, cheaper, and slightly more efficient. The older construction approach was to use three separate transformers. That approach had the advantage that each unit in the bank could be replaced individually in the event of trouble, but that does not outweigh the advantages of a combined three

phase unit for most applications. However, there are still a great many installations consisting of three single-phase units in service.



**Fig: A three-phase transformer bank composed of independent transformers.**



**Fig: A three-phase transformer wound on a single three-legged core.**

### Three-Phase Transformer Connections:

A three-phase transformer consists of three transformers, either separate or combined on one core. The primaries and secondaries of any three-phase transformer can be independently connected in either a Wye (Y) or a Delta ( $\Delta$ ). This gives a total of four possible connections for a three-phase transformer bank:

1. Wye(star)-Wye(star) (Y-Y)
2. Delta-Delta ( $\Delta$ - $\Delta$ )
3. (star)Wye-Delta (Y - $\Delta$ )

#### 4. Delta-Wye(star) ( $\Delta$ -Y)

The key to analyzing any three-phase transformer bank is to look at a single transformer in the bank. *Any single transformer in the bank behaves exactly like the single-phase transformers already studied.* The impedance, voltage regulation, efficiency, and similar calculations for three-phase transformers are done on a *per-phase basis*, using exactly the same techniques already developed for single-phase transformers.

The advantages and disadvantages of each type of three-phase transformer connection are explained below along with the relevant connection diagrams.

#### WYE-WYE CONNECTION:

The Y-Y connection details of three-phase transformers are shown in the figure below.

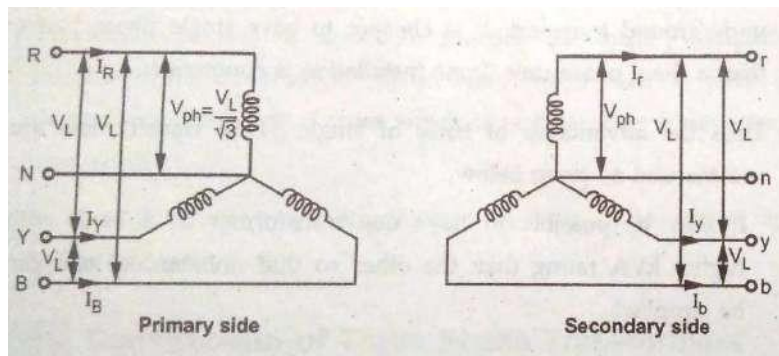
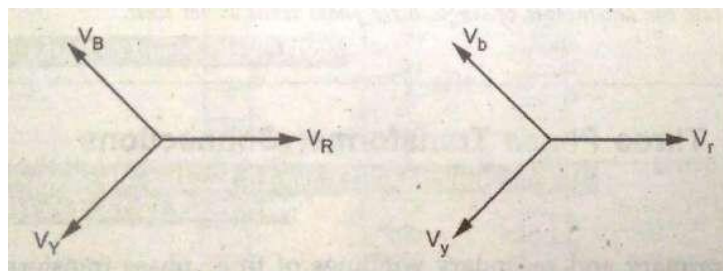
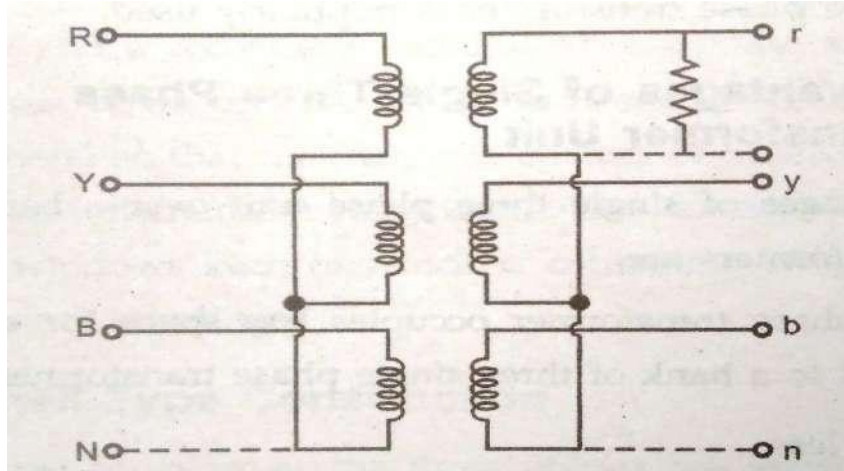


Figure (a): Star-Star(Y-Y) connection representation



**Figure (b): Star-Star(Y-Y) connection Phasor diagram**



**Figure (c): Star-Star(Y-Y) connection wiring diagram**

In a Y-Y connection, the primary voltage on each phase of the transformer is given by  $V_{\phi P} = V_{LP} / \sqrt{3}$ . The primary-phase voltage is related to the secondary-phase voltage by the turns ratio of the transformer. The phase voltage on the secondary is then related to the line voltage on the secondary by  $V_{LS} = \sqrt{3}V_{\phi S}$ . Therefore, overall the voltage ratio 'a' of the transformer is then given by:

$$\frac{V_{LP}}{V_{LS}} = \frac{\sqrt{3}V_{\phi P}}{\sqrt{3}V_{\phi S}} = a$$

**Advantage/Application:** This is useful and economical for low power high voltage transformers because the phase voltage is  $1/\sqrt{3}$  times the line voltage. Hence the number of turns per phase and the strength of insulation required would be less.

**Disadvantages:**

1. If loads on the transformer circuit are unbalanced, then the voltages on the phases of the transformer can become severely unbalanced.
2. Third-harmonic voltages can be large.

When a three-phase set of voltages is applied to a Y - Y transformer, the voltages in any phase will be  $120^\circ$  apart from the voltages in any other phase. However, *the third-harmonic components of each of the three phases will be in phase with each other*, since there are three cycles in the third harmonic for each cycle of the fundamental frequency. There will always be some third-harmonic components in a transformer because of the nonlinearity of the core, and these components add up.

The result is a very large third-harmonic component of voltage on top of the 50 or 60-Hz fundamental voltage. This third-harmonic voltage can be larger than the fundamental voltage itself.

Both the unbalance problem and the third-harmonic problem can be solved using one of the two following techniques:

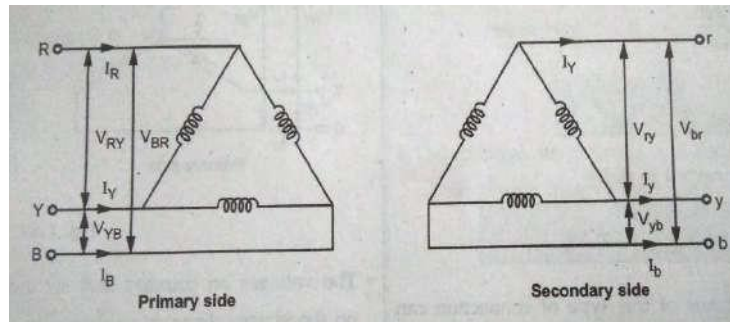
1. *Solidly ground the neutrals of the transformers*, especially the primary winding's neutral. This connection permits the additive third-harmonic components to cause a current flow in the neutral instead of building up large voltages. The neutral also provides a return path for any current imbalances in the load.
2. *Add a third (tertiary) winding connected in  $\Delta$*  to the transformer bank. If a third  $\Delta$  connected winding is added to the transformer, then the third-harmonic components of voltage in the  $\Delta$  will add up, causing a circulating current flow within the winding. This suppresses the third-harmonic components of voltage in the same manner as grounding the transformer neutrals.

The  $\Delta$  connected tertiary windings need not even be brought out of the transformer case, but they often are used to supply lights and auxiliary power within the substation where it is located. The tertiary windings must be large enough to handle the circulating currents, so they are usually made about one-third the power rating of the two main windings.

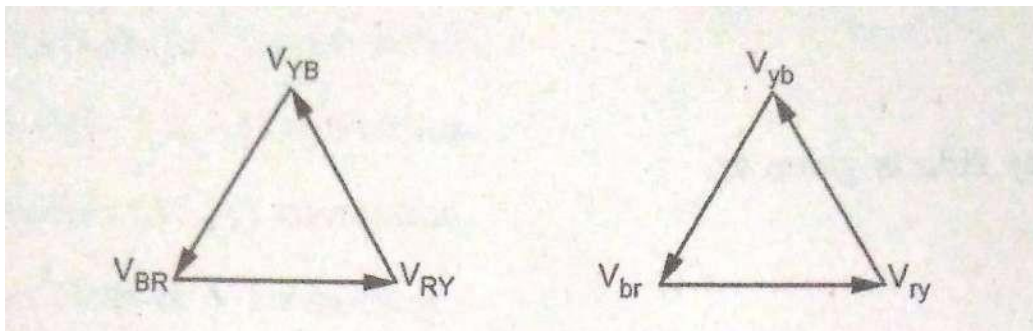
One or the other of these correction techniques *must* be used any time a Y-Y transformer is installed. In practice, very few Y-Y transformers are used, since the same jobs can be done by one of the other types of three-phase transformers.

**DELTA-DELTA CONNECTION:**

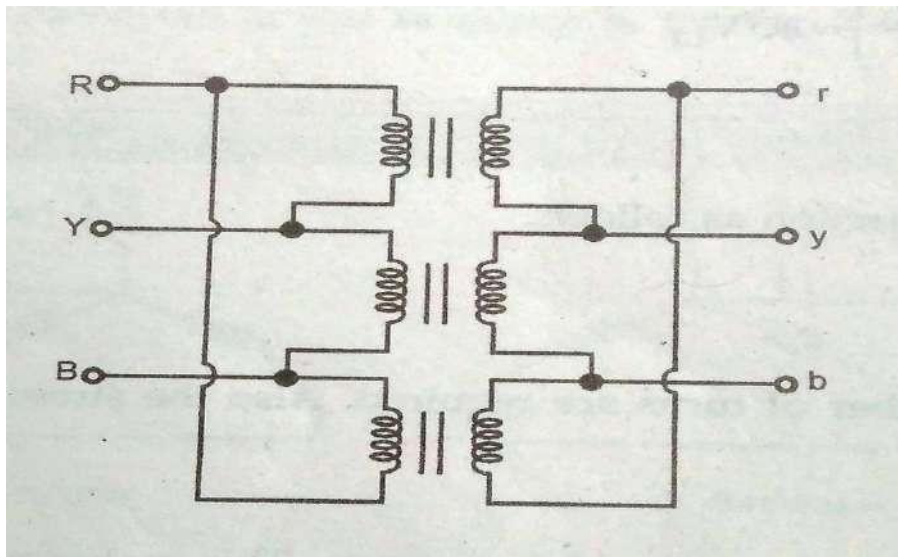
The  $\Delta$ -  $\Delta$  connection details are shown in the figure below.



**Figure (a): Delta-Delta ( $\Delta$ - $\Delta$ ) connection representation**



**Figure (b): Delta-Delta ( $\Delta$ - $\Delta$ ) connection Phasor diagram**



**Figure (c): Delta-Delta ( $\Delta$ - $\Delta$ ) connection wiring diagram**

In a  $\Delta$ -  $\Delta$  connection,  $V_{LP} = V_{\phi P}$  and  $V_{LS} = V_{\phi S}$ , so the relationship between primary and secondary line voltages is given by:

$$\frac{V_{LP}}{V_{LS}} = \frac{V_{\phi P}}{V_{\phi S}} = a \quad \Delta\text{-}\Delta$$

**Advantages/Application:**

- Transformers with this configuration are economical for high power low voltage application since the number of turns required for a given line voltage are more (since line voltage is same as phase voltage)
- This transformer has no phase shift associated with it and no problems with unbalanced loads or harmonics.
- For the secondary voltage to be perfect sinusoidal the magnetizing currents must contain third harmonic components. The Delta configuration provides a closed path for the circulation of third harmonic components of current. Hence the flux remains sinusoidal thus resulting in better sinusoidal voltages.

- The phase current is lesser than the Line current (by  $1/\sqrt{3}$ ). Hence the conductor cross sectional area can be smaller thus resulting in saving of conductor material.

**Disadvantages:**

- Due to the nonavailability of the Neutral point this configuration is not suitable for three phase four wire systems

**WYE-DELTA CONNECTION:**

The Y -  $\Delta$  connection details of three-phase transformers is shown in the figures below. In this connection, the primary line voltage is related to the primary phase voltage by  $V_{LP} = \sqrt{3}V_{\phi P}$  while the secondary line voltage is equal to the secondary phase voltage  $V_{LS} = V_{\phi S}$ . The voltage ratio of each phase is:

$$V_{\phi P} / V_{\phi S} = a$$

so the overall relationship between the line voltage on the primary side of the bank and the line voltage on the secondary side of the bank is:

$$V_{LP} / V_{LS} = \sqrt{3}V_{\phi P} / V_{\phi S} = \sqrt{3} a$$

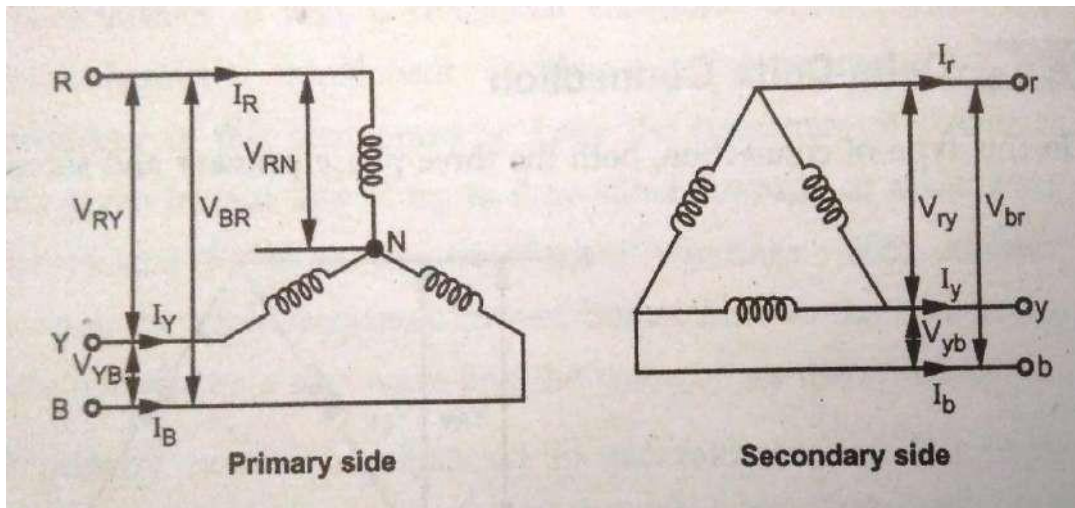


Figure (a): Wye-Delta (Y-Δ) connection representation

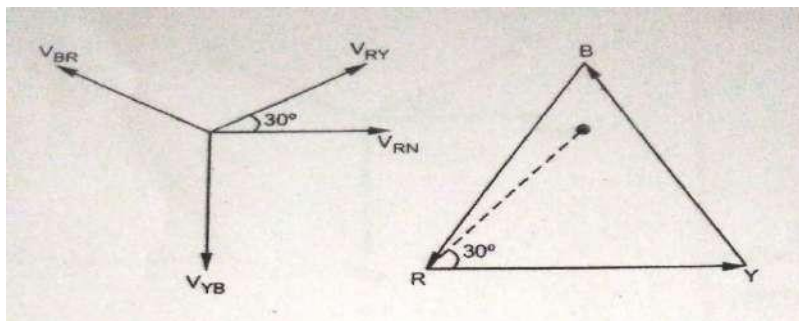
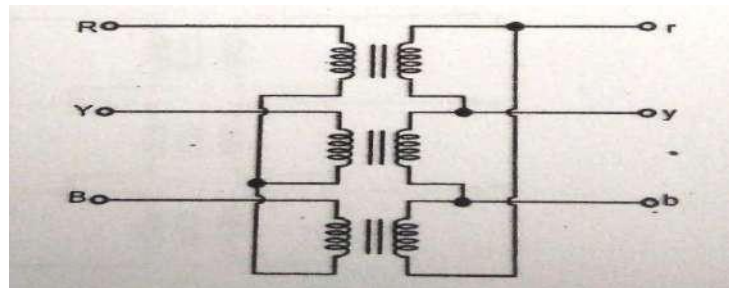


Figure (b): Wye -Delta (Y-Δ) connection Phasor diagram



**Figure (c): Wye -Delta (Y- $\Delta$ ) connection wiring diagram**

**Advantages/Application:**

- This connection is advantageous/economical for high power high voltage step **down** power transformers. Primary in star configuration can be used for higher voltage since line voltage is  $\sqrt{3}$  times the phase voltage and thus the number of turns required per phase will be lesser for a higher line voltage. The delta side with lower line voltage (line voltage being equal to phase voltage) can be used as secondary.
- The neutral available in primary can be earthed to avoid distortion
- Hence transformers with this type of connection are used in the main receiving end of a transmission line where a step **down** transformer is required.
- The Y -  $\Delta$  connection has no problem with third-harmonic components in its voltages, since they are consumed in a circulating current on the  $\Delta$  side.
- This connection is also more stable with respect to unbalanced loads, since the  $\Delta$  partially redistributes any imbalance that occurs. i.e. Load side (secondary) large unbalanced loads can be handled satisfactorily.

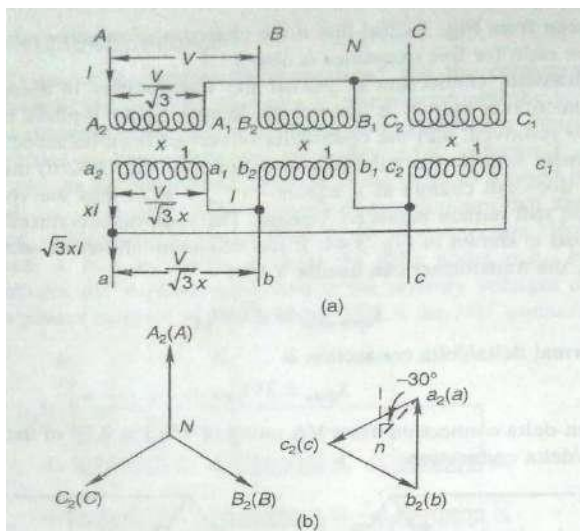
**Disadvantages:**

- This arrangement has one problem. Because of this type of connection, the secondary voltage is shifted  $30^\circ$  relative to the primary voltage of the transformer which can be further positive shift or negative shift. The fact that a phase shift has occurred can cause problems in paralleling the secondaries of two transformer banks together. The phase angles of transformer secondaries must be equal if they are to be paralleled, which means that attention must be paid to the direction of the  $30^\circ$  phase shift occurring in each transformer bank to be paralleled together.

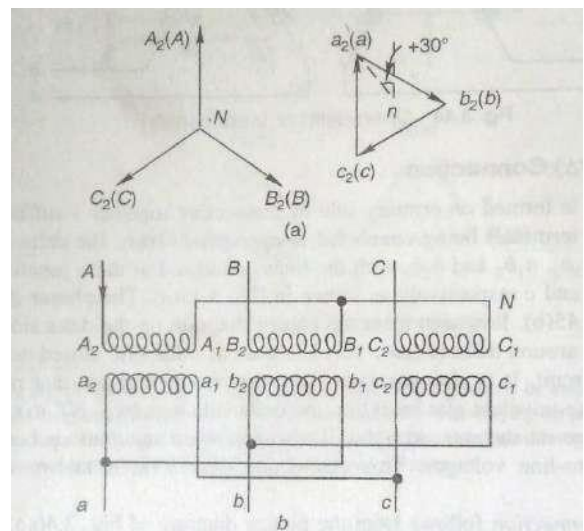
***Star/Delta (Y/D) Connection (Alternate explanation for Phase grouping):***

*Star connection is formed on primary side by connecting together 1 suffixed terminals with 2 suffixed terminals connected to appropriate lines. The delta is*

formed by connecting  $c_1a_2$ ,  $a_1b_2$  and  $b_1c_2$  with the lines connected to these junctions being labeled as  $a$ ,  $b$  and  $c$  respectively as shown in Fig. (a). The phasor diagram is drawn in Fig. (b). It is seen from the phasor diagram on the delta side that the sum of voltages around delta is zero. This is a must as otherwise closed delta would mean a short circuit. It is also observed from the phasor diagram that phase  $a$  to neutral voltage (equivalent star basis) on the delta side lags by  $-30^\circ$  to the phase-to-neutral voltage on the star side. This is also the phase relationship between the respective line to-line voltages. This connection, therefore, is known as  $-30^\circ$ -connection. Or  $YD1$  representing  $1^0$  clock position. With this notation secondary Delta lags the primary star by  $30^\circ$ .



**Fig:  $-30^\circ$  connection**



**Fig:  $+30^\circ$  connection**

The  $+30^\circ$ -connection follows from the phasor diagram of Fig. (a) above with the corresponding connection diagram shown in Fig. (b).

**Delta/Star (D/Y) Connection (Alternate explanation for Phase grouping):**

This connection is simply the interchange of primary and secondary roles in the star/delta connection. One just interchanges capital and small letter suffixing in

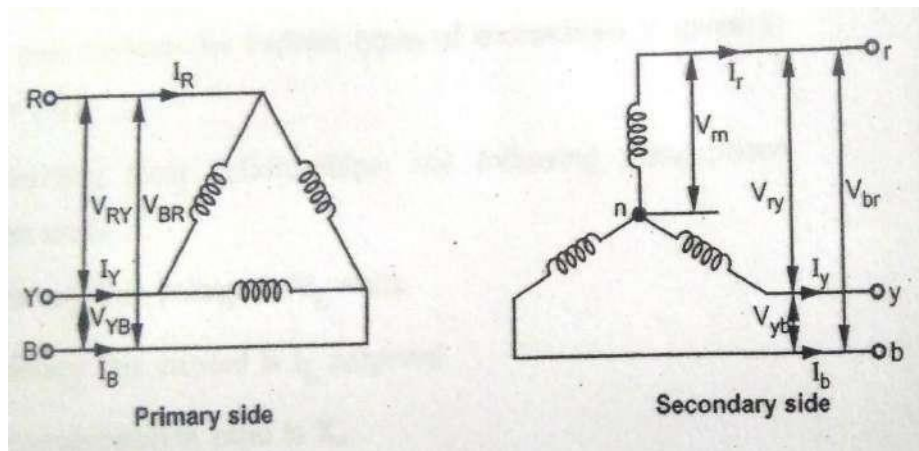
the above figures .But what was the  $- 30^\circ$ -connection will now be the  $+ 30^\circ$ -connection and vice versa.

**DELTA-WYE CONNECTION:**

$\Delta$ -Y connection details of three-phase transformers are shown in the figures below. In a  $\Delta$ -Y connection, the primary line voltage is equal to the primary-phase voltage  $V_{LP} = V_{\phi P}$  , while the secondary voltages are related by  $V_{LS} = \sqrt{3}V_{\phi S}$  . Therefore, the line-to-line voltage ratio of this transformer connection is given by :

$$\frac{V_{LP}}{V_{LS}} = \frac{V_{\phi P}}{\sqrt{3}V_{\phi S}}$$

$$\frac{V_{LP}}{V_{LS}} = \frac{a}{\sqrt{3}} \quad \Delta\text{-Y}$$



**Figure (a): Delta - Wye ( $\Delta$ -Y) connection representation**

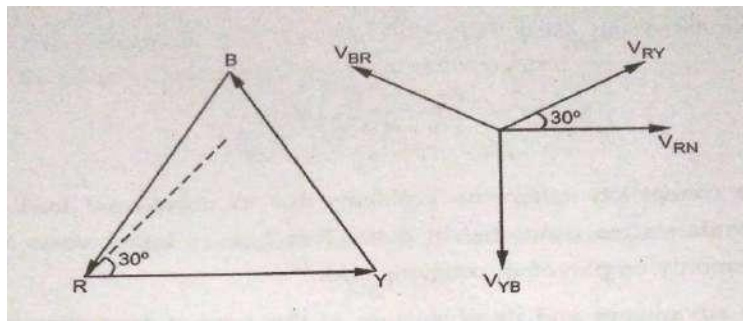


Figure (b): Delta - Wye ( $\Delta$ -Y) connection Phasor diagram

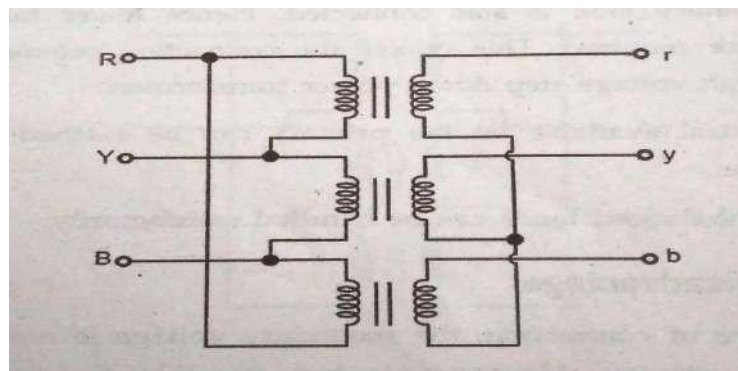


Figure (c): Delta - Wye ( $\Delta$ -Y) connection wiring diagram

**Advantages/Application:**

- This connection is advantageous/economical for high power high voltage *step up power transformers*. Primary in Delta configuration can be used for lower voltage (line voltage being equal to phase voltage) And secondary in Wye (star) configuration can be used for higher voltage since line voltage is  $\sqrt{3}$  times the phase voltage and thus the number of turns required per phase will be lesser for a higher line voltage.
- Hence transformers with this type of connection are used at the starting (Generating station) end of a transmission line where a *step up transformer* is required.

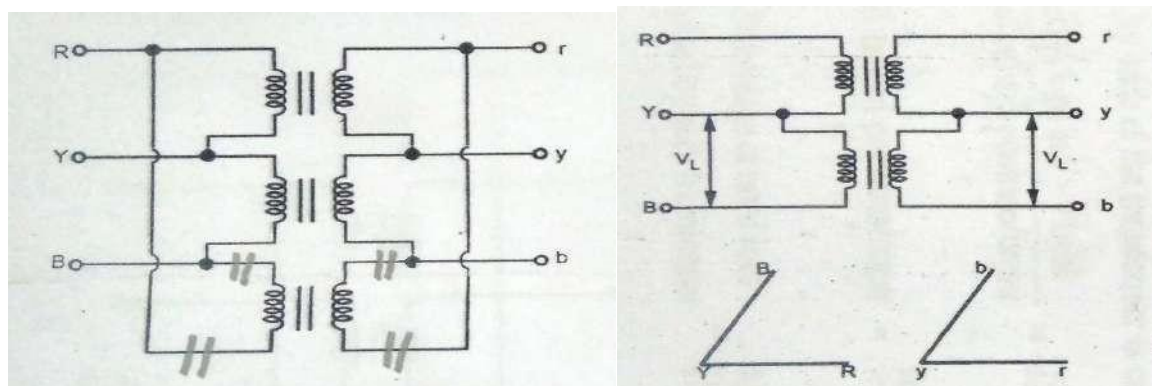
**Disadvantages:**

This connection has the same disadvantages and the same phase shift as the Y -  $\Delta$  transformer. The connection shown in the figure above makes the secondary voltage differ the primary voltage by  $30^\circ$  as in Y -  $\Delta$ .

**The Open  $\Delta$  (or V-V) Connection:**

In some situations a full transformer bank may not be used to accomplish three phase transformation. For example, suppose that a  $\Delta$  -  $\Delta$  transformer bank consisting of three separate transformers has a damaged phase which has to be removed for repair.

The resulting configuration is known as open  $\Delta$  (or V-V) Connection and is shown in the figure below.



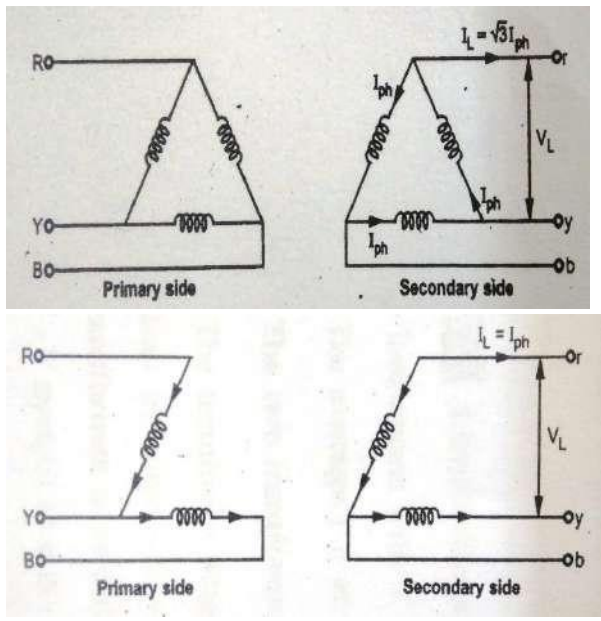
**Figure: Third transformer ( $V_{BR}$ ) removed from the Three transformer Bank and the corresponding Phasor diagram**

If the open Delta primary is now excited from a balanced three phase supply, then the voltage across the gap where the third transformer used to be would exactly be the same voltage that would be present if the third transformer were still there .

Thus, the open-delta connection lets a transformer bank work as a three phase transformer with only two transformers, allowing some *reduced* power flow to continue even with a damaged phase removed.

**Power Delivered in Open Delta configuration:**

How much apparent power can the bank supply with one of its three transformers removed? At first, it seems that it could supply two-thirds of its rated apparent power, since two-thirds of the transformers are still present. Things are not that simple. To understand what happens when a transformer is removed, let us see the figures (a) and (b) below. Figure (a) shows  $\Delta - \Delta$  connection and figure (b) shows  $V-V$  connection.



**Fig(a) :  $\Delta$  -  $\Delta$  connection**

**Fig(a) : V- V**

We know that the power output from a three phase system is  $\sqrt{3} V_L I_L \cos \phi$  where  $\cos \phi$  is the power factor. Hence in figure (a)  $\Delta$  -  $\Delta$  capacity =  $\sqrt{3} V_L I_L \cos \phi = \sqrt{3} V_L \sqrt{3} I_{Ph} \cos \phi$  (since  $I_L = \sqrt{3} I_{Ph}$ )

$$= 3V_L I_{Ph} \cos \phi$$

But in figure (b) V-V capacity =  $\sqrt{3} V_L I_L \cos \phi = \sqrt{3} V_L I_{Ph} \cos \phi$  (since  $I_L = I_{Ph}$ )

Therefore V-V capacity /  $\Delta$  -  $\Delta$  capacity =  $\sqrt{3} V_L I_{Ph} \cos \phi / 3 V_L I_{Ph} \cos \phi = 1/\sqrt{3}$   
= **0.577=57.7%**

**Summary conclusion:**

The total load carried by an open  $\Delta$  (V- V) connection is 57.7 % of the total capacity of the  $\Delta$ -  $\Delta$  and not  $2/3^{\text{rd}}$  (66.6 % ) of the total capacity of the  $\Delta$ -  $\Delta$  as might be expected. Thus for example, in a  $\Delta$ -  $\Delta$  bank of three transformers each of 100kVA if one transformer is removed, then the total capacity of the resulting V-V bank becomes 57.7 % of 300kVA i.e. 173.2kVA and not 200kVA.

In other words in V-V configuration the resultant capacity becomes 86.6% of the rated capacity of the two transformers put together. i.e.  $0.866 \times 200\text{kVA} = 173.2 \text{ kVA}$  .

The factor 0.866 is called **utility factor**

= (operating capacity of the transformers in V-V) / (Available capacity of the transformers in V-V)

**V – V Configuration is intentionally employed in the following applications:**

- When the three phase load is too small to warrant the installation of full three phase transformer bank.
- When one of the three transformers in a  $\Delta - \Delta$  bank is disabled service is at a reduced capacity is adequate, till the faulty transformer is repaired and restored.
- When it is expected that in the future, the total load will increase necessitating the closing of the open  $\Delta$ .

### Illustrative examples:

**Example1:** The following test results were obtained for a 20 kVA, 2400/240 V distribution transformer : O.C test (l.v side) 240 V, 1.066 A, 126.6 W. S.C test (h.v side) 57.5V, 8.34 A, 284 W. Calculate a) Equivalent circuit parameters when referred to h.v side and draw it. b) Efficiency of the transformer at half full-load with 0.8 power factor lagging. c) Regulation at full load 0.8 power factor lagging.

( JNTU May-15)

**Solution :** From O.C test,  $V_0 = 240$  V,  $I_0 = 1.066$  A,  $W_0 = 126.6$  W, Measurements are on LV, side i.e. secondary hence results will give parameters referred to secondary.

$$\therefore \cos\phi_0 = \frac{W_0}{V_0 I_0} = \frac{126.6}{240 \times 1.066} = 0.4948, \phi_0 = 60.34^\circ$$

$$i) \quad I_C = I_0 \cos\phi_0 = 1.066 \times 0.4948 = 0.5275 \text{ A}$$

$$I_m = I_0 \sin\phi_0 = 1.066 \times 0.8689 = 0.9263 \text{ A}$$

$$R'_0 = \frac{V_0}{I_0} = \frac{240}{0.5275} = 454.97 \Omega$$

$$X'_0 = \frac{V_0}{I_m} = 259.095 \Omega$$

$$K = \frac{V_1}{V_2} = \frac{2400}{240} = 10$$

$$R_0 = R'_0 \times K^2 = 45.497 \text{ k}\Omega$$

$$X_0 = X_0' x K^2 = 25.91 k\Omega \dots \dots \dots \text{Referred to h.v side}$$

From S.C. test,  $V_{sc} = 57.5 \text{ V}$ ,  $I_{sc} = 8.34 \text{ A}$ ,  $W_{sc} = 284 \text{ W}$

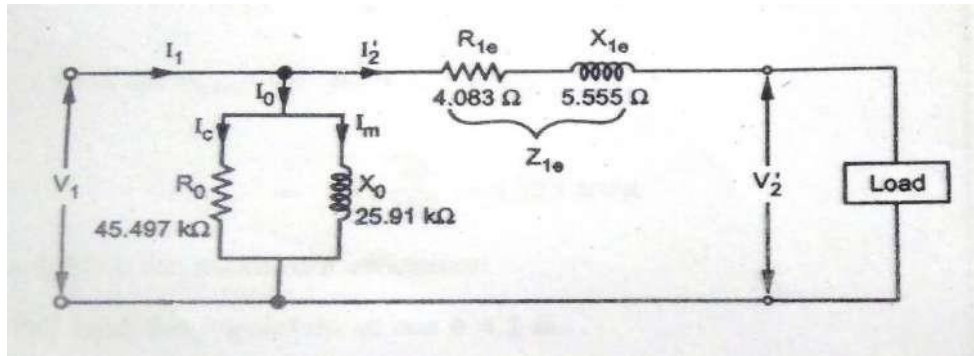
The meters are on h.v. side hence we get parameters referred to h.v. side.

$$R_{1e} = \frac{W_{sc}}{I_{sc}^2} = \frac{284}{(8.34)^2} = 4.083 k\Omega$$

$$\therefore Z_{1e} = \frac{V_{sc}}{I_{sc}} = 6.8944 \Omega$$

$$X_{1e} = \sqrt{Z_{1e}^2 - R_{1e}^2} = 5.555 \Omega$$

The equivalent circuits referred to h.v. side is shown in the figure below.



ii) From O.C test,

$$P_i = \text{iron loss} = 126.6 \text{ W}$$

From S.C test  $P_{cu} = 284 \text{ W}$  for  $I_{sc} = 8.34 \text{ A}$

$$I_1(FL) = \frac{VA}{V_1} = \frac{20 \times 10^3}{2400} = 8.34 \text{ A}$$

$$\text{As } I_1(FL) = I_{sc}, W_{sc} = P_{cu}(FL) = 284 \text{ W}$$

$$\therefore \% \eta_{HL} = \frac{nVA \cos \Phi}{nVA \cos \Phi + P_i + N^2 P_{cu}(FL)} \times 100 \dots \dots n = 0.5 \text{ on Half load}$$

$$= \frac{0.5 \times 20 \times 10^3 \times 0.8}{0.5 \times 20 \times 10^3 \times 0.8 + 126.6 + [0.5^2 \times 284]} \times 100$$

$$= \mathbf{97.589 \%}$$

iii)  $I_1(\text{FL}) = 8.34 \text{ A}, V_1 = 2400 \text{ V}, \cos \phi = 0.8$

$$\therefore \%R = \frac{I_1(\text{FL})[R_{1e} \cos \phi + X_{1e} \sin \phi]}{V_1} \times 100$$

$$= \frac{8.34[4.083 \times 0.8 + 5.55 \times 0.6]}{2400} \times 100$$

$$= \mathbf{2.293 \%}$$

**Example 2 :** Two similar 200 kVA, single phase transformers gave the following results in Sumpner's test : Mains wattmeter  $W_1 = 4 \text{ kW}$ , Series wattmeter  $W_2 = 6 \text{ kW}$  at full load current. Find out individual transformer efficiencies at i) Full load at unity p.f. and ii) Half load at 0.8 p.f. lead. (JNTUK April-12)

**Solution :** The given values are: Rating = 200 kVA,  $W_1 = 4 \text{ kW}$ ,  $W_2 = 6 \text{ kW}$

$W_1 =$  iron loss of both the transformers = 4 kW

$$\therefore P_i = \text{Iron loss for each transformer} = \frac{W_1}{2} = \frac{4}{2} = 2 \text{ kW}$$

$W_2 =$  Full load copper loss for both the transformers = 6 kW

$$\therefore (P_{cu})_{\text{F.L.}} = \text{Full load copper loss for each transformer} = \frac{W_2}{2} = \frac{6}{2} = 3 \text{ kW}$$

i) At full load,

$$\% \eta = \frac{VA_{\text{rating}} \cos \phi_2}{VA_{\text{rating}} \cos \phi_2 + P_i + (P_{cu})_{\text{F.L.}}} \times 100 \text{ with } \cos \phi_2 = 1$$

$$= \frac{200 \times 10^3 \times 1}{200 \times 10^3 \times 1 + 2 \times 10^3 + 3 \times 10^3} \times 100$$

$$= \mathbf{97.56 \%}$$

ii) At half load,  $\cos \phi_2 = 0.8$  and  $n = \frac{1}{2} = 0.5$

$$\therefore \% \eta = \frac{n \times (VA_{rating}) \times \cos \phi_2}{n \times (VA_{rating}) \times \cos \phi_2 + P_i + n^2 \times (P_{cu})_{F.L.}} \times 100$$

$(P_{cu})_{H.L.} = n^2 \times (P_{cu})_{F.L.}$  where  $n =$  Fraction of full load

$$\therefore \% \eta = \frac{0.5 \times 200 \times 10^3 \times 0.8}{0.5 \times 200 \times 10^3 \times 0.8 + 2 \times 10^3 + (0.5)^2 \times 3 \times 10^3} \times 100$$

$$= \mathbf{96.67 \%}$$

**Example 3:** A 200 V, 60 Hz single phase transformer has hysteresis and eddy current losses of 250 watts and 90 watts respectively. If the transformer is now energised from 230 V, 50 Hz supply. Calculate its core losses. Assume Steinmetz's constant equal to 1.6. (JNTUK April-12)

**Solution :** Let  $P_h =$  Hysteresis loss and  $P_e =$  Eddy current loss

Then they are given by  $P_h = K_h f B_m^x$  where  $x =$  Steinmetz's constant  $= 1.6$

and  $P_e = K_e f^2 B_m^2$

$$E = 4.44 f N B_m A \quad \text{where } \phi_m = B_m A$$

$$\therefore B_m \propto \frac{E}{f} \quad \text{i.e.} \quad P_h = K_h f \left(\frac{E}{f}\right)^{1.6}$$

$$\text{and } P_e = K_e f^2 \left(\frac{E}{f}\right)^2$$

For  $E = 200 \text{ V}$ ,  $f = 60 \text{ Hz}$ ,  $P_h = 250 \text{ W}$  and  $P_e = 90 \text{ W}$

$$250 = K_h \times 60 \times \left(\frac{200}{60}\right)^{1.6} \quad \text{i.e.} \quad K_h = 0.607$$

$$90 = K_e \times 60^2 \times \left(\frac{200}{60}\right)^2 \quad \text{i.e.} \quad K_e = 0.00225$$

Now  $E = 230 \text{ V}$ ,  $f = 50 \text{ Hz}$

$$\therefore P_h = 0.607 \times 50 \times \left(\frac{230}{50}\right)^{1.6} = 348.795 \text{ W}$$

$$P_e = 0.00225 \times 50^2 \times \left(\frac{230}{50}\right)^2 = 119.025 \text{ W}$$

$\therefore$  Core loss =  $P_h + P_e = 467.82 \text{ W}$  at  $230 \text{ V}$ ,  $50 \text{ Hz}$

**Example 4:** In a test for determination of the losses of a  $440\text{V}$ ,  $50 \text{ Hz}$  transformer, the total iron losses were found to be  $2500 \text{ W}$  at normal voltage and frequency. When the applied voltage and frequency were  $220 \text{ V}$  and  $25 \text{ Hz}$ , the iron losses were found to be  $850\text{W}$ . Calculate the eddy current loss at normal voltage and frequency. (JNTU Feb-10, May-15)

**Solution :** We have the data:  $V_1 = 440 \text{ V}$ ,  $f_1 = 50 \text{ Hz}$ ,  $V_2 = 220 \text{ V}$ ,  $f_2 = 25 \text{ Hz}$

$$\therefore \frac{V_1}{f_1} = 8.8 \text{ and } \frac{V_2}{f_2} = 8.8$$

Thus  $V/f$  is constant hence flux density  $B_m$  remains constant.

$$\therefore P_h = A f \quad \text{and} \quad P_e = B f^2$$

$$\therefore P_i = P_h + P_e = A f + B f^2$$

$$\therefore \text{In test - 1: } 2500 = A \times 50 + B \times 2500 \quad \dots\dots\dots(1)$$

$$\text{And in test - 2 } 850 = A \times 25 + B \times 625 \dots\dots\dots(2)$$

Solving we get  $A = 18$  and  $B = 0.64$

Thus eddy current loss a normal voltage and frequency is,

$$P_e = B f^2 = 0.64 \times (50)^2 = \mathbf{1600 \text{ W}}$$

**Example 5:** Two single phase transformer with equal turns have impedance of  $(0.5 + j3)$  ohm and  $(0.6 + j10)$  ohm with respect to the secondary. If they operate in parallel, determine how they will share a total load of 100 kW at p.f. 0.8 lagging ?

( JNTU April-04, Nov-04)

**Solution :** We have the data:  $Z_1 = 0.5 + j3 \Omega$ ,  $Z_2 = 0.6 + j10 \Omega$ ,

Total load = 100 kW

p.f. of load = 0.8 lag.

$$\cos \phi = 0.8, \phi = \cos^{-1} 0.8 = 36.86^\circ \text{ lag}$$

$$\text{kVA of load} = \frac{100}{0.8} = 125$$

$$\text{Hence } Q = 125 \angle -36.86^\circ \text{ kVA}$$

$$\begin{aligned} \text{Load shared by transformer 1} &= Q \left( \frac{Z_2}{Z_1 + Z_2} \right) \\ &= [125 \angle -36.86^\circ] \left[ \frac{0.6 + j10}{(0.5 + j3) + (0.6 + j10)} \right] \\ &= \frac{(125 \angle -36.86^\circ)(10.017 \angle -86.56^\circ)}{1.1 + j13} \\ &= \frac{1252.125 \angle -49.7^\circ}{13.046 \angle 85.16^\circ} = \mathbf{95.97 \angle -35.46^\circ \text{ kVA}} \end{aligned}$$

$$\text{p.f.} = \cos 35.46^\circ = \mathbf{0.8145 \text{ lag}}$$

$$\text{Load shared by transformer 2} = Q \left( \frac{Z_1}{Z_1 + Z_2} \right)$$

$$\begin{aligned}
 &= [125 \angle -36.86^\circ] \left[ \frac{0.5 + j3}{(0.5 + j3) + (0.6 + j10)} \right] \\
 &= \frac{(125 \angle -36.86^\circ)(3.041 \angle -80.53^\circ)}{13.046 \angle 85.16^\circ} \\
 &= \frac{380.125 \angle -43.67^\circ}{13.046 \angle 85.16^\circ} = \mathbf{29.13 \angle -41.49^\circ \text{ kVA}}
 \end{aligned}$$

**Example 6:** Two single phase transformer A and B of equal voltage ratio are running in parallel and supplying a load of 1000 A at 0.8 p.f. lag. The equivalent impedance of the two transformers are  $(2+j3)$  and  $(2.5+j5)$  ohms respectively. Calculate the current supplied by each transformer and the ratio of the kW output of the two transformer.

**Solution :** For transformer A,  $Z_A = 2 + j 3 \Omega$

For transformer B,  $Z_B = 2.5 + j 5 \Omega$

Current supplied by transformer A,

$$I_A = \frac{I \cdot Z_B}{Z_A + Z_B} \quad \dots\dots(1)$$

Current supplied by transformer B,

$$I_B = \frac{I \cdot Z_A}{Z_A + Z_B} \quad \dots\dots(2)$$

Taking ratio of equations (1) and (2) we have,

$$\begin{aligned}
 \frac{I_A}{I_B} &= \frac{Z_B}{Z_A} = \frac{2.5 + j5}{2 + j3} = \frac{5.5901 \angle 63.43^\circ}{3.060555 \angle 56.30^\circ} \\
 &= 1.5504 \angle 7.13^\circ
 \end{aligned}$$

$$\therefore \frac{I_A}{I_B} = 1.54 + j 0.1924 \quad I_A = (1.54 + j 0.1924) I_B$$

Total current  $I = 1000 \text{ A}$  and p.f. of total current = 0.8 lag

$$\therefore \cos \phi = 0.8, \phi = \cos^{-1} 0.8 = 36.86^\circ$$

$$\therefore I = 1000 \angle -36.86^\circ \text{ A} = (800 - j600) \text{ A}$$

We have,  $I = I_A + I_B$

$$\therefore 800 - j600 = [1.54 + j0.1924] I_B + I_B$$

$$= [2.54 + j0.1924] I_B$$

$$\therefore I_B = \frac{800 - j600}{2.54 + j0.1924} = \frac{1000 \angle -36.86^\circ}{2.5472 \angle -4.33^\circ}$$

$$= 392.58 \angle -41.19^\circ \text{ A}$$

$$\therefore I_B = 392.58 \angle -41.19^\circ \text{ A} = 295.42 - j258.53 \text{ A}$$

Now,  $I_A = (1.54 + j0.1924) I_B$

$$= [1.5519 \angle 7.12^\circ][392.58 \angle -41.19^\circ]$$

$$\therefore I_A = 609.24 \angle -34.07^\circ \text{ A} = 504.66 - j341.29 \text{ A}$$

The ratio of kW outputs is nothing but the ratio of inphase components of the two currents.

$$\frac{\text{output of Transformer A}}{\text{output of Transformer B}} = \frac{504.66}{295.42} = 1.7$$

**Example 7:** Two transformers A and B are connected in parallel to a load of  $(2 + j1.5) \Omega$ . Their impedances in secondary terms are  $Z_A = (0.15 + j0.5) \Omega$  and  $Z_B = (0.1 + j0.6) \Omega$ . Their no load terminal voltages are  $E_A = 207 \angle 0^\circ \text{ V}$ ,  $E_B = 205 \angle 0^\circ \text{ volts}$ . Find the power output and power factor of each transformer.

**Solution :** We have the data :  $Z_L = (2 + j1.5) \Omega$ ,  $Z_A = (0.15 + j0.5) \Omega$ ,  $Z_B = (0.1 + j0.6) \Omega$ ,  $E_A = 207 \angle 0^\circ \text{ volts}$  and  $E_B = 205 \angle 0^\circ \text{ volts}$ .  
Using the formulae for  $I_A$  and  $I_B$

$$I_A = \frac{E_A Z_B + Z_L (E_A - E_B)}{Z_A Z_B + Z_L (Z_A + Z_B)}$$

And  $I_B = \frac{E_B Z_A - Z_L (E_A - E_B)}{Z_A Z_B + Z_L (Z_A + Z_B)}$

$$I_A = \frac{[207/_0^0][0.1 + j0.6] + (2 + j1.5)[207/_0^0 - 205/_0^0]}{(0.15 + j0.5)(0.1 + j0.6) + (2 + j1.5)[(0.15 + j0.5) + (0.1 + j0.6)]}$$

Simplifying we get  $I_A = (42.196 \angle -38.84^0) \text{ A} = (32.866 - j26.463) \text{ A}$

and similarly we get  $I_B = \frac{E_B Z_A - Z_L (E_A - E_B)}{Z_A Z_B + Z_L (Z_A - Z_B)}$

$$= \frac{[205/_0^0][0.15 + j0.5] + (2 + j1.5)[207/_0^0 - 205/_0^0]}{(0.15 + j0.5)(0.1 + j0.6) + (2 + j1.5)[(0.15 + j0.5) + (0.1 + j0.6)]}$$

Solving,  $I_B = (33.5534 \angle -42.89^0) \text{ A}$   
 $= (24.5832 - j 22.8362) \text{ A}$

Now total current is given by,

$$\begin{aligned} I_L &= I_A + I_B \\ &= (32.866 - j 26.463) + (24.5832 - j 22.8362) \\ &= (57.4492 - j 49.2992) \text{ A} \\ &= 75.70 \angle -40.63^0 \text{ A} \end{aligned}$$

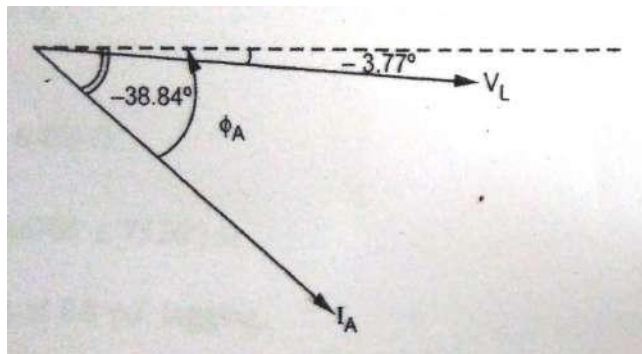
The load voltage,  $V_L = I_L Z_L = (75.70 \angle -40.63^0)(2 + j 1.5)$   
 $= (75.70 \angle -40.63^0) (2.5 \angle 36.86^0)$   
 $= 189.25 \angle -3.770 \text{ volts}$

The angle between  $V_L$  and  $I_A$  can be calculated as,

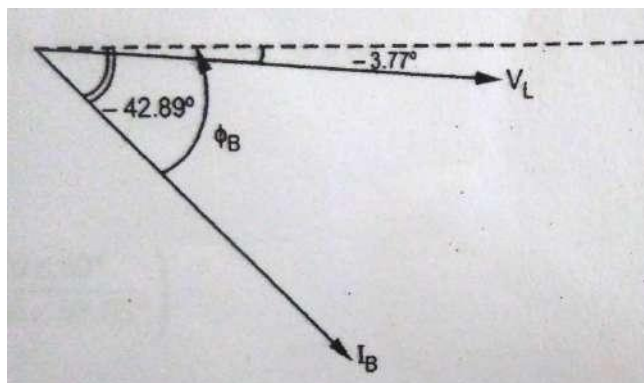
$$\begin{aligned} \phi_A &= (-38.84^0) - (-3.77^0) = -35.07^0 \\ \therefore \text{p.f.} &= \cos \phi_A = \cos (35.07) \\ &= \mathbf{0.8184 \text{ (lagging)}} \end{aligned}$$

The angle between  $V_L$  and  $I_B$  can be Calculated as,

$$\begin{aligned} \phi_B &= (-42.89^0) - (-3.77^0) = -39.12^0 \\ \text{p.f.} &= \cos \phi_B = \cos (39.12) \\ &= \mathbf{0.7758 \text{ (lagging)}} \end{aligned}$$



$$\begin{aligned}\text{Power output of transformer A} &= V_L I_A \cos \phi_A \\ &= 189.25 \times 42.196 \times 0.8184 \\ &= \mathbf{6535.40 \text{ W} = 6.5354 \text{ Kw}}\end{aligned}$$



$$\begin{aligned}\text{Power output of transformer B} &= V_L I_B \cos \phi_B \\ &= (189.25) (33.5534) (0.7758) \\ &= \mathbf{4926.31 \text{ W} = 49263 \text{ kW}}\end{aligned}$$

### Illustrative Examples on three phase transformers:

**Example 1:** An ideal 3- $\phi$  step down transformer connected in delta/star delivers power to a balanced 3 -  $\phi$  load of 120 Kva at 0.8 pf. The input line voltage is 11 Kv and the turn's ratio of transformer (Phase to Phase ) is 10. Determine the line voltage, line currents, and phase voltages, phase currents on both primary and secondary sides.

**Solution:**

$$\therefore K = \frac{N_1}{N_2} = 10 = \frac{V_{ph1}}{V_{ph2}} \quad \text{and } V_{ph1} = 11 \text{ Kv}$$

$$\therefore V_{ph2} = \frac{V_{ph1}}{10} = \frac{11 \times 10^3}{10} = 1100 \text{ V}$$

$$\therefore V_{L2} = \sqrt{3} V_{ph2} = 1.9052 \text{ kV}$$

Load  $VA = \sqrt{3} V_{L2} I_{L2}$  and is given as 120kVA

$$\text{i.e. } I_{L2} = \frac{120 \times 10^3}{\sqrt{3} \times 1.9052 \times 10^3}$$

$$\therefore I_{L2} = 36.36 \text{ A}$$

$$\therefore I_{ph2} = I_{L2} = 36.36 \text{ A}$$

$$K = \frac{I_{ph2}}{I_{ph1}} = 10 \text{ i.e. } I_{ph1} = \frac{36.36}{10} = 3.636 \text{ A}$$

$$\therefore I_{L1} = \sqrt{3} I_{ph1} = \sqrt{3} \times 3.636 = 6.298 \text{ A}$$

**Example 2:** A bank of three single phase transformers has its h.v. terminals connected to 3 wire, 3-phase, 11kV system. It's l.v. terminals are connected to a 3 wire, 3-phase load rated at 1500 kVA , 2200 V. specify the voltage, current and kVA ratings of each transformer for both h.v

and l.v windings for the following connections.

i) Y -  $\Delta$    ii)  $\Delta$  - Y   iii) Y - Y

**Solution:**

*The load is 1500 kVA hence the rating of each transformer is (1500/3) = 500 kVA and is same for all configurations. Since input is 11kV and output is 2200V, in all configurations the line voltages are also same on both primary side and secondary side. . i.e.  $V_{L1}=11\text{kV}$  and  $V_{L2} = 2,200 \text{ V}$ . The voltage and current ratings are specified on Phase basis only and hence lets us find out  $V_{Ph}$  and  $I_{Ph}$  on both HV side and LV side for the given four configurations.*

i) 11 kV Y and 2200 V Δ

$$V_{ph1} = \frac{11 \times 10^3}{\sqrt{3}} = 6350.8529 \text{ V}$$

$$V_{ph1} I_{ph1} = 500 \times 10^3 \quad \text{i.e. } I_{ph1} = 78.729 \text{ A}$$

$$V_{ph2} = 2200 \text{ V}$$

$$V_{ph2} I_{ph2} = 500 \times 10^3 \quad \text{i.e. } I_{ph2} = 227.2727 \text{ A}$$

ii) 11 kV Δ and 2200 V Y

$$V_{ph1} = 11 \text{ kV} \quad \text{and} \quad V_{ph2} = \frac{2200}{\sqrt{3}} = 1270.1705 \text{ V}$$

$$\therefore I_{ph1} = \frac{500 \times 10^3}{11 \times 10^3} = 45.45 \text{ A}$$

$$\therefore I_{ph2} = \frac{500 \times 10^3}{1270.1705} = 393.6479 \text{ A}$$

iii) 11 kV Y and 2200 V Y

$$V_{ph1} = \frac{11 \times 10^3}{\sqrt{3}} = 6350.8529 \text{ V}, \quad V_{ph2} = \frac{2200}{\sqrt{3}} = 1270.1705 \text{ V}$$

$$\therefore I_{ph1} = \frac{500 \times 10^3}{6350.8529} = 78.729 \text{ A}$$

$$\therefore I_{ph2} = \frac{500 \times 10^3}{1270.1705} = 393.6479 \text{ A}$$

**Example 3 :** A 3-Ø, 1200 kVA, 6.6/1.1kV transformer has Delta/Star connection. The per phase resistance is 2 Ω and 0.03 Ω on primary and secondary respectively. Calculate the efficiency on full load at 0.9 p.f. lagging, if iron losses are 20 kW.

**Solution :**  $V_{L1} = 6.6 \text{ kV}$ ,  $V_{L2} = 1.1 \text{ kV}$ ,  $V_{ph1} = V_{L1}$ ,  $V_{ph2} = V_{L2}/\sqrt{3}$

$$I_1(\text{FL}) = \frac{1200 \times 10^3}{\sqrt{3} V_{L1}} = \frac{1200 \times 10^3}{\sqrt{3} \times 6.6 \times 10^3} = 104.9727 \text{ A (line value)}$$

$$\therefore I_1(\text{Ph}) = \frac{I_1(\text{FL})}{\sqrt{3}} = 60.606 \text{ A}$$

$$K = (V_{Ph1}/V_{Ph2}) = (6.6 \times 10^3) / (1.1 \times 10^3/\sqrt{3}) = 10.393$$

$$\therefore R_{1e} = R_1 + R'_2 = R_1 + K^2 R_2 = 2 + (10.393)^2 (0.03) \times = 5.24 \Omega$$

$$\therefore P_{cu}(FL) = 3 \times [I_1(ph)]^2 \times R_{1e} = 3 \times (60.606)^2 \times 5.24 = 57740.9313 \text{ W}$$

$$\begin{aligned} \therefore \% \eta_{FL} &= \frac{VA \cos \Phi}{VA \cos \Phi + P_i + P_{cu}(FL)} \times 100 \\ &= \frac{1200 \times 10^3 \times 0.9}{1200 \times 10^3 \times 0.9 + 20 \times 10^3 + 57740.9313} \times 100 = \mathbf{93.285\%} \end{aligned}$$

**Example 4:** A 5000 kVA, 3 phase transformer 6.6 / 33 kV,  $\Delta$ / Y has a no load loss of 15 kW and full load of 50 kW. The impedance drop at full load is 7%. Calculate the primary voltage when a load of 3200 kW at 0.8 p.f. is delivered at 33 kV.

**Solution:** Secondary is star connected with  $V_{L2} = 33 \text{ kV}$ .

$$\therefore I_{L2} = \frac{VA}{\sqrt{3} V_{L2}} = \frac{5000 \times 10^3}{\sqrt{3} \times 33 \times 10^3} = 87.4773 \text{ A} = I_{ph2}$$

$$\text{Impedance drop per phase} = 7 \% \text{ of } V_{Ph2} = \frac{7}{100} \times \frac{33 \times 10^3}{\sqrt{3}} = 1333.6791 \text{ V}$$

$$\therefore Z_{2e} = \frac{1333.6791}{I_{ph2}} = \frac{1333.6791}{87.4773} = 15.2459 \Omega/\text{ph}$$

$$P_{cu}(FL) = \text{Total loss} - \text{No load loss} = 50 - 15 = 35 \text{ kW}$$

$$\text{But, } P_{cu}(FL) = 3 \times I_{2ph}^2 R_{2e} \text{ i.e. } R_{2e} = \frac{35 \times 10^3}{3 \times (87.4773)^2} = 1.524 \Omega$$

$$\therefore X_{2e} = \sqrt{Z_{2e}^2 - R_{2e}^2} = 15.169 \Omega/\text{ph}$$

When load is  $P_2 = 3200 \text{ kW}$ ,  $\cos \phi = 0.8$ ,  $V_{L2} = 33 \text{ kV}$

$$I_{L2} = \frac{P_2}{\sqrt{3}V_{L2} \cos\Phi} = \frac{3200 \times 10^3}{\sqrt{3} \times 33 \times 10^3 \times 0.8} = 69.98 \text{ A} = I_{2ph}$$

$$\begin{aligned} \therefore \% R &= \frac{I_{2ph} [R_{2e} \cos\Phi + X_{2e} \sin\Phi]}{V_{2ph}} \times 100 \\ &= \frac{69.98 [1.524 \times 0.8 + 15.169 \times 0.6]}{(33 \times 10^3 / \sqrt{3})} \times 100 = 3.79 \% \end{aligned}$$

Thus primary voltage must be increased by 3.79 % to maintain 33 kV at the secondary.

$$\therefore V_1 = 6.6 + 3.79 \% \text{ of } 6.6 = \mathbf{6.8501 \text{ kV}}$$

**Example 5 :** A 500 kVA, 3-phase, 50 Hz transformer has a voltage ratio (line voltage) of 33/11 kV and is delta/star connected. The resistances per phase are : high voltage 35  $\Omega$ , low voltage 0.876  $\Omega$  and the iron loss is 3050 W. Calculate the value of efficiency at full load and one-half of full load with 0.8 lagging power factor.

**Solution :** Since primary is Delta :  $V_{L1} = V_{ph1} = 33 \text{ kV}$

Since Secondary is Star:  $V_{L2} = 11 \text{ kV}$ ,  $V_{ph2} = \frac{11}{\sqrt{3}} = 6.35 \text{ kV}$

$$\therefore K = \frac{V_{ph1}}{V_{ph2}} = \frac{33}{6.35} = 5.1975$$

$$R_1 = 35 \Omega, R_2 = 0.876 \Omega$$

$$\therefore R_{1e} = R_1 + R'_2 = R_1 + K^2 R_2 = 58.6643 \Omega \quad (R'_2 = R_2 \text{ referred to primary})$$

$$I_1(\text{FL}) = \frac{VA}{\sqrt{3}V_{L1}} = \frac{500 \times 10^3}{\sqrt{3} \times 33 \times 10^3} = 8.7477 \text{ A} \quad \text{Therefore : } I_{1ph}(\text{FL}) = \frac{I(\text{FL})}{\sqrt{3}}$$

$$\begin{aligned}\therefore P_{cu}(FL) &= 3 \times I_{1ph}^2 (FL) \times R_{1e} \quad \text{and } I_{1ph} = \frac{I_1 (FL)}{\sqrt{3}} \\ &= 3 \times \left( \frac{8.7477}{\sqrt{3}} \right)^2 \times 58.6643 = 4489.1245 \text{ W}\end{aligned}$$

$$\begin{aligned}\therefore \% \eta_{FL} &= \frac{VA \cos \Phi}{VA \cos \Phi + P_i + P_{cu} (FL)} \times 100 \\ &= \frac{500 \times 10^3 \times 0.8}{500 \times 10^3 \times 0.8 + 3050 + 4489.12} \times 100 \\ &= \mathbf{98.15 \%}\end{aligned}$$

$$\begin{aligned}\% \eta_{HL} &= \frac{0.5 \times VA \cos \Phi}{0.5 \times VA \cos \Phi + P_i + [(0.5)^2 \times P_{cu} (FL)]} \times 100 \\ &= \frac{0.5 \times VA \cos \Phi}{0.5 \times VA \cos \Phi + 3050 + [(0.5)^2 \times 4489.12]} \times 100 = \mathbf{97.956 \%}\end{aligned}$$