POWER SYSTEM OPERATION AND CONTROL

(R18- R18A0214)

LECTURE NOTES

B.TECH

(III YEAR – II SEM)

(2019-20)

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MALLA REDDY COLLEGE OF ENGINEERING & TECHNOLOGY
(Autonomous Institution – UGC, Govt. of India)

(Affiliated to JNTU, Hyderabad, Approved by AICTE - Accredited by NBA & NAAC – 'A' Grade - ISO 9001:2015 Certified)

(R18A0214) POWER SYSTEM OPERATION AND CONTROL

Pre-requisites: Power System-I, Power System-II

COURSE OBJECTIVES:

- To give the knowledge on per unit system and faults.
- Give the knowledge of iterative method in power systems.
- To understand the concepts load flow studies.
- To understand Power systems stability
- To understand PF and Computer control in power systems

UNIT- I:

SYMMETRICAL COMPONENTS AND FAULT CALCULATIONS: Significance of positive, negative and zero sequence components, sequence impedances and sequence networks, fault calculations, sequence network equations, single line to ground fault, line to line fault, double line to ground fault, three phase faults, faults with fault impedance.

UNIT-II:

PER UNIT REPRESENTATION OF POWER SYSTEMS: The one line diagram, impedance and reactance diagrams, per unit quantities, changing the base of per unit quantities, advantages of per unit system.

LOAD FLOW STUDIES I: Bus Incidence Matrix, Y-bus formation by Direct Methods, Numerical Problems. Derivation of Static load flow equations. Load Flow Solutions Using Gauss Seidel Method: Load flow solution with and without P-V buses.

UNIT-III:

LOAD FLOW STUDIES II: Numerical Load flow Solution for Simple Power Systems (Max. 3-Buses): Determination of Bus Voltages, Injected Active and Reactive Powers (One Iteration only) and finding Line Flows/Losses for the given Bus Voltages, Newton Raphson Method: Load Flow Solution with and without P-V Buses, Derivation of Jacobian Elements, Fast Decoupled Method.

UNIT-IV:

POWER SYSTEM STABILITY: The stability problem – Concepts of Steady state stability, transient stability and Dynamic Stability - Swing equation. Equal area criterion of stability - Applications of Equal area criterion, Step by step solution of swing equation - Factors affecting transient stability.

UNIT-V:

PF CONTROL: Basics of speed governing mechanism and modeling – Control area concept, LFC control of a single area system, two area system.
COMPUTER CONTROL OF POWER SYSTEMS: System monitoring - data acquisition and control. System hardware configuration – SCADA and EMS functions.

TEXT BOOKS:

1. C.L.Wadhwa, Electrical Power Systems, 3rd Edn, New Age International Publishing Co., 2001.
2. D.P.Kothari and I.J.Nagrath, Modern Power System Analysis, 4th Edn, Tata McGraw Hill Education Private Limited2011.

REFERENCE BOOK:

1. D. P. Kothari: Modern Power System Analysis-Tata McGraw Hill Pub. Co. 2003.

COURSE OUTCOMES:

At the end of the course the student will be able to:

- Understand the concept of per unit system and faults in power systems.
- Evaluate the admittance matrix of a given power systems.
- Analyze the power system using iterative methods.
- Understand the concept of stability in power system.
- Understand the PF and computer control in power system.

UNIT-1

SYMMETRICAL COMPONENTS AND FAULT CALCULATIONS

1. INTRODUCTION

- A fault is any abnormal condition in a power system. The steady state operating mode of a power system is balanced 3-phase a.c. However, due to sudden external or internal changes in the system, this condition is disrupted.
- When the insulation of the system fails at one or more points or a conducting object comes into contact with a live point, a short circuit or a fault occurs.

1.1 CAUSES OF POWER SYSTEM FAULTS

The causes of faults are numerous, e.g.

- Lightning
- Heavy winds
- Trees falling across lines
- Vehicles colliding with towers or poles
- Birds shorting lines
- Aircraft colliding with lines
- Vandalism
- Small animals entering switchgear
- Line breaks due to excessive loading

1.2 COMMON POWER SYSTEM FAULTS

Power system faults may be categorized as one of four types; in order of frequency of occurrence, they are:

- Single line to ground fault
- Line to line fault
- Double line to ground fault
- Balanced three phase fault

The first three types constitutes severe unbalanced operating conditions which involves only one or two phases hence referred to as unsymmetrical faults. In the fourth type, a fault involving all the three phases occurs therefore referred to as symmetrical (balanced) fault.

1.3 EFFECTS OF POWER SYSTEM FAULTS

Faults may lead to fire breakout that consequently results into loss of property, loss of life and destruction of a power system network. Faults also leads to cut of supply in areas beyond the fault point in a transmission and distribution network leading to power blackouts; this interferes with industrial and commercial activities that supports economic growth, stalls learning activities in institutions, work in offices, domestic applications and creates insecurity at night.

1.4 SYMMETRICAL COMPONENTS

The majority of faults in power systems are asymmetrical. To analyze an asymmetrical fault, an unbalanced 3- phase circuit has to be solved. Since the direct solution of such a circuit is very difficult, the solution can be more easily obtained by using symmetrical components since this yields three (fictitious) single phase networks, only one of which contains a driving emf. Since the system reactances are balanced the thee fictitious networks have no mutual coupling between them, a fact that is making this method of analysis quite simple.

1.4.1 General principles

Any set of unbalanced 3-phase voltages (or current) can be transformed into 3 balanced sets. These are:

1. A positive sequence set of three symmetrical voltages (i.e. all numerically equal and all displaced from each other by $120^{0)}$ having the same phase sequence *abc* as the original set and denoted by V_{a1},V_{b1},V_{c1} as shown in the fig(1a)

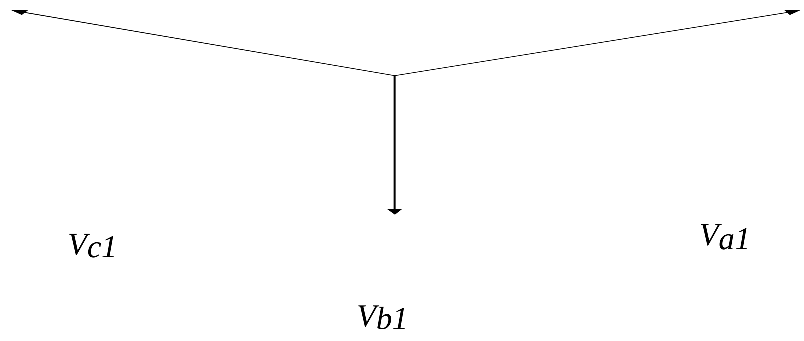


Fig. (a)

2. A negative sequence set of three symmetrical voltages having the phase sequence opposite to that of the original set and denoted by V_{a2}, V_{b2}, V_{c2} as shown in fig(1b)

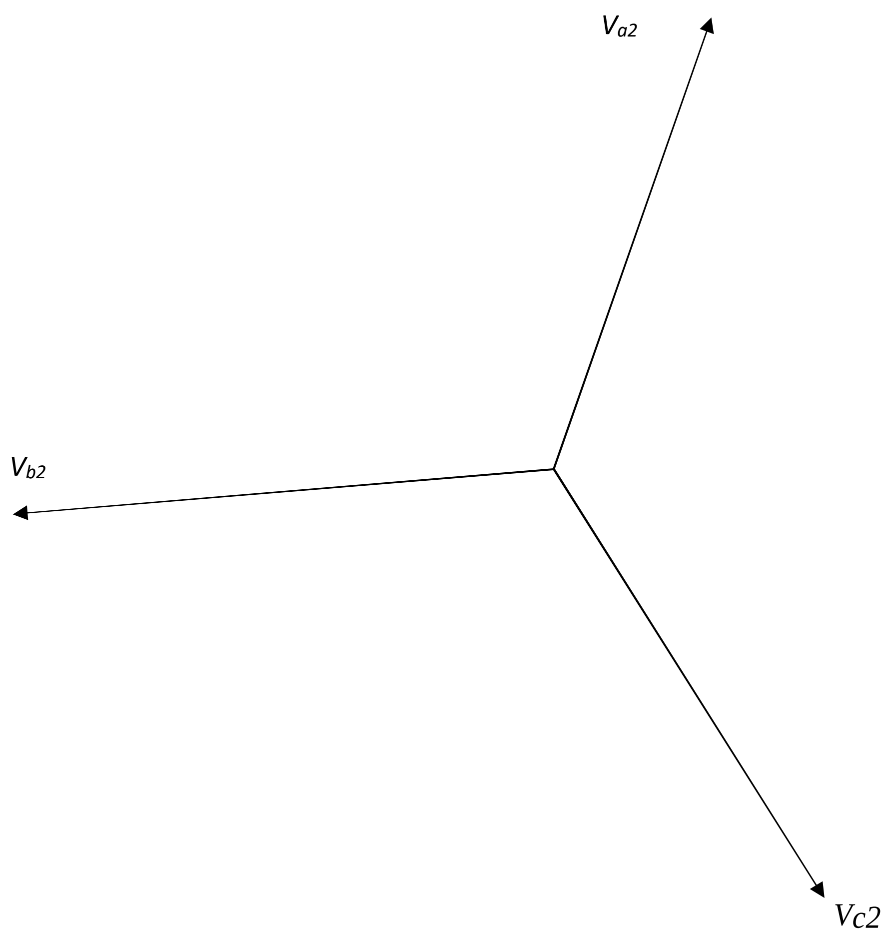


Fig. 1 (b)

3. A zero sequence set of three voltages, all equal in magnitude and in phase with each other and denoted by Va0, Vb0, Vc0 as shown in fig (1c) below:

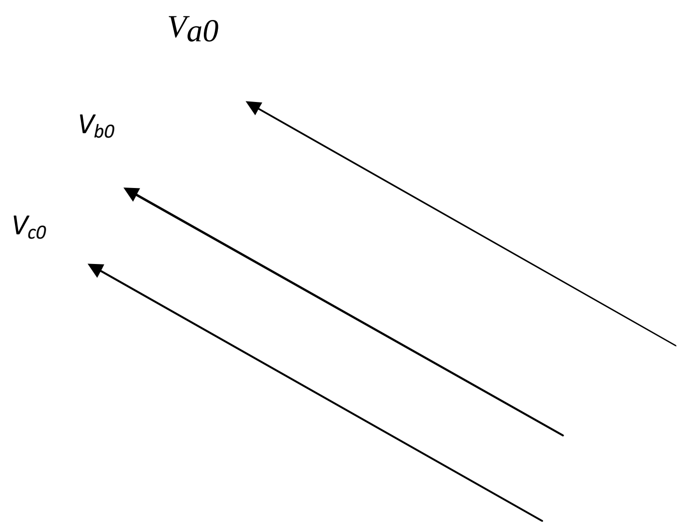


Fig. 1 (c)

The positive, negative and zero sequence sets above are known as symmetrical components. Thus we have,

$V_a = V_{a1} + V_{a2} + V_{a0}$

$V_b = V_{b1} + V_{b2} + V_{b0}$

$V_c = V_{c1} + V_{c2} + V_{c0}$

The symmetrical components application to power system analysis is of fundamental importance since it can be used to transform arbitrarily unbalanced condition into symmetrical components, compute the system response by straightforward circuit analysis on simple circuit models and transform the results back to the original phase variables.

Generally the subscripts 1, 2 and 0 are used to indicate positive sequence, negative sequence and zero sequence respectively.

The symmetrical components do not have separate existence; they are just mathematical components of unbalanced currents (or voltages) which actually flow in the system.

1.4.2 The "a" operator

The operator "a" as used in symmetrical components is one in which when multiplied to a vector, rotates the vector through 120^0 in a positive (anticlockwise) direction without changing the magnitude.

The operator "a" is defined as $1\angle 120^0$

1.5 THREE-SEQUENCE IMPEDANCES AND SEQUENCE NETWORKS

Positive sequence currents give rise to only positive sequence voltages, the negative sequence currents give rise to only negative sequence voltages and zero sequence currents give rise to only zero sequence voltages, hence each network can be regarded as flowing within in its own network through impedances of its own sequence only.

In any part of the circuit, the voltage drop caused by current of a certain sequence depends on the impedance of that part of the circuit to current of that sequence.

The impedance of any section of a balanced network to current of one sequence may be different from impedance to current of another sequence.

The impedance of a circuit when positive sequence currents are flowing is called impedance,

When only negative sequence currents are flowing the impedance is termed as negative sequence impedance.

With only zero sequence currents flowing the impedance is termed as zero sequence impedance.

The analysis of unsymmetrical faults in power systems is carried out by finding the symmetrical components of the unbalanced currents. Since each sequence current causes a voltage drop of that sequence only, each sequence current can be considered to flow in an independent network composed of impedances to current of that sequence only.

The single phase equivalent circuit composed of the impedances to current of any one sequence only is called the sequence network of that particular sequence.

The sequence networks contain the generated emfs and impedances of like sequence.

Therefore for every power system we can form three- sequence network s. These sequence networks, carrying current I_{a1}, I_{a2} and I_{a0} are then inter-connected to represent the different fault conditions.

1.6 PHYSICAL SIGNIFICANCE OF SEQUENCE COMPONENTS

This is achieved by considering the fields which results when these sequence voltages are applied to the stator of a 3-phase machine e.g. an induction motor.

If a positive sequence set of voltages is applied to the terminals a, b, c of the machine, a magnetic field revolving in a certain direction will be set up. If now the voltages to the terminals band c are changed by interchanging the leads to terminals b and c, it is known from induction motor theory that the direction of magnetic field would be reversed.

It is noted that for this condition, the relative phase positions of the voltages applied to the motor are the same as for the negative sequence set.

Hence, a negative sequence set of voltages produces a rotating field rotating in an opposite direction to that of positive sequence.

For both positive and negative sequence components, the standard convention of counter clockwise rotation is followed.

The application of zero sequence voltages does not produce any field because these voltages are in phase and the three -phase windings are displaced by 120^0.The positive and the negative sequence set are the balanced one. Thus, if only positive and negative sequence currents are flowing, the phasor sum of each will be zero and there will be no residual current. However, the zero sequence components of currents in the three phases are in phase and the residual current will be three times the zero sequence current of one phase. In the case of a fault involving ground, the positive and negative sequence currents are in equilibrium while the zero sequence currents flow through the ground and overhead ground wires.

SEQUENCE NETWORKS OF SYNCHRONOUS MACHINES

An unloaded synchronous machine having its neutral earthed through impedance, z_n, is shown in fig. 2(a) below.

A fault at its terminals causes currents I_a, I_b and I_c to flow in the lines. If fault involves earth, a current I_n flows into the neutral from the earth. This current flows through the neutral impedance Z_n.

Thus depending on the type of fault, one or more of the line currents may be zero.I_{a1}

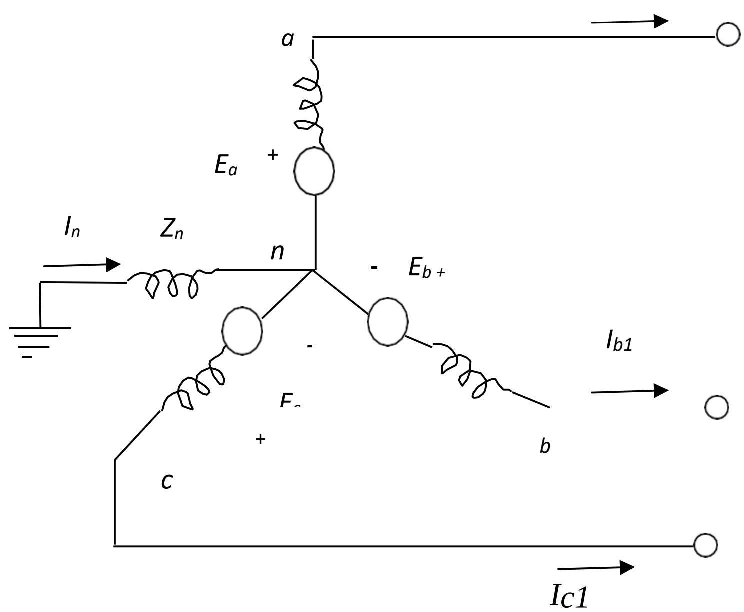


Fig.2 (a)

Positive sequence network

The generated voltages of a synchronous machine are of positive sequence only since the windings of a synchronous machine are symmetrical.

The positive sequence network consists of an emf equal to no load terminal voltages and is in series with the positive sequence impedance Z_1 of the machine. Fig.2 (b) and fig.2(c) shows the paths for positive sequence currents and positive sequence network respectively on a single phase basis in the synchronous machine. The neutral impedance Z_n does not appear in the circuit because the phasor sum of I_{a1}, I_{b1} and I_{c1} is zero and no positive sequence current can flow through Z_n. Since its a balanced circuit, the positive sequence N

The reference bus for the positive sequence network is the neutral of the generator.

The positive sequence impedance Z_1 consists of winding resistance and direct axis reactance. The reactance is the sub-transient reactance X''_d or transient reactance X'_d or synchronous reactance X_d depending on whether sub-transient, transient or steady state conditions are being studied.

From fig.2 (b), the positive sequence voltage of terminal *a* with respect to the reference bus is given by:

$V_{a1} = E_a - Z_1 I_{a1}$

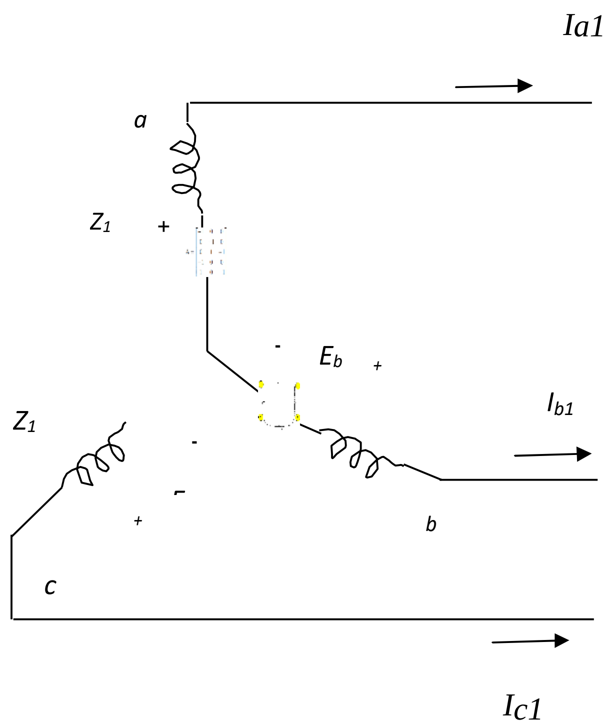


Fig.2 (b)

Reference bus

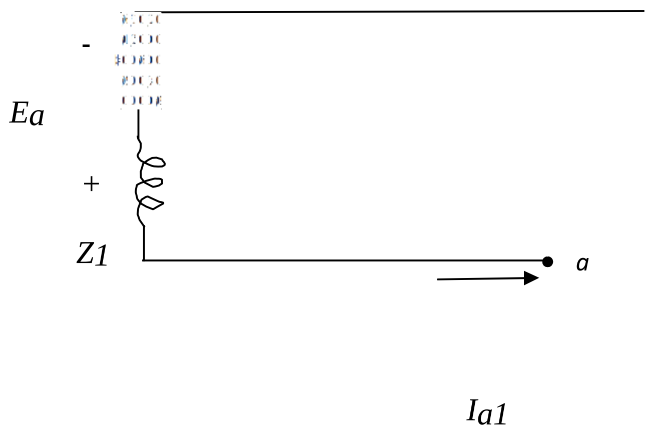


Fig.2(c)

2.02 Negative sequence network

A synchronous machine does not generate any negative sequence voltage. The flow of negative sequence currents in the stator windings creates an mmf which rotates at synchronous speed in a direction opposite to the direction of rotor, i.e., at twice the synchronous speed with respect to rotor.

Thus the negative sequence mmf alternates past the direct and quadrature axis and sets up a varying armature reaction effect. Thus, the negative sequence reactance is taken as the average of direct axis and quadrature axis sub-transient reactance, i.e.,

$X_2 = 0.5\,(X''_d + X''_q)$.

It not necessary to consider any time variation of X2 *during* transient conditions because there is no normal constant armature reaction to be effected. For more accurate calculations, the negative sequence resistance should be considered to account for power dissipated in the rotor poles or damper winding by double supply frequency induced currents.

The fig.2 (d) and fig.2 (e) shows the negative sequence currents paths and the negative sequence network respectively on a single phase basis of a synchronous machine.

The reference bus for the negative sequence network is the neutral of the machine. Thus, the negative sequence voltage of terminal *a* with respect to the reference bus is given by:

$V_{a2} = -Z_2 I_{a2}$

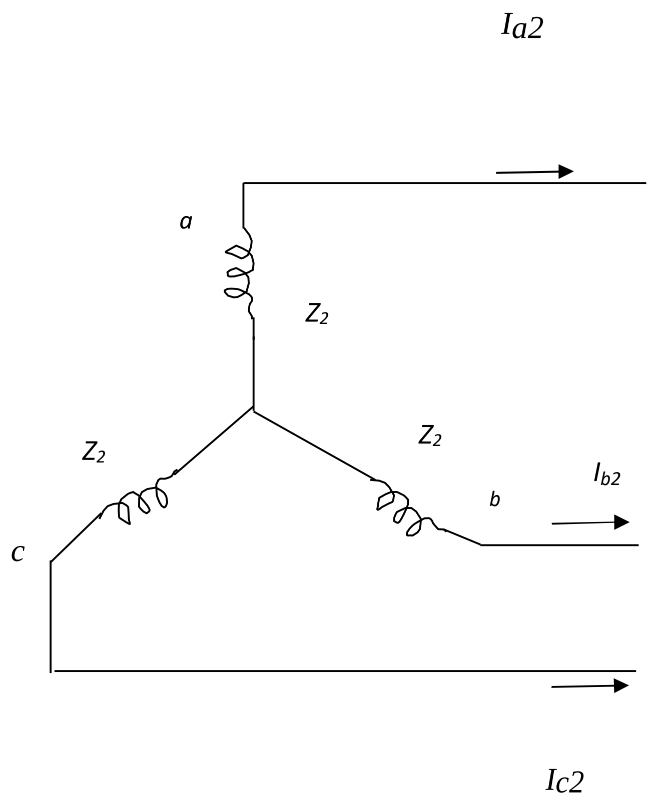


Fig.2 (d)

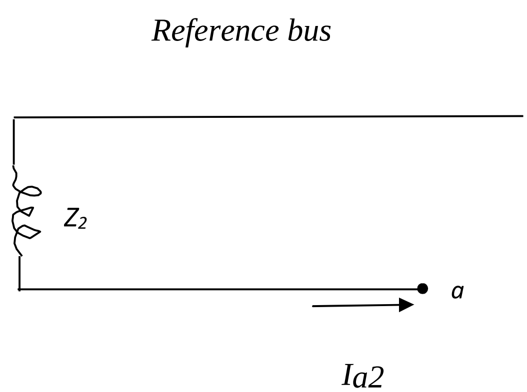


Fig.2 (e)

2.0.3 Zero sequence network

No zero sequence voltage is induced in a synchronous machine. The flow of zero sequence currents in the stator windings produces three mmf which are in time phase. If each phase winding produced a sinusoidal space mmf, then with the rotor removed, the flux at a point on the axis of the stator due to zero sequence current would be zero at every instant.

When the flux in the air gap or the leakage flux around slots or end connections is considered, no point in these regions is equidistant from all the three –phase windings of the stator.

The mmf produced by a phase winding departs from a sine wave, by amounts which depend upon the arrangement of the winding.

The zero sequence currents flow through the neutral impedance Z_n and the current flowing through this impedance is $3I_{a0}$.

Fig.2(f) and fig.2(g) shows the zero sequence current paths and zero sequence network respectively, and as can be seen, the zero sequence voltage drop from point *a* to ground is $-3I_{a0}Z_n - I_{a0}Z_{g0}$ where Z_{g0} is the zero sequence impedance per phase of the generator.

Since the current in the zero sequence network is I_{a0} this network must have an impedance of $3Z_n + Z_{g0}$. Thus,

$$Z_0 = 3Z_n + Z_{g0}$$

The zero sequence voltage of terminal *a* with respect to the reference bus is thus:

$$V_{a0} = -I_{a0}Z_0$$

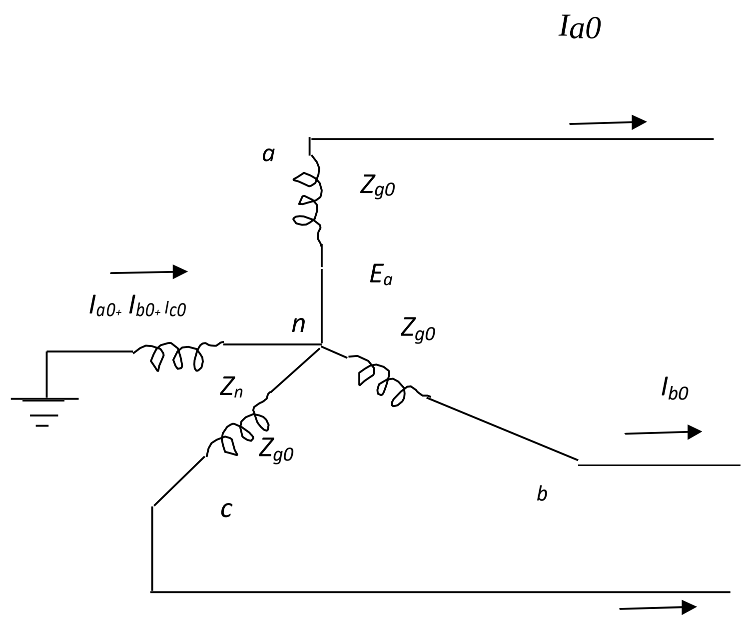


Fig.2 (f) Ic0

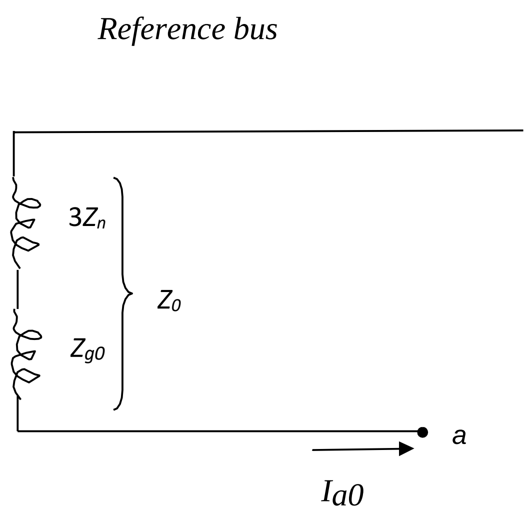


Fig.2 (g)

SEQUENCE IMPEDANCES OF TRANSMISSION LINE

The positive and negative sequence impedances of linear symmetrical static circuits do not depend on the phase sequence and are, therefore equal. When only zero sequence currents flow in the lines, the currents in all the phases are identical. These currents return partly through the ground and partly through overhead ground wires.

The magnetic field due to the flow of zero sequence currents through line, ground and round wires is very different from the magnetic field due to positive sequence currents. The zero sequence reactance of lines is about 2 to 4 times the positive sequence reactance.

SEQUENCE IMPEDANCES OF TRANSFORMERS

A power system network has a number of transformers for stepping up and stepping down the voltage levels.

A transformer for a 3-phase circuit may consist of three single phase transformers with windings suitably connected in star or delta or it may be a 3-phase unit.

Modern transformers are invariably three-phase units because of their lower cost, lesser space requirements and higher efficiency. The positive sequence impedance of a transformer equals its leakage impedance. The resistance of the windings is usually small as compared to leakage reactance.

For transformers above 1 MVA rating, the reactance and impedance are almost equal. Since the transformer is a static device, the negative sequence impedance is equal to the positive sequence impedance.

The zero sequence impedance of 3-phase units is slightly different from positive sequence impedance. However the difference is very slight and the zero sequence impedance is also assumed to be the same as the positive sequence impedance.

The flow of zero sequence currents through a transformer and hence in the system depends greatly on the winding connections. The zero sequence currents can flow through the winding connected in star only if the star point is grounded. If the star point isolated zero sequence currents cannot flow in the winding.

The zero sequence currents cannot flow in the lines connected to a delta connected winding because no return path is available for these zero sequence currents. However, the zero sequence currents caused by the presence of zero sequence voltages can circulate through the delta connected windings.

FORMATION OF SEQUENCE NETWORKS

A power system network consists of synchronous machines, transmission lines and transformers.

The positive sequence network is the same as the single line reactance diagram used for the calculation of symmetrical fault current. The reference bus for positive sequence network is the system neutral.

The negative sequence network is similar to the positive sequence network except that the negative sequence network does not contain any voltage source. The negative sequence impedances for transmission line and transformers are the same as the positive sequence impedances. But the negative sequence impedance of a synchronous machine may be different from its positive sequence impedance.

Any impedance connected between a neutral and ground is not included in the positive and negative sequence networks because the positive and the negative sequence currents cannot flow through such impedance.

The zero sequence network also does not contain any voltage source. Any impedance included between neutral and ground becomes three times its value in a zero sequence network.

The following are the summary of the rules for the formation of sequence networks:-

- The positive sequence network is the same as single line impedance or reactance diagram used in symmetrical fault analysis. The reference bus for this network is the system neutral.

- The generators in power system produce balanced voltages. Therefore only positive sequence network has voltage source. There are no voltage sources in negative and zero sequence networks.
- The positive sequence current can cause only positive sequence voltage drop. Similarly negative sequence current can cause only negative sequence voltage drop and zero sequence current can cause only zero sequence voltage drop.
- The reference for negative sequence network is the system neutral. However, the reference for zero sequence network is the ground. Zero sequence current can flow only if the neutral is grounded.
- The neutral grounding impedance Z_n appears as $3Z_n$ in the zero sequence network.
- The three sequence networks are independent and are interconnected suitably depending on the type of fault.

UNSYMMETRICAL FAULTS

The basic approach to the analysis of unsymmetrical faults is to consider the general situation shown in the fig.3.0 which shows the three lines of the three- phase power system at the point of fault.

The general terminals brought out are for purposes of external connections which simulate the fault. Appropriate connections of the three stubs represent the different faults, e.g., connecting stub `*a'* to ground produces a single line to ground fault, through zero impedance, on phase

`*a'*. The currents in stubs *b* and *c* are then zero and I_a is the fault current.

Similarly, the connection of stubs b and *c* produces a line to line fault, through zero impedance, between phases *b* and *c*, the current in stub *a* is then zero and I_b is equal to I_c.The positive assignment of phase quantities is important. It is seen that the currents flow out of the system.

The three general sequence circuits are shown in fig.3.1 (a). The ports indicated correspond to the general 3- phase entry port of fig.3.1. A suitable inter- connection of the three- sequence networks depending on the type fault yields the solution to the problem.

The sequence networks of fig.3.1 (a) can be replaced by equivalent sequence networks of fig.3.1 (b) . Z_0, Z_1 and Z_2 indicate the sequence impedances of the network looking into the fault.

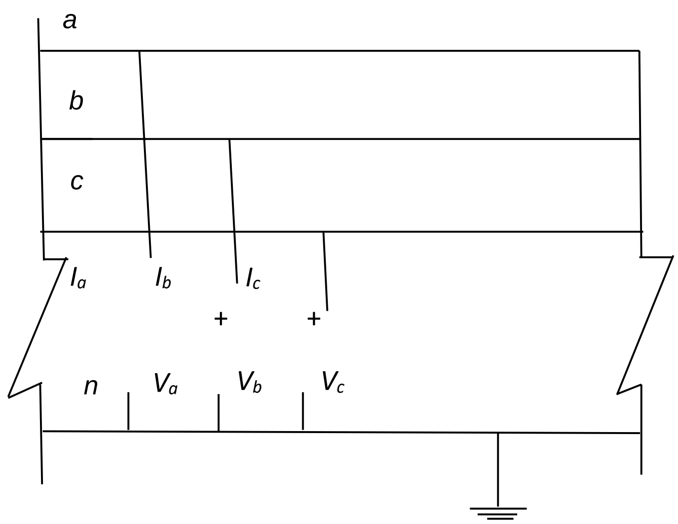


Fig.3.0 General 3- phase access port

General sequence networks

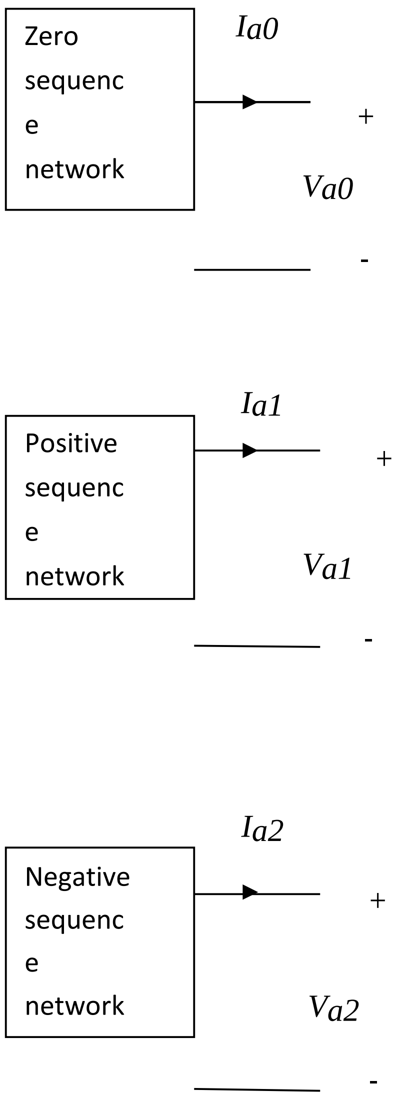


Fig.3.1 (a)

Equivalent sequence networks

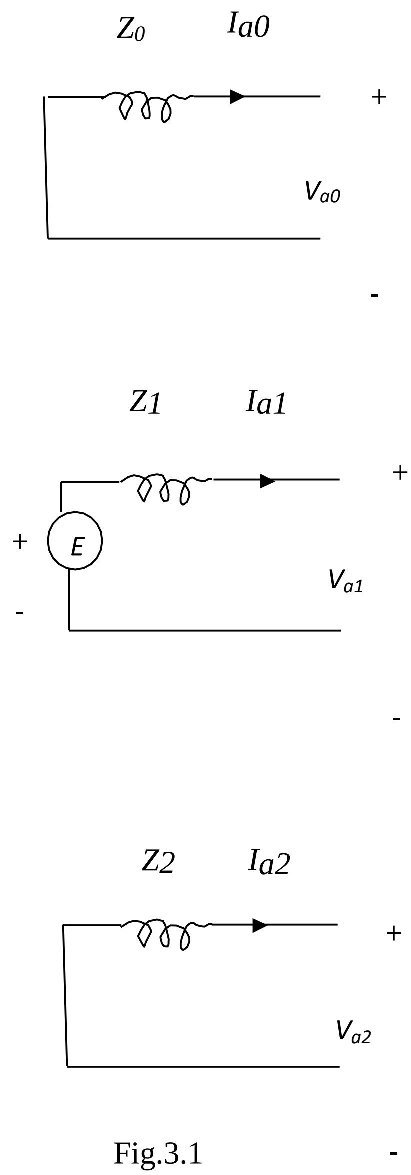


Fig.3.1 (b)

SINGLE LINE TO GROUND FAULT

The termination of the three- phase access port as shown in fig. 3.2 brings about a condition of single line to ground fault through a fault impedance Zf .

Typically Z_f is set to zero in all fault studies. I include Z_f in the analysis for the sake of generality. The terminal conditions at the fault point give the following equations:

$I_b = 0$

$I_c = 0$

$V_a = I_a Z_f$

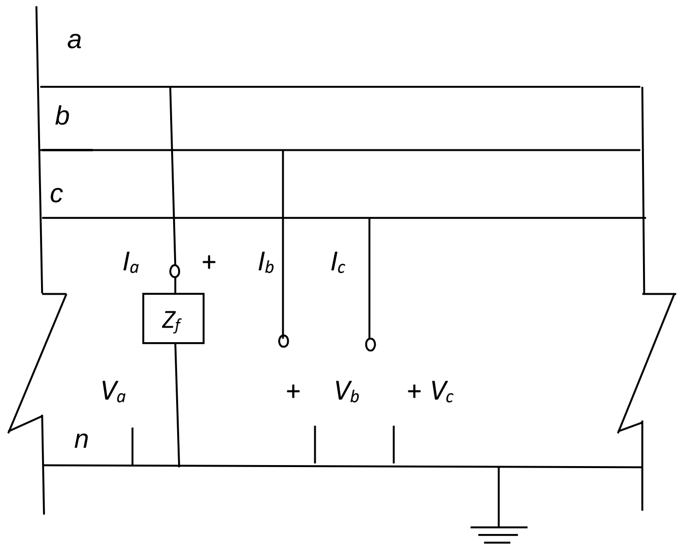


Fig. 3.2

Connections of sequence networks for a single line to ground fault and its simplified equivalent circuit are shown in the fig. 3.3(a) and fig. 3.3 (b) below:

General sequence networks

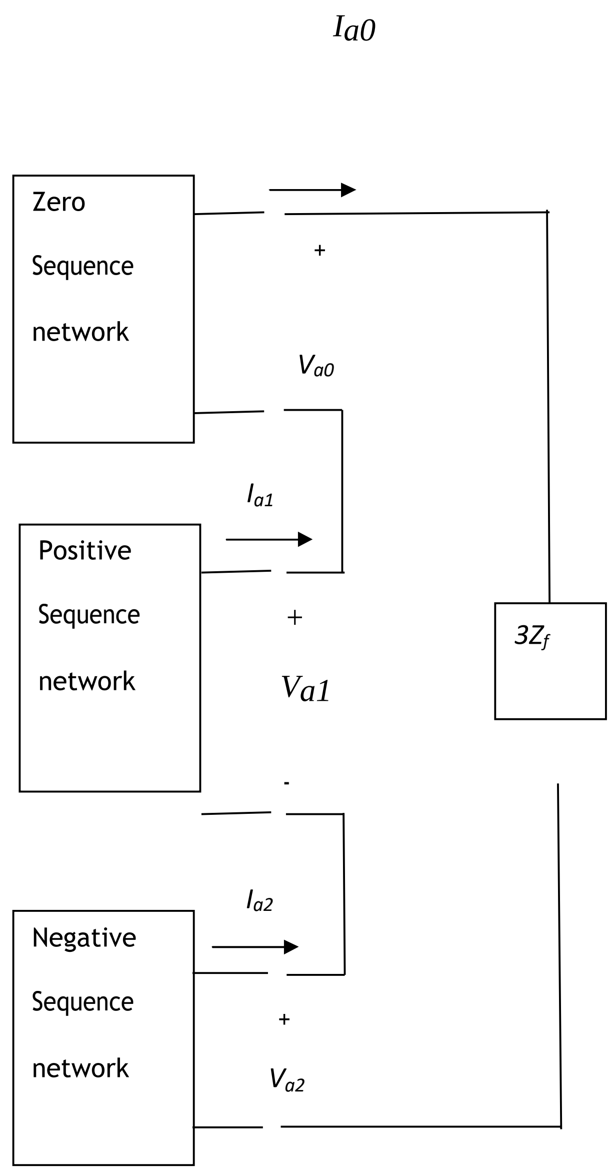


Fig.3.3 (a)

Equivalent sequence networks

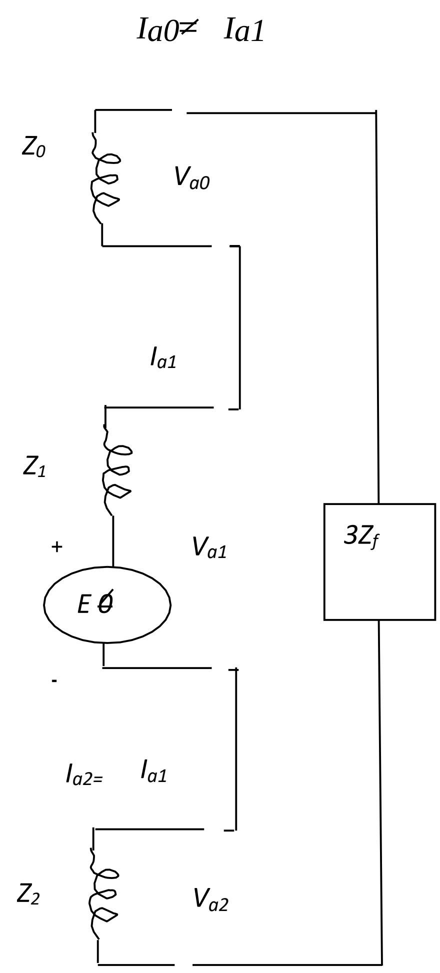


Fig.3.3 (b)

LINE TO LINE FAULT

The termination of the three- phase access port as in the fig.3.4 below simulates a line to line fault through a fault impedance Z_f .

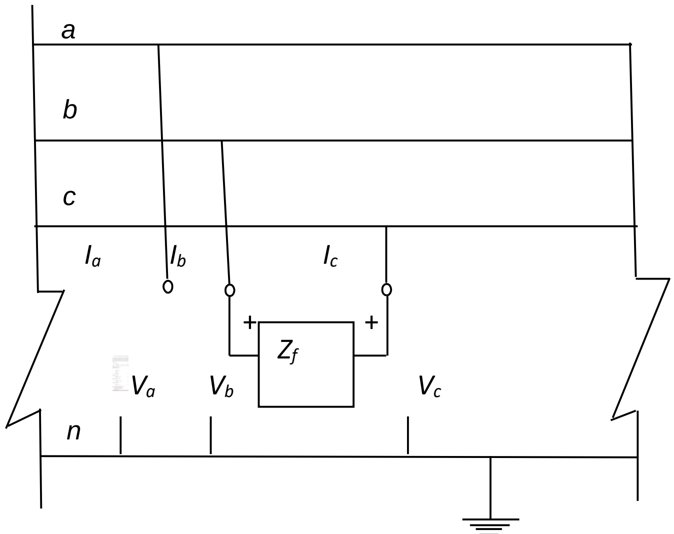


Fig. 3.4

The terminal conditions at the fault point give the following equations,

$I_a = 0$

$I_b = -I_c$

$V_b = V_c + Z_f I_b$

$I_b = -I_c = I_{a0} + a^2 I_{a1} + a I_{a2}$

Connection of sequence networks for a line to line fault and its simplified equivalent circuit are shown in the fig.3.5 (a) and fig.(b) below.

Equivalent sequence networks

General sequence networks

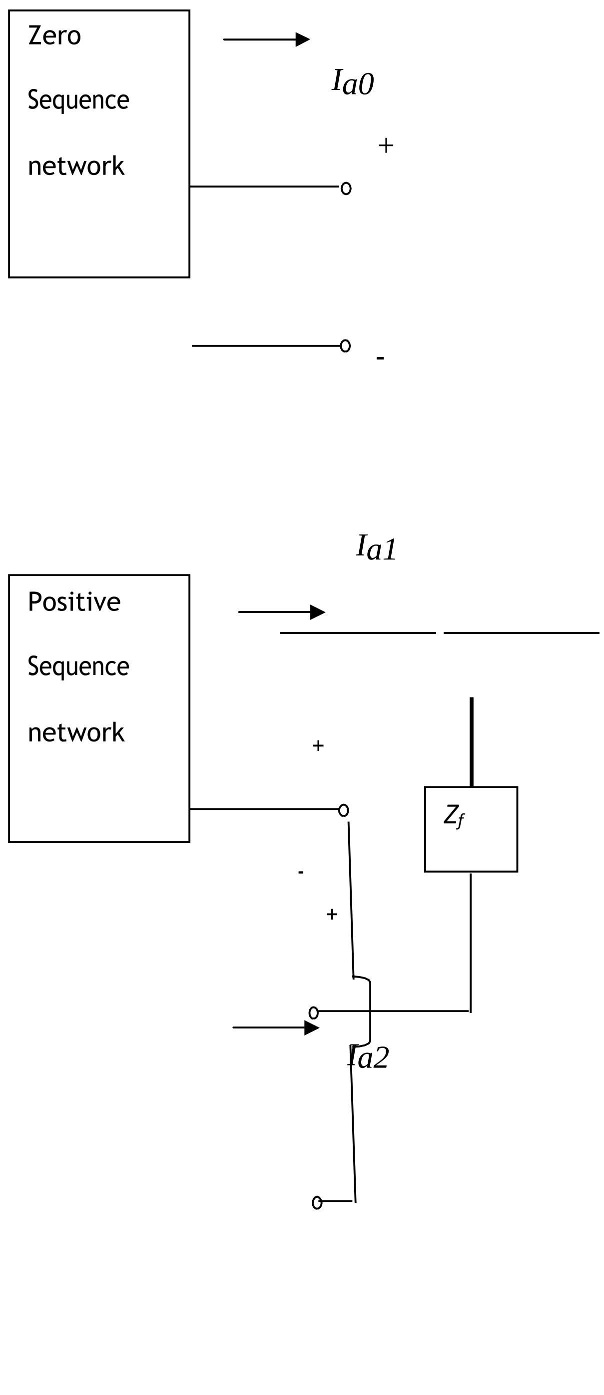


Fig. 3.5 (a)

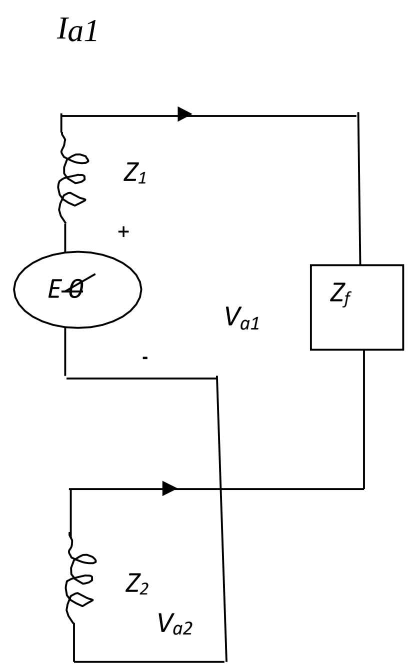


Fig.3.5 (b)

DOUBLE LINE TO GROUND FAULT

The termination of the three- phase access port as shown in fig.3.6 simulates a double line to ground fault through fault impedance Z_f.

The terminal conditions at the fault point give the following equations,

$I_a = 0$

$V_b = V_c = (I_b + I_c) Z_f$

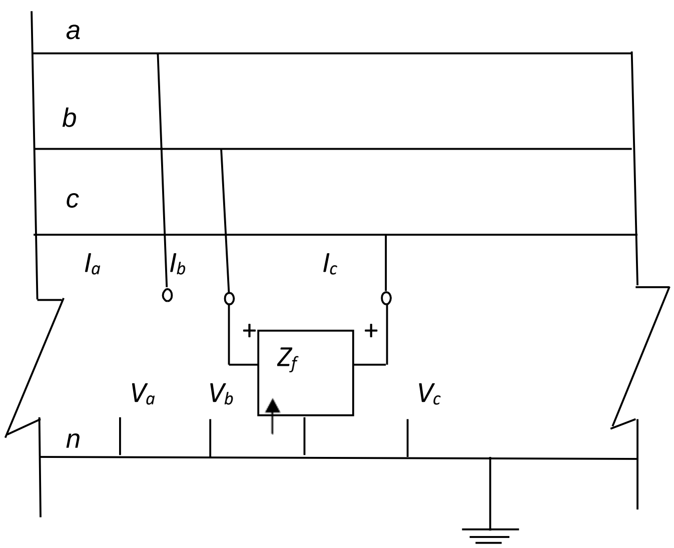


Fig. 3.6

The sequence networks and the equivalent circuit are shown by the Fig.3.7 (a) and Fig. 3.7 (b) below

General sequencenetworks networks

Equivalent sequence

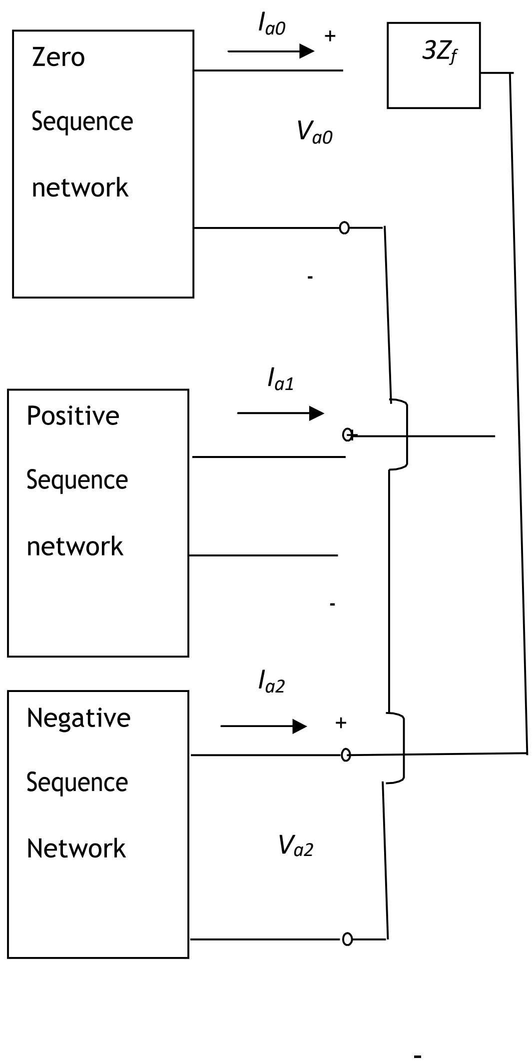


Fig. 3.7(a)

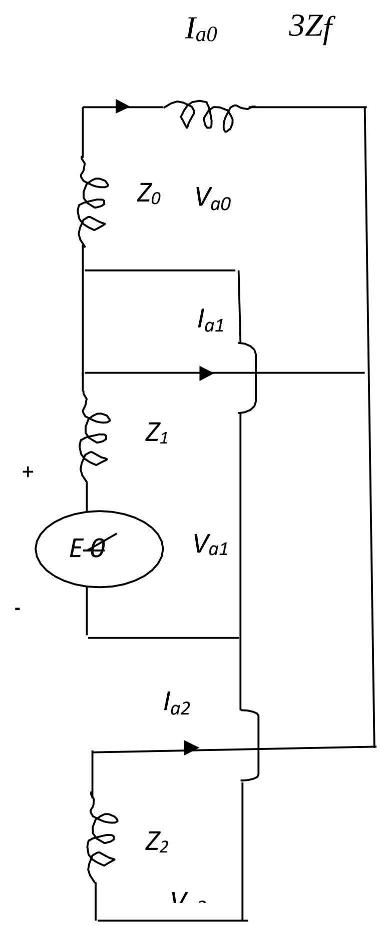


Fig.3.7 (b)

UNIT-II

(PART-I)

PER UNIT REPRESENTATION OF POWER SYSTEMS

One Line Diagram

In practice, electric power systems are very complex and their size is unwieldy. It is very difficult to represent all the components of the system on a single frame. The complexities could be in terms of various types of protective devices, machines (transformers, generators, motors, etc.), their connections (star, delta, etc.), etc. Hence, for the purpose of power system analysis, a simple single phase equivalent circuit is developed called, the one line diagram (OLD) or the single line diagram (SLD). *An SLD is thus, the concise form of representing a given power system.* It is to be noted that a given SLD will contain only such data that are relevant to the system analysis/study under consideration. For example, the details of protective devices need not be shown for load flow analysis nor it is necessary to show the details of shunt values for stability studies.

Symbols used for SLD

Various symbols are used to represent the different parameters and machines as single phase equivalents on the SLD,. Some of the important symbols used are as listed in the table of Figure 1.

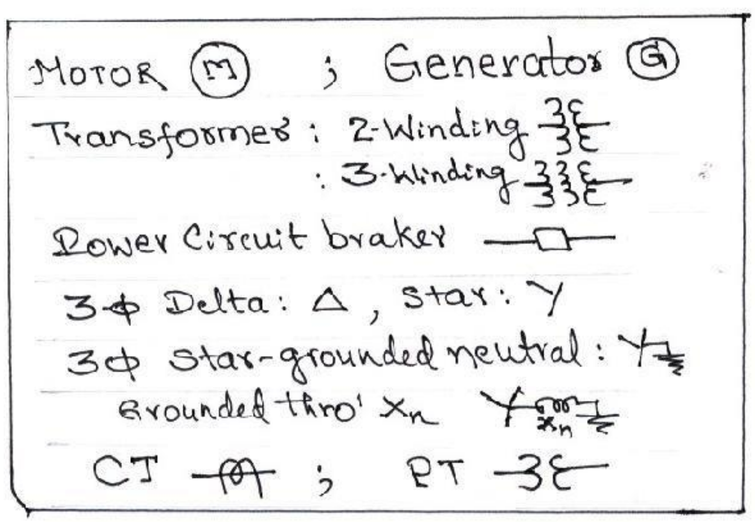


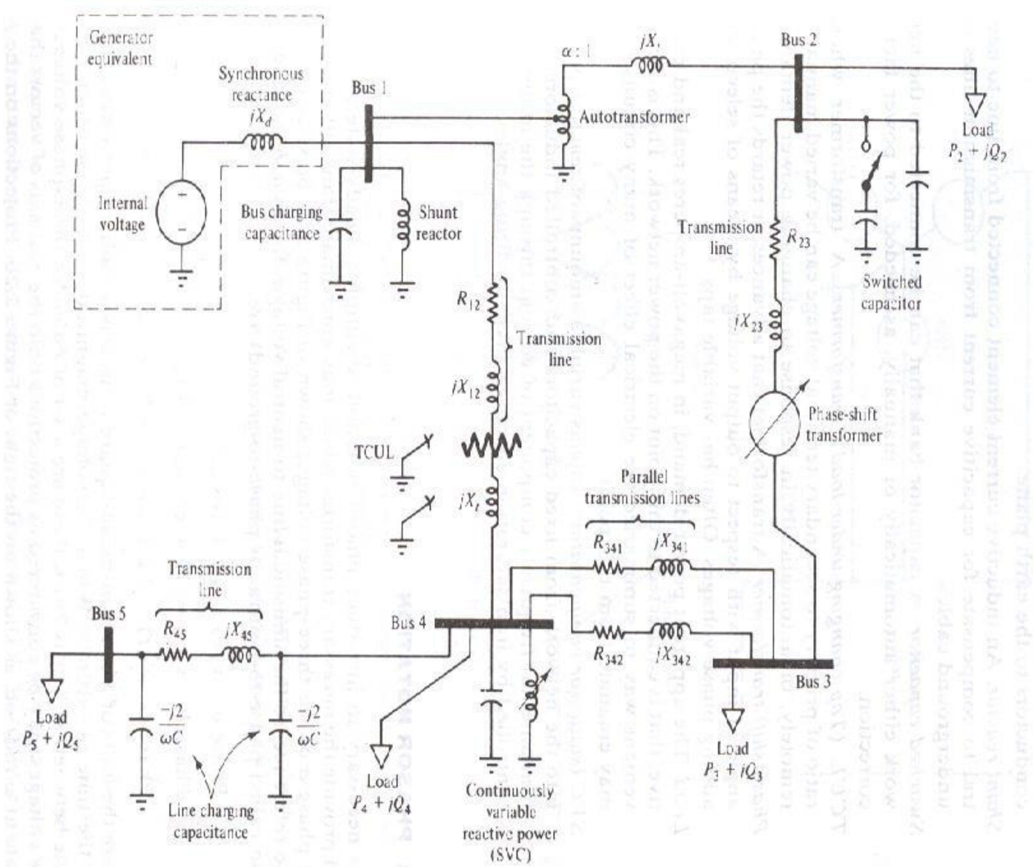
Figure 1. TABLE OF SYMBOLS FOR USE ON SLDS

Example system

Consider for illustration purpose, a sample example power system and data as under:

Generator 1: 30 MVA, 10.5 KV, X"= 1.6 ohms, Generator 2: 15 MVA, 6.6 KV, X"=

ohms, Generator 3: 25 MVA, 6.6 KV, X"= 0.56 ohms, Transformer 1 (3-phase): 15 MVA, 33/11 KV, X=15.2 ohms/phase on HT side, Transformer 2 (3-phase): 15 MVA, 33/6.2 KV, X=16.0 ohms/phase on HT side, Transmission Line: 20.5 ohms per phase, Load A: 15 MW, 11 KV, 0.9 PF (lag); and Load B: 40 MW, 6.6 KV, 0.85 PF (lag). The corresponding SLD incorporating the standard symbols can be shown as in figure 2.



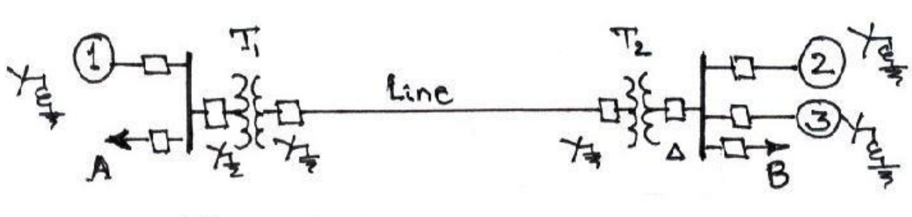


Figure 2. SAMPLE SYSTEM OLD

It is observed here, that the generators are specified in 3-phase MVA, L-L voltage and per phase Y-equivalent impedance, transformers are specified in 3-phase MVA, L-L voltage transformation ratio and per phase Y-equivalent impedance on any one side and the loads are specified in 3-phase MW, L-L voltage and power factor.

Impedance Diagram

The impedance diagram on single-phase basis for use under balanced conditions can be easily drawn from the SLD. The following assumptions are made in obtaining the impedance diagrams.

Assumptions:

1. The single phase transformer equivalents are shown as ideals with impedances on appropriate side (LV/HV),
2. The magnetizing reactances of transformers are negligible,
3. The generators are represented as constant voltage sources with series resistance or reactance,
4. The transmission lines are approximated by their equivalent π-Models,
5. The loads are assumed to be passive and are represented by a series branch of resistance or reactance and
6. Since the balanced conditions are assumed, the neutral grounding impedances do not appear in the impedance diagram.

Example system

As per the list of assumptions as above and with reference to the system of figure 2, the impedance diagram can be obtained as shown in figure 3.

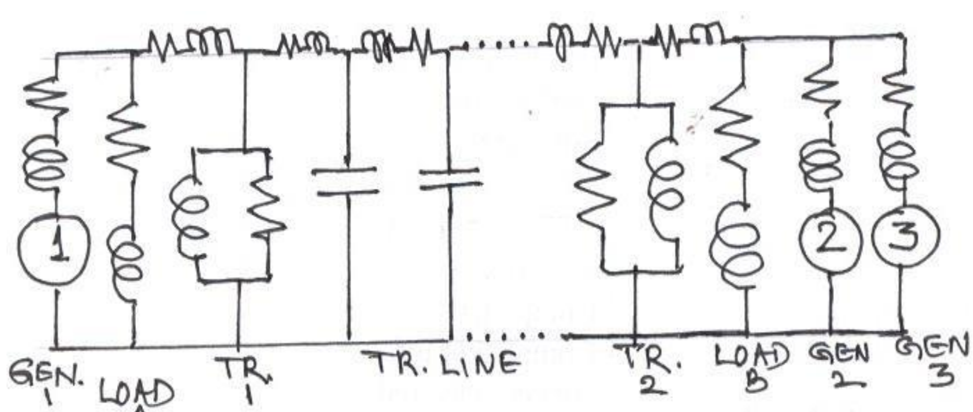


Figure 3. IMPEDANCE DIAGRAM

Reactance Diagram

With some more additional and simplifying assumptions, the impedance diagram can be simplified further to obtain the corresponding reactance diagram. The following are the assumptions made.

Additional assumptions:

- The resistance is often omitted during the fault analysis. This causes a very negligible error since, resistances are negligible
- Loads are Omitted
- Transmission line capacitances are ineffective &
- Magnetizing currents of transformers are neglected.

Example system

as per the assumptions given above and with reference to the system of figure 2 and figure 3, the reactance diagram can be obtained as shown in figure 4.

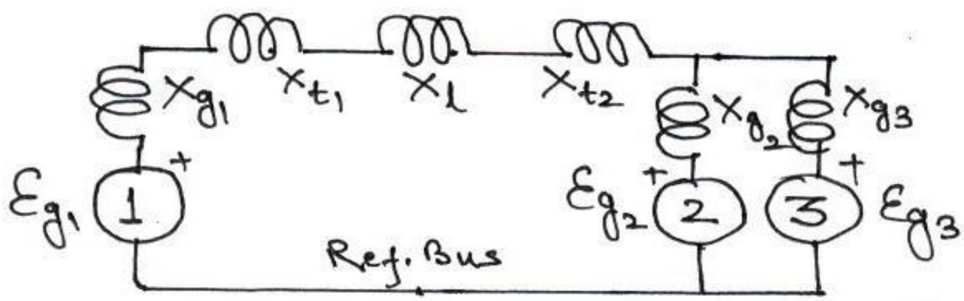


Figure 4. REACTANCE DIAGRAM

Note: These impedance & reactance diagrams are also refered as the *Positive Sequence Diagrams/ Networks.*

Per Unit Quantities

during the power system analysis, it is a usual practice to represent current, voltage, impedance, power, etc., of an electric power system in per unit or percentage of the base or reference value of the respective quantities. The numerical per unit (pu) value of any quantity is its ratio to a chosen base value of the same dimension. Thus a pu value is a normalized quantity with respect to the chosen base value.

Definition: Per Unit value of a given q*uantity is the ratio of the* ***actual value*** *in any given unit to the* ***base value*** *in the same unit.* The percent value is 100 times the pu value. Both the pu and percentage methods are simpler than the use of actual values. Further, the main advantage in using the pu system of computations is that the result that comes out of the sum, product, quotient, etc. of two or more pu values is expressed in per unit itself.

In an electrical power system, the parameters of interest include the current, voltage, complex power (VA), impedance and the phase angle. Of these, the phase angle is dimensionless and the other four quantities can be described by knowing any two of them. Thus clearly, an arbitrary choice of any two base values will evidently fix the other base values.

Normally the nominal voltage of lines and equipment is known along with the complex power rating in MVA. Hence, in practice, the base values are chosen for complex power (MVA) and line voltage (KV). The chosen base MVA is the same for all the parts of the system. However, the base voltage is chosen with reference to a particular section of the system and the other base voltages (with reference to the other sections of the systems, these sections caused by the presence of the transformers) are then related to the chosen one by the turns-ratio of the connecting transformer.

If I_b is the base current in kilo amperes and V_b, the base voltage in kilovolts, then the base MVA is, $S_b = (V_b I_b)$. Then the base values of current & impedance are given by

$$\text{Base current (kA)}, I_b = MVA_b/KV_b = S_b/V_b \quad (1.1)$$

$$\text{Base impedance}, Zb = (V_b/I_b) = (KV_b^2 / MVA_b) \quad (1.2)$$

Hence the per unit impedance is given by

$$Z_{pu} = Z_{ohms}/Z_b = Z_{ohms} (MVA_b/KV_b^2) \quad (1.3)$$

In 3-phase systems, KV_b is the line-to-line value & MVA_b is the 3-phase MVA. [1-phase MVA = (1/3) 3-phase MVA].

Changing the base of a given pu value:

It is observed from equation (3) that the pu value of impedance is proportional directly to the base MVA and inversely to the square of the base KV. If Z_{pu}^{new} is the pu impedance required to be calculated on a new set of base values: MVA_b^{new} & KV_b^{new} from the already given per unit impedance Z_{pu}^{old}, specified on the old set of base values, MVA_b^{old} & KV_b^{old} , then we have

$$Z_{pu}^{new} = Z_{pu}^{old} (MVA_b^{new}/MVA_b^{old}) (KV_b^{old}/KV_b^{new})^2 \quad (1.4)$$

On the other hand, the change of base can also be done by first converting the given pu impedance to its ohmic value and then calculating its pu value on the new set of base values.

Merits and Demerits of pu System

Following are the advantages and disadvantages of adopting the pu system of computations in electric power systems:

Merits

- The pu value is the same for both 1-phase and & 3-phase systems
- The pu value once expressed on a proper base, will be the same when refereed to either side of the transformer. Thus the presence of transformer is totally eliminated
- The variation of values is in a smaller range 9nearby unity). Hence the errors involved in pu computations are very less.
- Usually the nameplate ratings will be marked in pu on the base of the name plate ratings, etc.

Demerits:

- If proper bases are not chosen, then the resulting pu values may be highly absurd (such as 5.8 pu, -18.9 pu, etc.). This may cause confusion to the user. However, this problem can be avoided by selecting the base MVA near the high-rated equipment and a convenient base KV in any section of the system.

pu Impedance / Reactance Diagram

for a given power system with all its data with regard to the generators, transformers, transmission lines, loads, etc., it is possible to obtain the corresponding impedance or reactance diagram as explained above. If the parametric values are shown in pu on the properly selected base values of the system, then the diagram is refered as the per unit impedance or reactance diagram. In forming a pu diagram, the following are the procedural steps involved:

1. Obtain the one line diagram based on the given data
2. Choose a common base MVA for the system
3. Choose a base KV in any one section (Sections formed by transformers)
4. Find the base KV of all the sections present
5. Find pu values of all the parameters: R,X, Z, E, etc.
6. Draw the pu impedance/ reactance diagram.

UNIT-II

PER UNIT REPRESENTATION OF POWER SYSTEMS

One Line Diagram

In practice, electric power systems are very complex and their size is unwieldy. It is very difficult to represent all the components of the system on a single frame. The complexities could be in terms of various types of protective devices, machines (transformers, generators, motors, etc.), their connections (star, delta, etc.), etc. Hence, for the purpose of power system analysis, a simple single phase equivalent circuit is developed called, the one line diagram (OLD) or the single line diagram (SLD). *An SLD is thus, the concise form of representing a given power system.* It is to be noted that a given SLD will contain only such data that are relevant to the system analysis/study under consideration. For example, the details of protective devices need not be shown for load flow analysis nor it is necessary to show the details of shunt values for stability studies.

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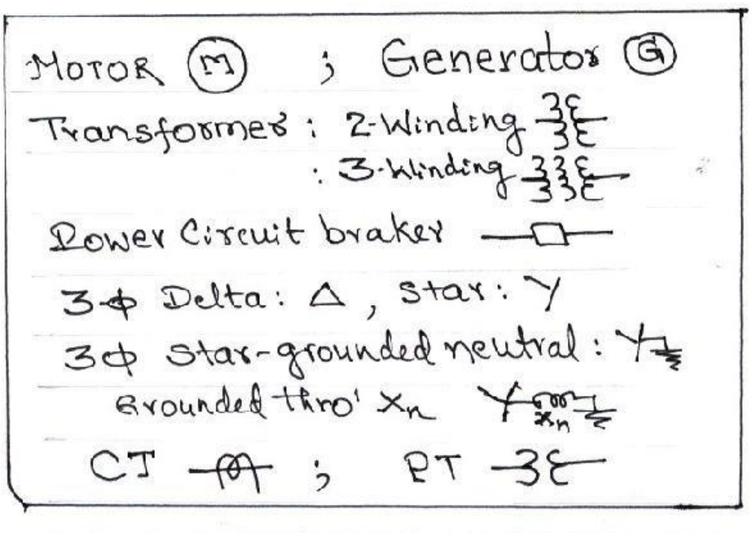

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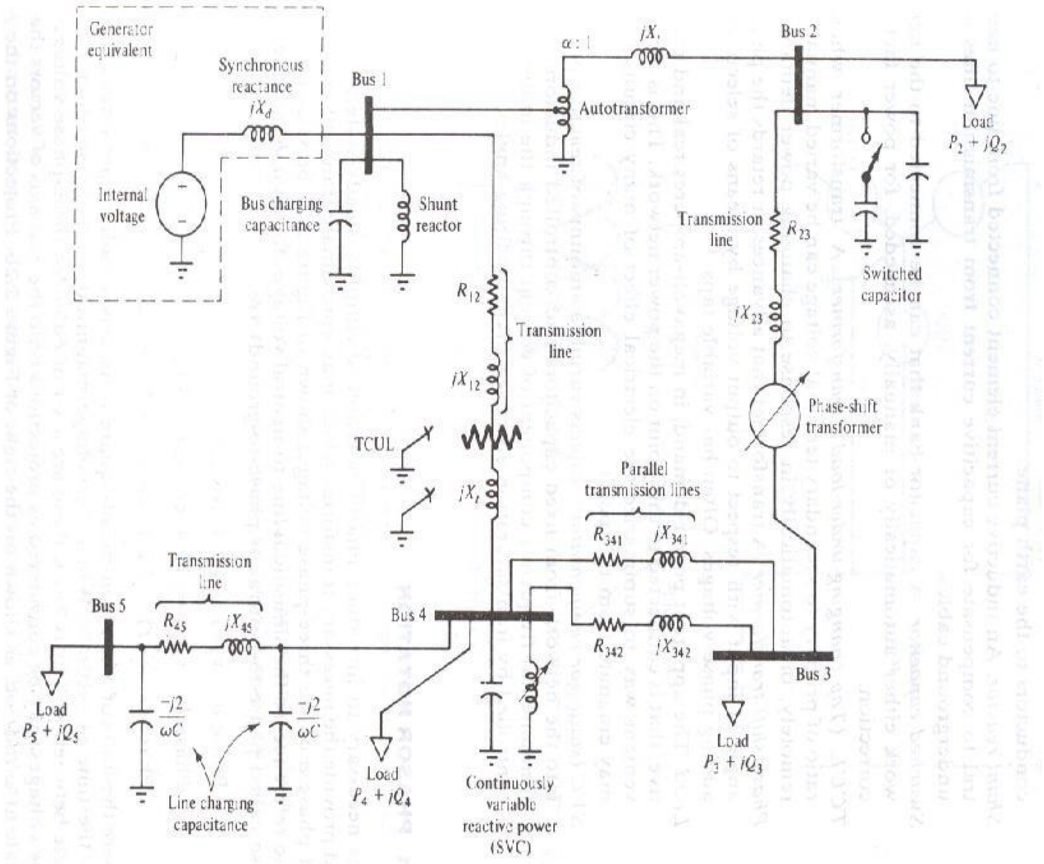

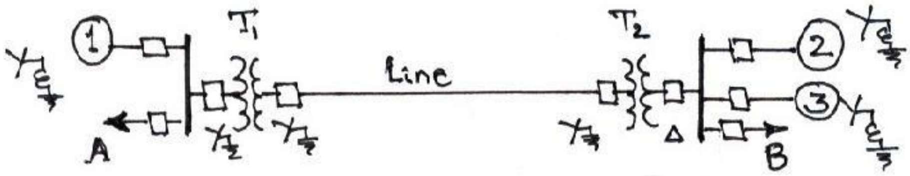

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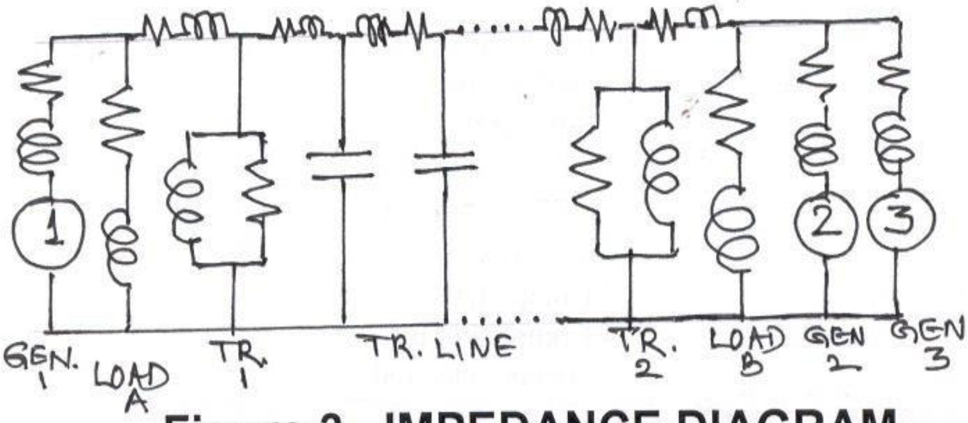

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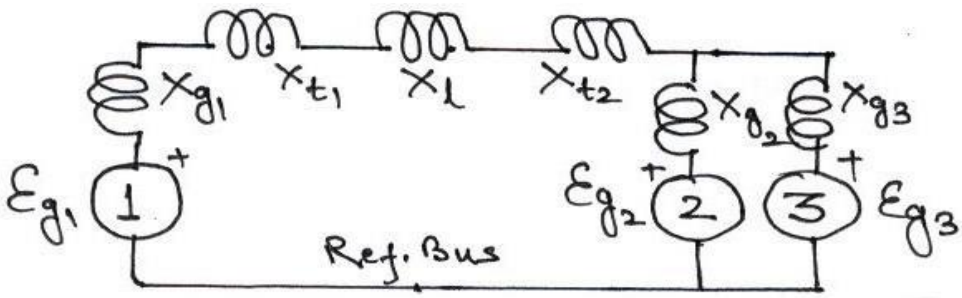

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4. Find the base KV of all the sections present
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LOAD FLOW STUDIES 1

FORMATION OF YBUS AND ZBUS

The bus admittance matrix, YBUS plays a very important role in computer aided power system analysis. It can be formed in practice by either of the methods as under:

1. Rule of Inspection
2. Singular Transformation
3. Non-Singular Transformation
4. ZBUS Building Algorithms, etc.

The performance equations of a given power system can be considered in three different frames of reference as discussed below:

Frames of Reference:

Bus Frame of Reference: There are b independent equations (b = no. of buses) relating the bus vectors of currents and voltages through the bus impedance matrix and bus admittance matrix:

EBUS = ZBUS IBUS
IBUS = YBUS EBUS

Branch Frame of Reference: There are b independent equations (b = no. of branches of a selected Tree sub-graph of the system Graph) relating the branch vectors of currents and voltages through the branch impedance matrix and branch admittance matrix:

EBR = ZBR IBR
IBR = YBR EBR

Loop Frame of Reference: There are b independent equations (b = no. of branches of a selected Tree sub-graph of the system Graph) relating the branch vectors of currents and voltages through the branch impedance matrix and branch admittance matrix:

ELOOP = ZLOOP ILOOP
ILOOP = YLOOP ELOOP

Of the various network matrices refered above, the bus admittance matrix (YBUS) and the bus impedance matrix (ZBUS) are determined for a given power system by the rule of inspection as explained next.

Rule of Inspection

Consider the 3-node admittance network as shown in figure5. Using the basic branch relation: I = (YV), for all the elemental currents and applying Kirchhoff‟s Current Law principle at the nodal points, we get the relations as under:

At node 1: I1 =Y1V1 + Y3 (V1-V3) + Y6 (V1 – V2)
At node 2: I2 =Y2V2 + Y5 (V2-V3) + Y6 (V2 – V1)

At node 3: 0 = Y3 (V3-V1) + Y4V3 + Y5 (V3 – V2) **(12)**

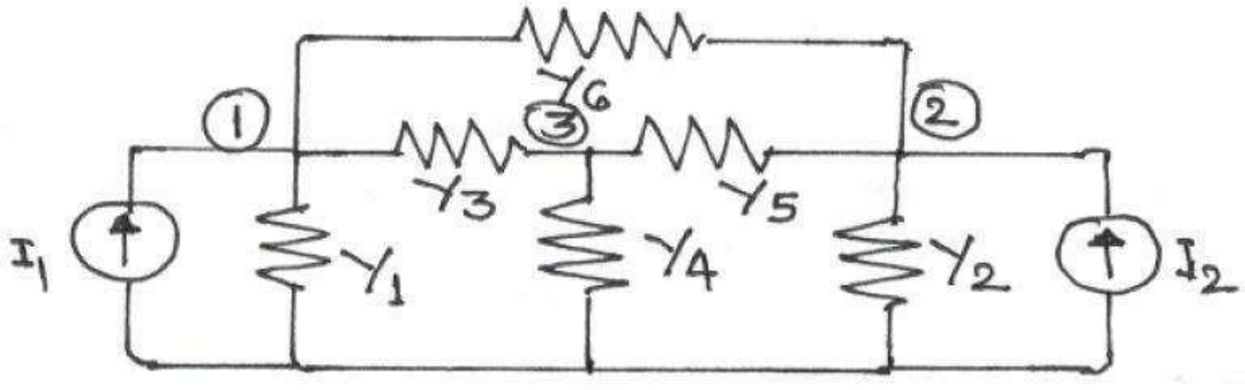


Fig. 3 Example System for finding YBUS

These are the performance equations of the given network in admittance form and they can be represented in matrix form as:

$$\begin{vmatrix} I_1 \\ I_2 \\ 0 \end{vmatrix} = \begin{vmatrix} (Y_1+Y_3+Y_6) & -Y_6 & -Y_3 \\ -Y_6 & (Y_2+Y_5+Y_6) & -Y_5 \\ -Y_3 & -Y_5 & (Y_3+Y_4+Y_5) \end{vmatrix} \begin{vmatrix} V_1 \\ V_2 \\ V_3 \end{vmatrix} \quad \textbf{(13)}$$

In other words, the relation of equation (9) can be represented in the form

IBUS = YBUS EBUS **(14)**

Where, YBUS is the bus admittance matrix, IBUS & EBUS are the bus current and bus voltage vectors respectively. By observing the elements of the bus admittance matrix, YBUS of equation (13), it is observed that the matrix elements can as well be obtained by a simple inspection of the given system diagram:

Diagonal elements: A diagonal element (Yii) of the bus admittance matrix, YBUS, is equal to the sum total of the admittance values of all the elements incident at the bus/node i,

Off Diagonal elements: An off-diagonal element (Yij) of the bus admittance matrix, YBUS, is equal to the negative of the admittance value of the connecting element present between the buses I and j, if any. This is the principle of the rule of inspection. Thus the algorithmic equations for the rule of inspection are obtained as:

Yii = S yij (j = 1,2,…….n)

Yij = - yij (j = 1,2,…….n) **(15)**

For i = 1,2,….n, n = no. of buses of the given system, yij is the admittance of element connected between buses i and j and yii is the admittance of element connected between bus i and ground (reference bus).

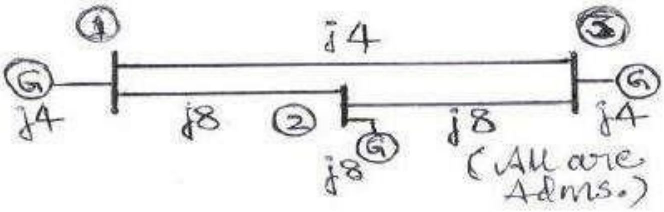
Bus impedance matrix

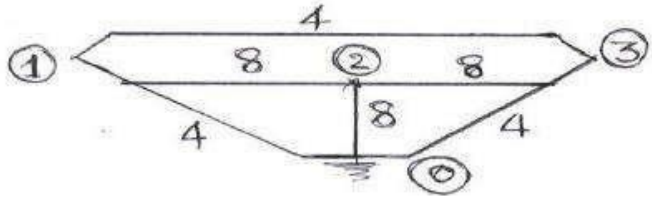
In cases where, the bus impedance matrix is also required, it cannot be formed by direct inspection of the given system diagram. However, the bus admittance matrix determined by the rule of inspection following the steps explained above, can be inverted to obtain the bus impedance matrix, since the two matrices are interinvertible.
Note: It is to be noted that the rule of inspection can be applied only to those power systems that do not have any mutually coupled elements.

Examples on Rule of Inspection:

Example 6: Obtain the bus admittance matrix for the admittance network shown aside by the rule of inspection

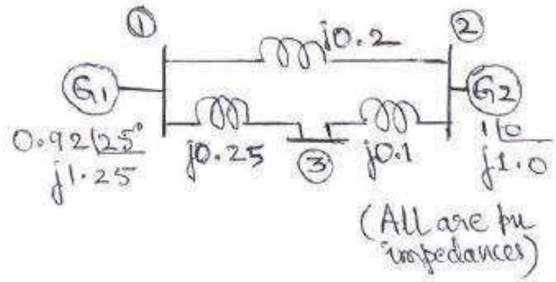
$$Y_{BUS} = j\begin{vmatrix} 16 & -8 & -4 \\ -8 & 24 & -8 \\ -4 & -8 & 16 \end{vmatrix}$$



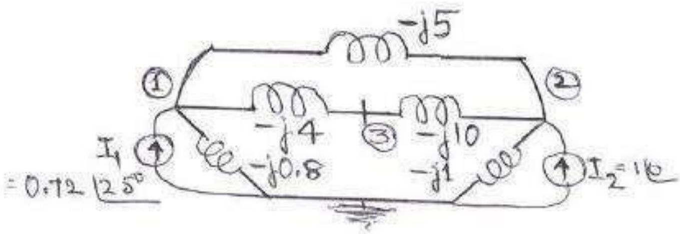


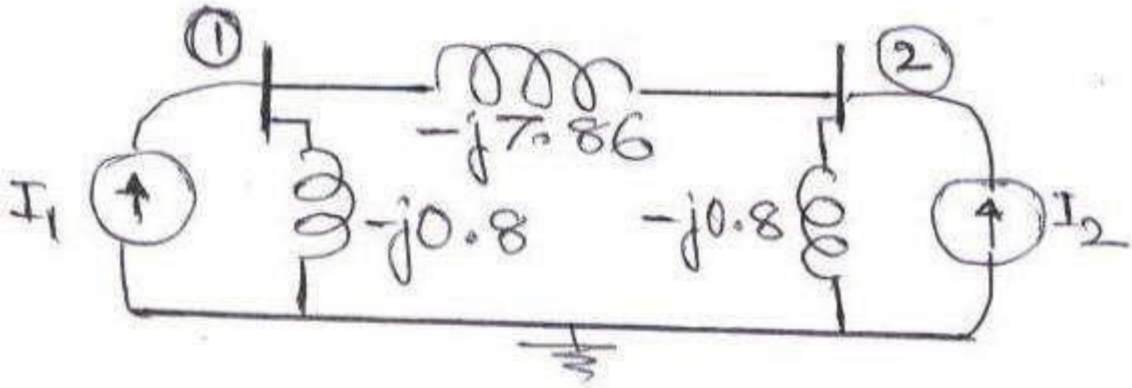
Example 7: Obtain YBUS for the impedance network shown aside by the rule of inspection. Also, determine YBUS for the reduced network after eliminating the eligible unwanted node. Draw the resulting reduced system diagram.

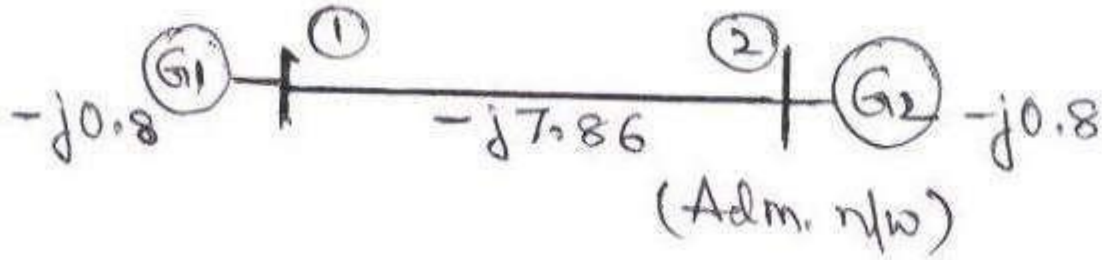
$$Y_{BUS} = j\begin{vmatrix} -9.8 & 5 & 4 \\ 5 & -16 & 10 \\ 4 & 10 & -14 \end{vmatrix}$$



$$Z_{BUS} = Y_{BUS}^{-1}$$







$$Y_{BUS}^{New} = Y_A - Y_B Y_D^{-1} Y_C$$

$$Y_{BUS} = j\begin{vmatrix} -8.66 & 7.86 \\ 7.86 & -8.66 \end{vmatrix}$$

SINGULAR TRANSFORMATIONS

The primitive network matrices are the most basic matrices and depend purely on the impedance or admittance of the individual elements. However, they do not contain any information about the behaviour of the interconnected network variables. Hence, it is necessary to transform the primitive matrices into more meaningful matrices which can relate variables of the interconnected network.

Bus admittance matrix, YBUS and Bus impedance matrix, ZBUS

In the bus frame of reference, the performance of the interconnected network is described by *n* independent nodal equations, where *n* is the total number of buses (n+1nodes are present, out of which one of them is designated as the reference node).
For example a 5-bus system will have 5 external buses and 1 ground/ ref. bus). The performance equation relating the bus voltages to bus current injections in bus frame of reference in admittance form is given by

IBUS = YBUS EBUS **(17)**

Where EBUS = vector of bus voltages measured with respect to reference bus
IBUS = Vector of currents injected into the bus
YBUS = bus admittance matrix

The performance equation of the primitive network in admittance form is given by
i + j = [y] v

Pre-multiplying by At (transpose of A), we obtain

At i +At j = At [y] v **(18)**

However, as per equation (4),

At i =0,

since it indicates a vector whose elements are the algebraic sum of element currents incident at a bus, which by Kirchhoff‟s law is zero. Similarly, At j gives the algebraic sum of all source currents incident at each bus and this is nothing but the total current injected at the bus. Hence,

At j = IBUS **(19)**

Thus from (18) we have, IBUS = At [y] v **(20)**

However, from (5), we have
v =A EBUS
And hence substituting in (20) we get,

IBUS = At [y] A EBUS **(21)**

Comparing (21) with (17) we obtain,

YBUS = At [y] A **(22)**

The bus incidence matrix is rectangular and hence singular. Hence, (22) gives a singular transformation of the primitive admittance matrix [y]. The bus impedance matrix is given by ,

ZBUS = $YBUS^{-1}$ **(23)**

Note: This transformation can be derived using the concept of power invariance, however, since the transformations are based purely on KCL and KVL, the transformation will obviously be power invariant.

Examples on Singular Transformation:

Example 8: For the network of Fig E8, form the primitive matrices [z] & [y] and obtain the bus admittance matrix by singular transformation. Choose a Tree T(1,2,3). The data is given in Table E8.

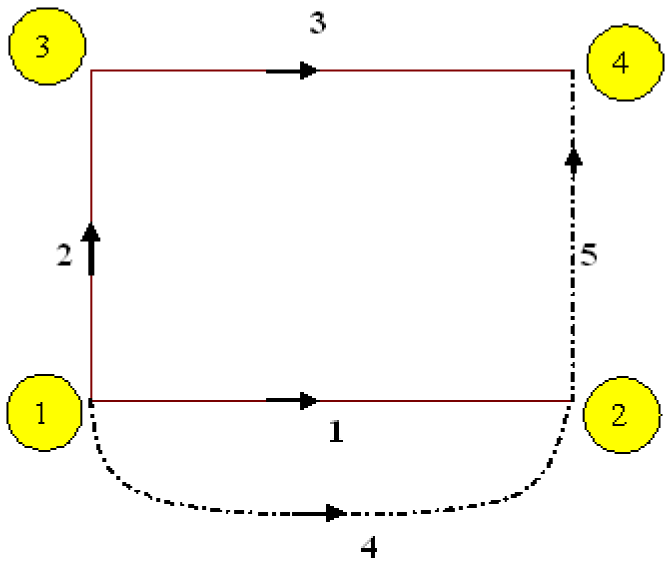


Fig E8 System for Example-8

Table E8: Data for Example-8

Elements	Self impedance	Mutual impedance
1	j 0.6	-
2	j 0.5	j 0.1(with element 1)
3	j 0.5	-
4	j 0.4	j 0.2 (with element 1)
5	j 0.2	-

Solution:
The bus incidence matrix is formed taking node 1 as the reference bus.

$$A = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 0 \\ 1 & 0 & -1 \end{bmatrix}$$

The primitive incidence matrix is given by

$$[z] = \begin{bmatrix} j0.6 & j0.1 & 0.0 & j0.2 & 0.0 \\ j0.1 & j0.5 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & j0.5 & 0.0 & 0.0 \\ j0.2 & 0.0 & 0.0 & j0.4 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & j0.2 \end{bmatrix}$$

The primitive admittance matrix [y] = [z]-1 and given by,

$$[y] = \begin{bmatrix} -j2.0833 & j0.4167 & 0.0 & j1.0417 & 0.0 \\ j0.4167 & -j2.0833 & 0.0 & -j0.2083 & 0.0 \\ 0.0 & 0.0 & -j2.0 & 0.0 & 0.0 \\ j1.0417 & -j0.2083 & 0.0 & -j3.0208 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & -j5.0 \end{bmatrix}$$

The bus admittance matrix by singular transformation is obtained as

$$Y_{BUS} = A^t\,[y]\,A = \begin{bmatrix} -j8.0208 & j0.2083 & j5.0 \\ j0.2083 & -j4.0833 & j2.0 \\ j5.0 & j2.0 & -j7.0 \end{bmatrix}$$

$$Z_{BUS} = Y_{BUS}^{-1} = \begin{bmatrix} j0.2713 & j0.1264 & j0.2299 \\ j0.1264 & j0.3437 & j0.1885 \\ j0.2299 & j0.1885 & j0.3609 \end{bmatrix}$$

SUMMARY

The formulation of the mathematical model is the first step in obtaining the solution of any electrical network. The independent variables can be either currents or voltages. Correspondingly, the elements of the coefficient matrix will be impedances or admittances.

Network equations can be formulated for solution of the network using graph theory, independent of the nature of elements. In the graph of a network, the tree-branches and links are distinctly identified. The complete information about the interconnection of the network, with the directions of the currents is contained in the bus incidence matrix.

The information on the nature of the elements which form the interconnected network is contained in the primitive impedance matrix. A primitive element can be represented in impedance form or admittance form. In the bus frame of reference, the performance of the interconnected system is described by $(n\text{-}1)$ nodal equations, where n is the number of nodes. The bus admittance matrix and the bus impedance matrix relate the bus voltages and currents. These matrices can be obtained from the primitive impedance and admittance matrices.

REVIEW OF NUMERICAL SOLUTION OF EQUATIONS

The numerical analysis involving the solution of algebraic simultaneous equations forms the basis for solution of the performance equations in computer aided electrical power system analyses, such as during linear graph analysis, load flow analysis (nonlinear equations), transient stability studies (differential equations), etc. Hence, it is necessary to review the general forms of the various solution methods with respect to all forms of equations, as under:

1. Solution Linear equations:
* **Direct methods**:

- Cramer‟s (Determinant) Method,
- Gauss Elimination Method (only for smaller systems),
- LU Factorization (more preferred method), etc.

*** Iterative methods**:
- Gauss Method
- Gauss-Siedel Method (for diagonally dominant systems)

3. Solution of Nonlinear equations: Iterative methods only:
- Gauss-Siedel Method (for smaller systems)
- Newton-Raphson Method (if corrections for variables are small)

4. Solution of differential equations:
Iterative methods only:
- Euler and Modified Euler method,
- RK IV-order method,
- Milne‟s predictor-corrector method, etc.

It is to be observed that the nonlinear and differential equations can be solved only by the iterative methods. The iterative methods are characterized by the various performance features as under:
_ Selection of initial solution/ estimates
_ Determination of fresh/ new estimates during each iteration
_ Selection of number of iterations as per tolerance limit
_ Time per iteration and total time of solution as per the solution method selected
_ Convergence and divergence criteria of the iterative solution
_ Choice of the Acceleration factor of convergence, etc.

A comparison of the above solution methods is as under:
In general, the direct methods yield exact or accurate solutions. However, they are suited for only the smaller systems, since otherwise, in large systems, the possible round-off errors make the solution process inaccurate. The iterative methods are more useful when the diagonal elements of the coefficient matrix are large in comparison with the off diagonal elements. The round-off errors in these methods are corrected at the successive steps of the iterative process.The Newton-Raphson method is very much useful for solution of non –linear equations, if all the values of the corrections for the unknowns are very small in magnitude and the initial values of unknowns are selected to be reasonably closer to the exact solution.

LOAD FLOW STUDIES

Introduction: Load flow studies are important in planning and designing future expansion of power systems. The study gives steady state solutions of the voltages at all the buses, for a particular load condition. Different steady state solutions can be obtained, for different operating conditions, to help in planning, design and operation of the power system. Generally, load flow studies are limited to the transmission system, which involves bulk power transmission. The load at the buses is assumed to be known. Load flow studies throw light on some of the important aspects of the system operation, such as: violation of voltage magnitudes at the buses, overloading of lines, overloading of generators, stability margin reduction, indicated by power angle differences between buses linked by a line, effect of contingencies like line voltages, emergency shutdown of generators, etc. Load flow studies are required for deciding the economic operation of the power system. They are also required in transient stability studies. Hence, load flow studies play a vital role in power system studies. Thus the load flow problem consists of finding the power flows (real and reactive) and voltages of a network for given bus conditions. At each bus, there are four quantities of interest to be known for further analysis: the real and reactive power, the voltage magnitude and its phase angle. Because of the nonlinearity of the algebraic equations, describing the given power system, their solutions are obviously, based on the iterative methods only. The constraints placed on the load flow solutions could be:
_ The Kirchhoff"s relations holding good,
_ Capability limits of reactive power sources,
_ Tap-setting range of tap-changing transformers,
_ Specified power interchange between interconnected systems,
_ Selection of initial values, acceleration factor, convergence limit, etc.

Classification of buses for LFA: Different types of buses are present based on the specified and unspecified variables at a given bus as presented in the table below:

Table 1. Classification of buses for LFA

Sl. No.	Bus Types	Specified Variables	Unspecified variables	Remarks
1	Slack/ Swing Bus	$\|V\|$, δ	P_G, Q_G	$\|V\|$, δ: are assumed if not specified as 1.0 and 0^0
2	Generator/ Machine/ PV Bus	P_G, $\|V\|$	Q_G, δ	A generator is present at the machine bus
3	Load/ PQ Bus	P_G, Q_G	$\|V\|$, δ	About 80% buses are of PQ type
4	Voltage Controlled Bus	P_G,Q_G, $\|V\|$	δ, a	'a' is the % tap change in tap-changing transformer

Importance of swing bus: The slack or swing bus is usually a PV-bus with the largest capacity generator of the given system connected to it. The generator at the swing bus supplies the power difference between the "specified power into the system at the other buses" and the "total system output plus losses". Thus swing bus is needed to supply the additional real and reactive power to meet the losses. Both the magnitude and phase angle of voltage are specified at the swing bus, or otherwise, they are assumed to be equal to 1.0 p.u. and 00 , as per flat-start procedure of iterative
solutions. The real and reactive powers at the swing bus are found by the computer routine as part of the load flow solution process. It is to be noted that the source at the swing bus is a perfect one, called the swing machine, or slack machine. It is voltage regulated, i.e., the magnitude of voltage fixed. The phase angle is the system reference phase and hence is fixed. The generator at the swing bus has a torque angle and excitation which vary or swing as the demand changes. This variation is such as to produce fixed voltage.

Importance of YBUS based LFA:

The majority of load flow programs employ methods using the bus admittance matrix, as this method is found to be more economical. The bus admittance matrix plays a very important role in load flow analysis. It is a complex, square and symmetric matrix and hence only n(n+1)/2 elements of YBUS need to be stored for a n-bus system. Further, in the YBUS matrix, Yij = 0, if an incident element is not present in the system connecting the buses „i‟ and „j‟. since in a large power system, each bus is connected only to a fewer buses through an incident element, (about 6-8), the coefficient matrix, YBUS of such systems would be highly sparse, i.e., it will have many zero valued elements in it. This is defined by the sparsity of the matrix, as under:

$$\text{Percentage sparsity of a given matrix of } n^{th} \text{ order:} = \frac{\text{Total no. of zero valued elements of } Y_{BUS}}{\text{Total no. of entries of } Y_{BUS}}$$

$$S = (Z / n^2) \times 100 \ \% \qquad (1)$$

The percentage sparsity of YBUS, in practice, could be as high as 80-90%, especially for very large, practical power systems. This sparsity feature of YBUS is extensively used in reducing the load flow calculations and in minimizing the memory required to store the coefficient matrices. This is due to the fact that only the non-zero elements YBUS can be stored during the computer based implementation of the schemes, by adopting the suitable optimal storage schemes. While YBUS is thus highly sparse, it‟s inverse, ZBUS, the bus impedance matrix is not so. It is a FULL matrix, unless the optimal bus ordering schemes are followed before proceeding for load flow analysis.

THE LOAD FLOW PROBLEM

Here, the analysis is restricted to a balanced three-phase power system, so that the analysis can be carried out on a single phase basis. The per unit quantities are used for all quantities. The first step in the analysis is the formulation of suitable equations for the power flows in the system. The power system is a large interconnected system, where various buses are connected by transmission lines. At any bus, complex power is injected into the bus by the generators and complex power is drawn by the loads. Of course at any bus, either one of them may not be present. The power is transported from one bus to other via the transmission lines. At any bus *i*, the complex power Si (injected), shown in figure 1, is defined as

$$S_i = S_{Gi} - S_{Di} \quad (2)$$

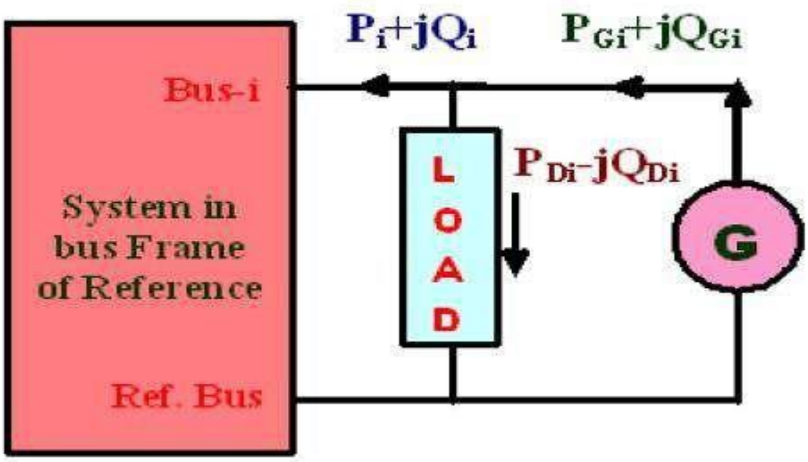


Fig.1 power flows at a bus-i

where Si = net complex power injected into bus *i*, SGi = complex power injected by the generator at bus *i*, and SDi = complex power drawn by the load at bus *i*. According to conservation of complex power, at any bus i, the complex power injected into the bus must be equal to the sum of complex power flows out of the bus via the transmission lines. Hence,

Si = _Sij " i = 1, 2,.............. n **(3)**

where Sij is the sum over all lines connected to the bus and *n* is the number of buses in the system (excluding the ground). The bus current injected at the bus-i is defined as

Ii = IGi – IDi " i = 1, 2, ………..n **(4)** where IGi is the current injected by the generator at the bus and IDi is the current drawn by the load (demand) at that bus. In the bus frame of reference

IBUS = YBUS VBUS
(5)

where

$$I_{BUS} = \begin{bmatrix} I_1 \\ I_2 \\ . \\ . \\ I_n \end{bmatrix}$$ is the vector of currents injected at the buses,

Y_{BUS} is the bus admittance matrix, and

$$V_{BUS} = \begin{bmatrix} V_1 \\ V_2 \\ . \\ . \\ V_n \end{bmatrix}$$ is the vector of complex bus voltages.

Equation (5) can be considered as

$$I_i = \sum_{j=1}^{n} Y_{ij} V_j \qquad \forall\ i = 1, 2, \ldots\ldots\ldots.n \qquad \textbf{(6)}$$

The complex power S_i is given by

$$S_i = V_i\, I_i^*$$

$$= V_i \left(\sum_{j=1}^{n} Y_{ij}\, V_j \right)^*$$

$$= V_i \left(\sum_{j=1}^{n} Y_{ij}^* V_j^* \right) \qquad \textbf{(7)}$$

Let $\qquad V_i \underline{\underline{\Delta}} |V_i| \angle\, \delta_i = |V_i| (\cos\ \delta_i + j \sin\ \delta_i)$

$$\delta_{ij} = \delta_i - \delta_j$$

$$Y_{ij} = G_{ij} + jB_{ij}$$

Hence from (7), we get,

$$S_i = \sum_{j=1}^{n} |V_i| \, |V_j| \, (\cos\delta_{ij} + j\sin\delta_{ij})(G_{ij} - jB_{ij}) \quad \textbf{(8)}$$

Separating real and imaginary parts in (8) we obtain,

$$P_i = \sum_{j=1}^{n} |V_i| \, |V_j| \, (G_{ij}\cos\delta_{ij} + B_{ij}\sin\delta_{ij}) \quad \textbf{(9)}$$

$$Q_i = \sum_{j=1}^{n} |V_i| \, |V_j| \, (G_{ij}\sin\delta_{ij} - B_{ij}\cos\delta_{ij}) \quad \textbf{(10)}$$

An alternate form of P_i and Q_i can be obtained by representing Y_{ik} also in polar form

as $\quad Y_{ij} = |Y_{ij}| \angle\theta_{ij} \quad$ **(11)**

Again, we get from (7),

$$S_i = |V_i| \angle\delta_i \sum_{j=1}^{n} |Y_{ij}| \angle -\theta_{ij} \, |V_j| \angle -\delta_j \quad \textbf{(12)}$$

The real part of (12) gives P_i.

$$P_i = |V_i| \sum_{j=1}^{n} |Y_{ij}| \, |V_j| \cos(-\theta_{ij} + \delta_i - \delta_j)$$

$$= |V_i| \sum_{j=1}^{n} |Y_{ij}| \, |V_j| \cos - (\theta_{ij} - \delta_i + \delta_j) \qquad \text{or}$$

$$P_i = \sum_{j=1}^{n} |V_i| |V_j| |Y_{ij}| \cos(\theta_{ij} - \delta_i + \delta_j) \qquad \forall\ i = 1, 2, \ldots\ldots\ldots. n, \quad \textbf{(13)}$$

Similarly, Q_i is imaginary part of (12) and is given by

$$Q_i = |V_i| \sum_{j=1}^{n} |Y_{ij}| \, |V_j| \sin - (\theta_{ij} - \delta_i + \delta_j) \qquad \text{or}$$

$$Q_i = -\sum_{j=1}^{n} |V_i| |V_j| |Y_{ij}| \sin(\theta_{ij} - \delta_i + \delta_j) \qquad \forall\ i = 1, 2, \ldots\ldots\ldots. n \quad \textbf{(14)}$$

Equations (9)-(10) and (13)-(14) are the „power flow equations" or the „load flow equations" in two alternative forms, corresponding to the n-bus system, where each bus-*i* is characterized by four variables, Pi, Qi, |Vi|, and di. Thus a total of 4n variables are involved in these equations. The load flow equations can be solved for any 2n unknowns, if the other 2n variables are specified. This establishes the need for classification of buses of the system for load flow analysis into: PV bus, PQ bus, etc.

DATA FOR LOAD FLOW

Irrespective of the method used for the solution, the data required is common for any load flow. All data is normally in pu. The bus admittance matrix is formulated from these data. The various data required are as under:

System data: It includes: number of buses-*n*, number of PV buses, number of loads, number of transmission lines, number of transformers, number of shunt elements, the slack bus number, voltage magnitude of slack bus (angle is generally taken as 0o), tolerance limit, base MVA, and maximum permissible number of iterations.

Generator bus data: For every PV bus *i*, the data required includes the bus number, active power generation PGi, the specified voltage magnitude *i sp V* , , minimum reactive power limit Qi,min, and maximum reactive power limit Qi,max.

Load data: For all loads the data required includes the the bus number, active power demand PDi, and the reactive power demand QDi.

Transmission line data: For every transmission line connected between buses *i* and *k* the data includes the starting bus number *i*, ending bus number *k*,.*r*esistance of the line, reactance of the line and the half line charging admittance.

Transformer data:
For every transformer connected between buses *i* and *k* the data to be given includes: the starting bus number *i*, ending bus number *k*, resistance of the transformer, reactance of the transformer, and the off nominal turns-ratio *a*.

Shunt element data: The data needed for the shunt element includes the bus number where element is connected, and the shunt admittance (Gsh + j Bsh).

GAUSS – SEIDEL (GS) METHOD
The GS method is an iterative algorithm for solving non linear algebraic equations. An initial solution vector is assumed, chosen from past experiences, statistical data or from practical considerations. At every subsequent iteration, the solution is updated till convergence is reached. The GS method applied to power flow problem is as discussed below.

Case (a): Systems with PQ buses only:
Initially assume all buses to be PQ type buses, except the slack bus. This means that (n–1) complex bus voltages have to be determined. For ease of programming, the slack bus is generally numbered as bus-1. PV buses are numbered in sequence and PQ buses are ordered next in sequence. This makes programming easier, compared to random ordering of buses. Consider the expression for the complex power at bus-i, given from (7), as:

$$S_i = V_i \left(\sum_{j=1}^{n} Y_{ij} V_j \right)^*$$

This can be written as

$$S_i^* = V_i^* \left(\sum_{j=1}^{n} Y_{ij} V_j \right) \qquad \textbf{(15)}$$

Since $S_i^* = P_i - jQ_i$, we get,

$$\frac{P_i - jQ_i}{V_i^*} = \sum_{j=1}^{n} Y_{ij} V_j$$

So that,

$$\frac{P_i - jQ_i}{V_i^*} = Y_{ii} V_i + \sum_{\substack{j=1 \\ j \neq i}}^{n} Y_{ij} V_j \qquad \textbf{(16)}$$

Rearranging the terms, we get,

$$V_i = \frac{1}{Y_{ii}} \left[\frac{P_i - jQ_i}{V_i^*} - \sum_{\substack{j=1 \\ j \neq i}}^{n} Y_{ij} V_j \right] \qquad \forall\, i = 2, 3, \ldots\ldots\ldots n \qquad \textbf{(17)}$$

Equation (17) is an implicit equation since the unknown variable, appears on both sides of the equation. Hence, it needs to be solved by an iterative technique. Starting from an initial estimate of all bus voltages, in the RHS of (17) the most recent values of the bus voltages is substituted. One iteration of the method involves computation of all the bus voltages. In Gauss–Seidel method, the value of the updated voltages are used in the computation of subsequent voltages in the same iteration, thus speeding up convergence. Iterations are carried out till the magnitudes of all bus voltages do not change by more than the tolerance value. Thus the algorithm for GS method is as under:

Algorithm for GS method

1. Prepare data for the given system as required.
2. Formulate the bus admittance matrix YBUS. This is generally done by the rule of inspection.
3. Assume initial voltages for all buses, 2,3,…n. In practical power systems, the magnitude of the bus voltages is close to 1.0 p.u. Hence, the complex bus voltages at all (n-1) buses (except slack bus) are taken to be $1.0\square 0^0$. This is normally refered as the ***flat start*** solution.
4. Update the voltages. In any (k +1)*st* iteration, from (17) the voltages are given by

$$V_i^{(k+1)} = \frac{1}{Y_{ii}}\left[\frac{P_i - jQ_i}{(V_i^{(k)})^*} - \sum_{j=1}^{i-1} Y_{ij} V_j^{(k+1)} - \sum_{j=i+1}^{n} Y_{ij} V_j^{(k)}\right] \quad \forall\ i=2,3,\ldots n \qquad \textbf{(18)}$$

Here note that when computation is carried out for bus-i, updated values are already available for buses 2,3….(i-1) in the current *(k+1)st* iteration. Hence these values are used. For buses *(i+1)…..n,* values from previous, *kth* iteration are used.

$$\left|\Delta V_i^{(k+1)}\right| = \left|V_i^{(k+1)} - V_i^{(k)}\right| < \varepsilon \qquad \forall\ i = 2,3,\ldots n \qquad \textbf{(19)}$$

Where,e is the tolerance value. Generally it is customary to use a value of 0.0001 pu. Compute slack bus power after voltages have converged using (15) [assuming bus 1 is slack bus].

$$S_1^* = P_1 - jQ_1 = V_1^*\left(\sum_{j=1}^{n} Y_{1j} V_j\right) \qquad \textbf{(20)}$$

7. Compute all line flows.
8. The complex power loss in the line is given by Sik + Ski. The total loss in the system is calculated by summing the loss over all the lines.

Case (b): Systems with PV buses also present:
At PV buses, the magnitude of voltage and not the reactive power is specified. Hence it is needed to first make an estimate of Qi to be used in (18). From (15) we have

$$Q_i = -\operatorname{Im}\left\{V_i^* \sum_{j=1}^{n} Y_{ij} V_j\right\}$$

Where *Im* stands for the imaginary part. At any *(k+1)*st iteration, at the PV bus-*i*,

$$Q_i^{(k+1)} = -\operatorname{Im}\left\{(V_i^{(k)})^* \sum_{j=1}^{i-1} Y_{ij} V_j^{(k+1)} + (V_i^{(k)})^* \sum_{j=i}^{n} Y_{ij} V_j^{(k)}\right\} \qquad \textbf{(21)}$$

The steps for i^{th} PV bus are as follows:

1. Compute $Q_i^{(k+1)}$ using (21)
2. Calculate V_i using (18) with $Q_i = Q_i^{(k+1)}$
3. Since $|V_i|$ is specified at the PV bus, the magnitude of V_i obtained in step 2

has to be modified and set to the specified value $|V_{i,sp}|$. Therefore,

$$V_i^{(k+1)} = |V_{i,sp}| \angle \delta_i^{(k+1)} \qquad \textbf{(22)}$$

The voltage computation for PQ buses does not change.

Case (c): Systems with PV buses with reactive power generation limits specified:
In the previous algorithm if the Q limit at the voltage controlled bus is violated during any iteration, i.e $(k +1)$ *i Q* computed using (21) is either less than Qi, min or greater than Qi,max, it means that the voltage cannot be maintained at the specified value due to lack of reactive power support. This bus is then treated as a PQ bus in the *(k+1)st* iteration and the voltage is calculated with the value of Qi set as follows:

If $Q_i < Q_{i,min}$	If $Q_i > Q_{i,max}$
Then $Q_i = Q_{i,min}$.	Then $Q_i = Q_{i,max}$.

(23)

If in the subsequent iteration, if Qi falls within the limits, then the bus can be switched back to PV status.

Acceleration of convergence
It is found that in GS method of load flow, the number of iterations increase with increase in the size of the system. The number of iterations required can be reduced if the correction in voltage at each bus is accelerated, by multiplying with a constant α, called the acceleration factor. In the (k+1)st iteration we can let

$$V_i^{(k+1)}(accelerate\ d) = V_i^{(k)} + \alpha\left(V_i^{(k+1)} - V_i^{(k)}\right) \qquad \textbf{(24)}$$

where □ is a real number. When □ =1, the value of (*k* +1) is the computed value. If 1<□ <2 then the value computed is extrapolated. Generally _ is taken between 1.2 to 1.6, for GS load flow procedure. At PQ buses (pure load buses) if the voltage magnitude violates the limit, it simply means that the specified reactive power demand cannot be supplied, with the voltage maintained within acceptable limits.

Examples on GS load flow analysis:

Example-1: *Obtain the voltage at bus 2 for the simple system shown in Fig 2, using the Gauss–Seidel method, if $V_1 = 1\angle 0^0$ pu.*

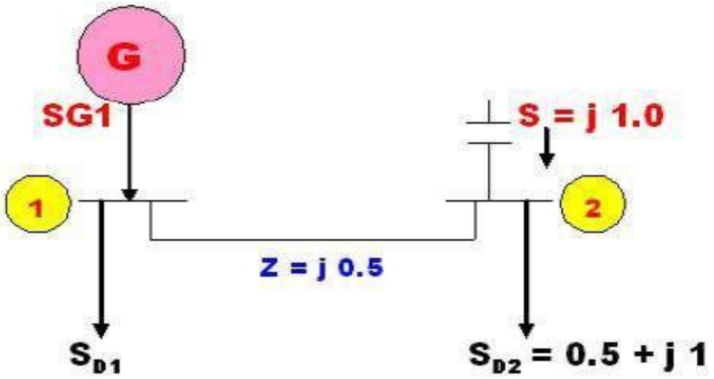


Fig : System of Example 1

Solution:

Here the capacitor at bus 2, injects a reactive power of 1.0 pu. The complex power injection at bus 2 is

$$S_2 = j1.0 - (0.5 + j\,1.0) = -\,0.5 \text{ pu.}$$

$$V_1 = 1\angle 0^0$$

$$Y_{BUS} = \begin{bmatrix} -j2 & j2 \\ j2 & -j2 \end{bmatrix}$$

$$V_2^{(k+1)} = \frac{1}{Y_{22}}\left[\frac{P_2 - jQ_2}{\left(V_2^{(k)}\right)^*} - Y_{21}\,V_1\right]$$

Since V_1 is specified it is a constant through all the iterations. Let the initial voltage at bus 2, $V_2^0 = 1 + j\,0.0 = 1\angle 0^0$ pu.

$$V_2^1 = \frac{1}{-j2}\left[\frac{-0.5}{1\angle 0^0} - \left(j2 \times 1\angle 0^0\right)\right]$$

$$= 1.0 - j0.25 = 1.030776 \ \angle - 14.036^0$$

$$V_2^2 = \frac{1}{-j2}\left[\frac{-0.5}{1.030776\angle 14.036^0} - \left(j2 \times 1\angle 0^0\right)\right]$$

$$= 0.94118 - j\ 0.23529 \ = 0.970145 \ \angle -14.036^0$$

$$V_2^3 = \frac{1}{-j2}\left[\frac{-0.5}{0.970145\angle 14.036^0} - \left(j2 \times 1\angle 0^0\right)\right]$$

$$= 0.9375 - j\ 0.249999 \ = 0.970261 \ \angle -14.931^0$$

$$V_2^4 = \frac{1}{-j2}\left[\frac{-0.5}{0.970261\angle 14.931^0} - \left(j2 \times 1\angle 0^0\right)\right]$$

$$= 0.933612 - j\ 0.248963 = 0.966237 \ \angle -14.931^0$$

$$V_2^5 = \frac{1}{-j2}\left[\frac{-0.5}{0.966237\angle 14.931^0} - \left(j2 \times 1\angle 0^0\right)\right]$$

$$= 0.933335 - j\ 0.25 \ = 0.966237 \ \angle -14.995^0$$

Since the difference in the voltage magnitudes is less than 10-6 pu, the iterations can be stopped. To compute line flow

$$I_{12} = \frac{V_1 - V_2}{Z_{12}} = \frac{1\angle 0^0 - 0.966237\angle -14.995^0}{j0.5}$$

$$= 0.517472 \ \angle -14.931^0$$

$$S_{12} = V_1 I_{12}^* = 1\angle 0^0 \times 0.517472 \ \angle \ 14.931^0$$

$$= 0.5 + \text{j } 0.133329 \text{ pu}$$

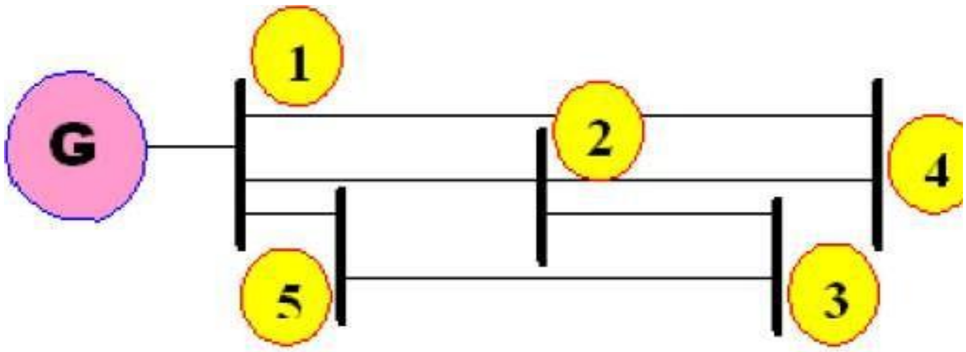
$$I_{21} = \frac{V_2 - V_1}{Z_{12}} = \frac{0.966237\angle -14.995^0 - 1\angle 0^0}{j0.5}$$

$$= 0.517472 \ \angle -194.93^0$$

$$S_{21} = V_2 I_{21}^* = -0.5 + \text{j } 0.0 \text{ pu}$$

The total loss in the line is given by S12 + S21 = j 0.133329 pu Obviously, it is observed that there is no real power loss, since the line has no resistance.

Example-2:
For the power system shown in fig. below, with the data as given in tables below, obtain the bus voltages at the end of first iteration, by applying GS method.



Power System of Example 2

Line data of example 2

SB	EB	R (pu)	X (pu)	$\frac{B_C}{2}$
1	2	0.10	0.40	-
1	4	0.15	0.60	-
1	5	0.05	0.20	-
2	3	0.05	0.20	-
2	4	0.10	0.40	-
3	5	0.05	0.20	-

Bus data of example 2

Bus No.	P_G (pu)	Q_G (pu)	P_D (pu)	Q_D (pu)	$\|V_{SP}\|$ (pu)	δ
1	-	-	-	-	1.02	0^o
2	-	-	0.60	0.30	-	-
3	1.0	-	-	-	1.04	-
4	-	-	0.40	0.10	-	-
5	-	-	0.60	0.20	-	-

Solution: In this example, we have,

- Bus 1 is slack bus, Bus 2, 4, 5 are PQ buses, and Bus 3 is PV bus
- The lines do not have half line charging admittances

$P_2 + jQ_2 = P_{G2} + jQ_{G2} - (P_{D2} + jQ_{D2}) = -0.6 - j0.3$

$P_3 + jQ_3 = P_{G3} + jQ_{G3} - (P_{D3} + jQ_{D3}) = 1.0 + jQ_{G3}$

Similarly $P_4 + jQ_4 = -0.4 - j0.1$, $P_5 + jQ_5 = -0.6 - j0.2$

The Y_{bus} formed by the rule of inspection is given by:

Y_{bus} =

2.15685 -j8.62744	-0.58823 +j2.35294	0.0+j0.0	-0.39215 +j1.56862	-1.17647 +j4.70588
-0.58823 +j2.35294	2.35293 -j9.41176	-1.17647 +j4.70588	-0.58823 +j2.35294	0.0+j0.0
0.0+j0.0	-1.17647 +j4.70588	2.35294 -j9.41176	0.0+j0.0	-1.17647 +j4.70588
-0.39215 +j1.56862	-0.58823 +j2.35294	0.0+j0.0	0.98038 -j3.92156	0.0+j0.0
-1.17647 +j4.70588	0.0+j0.0	-1.17647 +j4.70588	0.0+j0.0	2.35294 -j9.41176

The voltages at all PQ buses are assumed to be equal to 1+j0.0 pu. The slack bus voltage is taken to be V_1^0 = 1.02+j0.0 in all iterations.

$$V_2^1 = \frac{1}{Y_{22}}\left[\frac{P_2 - jQ_2}{V_2^{o*}} - Y_{21}V_1^o - Y_{23}V_3^0 - Y_{24}V_4^0 - Y_{25}V_5^0\right]$$

$$= \frac{1}{Y_{22}}\left[\frac{-0.6 + j0.3}{1.0 - j0.0} - \{(-0.58823 + j2.35294)\times 1.02\angle 0^o\}\right.$$

$$\left. - \{(-1.17647 + j4.70588)\times 1.04\angle 0^o\} - \{(-0.58823 + j2.35294)\times 1.0\angle 0^o\}\right]$$

$$= 0.98140\angle -3.0665^o = 0.97999 - j0.0525$$

Bus 3 is a PV bus. Hence, we must first calculate Q_3. This can be done as under:

$$Q_3 = |V_3||V_1|(G_{31}\sin\delta_{31} - B_{31}\cos\delta_{31}) + |V_3||V_2|(G_{32}\sin\delta_{32} - B_{32}\cos\delta_{32})$$

$$+ |V_3|^2(G_{33}\sin\delta_{33} - B_{33}\cos\delta_{33}) + |V_3||V_4|(G_{34}\sin\delta_{34} - B_{34}\cos\delta_{34})$$

$$+ |V_3||V_5|(G_{35}\sin\delta_{35} - B_{35}\cos\delta_{35})$$

We note that $\delta_1 = 0^o$; $\delta_2 = -3.0665^o$; $\delta_3 = 0^o$; $\delta_4 = 0^o$ and $\delta_5 = 0^o$

$\therefore \delta_{31} = \delta_{33} = \delta_{34} = \delta_{35} = 0^o$ ($\delta_{ik} = \delta_i - \delta_k$); $\delta_{32} = 3.0665^o$

$Q_3 = 1.04\,[1.02\,(0.0+j0.0) + 0.9814\,\{-1.17647 \times \sin(3.0665^o) - 4.70588$
$\times\cos(3.0665^o)\} + 1.04\{-9.41176 \times\cos(0^o)\} + 1.0\,\{0.0 + j0.0\} + 1.0\{-4.70588\times\cos(0^0)\}]$

$$= 1.04\,[-4.6735 + 9.78823 - 4.70588] = 0.425204 \text{ pu.}$$

$$V_3^1 = \frac{1}{Y_{33}}\left[\frac{P_3 - jQ_3}{V_3^{o*}} - Y_{31}V_1^o - Y_{32}V_2^1 - Y_{34}V_4^0 - Y_{35}V_5^0\right]$$

$$= \frac{1}{Y_{33}}\left[\frac{1.0 - j0.425204}{1.04 - j0.0} - \{(-1.7647 + j4.70588) \times (0.98140\angle -3.0665^o)\}\right.$$

$$\left. - \{(-1.17647 + j4.70588) \times (1\angle 0^o)\}\right]$$

$$= 1.05569\angle 3.077^o = 1.0541 + j0.05666 \text{ pu.}$$

Since it is a PV bus, the voltage magnitude is adjusted to specified value and V_3^1 is computed as: $V_3^1 = 1.04\angle 3.077^0$ pu

$$V_4^1 = \frac{1}{Y_{44}}\left[\frac{P_4 - jQ_4}{V_4^{o*}} - Y_{41}V_1^o - Y_{42}V_2^1 - Y_{43}V_3^1 - Y_{45}V_5^0\right]$$

$$= \frac{1}{Y_{44}}\left[\frac{-0.4 + j0.1}{1.0 - j0.0} - \{(-0.39215 + j1.56862) \times 1.02\angle 0^o\}\right.$$

$$\left. - \{(-0.58823 + j2.35294) \times (0.98140\angle -3.0665^o)\}\right]$$

$$= \frac{0.45293 - j3.8366}{0.98038 - j3.92156} = 0.955715\angle -7.303^o \text{ pu} = 0.94796 - j0.12149$$

$$V_5^1 = \frac{1}{Y_{55}}\left[\frac{P_5 - jQ_5}{V_5^{o*}} - Y_{51}V_1^o - Y_{52}V_2^1 - Y_{53}V_3^1 - Y_{54}V_4^1\right]$$

$$= \frac{1}{Y_{55}}\left[\frac{-0.6 + j0.2}{1.0 - j0.0} - \{(-1.17647 + j4.70588) \times 1.02\angle 0^o\}\right.$$

$$\left. - \{(-1.17647 + j4.70588) \times 1.04\angle 3.077^o\}\right]$$

$$= 0.994618\angle -1.56^o = 0.994249 - j0.027$$

Thus at end of 1st iteration, we have,

$V_1 = 1.02\angle 0^0$ pu $\quad\quad V_2 = 0.98140\ \angle -3.066^0$ pu

$V_3 = 1.04\angle 3.077^0$ pu $\quad\quad V_4 = 0.955715\angle -7.303^0$ pu

and $\quad V_5 = 0.994618\angle -1.56^0$ pu

Example-3:

Obtain the load flow solution at the end of first iteration of the system with data as given below. The solution is to be obtained for the following cases

(i) All buses except bus 1 are PQ Buses

(ii) Bus 2 is a PV bus whose voltage magnitude is specified as 1.04 pu

(iii) Bus 2 is PV bus, with voltage magnitude specified as 1.04 and 0.25_Q2_1.0 pu.

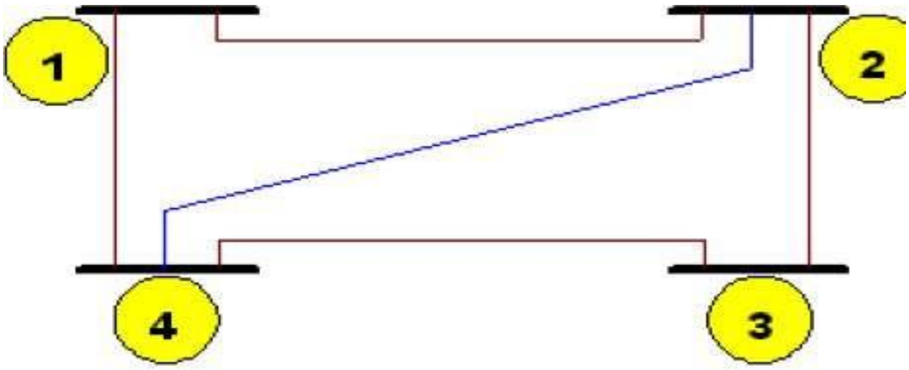


Fig. System for Example 3

Table: Line data of example 3

SB	EB	R (pu)	X (pu)
1	2	0.05	0.15
1	3	0.10	0.30
2	3	0.15	0.45
2	4	0.10	0.30
3	4	0.05	0.15

Table: Bus data of example 3

Bus No.	P_i (pu)	Q_i (pu)	V_i
1	–	–	$1.04 \angle 0^0$
2	0.5	– 0.2	–
3	– 1.0	0.5	–
4	– 0.3	– 0.1	–

Solution: Note that the data is directly in terms of injected powers at the buses. The bus admittance matrix is formed by inspection as under:

Y_{BUS} =

3.0 – j9.0	–2.0 + j6.0	– 1.0 + j3.0	0
–2.0 + j6.0	3.666 – j11.0	– 0.666 + j2.0	– 1.0 + j3.0
–1.0 + j3.0	–0.666 + j2.0	3.666 – j11.0	–2.0 + j6.0
0	–1.0 + j3.0	– 2.0 + j6.0	3.0 – j9.0

Case(i): All buses except bus 1 are PQ Buses

Assume all initial voltages to be $1.0 \angle 0^0$ pu.

$$V_2^1 = \frac{1}{Y_{22}} \left[\frac{P_2 - jQ_2}{V_2^{o*}} - Y_{21} V_1^o - Y_{23} V_3^0 - Y_{24} V_4^0 \right]$$

$$= \frac{1}{Y_{22}}\left[\frac{0.5 + j0.2}{1.0 - j0.0} - \{(-2.0 + j6.0)\times(1.04\angle 0^o)\}\right.$$

$$\left. - \{(-0.666 + j2.0)\times(1.0\angle 0^o)\} - \{(-1.0 + j3.0)\times(1.0\angle 0^0)\}\right]$$

$$= 1.02014 \angle 2.605^o$$

$$V_3^1 = \frac{1}{Y_{33}}\left[\frac{P_3 - jQ_3}{V_3^{o*}} - Y_{31} V_1^o - Y_{32} V_2^1 - Y_{34} V_4^0\right]$$

$$= \frac{1}{Y_{33}}\left[\frac{-1.0 - j0.5}{1.0 - j0.0} - \{(-1.0 + j3.0)\times(1.04\angle 0.0^o)\}\right.$$

$$\left. - \{(-0.666 + j2.0)\times(1.02014\angle 2.605^o)\} - \{(-2.0 + j6.0)\times(1.0\angle 0^0)\}\right]$$

$$= 1.03108 \angle -4.831^o$$

$$V_4^1 = \frac{1}{Y_{44}}\left[\frac{P_4 - jQ_4}{V_4^{o*}} - Y_{41} V_1^o - Y_{42} V_2^1 - Y_{43} V_3^1\right]$$

$$= \frac{1}{Y_{44}}\left[\frac{0.3 + j0.1}{1.0 - j0.0} - \{(-1.0 + j3.0)\times(1.02014\angle 2.605^o)\}\right.$$

$$\left. - \{(-2.0 + j6.0)\times(1.03108\angle -4.831^o)\}\right]$$

$$= 1.02467 \angle -0.51^o$$

Hence

$V_1^1 = 1.04 \angle 0^0$ pu $\qquad V_2^1 = 1.02014 \angle 2.605^0$ pu

$V_3^1 = 1.03108 \angle -4.831^0$ pu $\qquad V_4^1 = 1.02467 \angle -0.51^0$ pu

Case(ii): Bus 2 is a PV bus whose voltage magnitude is specified as 1.04 pu

We first compute Q_2.

$$Q_2 = |V_2|\left[|V_1|\left(G_{21}\sin\delta_{21} - B_{21}\cos\delta_{21}\right) + |V_2|\left(G_{22}\sin\delta_{22} - B_{22}\cos\delta_{22}\right)\right.$$
$$\left. + |V_3|\left(G_{23}\sin\delta_{23} - B_{23}\cos\delta_{23}\right) + |V_4|\left(G_{24}\sin\delta_{24} - B_{24}\cos\delta_{24}\right)\right]$$

$$= 1.04\ [1.04\ \{-6.0\} + 1.04\ \{11.0\} + 1.0\{-2.0\} + 1.0\ \{-3.0\} = 0.208 \text{ pu.}$$

$$V_2^1 = \frac{1}{Y_{22}}\left[\frac{0.5 - j0.208}{1.04\angle 0^0} - \{(-2.0 + j6.0)\times(1.04\angle 0^o)\}\right.$$
$$\left. - \{(-0.666 + j2.0)\times(1.0\angle 0^o)\} - \{(-1.0 + j3.0)\times(1.0\angle 0^o)\}\right]$$
$$= 1.051288 + j0.033883$$

The voltage magnitude is adjusted to 1.04. Hence $V_2^1 = 1.04 \angle 1.846^0$

$$V_3^1 = \frac{1}{Y_{33}}\left[\frac{-1.0 - j0.5}{1.0\ \angle 0^0} - \{(-1.0 + j3.0)\times(1.04\angle 0.0^o)\}\right.$$
$$\left. - \{(-0.666 + j2.0)\times(1.04\angle 1.846^o)\} - \{(-2.0 + j6.0)\times(1.0\angle 0^0)\}\right]$$
$$= 1.035587 \angle -4.951^o \text{ pu.}$$

$$V_4^1 = \frac{1}{Y_{44}}\left[\frac{0.3 + j0.1}{1.0 - j0.0} - \{(-1.0 + j3.0)\times(1.04\angle 1.846^o)\}\right.$$
$$\left. - \{(-2.0 + j6.0)\times(1.035587\angle -4.951^o)\}\right]$$
$$= 0.9985 \angle -0.178^o$$

Hence at end of 1st iteration we have:

$V_1^1 = 1.04\ \angle 0^0$ pu $\qquad V_2^1 = 1.04\ \angle 1.846^0$ pu

$V_3^1 = 1.035587\ \angle -4.951^0$ pu $\qquad V_4^1 = 0.9985 \angle -0.178^0$ pu

Case (iii): Bus 2 is PV bus, with voltage magnitude specified as 1.04 & $0.25 \le Q_2 \le 1$ pu. If $0.25 \le Q_2 \le 1.0$ pu then the computed value of $Q_2 = 0.208$ is less than the lower limit. Hence, Q_2 is set equal to 0.25 pu. Iterations are carried out with this value of Q_2. The voltage magnitude at bus 2 can no longer be maintained at 1.04. Hence, there is no necessity to adjust for the voltage magnitude. Proceeding as before we obtain at the end of first iteration,

$$V_1^1 = 1.04 \angle 0^0 \text{ pu} \qquad V_2^1 = 1.05645 \angle 1.849^0 \text{ pu}$$

$$V_3^1 = 1.038546 \angle -4.933^0 \text{ pu} \qquad V_4^1 = 1.081446 \angle 4.896^0 \text{ pu}$$

Limitations of GS load flow analysis

GS method is very useful for very small systems. It is easily adoptable, it can be generalized and it is very efficient for systems having less number of buses. However, GS LFA fails to converge in systems with one or more of the features as under:

- Systems having large number of radial lines
- Systems with short and long lines terminating on the same bus
- Systems having negative values of transfer admittances
- Systems with heavily loaded lines, etc.

GS method successfully converges in the absence of the above problems. However, convergence also depends on various other set of factors such as: selection of slack bus, initial solution, acceleration factor, tolerance limit, level of accuracy of results needed, type and quality of computer/ software used, etc.

UNIT III

LOAD FLOW STUDIES-II

NEWTON –RAPHSON METHOD

Newton-Raphson (NR) method is used to solve a system of non-linear algebraic equations of the form f(x) =0. Consider a set of *n* non-linear algebraic equations given by

$$f_i(x_1, x_2 \ldots\ldots x_n) = 0 \qquad i = 1,2\ldots\ldots n \qquad \textbf{(25)}$$

Let $x_1^{\,0}, x_2^{\,0} \ldots\ldots\ldots x_n^{\,0}$, be the initial guess of unknown variables and $\Delta x_1^{\,0}, \Delta x_2^{\,0} \ldots\ldots \Delta x_n^{\,0}$ be the respective corrections. Therefore,

$$f_i(x_1^{\,0} + \Delta x_1^{\,0}, x_2^{\,0} + \Delta x_2^{\,0} \ldots\ldots\ldots x_n^{\,0} + \Delta x_n^{\,0}) = 0 \qquad i = 1,2\ldots\ldots n \qquad \textbf{(26)}$$

The above equation can be expanded using Taylor's series to give

$$f_i(x_1^{\,0}, x_2^{\,0} \ldots\ldots x_n^{\,0}) + \left[\left(\frac{\partial f_i}{\partial x_1}\right)^0 \Delta x_1^{\,0} + \left(\frac{\partial f_i}{\partial x_2}\right)\Delta x_2^{\,0} + \ldots\ldots + \left(\frac{\partial f_i}{\partial x_n}\right)\Delta x_n^{\,0}\right]$$

$$+ \text{Higher order terms} = 0 \qquad \forall\ i = 1,2\ldots\ldots n \qquad \textbf{(27)}$$

Where, $\left(\frac{\partial f_i}{\partial x_1}\right)^0, \left(\frac{\partial f_i}{\partial x_2}\right)^0, \ldots\ldots\ldots \left(\frac{\partial f_i}{\partial x_n}\right)^0$ are the partial derivatives of f_i with respect to $x_1, x_2 \ldots\ldots x_n$ respectively, evaluated at ($x_1^{\,0}, x_2^{\,0} \ldots\ldots\ldots x_0^{\,n}$). If the higher order terms are neglected, then (27) can be written in matrix form as

$$\begin{bmatrix} f_1^{\,0} \\ f_2^{\,0} \\ \cdot \\ \cdot \\ \cdot \\ f_n^{\,0} \end{bmatrix} + \begin{bmatrix} \left(\frac{\partial f_1}{\partial x_1}\right)^0 & \left(\frac{\partial f_1}{\partial x_2}\right)^0 & \cdot\ \cdot\ \cdot & \left(\frac{\partial f_1}{\partial x_n}\right)^0 \\ \left(\frac{\partial f_2}{\partial x_1}\right)^0 & \left(\frac{\partial f_2}{\partial x_2}\right)^0 & \cdot\ \cdot\ \cdot & \left(\frac{\partial f_2}{\partial x_n}\right)^0 \\ \cdot & \cdot & \cdot\ \cdot\ \cdot & \cdot \\ \cdot & \cdot & \cdot\ \cdot\ \cdot & \cdot \\ \cdot & \cdot & \cdot\ \cdot\ \cdot & \cdot \\ \left(\frac{\partial f_n}{\partial x_1}\right) & \left(\frac{\partial f_n}{\partial x_2}\right) & \cdot\ \cdot\ \cdot & \left(\frac{\partial f_n}{\partial x_n}\right) \end{bmatrix} \begin{bmatrix} \Delta x_1^{\,0} \\ \Delta x_2^{\,0} \\ \cdot \\ \cdot \\ \cdot \\ \Delta x_n^{\,0} \end{bmatrix} = 0 \qquad \textbf{(28)}$$

In vector form (28) can be written as

$$F^0 + J^0 \Delta X^0 = 0$$

Or $\qquad F^0 = -J^0 \Delta X^0$

Or $\qquad \Delta X^0 = -[J^0]^{-1} F^0 \qquad$ **(29)**

And $\qquad X^1 = X^0 + \Delta X^0 \qquad$ **(30)**

Here, the matrix [J] is called the **Jacobian** matrix. The vector of unknown variables is updated using (30). The process is continued till the difference between two successive iterations is less than the tolerance value.

NR method for load flow solution in polar coordinates

In application of the NR method, we have to first bring the equations to be solved, to the form $f_i(x_1, x_2, ... x_n) = 0$, where $x_1, x_2, ... x_n$ are the unknown variables to be determined. Let us assume that the power system has n_1 PV buses and n_2 PQ buses. In polar coordinates the unknown variables to be determined are:

(i) δ_i, the angle of the complex bus voltage at bus *i*, at all the PV and PQ buses. This gives us $n_1 + n_2$ unknown variables to be determined.

(ii) $|V_i|$, the voltage magnitude of bus *i*, at all the PQ buses. This gives us n_2 unknown variables to be determined.

Therefore, the total number of unknown variables to be computed is: $n_1 + 2n_2$, for which we need $n_1 + 2n_2$ consistent equations to be solved. The equations are given by,

$$\Delta P_i = P_{i,sp} - P_{i,cal} = 0 \quad \textbf{(31)}$$

$$\Delta Q_i = Q_{i,sp} - Q_{i,cal} = 0 \quad \textbf{(32)}$$

Where $P_{i,sp}$ = Specified active power at bus *i*

$Q_{i,sp}$ = Specified reactive power at bus *i*

$P_{i,cal}$ = Calculated value of active power using voltage estimates.

$Q_{i,cal}$ = Calculated value of reactive power using voltage estimates

ΔP = Active power residue

ΔQ = Reactive power residue

The real power is specified at all the PV and PQ buses. Hence (31) is to be solved at all PV and PQ buses leading to $n_1 + n_2$ equations. Similarly the reactive power is specified at all the PQ buses. Hence, (32) is to be solved at all PQ buses leading to n_2 equations.

We thus have $n_1 + 2n_2$ equations to be solved for $n_1 + 2n_2$ unknowns. (31) and (32) are of the form F(x) = 0. Thus NR method can be applied to solve them. Equations (31) and (32) can be written in the form of (30) as:

$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \begin{bmatrix} J_1 & J_2 \\ J_3 & J_4 \end{bmatrix} \begin{bmatrix} \Delta \delta \\ \Delta |V| \end{bmatrix} \qquad \textbf{(33)}$$

Where J_1, J_2, J_3, J_4 are the negated partial derivatives of ΔP and ΔQ with respect to corresponding δ and $|V|$. The negated partial derivative of ΔP, is same as the partial derivative of P_{cal}, since P_{sp} is a constant. The various computations involved are discussed in detail next.

Computation of P_{cal} and Q_{cal}:

The real and reactive powers can be computed from the load flow equations as:

$$P_{i,Cal} = P_i = \sum_{k=1}^{n} |V_i||V_k|(G_{ik} \cos\delta_{ik} + B_{ik} \sin\delta_{ik})$$

$$= G_{ii}|V_i|^2 + \sum_{\substack{k=1 \\ k\neq i}}^{n} |V_i||V_k|(G_{ik} \cos\delta_{ik} + B_{ik} \sin\delta_{ik}) \qquad \textbf{(34)}$$

$$Q_{i,Cal} = Q_i = \sum_{k=1}^{n} |V_i||V_k|(G_{ik} \sin\delta_{ik} - B_{ik} \cos\delta_{ik})$$

$$= -B_{ii}|V_i|^2 + \sum_{\substack{k=1 \\ k\neq i}}^{n} |V_i||V_k|(G_{ik} \sin\delta_{ik} - B_{ik} \cos\delta_{ik}) \qquad \textbf{(35)}$$

The powers are computed at any $(r+1)^{st}$ iteration by using the voltages available from previous iteration. The elements of the Jacobian are found using the above equations as:

Elements of J_1

$$\frac{\partial P_i}{\partial \delta_i} = \sum_{\substack{k=1 \\ k\neq i}}^{n} |V_i||V_k|\{G_{ik}(-\sin\delta_{ik}) + B_{ik}\cos\delta_{ik}\}$$

$$= -Q_i - B_{ii}|V_i|^2$$

$$\frac{\partial P_i}{\partial \delta_k} = |V_i||V_k|(G_{ik}(-\sin\delta_{ik})(-1) + B_{ik}(\cos\delta_{ik})(-1))$$

Elements of J_3

$$\frac{\partial Q_i}{\partial \delta_i} = \sum_{\substack{k=1 \\ k \neq i}}^{n} |V_i||V_k|(G_{ik} \cos\delta_{ik} + B_{ik} \sin\delta_{ik}) = P_i - G_{ii}|V_i|^2$$

$$\frac{\partial Q_i}{\partial \delta_k} = -|V_i||V_k|(G_{ik} \cos\delta_{ik} + B_{ik} \sin\delta_{ik})$$

Elements of J_2

$$\frac{\partial P_i}{\partial |V_i|}|V_i| = 2|V_i|^2 G_{ii} + |V_i| \sum_{\substack{k=1 \\ k \neq i}}^{n} |V_k|(G_{ik} \cos\delta_{ik} + B_{ik} \sin\delta_{ik}) = P_i + |V_i|^2 G$$

$$\frac{\partial P_i}{\partial |V_k|}|V_k| = |V_i||V_k|(G_{ik} \cos\delta_{ik} + B_{ik} \sin\delta_{ik})$$

Elements of J_4

$$\frac{\partial P_i}{\partial |V_i|}|V_i| = -2|V_i|^2 B_{ii} + \sum_{\substack{k=1 \\ k \neq i}}^{n} |V_i||V_k|(G_{ik} \sin\delta_{ik} - B_{ik} \cos\delta_{ik}) = Q_i - |V_i|^2$$

$$\frac{\partial Q_i}{\partial |V_k|}|V_k| = |V_i||V_k|(G_{ik} \sin\delta_{ik} - B_{ik} \cos\delta_{ik})$$

Thus, the linearized form of the equation could be considered agai

$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \begin{bmatrix} H & N \\ M & L \end{bmatrix} \begin{bmatrix} \Delta\delta \\ \dfrac{\Delta|V|}{|V|} \end{bmatrix}$$

The elements are summarized below:

(i) $H_{ii} = \dfrac{\partial P_i}{\partial \delta_i} = -Q_i - B_{ii}|V_i|^2$

(ii) $H_{ik} = \dfrac{\partial P_i}{\partial \delta_k} = a_k f_i - b_k e_i = |V_i||V_k|(G_{ik} \sin\delta_{ik} - B_{ik} \cos\delta_{ik})$

(iii) $N_{ii} = \dfrac{\partial P_i}{\partial |V_i|}|V_i| = P_i + G_{ii}|V_i|^2$

(iv) $N_{ik} = \dfrac{\partial P_i}{\partial |V_k|}|V_k| = a_k e_i + b_k f_i = |V_i||V_k|(G_{ik} \cos\delta_{ik} + B_{ik} \sin\delta_{ik})$

(v) $M_{ii} = \dfrac{\partial Q_i}{\partial \delta_i} = P_i - G_{ii}|V_i|^2$

DECOUPLED LOAD FLOW

In the NR method, the inverse of the Jacobian has to be computed at every iteration. When solving large interconnected power systems, alternative solution methods are possible, taking into account certain observations made of practical systems. These are,

- Change in voltage magnitude $|V_i|$ at a bus primarily affects the flow of reactive power Q in the lines and leaves the real power P unchanged. This observation implies that $\frac{\partial Q_i}{\partial |V_j|}$ is much larger than $\frac{\partial P_i}{\partial |V_j|}$. Hence, in the Jacobian, the elements of the sub-matrix $[N]$, which contains terms that are partial derivatives of real power with respect to voltage magnitudes can be made zero.

- Change in voltage phase angle at a bus, primarily affects the real power flow P over the lines and the flow of Q is relatively unchanged. This observation implies that $\frac{\partial P_i}{\partial \delta_j}$ is much larger than $\frac{\partial Q_i}{\partial \delta_j}$. Hence, in the Jacobian the elements of the sub-matrix $[M]$, which contains terms that are partial derivatives of reactive power with respect to voltage phase angles can be made zero.

These observations reduce the NRLF linearised form of equation to

$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \begin{bmatrix} H & 0 \\ 0 & L \end{bmatrix} \begin{bmatrix} \Delta\delta \\ \frac{\Delta|V|}{|V|} \end{bmatrix} \qquad \textbf{(37)}$$

From (37) it is obvious that the voltage angle corrections $\Delta\delta$ are obtained using real power residues ΔP and the voltage magnitude corrections $|\Delta V|$ are obtained from reactive power residues ΔQ. This equation can be solved through two alternate strategies as under:

Strategy-1

(i) Calculate $\Delta P^{(r)}, \Delta Q^{(r)}$ and $J^{(r)}$

(ii) Compute $\begin{bmatrix} \Delta\delta^{(r)} \\ \dfrac{\Delta|V^{(r)}|}{|V^{(r)}|} \end{bmatrix} = \left[J^{(r)}\right]^{-1} \begin{bmatrix} \Delta P^{(r)} \\ \Delta Q^{(r)} \end{bmatrix}$

(iii) Update δ and $|V|$.

(iv) Go to step (i) and iterate till convergence is reached.

Strategy-2

(i) Compute $\Delta P^{(r)}$ and Sub-matrix $H^{(r)}$. From (37) find $\Delta\delta^{(r)} = \left[H^{(r)}\right]^{-1} \Delta P^{(r)}$

(ii) Up date δ using $\delta^{(r+1)} = \delta^{(r)} + \Delta\delta^{(r)}$.

(iii) Use $\delta^{(r+1)}$ to calculate $\Delta Q^{(r)}$ and $L^{(r)}$

(iv) Compute $\dfrac{\Delta|V^{(r)}|}{|V^{(r)}|} = \left[L^{(r)}\right]^{-1} \Delta Q^{(r)}$

(v)Update, $|V^{(r+1)}| = |V^{(r)}| + |\Delta V^{(r)}|$

(vi) Go to step (i) and iterate till convergence is reached.

In the first strategy, the variables are solved simultaneously. In the second strategy the iteration is conducted by first solving for $\Delta\delta$ and using updated values of δ to calculate $\Delta|V|$. Hence, the second strategy results in faster convergence, compared to the first strategy.

FAST DECOUPLED LOAD FLOW

If the coefficient matrices are constant, the need to update the Jacobian at every iteration is eliminated. This has resulted in development of fast decoupled load Flow (FDLF). Here, certain assumptions are made based on the observations of practical power systems as under:

- $B_{ij} >> G_{ij}$ (Since the X/R ratio of transmission lines is high in well designed systems)

- The voltage angle difference $(\delta_i - \delta_j)$ between two buses in the system is very small. This means $\cos(\delta_i - \delta_j) \cong 1$ and $\sin(\delta_i - \delta_j) = 0.0$
- $Q_i << B_{ii}|V_i|^2$

With these assumptions the elements of the Jacobian become

$$H_{ik} = L_{ik} = -|V_i||V_k|B_{ik} \qquad (i \neq k)$$

$$H_{ii} = L_{ii} = -B_{ii}|V_i|^2$$

The matrix (37) reduces to

$$[\Delta P] = [|V_i||V_j|B'_{ij}][\Delta\delta]$$

$$[\Delta Q] = [|V_i||V_j|B''_{ij}]\left[\frac{\Delta|V|}{|V|}\right] \qquad \textbf{(38)}$$

Where B'_{ij} and B_{ij}'' are negative of the susceptances of respective elements of the bus admittance matrix. In (38) if we divide LHS and RHS by $|V_i|$ and assume $|V_j| \cong 1$, we get,

$$\left[\frac{\Delta P}{|V|}\right] = [B'_{ij}][\Delta\delta]$$

$$\left[\frac{\Delta Q}{|V|}\right] = [B''_{ij}]\left[\frac{\Delta|V|}{|V|}\right] \qquad \textbf{(39)}$$

Equations (39) constitute the Fast Decoupled load flow equations. Further simplification is possible by:

- Omitting effect of phase shifting transformers
- Setting off-nominal turns ratio of transformers to 1.0
- In forming B'_{ij}, omitting the effect of shunt reactors and capacitors which mainly affect reactive power
- Ignoring series resistance of lines in forming the Y_{bus}.

With these assumptions we obtain a loss-less network. In the FDLF method, the matrices $[B']$ and $[B'']$ are constants and need to be inverted only once at the beginning of the iterations.

REPRESENTATION OF TAP CHANGING TRANSFORMERS

Consider a tap changing transformer represented by its admittance connected in series with an ideal autotransformer as shown (a= turns ratio of transformer)

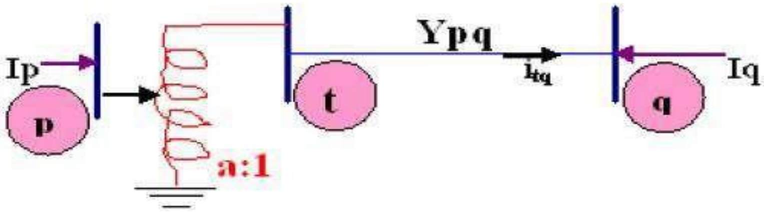


Fig. 2. Equivalent circuit of a tap setting transformer

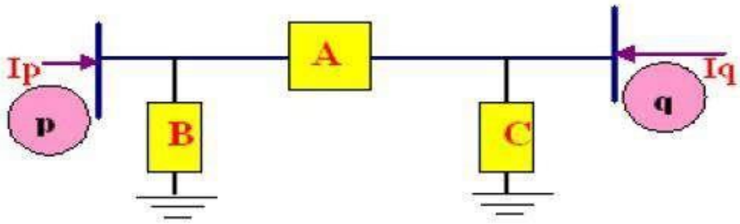


Fig. 3. π-Equivalent circuit of Fig.2 above.

By equating the bus currents in both the mutually equivalent circuits as above, it can be shown that the π-equivalent circuit parameters are given by the expressions as under:

(i) Fixed tap setting transformers (on no load)

A = Ypq/ a

B = 1/a (1/a -1) Ypq

C = (1-1/a) Ypq

(i) Tap changing under load (TCUL) transformers (on load)

A = Ypq

B = (1/a -1) (1/a + 1 – Eq/Ep) Ypq

C = (1-1/a) (Ep/Eq) Ypq

Thus, here, in the case of TCUL transformers, the shunt admittance values are observed to be a function of the bus voltages.

COMPARISON OF LOAD FLOW METHODS

The comparison of the methods should take into account the computing time required for preparation of data in proper format and data processing, programming ease, storage requirements, computation time per iteration, number of iterations, ease and time required for modifying network data when operating conditions change, etc. Since all the methods presented are in the bus frame of reference in admittance form, the data preparation is same for all the methods and the bus admittance matrix can be formed using a simple algorithm, by the rule of inspection. Due to simplicity of the equations, Gauss-Seidel method is relatively easy to program. Programming of NR method is more involved and becomes more complicated if the buses are randomly numbered. It is easier to program, if the PV buses are ordered in sequence and PQ buses are also ordered in sequence.

The storage requirements are more for the NR method, since the Jacobian elements have to be stored. The memory is further increased for NR method using rectangular coordinates. The storage requirement can be drastically reduced by using sparse matrix techniques, since both the admittance matrix and the Jacobian are sparse matrices. The time taken for a single iteration depends on the number of arithmetic and logical operations required to be performed in a full iteration. The Gauss –Seidel method requires the fewest number of operations to complete iteration. In the NR method, the computation of the Jacobian is necessary in every iteration. Further, the inverse of the Jacobian also has to be computed. Hence, the time per iteration is larger than in the GS method and is roughly about 7 times that of the GS method, in large systems, as depicted graphically in figure below. Computation time can be reduced if the Jacobian is updated once in two or three iterations. In FDLF method, the Jacobian is constant and needs to be computed only once. In both NR and FDLF methods, the time per iteration increases directly as the number of buses.

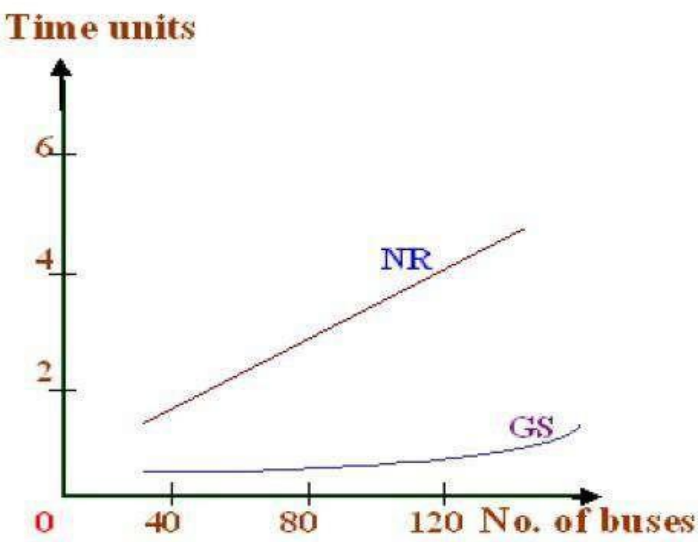


Figure 4. Time per Iteration in GS and NR methods

The number of iterations is determined by the convergence characteristic of the method. The GS method exhibits a linear convergence characteristic as compared to the NR method which has a quadratic convergence. Hence, the GS method requires more number of iterations to get a converged solution as compared to the NR method. In the GS method, the number of iterations increases directly as the size of the system increases. In contrast, the number of iterations is relatively constant in NR and FDLF methods. They require about 5-8 iterations for convergence in large systems. A significant increase in rate of convergence can be obtained in the GS method if an acceleration factor is used. All these variations are shown graphically in figure below. The number of iterations also depends on the required accuracy of the solution. Generally, a voltage tolerance of 0.0001 pu is used to obtain acceptable accuracy and the real power mismatch and reactive power mismatch can be taken as 0.001 pu. Due to these reasons, the NR method is faster and more reliable for large systems. The convergence of FDLF method is geometric and its speed is nearly 4-5 times that of NR method.

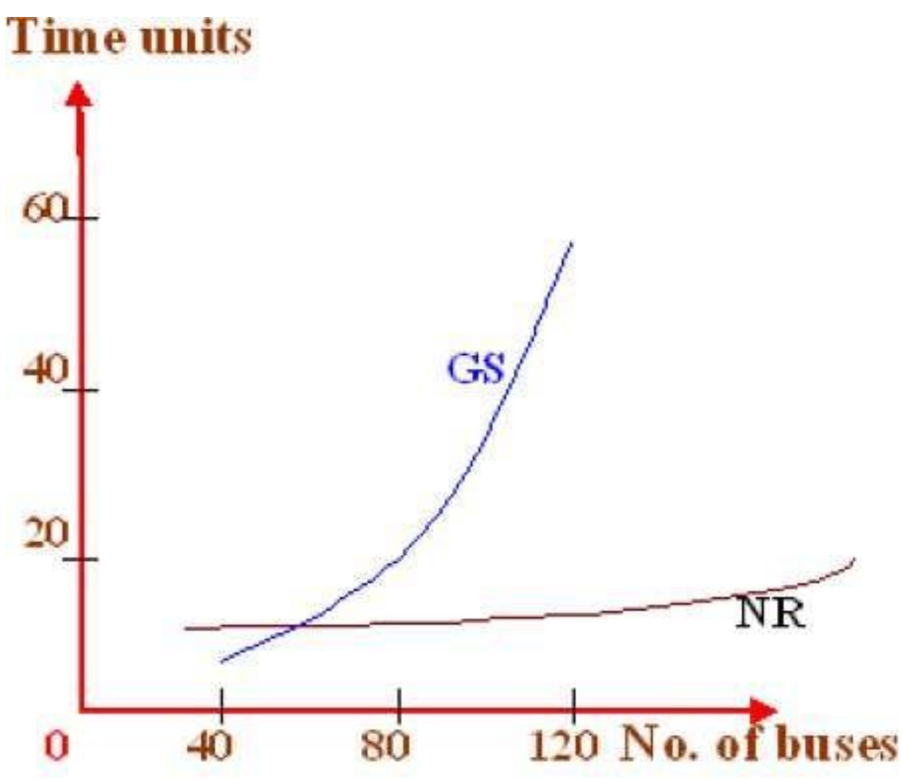


Figure 5. Total time of Iteration in GS and NR methods

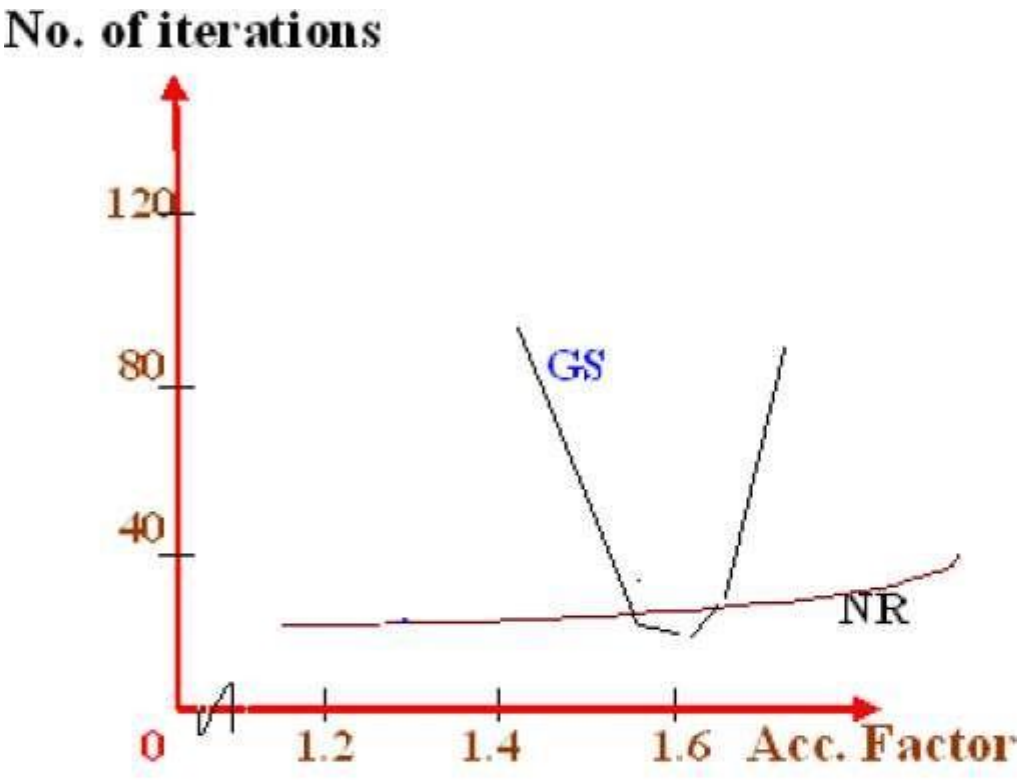


Figure 6. Influence of acceleration factor on load flow methods

FINAL WORD

In this chapter, the load flow problem, also called as the power flow problem, has been considered in detail. The load flow solution gives the complex voltages at all the buses and the complex power flows in the lines. Though, algorithms are available using the impedance form of the equations, the sparsity of the bus admittance matrix and the ease of building the bus admittance matrix, have made algorithms using the admittance form of equations more popular. The most popular methods are the Gauss-Seidel method, the Newton-Raphson method and the Fast Decoupled Load Flow method. These methods have been discussed in detail with illustrative examples. In smaller systems, the ease of programming and the memory requirements, make GS method attractive. However, the computation time increases with increase in the size of the system. Hence, in large systems NR and FDLF methods are more popular. There is a trade off between various requirements like speed, storage, reliability, computation time, convergence characteristics etc. No single method has all the desirable features.

UNIT-IV

POWER SYSTEM STABILITY:

INTRODUCTION

Power system stability of modern large inter-connected systems is a major problem for secure operation of the system. Recent major black-outs across the globe caused by system instability, even in very sophisticated and secure systems, illustrate the problems facing secure operation of power systems. Earlier, stability was defined as the ability of a system to return to normal or stable operation after having been subjected to some form of disturbance. This fundamentally refers to the ability of the system to remain in synchronism. However, modern power systems operate under complex interconnections, controls and extremely stressed conditions. Further, with increased automation and use of electronic equipment, the quality of power has gained utmost importance, shifting focus on to concepts of voltage stability, frequency stability,
inter-area oscillations etc.

The IEEE/CIGRE Joint Task Force on stability terms and conditions have proposed the following definition in 2004: "*Power System stability is the ability of an electric power system, for a given initial operating condition, to regain a state of operating equilibrium after being subjected to a physical disturbance, with most system variables bounded, so that practically the entire system remains intact".* The Power System is an extremely non-linear and dynamic system, with operating parameters continuously varying. Stability is hence, a function of the initial operating condition and the nature of the disturbance. Power systems are continually subjected to small disturbances in the form of load changes. The system must be in a position to be able to adjust to the changing conditions and operate satisfactorily. The system must also withstand large disturbances, which may even cause structural changes due to isolation of some faulted elements. A power system may be stable for a particular (large) disturbance and unstable for another disturbance. It is impossible to design a system which is stable under all disturbances. The power system is generally designed to be stable under those disturbances which have a high degree of occurrence. The response to a disturbance is extremely complex and involves practically all the equipment of the power system. For example, a short circuit leading to a line isolation by circuit breakers will cause variations in the power flows, network bus voltages and generators rotor speeds. The voltage variations will actuate the voltage regulators in the system and generator speed variations will actuate the prime mover governors; voltage and frequency variations will affect the system loads. In stable systems, practically all generators and loads remain connected, even though parts of the system may be isolated to preserve bulk operations. On the other hand, an unstable system condition could lead to cascading outages and a shutdown of a major portion of the power system.

ROTOR ANGLE STABILITY

Rotor angle stability refers to the ability of the synchronous machines of an interconnected power system to remain in synchronism after being subjected to a disturbance. Instability results in some generators accelerating (decelerating) and losing synchronism with other generators. Rotor angle stability depends on the ability of each synchronous machine to maintain equilibrium between electromagnetic torque and mechanical torque. Under steady state, there is equilibrium between the input mechanical torque and output electromagnetic torque of each generator, and its speed remains a constant. Under a disturbance, this equilibrium is upset and the generators accelerate/decelerate according to the mechanics of a rotating body. Rotor angle stability is further categorized as follows:

Small single (or small disturbance) rotor angle stability

It is the ability of the power system to maintain synchronism under small disturbances. In this case, the system equation can be linearized around the initial operating point and the stability depends only on the operating point and not on the disturbance. Instability may result in

(i) A non oscillatory or a periodic increase of rotor angle

(ii) (ii) Increasing amplitude of rotor oscillations due to insufficient damping. The first form of instability is largely eliminated by modern fast acting voltage regulators and the second form of instability is more common. The time frame of small signal stability is of the order of 10-20 seconds after a disturbance.

Large-signal rotor angle stability or transient stability

This refers to the ability of the power system to maintain synchronism under large disturbances, such as short circuit, line outages etc. The system response involves large excursions of the generator rotor angles. Transient stability depends on both the initial operating point and the disturbance parameters like location, type, magnitude etc. Instability is normally in the form of a periodic angular separation. The time frame of interest is 3-5 seconds after disturbance. The term dynamic stability was earlier used to denote the steady- state stability in the presence of automatic controls (especially excitation controls) as opposed to manual controls. Since all generators are equipped with automatic controllers today, dynamic stability has lost relevance and the Task Force has recommended against its usage.

MECHANICS OF ROTATORY MOTION

Since a synchronous machine is a rotating body, the laws of mechanics of rotating bodies are applicable to it. In rotation we first define the fundamental quantities. The angle _m is defined, with respect to a circular arc with its center at the vertex of the angle, as the ratio of the arc length *s* to radius *r*.

$$\theta_m = \frac{s}{r} \tag{1}$$

The unit is radian. Angular velocity ω_m is defined as

$$\omega_m = \frac{d\theta_m}{dt} \tag{2}$$

and angular acceleration as

$$\alpha = \frac{d\omega_m}{dt} = \frac{d^2\theta_m}{dt^2} \tag{3}$$

The torque on a body due to a tangential force F at a distance r from axis of rotation is given by

$$T = r\,F \qquad (4)$$

The total torque is the summation of infinitesimal forces, given by

$$T = \int r\,dF \qquad (5)$$

The unit of torque is N-m. When torque is applied to a body, the body experiences angular acceleration. Each particle experiences a tangential acceleration $a = r\alpha$, where r is the distance of the particle from axis of rotation. The tangential force required to accelerate a particle of mass *dm* is

$$dF = a\,dm = r\,\alpha\,dm \qquad (6)$$

The torque required for the particle is

$$dT = r\,dF = r^2\,\alpha\,dm \qquad (7)$$

and that required for the whole body is given by

$$T = \alpha \int r^2 dm = I\,\alpha \qquad (8)$$

Here

$$I = \int r^2 dm \qquad (9)$$

is called the moment of inertia of the body. The unit is $Kg - m^2$. If the torque is assumed to be the result of a number of tangential forces F, which act at different points of the body

$$T = \sum r\,F$$

Now each force acts through a distance

$$ds = r\,d\theta_m$$

The work done is $\sum F\,.\,ds$

$$dW = \sum F\,r\,d\theta_m = d\theta_m\,T$$

$$W = \int T\,d\theta_m \qquad (10)$$

and $$T = \frac{dW}{d\theta_m} \qquad (11)$$

Thus the unit of torque may also be Joule per radian.

The power is defined as rate of doing work. Using (11)

$$P = \frac{dW}{dt} = \frac{T\, d\theta_m}{dt} = T\,\omega_m \tag{12}$$

The angular momentum M is defined as

$$M = I\,\omega_m \tag{13}$$

and the kinetic energy is given by

$$KE = \frac{1}{2} I\,\omega_m^{\ 2} = \frac{1}{2} M\,\omega_m \tag{14}$$

From (14) we can see that the unit of M is seen to be J-sec/rad.

SWING EQUATION:

From (8)

$$I\alpha = T$$

or $$\frac{I\, d^2\theta_m}{dt^2} = T \tag{15}$$

Here T is the net torque of all torques acting on the machine, which includes the shaft torque (due to prime mover of a generator or load on a motor), torque due to rotational losses (friction, windage and core loss) and electromagnetic torque.

Let T_m = shaft torque or mechanical torque corrected for rotational losses

T_e = Electromagnetic or electrical torque

For a generator T_m tends to accelerate the rotor in positive direction of rotation and for a motor retards the rotor.

The accelerating torque for a generator

$$T_a = T_m - T_e \tag{16}$$

Under steady-state operation of the generator, T_m is equal to T_e and the accelerating torque is zero. There is no acceleration or deceleration of the rotor masses and the machines run at a constant synchronous speed. In the stability analysis in the following sections, T_m is assumed to be a constant since the prime movers (steam turbines or hydro turbines) do no act during the short time period in which rotor dynamics are of interest in the stability studies.

Now (15) has to be solved to determine θ_m as a function of time. Since θ_m measured with respect to a stationary reference axis on the stator, it is the measure of th absolute rotor angle and increases continuously with time even at constant synchronou speed. Since machine acceleration /deceleration is always measured relative t synchronous speed, the rotor angle is measured with respect to a synchronously rotatir reference axis. Let

$$\delta_m = \theta_m - \omega_{sm} t \tag{17}$$

where ω_{sm} is the synchronous speed in mechanical rad/s and δ_m is the angul displacement in mechanical radians.

Taking the derivative of (17) we get

$$\frac{d\delta_m}{dt} = \frac{d\theta_m}{dt} - \omega_{sm}$$

$$\frac{d^2\delta_m}{dt^2} = \frac{d^2\theta_m}{dt^2} \tag{18}$$

Substituting (18) in (15) we get

$$I\frac{d^2\delta_m}{dt^2} = T_a = T_m - T_e \quad \text{N-m} \tag{19}$$

Multiplying by ω_m on both sides we get

$$\omega_m I\frac{d^2\delta_m}{dt^2} = \omega_m (T_m - T_e) \quad \text{N-m} \tag{20}$$

From (12) and (13), we can write

$$M\frac{d^2\delta_m}{dt^2} = P_m - P_a \qquad W \tag{21}$$

where M is the angular momentum, also called inertia constant

P_m = shaft power input less rotational losses

P_e = Electrical power output corrected for losses

P_a = acceleration power

M depends on the angular velocity ω_m, and hence is strictly not a constant, because ω_m deviates from the synchronous speed during and after a disturbance. However, under stable conditions ω_m does not vary considerably and M can be treated as a constant. (21) is called the *"Swing equation"*. The constant M depends on the rating of the machine and varies widely with the size and type of the machine. Another constant called H constant (also referred to as inertia constant) is defined as

$$\text{H} = \frac{\begin{matrix} \textit{stored kinetic energy in mega joules} \\ \textit{at sychronous speed} \end{matrix}}{\textit{Machine rating in MVA}} \; MJ/MVA \tag{22}$$

H falls within a narrow range and typical values are given in Table 9.1.

If the rating of the machine is G MVA, from (22) the stored kinetic energy is GH Mega Joules. From (14)

$$\text{GH} = \frac{1}{2} M\omega_{sm} \;\; \text{MJ} \tag{23}$$

or

$$\text{M} = \frac{2GH}{\omega_{sm}} \quad \text{MJ-s/mech rad} \tag{24}$$

The swing equation (21) is written as

$$\frac{2H}{\omega_{sm}} \frac{d^2\delta_m}{dt^2} = \frac{P_a}{G} = \frac{P_{m-}P_e}{G} \tag{25}$$

In (.25) δ_m is expressed in mechanical radians and ω_{sm} in mechanical radians per second (the subscript *m* indicates mechanical units). If δ and ω have consistent units then mec

$$\frac{2H}{\omega_s} \frac{d^2\delta}{dt^2} = P_a = P_m - P_e \quad \text{pu} \tag{26}$$

Here ω_s is the synchronous speed in electrical rad/s ($\omega_s = \left(\frac{p}{2}\right)\omega_{sm}$) and P_a is acceleration power in per unit on same base as H. For a system with an electrical frequency f Hz, (26) becomes

$$\frac{H}{\pi f} \frac{d^2\delta}{dt^2} = P_a = P_m - P_e \quad \text{pu} \tag{27}$$

when δ is in electrical radians and

$$\frac{H}{180f}\frac{d^2\delta}{dt^2} = P_a = P_m - P_e \quad \text{pu} \tag{28}$$

when δ is in electrical degrees.

(27) and (28) also represent the swing equation. It can be seen that the swing equation is a second order differential equation which can be written as two first order differential equations:

$$\frac{2H}{\omega_s}\frac{d\omega}{dt} = P_m - P_e \quad \text{pu} \tag{29}$$

$$\frac{d\delta}{dt} = \omega - \omega_s \tag{30}$$

in which ω, ω_s and δ are in electrical units. In deriving the swing equation, damping has been neglected.

Table 1 : H constants of synchronous machines

Type of machine		H (MJ/MVA)
Turbine generator condensing	1800 rpm	9 – 6
	3600 rpm	7 – 4
Non condensing	3600 rpm	4 – 3
Water wheel generator		
	Slow speed < 200 rpm	2 – 3
	High speed > 200 rpm	2 – 4
Synchronous condenser		
	Large	1.25 } 25% less for hydrogen cooled
	Small	1.0 }
Synchronous motor with load varying from 1.0 to 5.0		2.0

In defining the inertia constant H, the MVA base used is the rating of the machine. In a multi machine system, swing equation has to be solved for each machine, in which case, a common MVA base for the system has to chosen. The constant H of each machine must be consistent with the system base.

Let

G_{mach} = Machine MVA rating (base)

G_{system} = System MVA base

In (9.25), H is computed on the machine rating $G = G_{mach}$

Multiplying (9.25) by $\frac{G_{mach}}{G_{system}}$ on both sides we get

$$\left(\frac{G_{mach}}{G_{system}}\right)\frac{2H}{\omega_{sm}}\frac{d^2\delta_m}{dt^2} = \frac{P_m - P_e}{G_{mach}}\left(\frac{G_{mach}}{G_{system}}\right) \quad (31)$$

$$\frac{2H_{system}}{\omega_{sm}}\frac{d^2\delta_m}{dt^2} = P_m - P_e \quad \text{pu (on system base)}$$

where $H_{system} = H\frac{G_{mach}}{G_{system}}$ (32)

In the stability analysis of a multi machine system, computation is considerably reduced if the number of swing equations to be solved is reduced. Machines within a plant normally swing together after a disturbance. Such machines are called coherent machines and can be replaced by a single equivalent machine, whose dynamics reflects the dynamics of the plant.

Example 1:

A 50Hz, 4 pole turbo alternator rated 150 MVA, 11 kV has an inertia constant of 9 MJ / MVA. Find the (a) stored energy at synchronous speed (b) the rotor acceleration if the input mechanical power is raised to 100 MW when the electrical load is 75 MW, (c) the speed at the end of 10 cycles if acceleration is assumed constant at the initial value.

Solution:

(a) Stored energy = GH = 150 × 9 = 1350 MJ

(b) $P_a = P_m - P_e = 100 - 75 = 25$ MW

$$M = \frac{GH}{180f} = \frac{1350}{180 \times 50} = 0.15 \text{ MJ} - \text{s}/^\circ\text{e}$$

$$0.15\frac{d^2\delta}{dt^2} = 25$$

$$\text{Acceleration } \alpha = \frac{d^2\delta}{dt^2} = \frac{25}{0.15} = 166.6 \ ^\circ\text{e/s}^2$$

$$= 166.6 \times \frac{2}{P} \ ^\circ\text{m/s}^2$$

$$= 166.6 \times \frac{2}{P} \times \frac{1}{360} \ rps/\text{s}$$

$$= 166.6 \times \frac{2}{P} \times \frac{1}{360} \times 60 \text{ rpm/s}$$

$$= 13.88 \text{ rpm/s}$$

* Note °e = electrical degree; °m = mechanical degree; P=number of poles.

(c) 10 cycles = $\frac{10}{50} = 0.2$ s

N_S = Synchronous speed = $\frac{120 \times 50}{4} = 1500$ rpm

Rotor speed at end of 10 cycles = $N_S + \alpha \times 0.2$

$= 1500 + 13.88 \times 0.2 = 1502.776$ rpm

Example 2:

Two 50 Hz generating units operate in parallel within the same plant, with the following ratings:

Unit 1: 500 MVA, 0.8 pf, 13.2 kV, 3600 rpm: H = 4 MJ/MVA

Unit 2: 1000 MVA, 0.9 pf, 13.8 kV, 1800 rpm: H = 5 MJ/MVA

Calculate the equivalent H constant on a base of 100 MVA.

Solution:

$$H_{1system} = H_{1mach} \times \frac{G_{1mach}}{G_{system}}$$

$$= 4 \times \frac{500}{100} = 20 \text{ MJ/MVA}$$

$$H_{2system} = H_{2mach} \times \frac{G_{2mach}}{G_{system}}$$

$$= 5 \times \frac{1000}{100} = 50 \text{ MJ/MVA}$$

$$H_{eq} = H_1 + H_2 = 20 + 50 = 70 \text{ MJ/MVA}$$

This is the equivalent inertia constant on a base of 100 MVA and can be used when the two machines swing coherently.

POWER–ANGLE EQUATION:

In solving the swing equation, certain assumptions are normally made (i) Mechanical power nput Pm is a constant during the period of interest, immediately after the disturbance (ii) Rotor speed changes are insignificant. (iii) Effect of voltage regulating loop during the transient is neglected i.e the excitation is assumed to be a constant. As discussed in section 9.4, the power–angle relationship plays a vital role in the solution of the swing equation.

POWER–ANGLE EQUATION FOR A NON–SALIENT POLE MACHINE:

The simplest model for the synchronous generator is that of a constant voltage behind an impedance. This model is called the classical model and can be used for cylindrical rotor (non–salient pole) machines. Practically all high–speed turbo alternators are of cylindrical rotor construction, where the physical air gap around the periphery of the rotor is uniform. This type of generator has approximately equal magnetic reluctance, regardless of the angular position of the rotor, with respect to the armature mmf.

The power output of the generator is given by the real part of $E_g I_a^*$.

$$I_a = \frac{E_g \angle\delta - V_t \angle 0°}{R_a + jx_d} \tag{38}$$

Neglecting R_a, $I_a = \dfrac{E_g \angle\delta - V_t \angle 0°}{jx_d}$

$$P = \Re \left\{ (E_g \angle\delta)\left(\frac{E_g \angle 90° - \delta}{x_d} - \frac{V_t \angle 90°}{x_d} \right)^* \right\}$$

$$= \frac{E_g^{\,2} \cos 90°}{x_d} - \frac{E_g V_t \cos(90° + \delta)}{x_d}$$

$$= \frac{E_g V_t \sin\delta}{x_d} \tag{39}$$

(Note- $\Re$ stands for real part of)

The maximum power that can be transferred for a particular excitation is given by $\dfrac{E_g V_t}{x_d}$ at $\delta = 90°$.

POWER ANGLE EQUATION FOR A SALIENT POLE MACHINE:

Here because of the salient poles, the reluctance of the magnetic circuit in which flows the flux produced by an armature mmf in line with the quadrature axis is higher than that of the magnetic circuit in which flows the flux produced by the armature mmf in line with the direct axis. These two components of armature mmf are proportional to the corresponding components of armature current. The component of armature current producing an mmf acting in line with direct axis is called the direct component, Id. The component of armature current producing an mmf acting in line with the quadrature axis is called the quadrature axis component, Iq.

Power output $P = V_t I_a \cos\theta$

$$= E_d I_d + E_q I_q \qquad (40)$$

$$E_d = V_t \sin\delta \qquad (41a)$$

$$E_q = V_t \cos\delta \qquad (41b)$$

$$I_d = \frac{E_g - E_q}{x_d} = I_a \sin(\delta + \theta) \qquad (41c)$$

$$I_q = \frac{E_d}{x_q} = I_a \cos(\delta + \theta) \qquad (41d)$$

Substituting (9.41c) and (9.41d) in (9.40), we obtain

$$P = \frac{E_g V_t \sin\delta}{x_d} + \frac{V_t^2 (x_d - x_q) \sin 2\delta}{2 x_d x_q} \qquad (42)$$

(9.42) gives the steady state power angle relationship for a salient pole machine. The second term does not depend on the excitation and is called the reluctance power component. This component makes the maximum power greater than in the classical model. However, the angle at which the maximum power occurs is less than 90°.

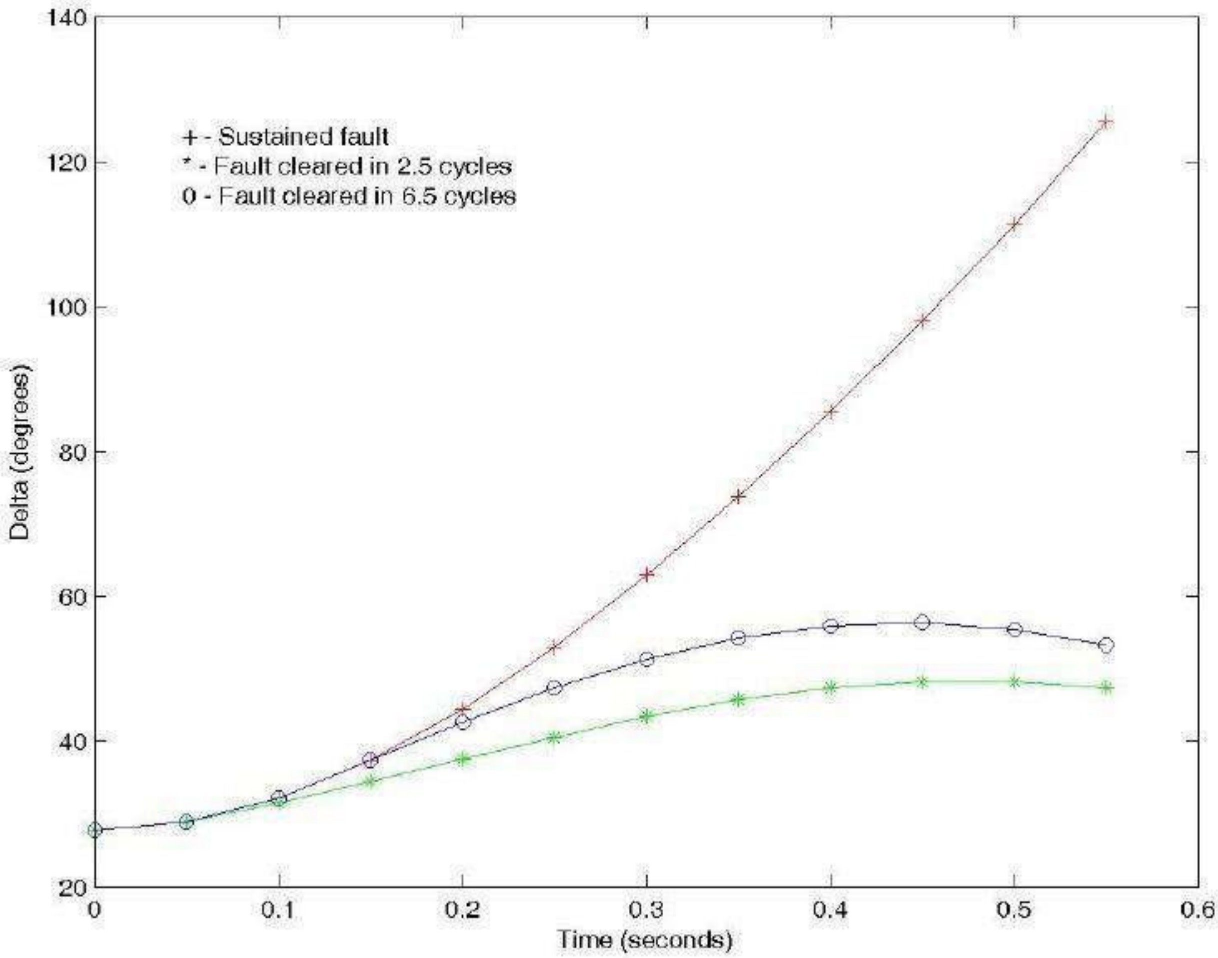
TRANSIENT STABILITY:

As defined earlier, transient stability is the ability of the system to remain stable under large disturbances like short circuits, line outages, generation or load loss etc. The evaluation of the transient stability is required offline for planning, design etc. and online for load management, emergency control and security assessment. Transient stability analysis deals with actual solution of the nonlinear differential equations describing the dynamics of the machines and their controls and interfacing it with the algebraic equations describing the interconnections through the transmission network. Since the disturbance is large, linearized analysis of the swing equation (which describes the rotor dynamics) is not possible. Further, the fault may cause structural changes in the network, because of which the power angle curve prior to fault, during the fault and post fault may be different. Due to these reasons, a general stability criteria for transient stability cannot be established, as was done in the case of steady state stability (namely PS > 0). Stability can be established, for a given fault, by actual solution of the swing equation. The time taken for the fault to be cleared (by the circuit breakers) is called the *clearing time*. If the fault is cleared fast enough, the probability of the system remaining stable after the clearance is more. If the fault persists for a longer time, likelihood of instability is increased. *Critical clearing time* is the maximum time available for clearing the fault, before the system loses stability. Modern circuit breakers are equipped with auto reclosure facility, wherein the breaker automatically recloses after two sequential openings. If the fault still persists, the breakers open permanently. Since most faults are transient, the first reclosure is in general successful. Hence, transient stability has been greatly enhanced by auto closure breakers.

Some common assumptions made during transient stability studies are as follows:

1. Transmission line and synchronous machine resistances are neglected. Since resistance introduces a damping term in the swing equation, this gives pessimistic results.
2. Effect of damper windings is neglected which again gives pessimistic results.
3. Variations in rotor speed are neglected.
4. Mechanical input to the generator is assumed constant. The governor control loop is neglected. This also leads to pessimistic results.
5. The generator is modeled as a constant voltage source behind a transient reactance, neglecting the voltage regulator action.
6. Loads are modeled as constant admittances and absorbed into the bus admittance matrix.

The above assumptions, vastly simplify the equations. A digital computer program for transient stability analysis can easily include more detailed generator models and effect of controls, the discussion of which is beyond the scope of present treatment. Studies on the transient stability of an SMIB system, can shed light on some important aspects of stability of larger systems. The figure below shows an example of how the clearing time has an effect on the swing curve of the machine.



Modified Euler's method:

Euler‟s method is one of the easiest methods to program for solution of differential equations using a digital computer . It uses the Taylor‟s series expansion, discarding all second–order and higher–order terms. Modified Euler‟s algorithm uses the derivatives at the beginning of a time step, to predict the values of the dependent variables at the end of the step (t1 = t0 +Δt). Using the predicted values, the derivatives at the end of the interval are computed. The average of the two derivatives is used in updating the variables.

Consider two simultaneous differential equations:

$$\frac{dx}{dt} = f_x(x, y, t)$$

$$\frac{dy}{dt} = f_y(x, y, t)$$

Starting from initial values x0, y0, t0 at the beginning of a time step and a step size *h* we solve as follows:

Let

$$D_x = f_x(x_0, y_0, t_0) = \left.\frac{dx}{dt}\right|_0$$

$$D_y = f_y(x_0, y_0, t_0) = \left.\frac{dy}{dt}\right|_0$$

$$\left.\begin{aligned} x^P &= x_0 + D_x\, h \\ y^P &= y_0 + D_y\, h \end{aligned}\right\} \quad \text{Predicted values}$$

$$D_{xP} = \left.\frac{dx}{dt}\right|_P = f_x(x^P, y^P, t_1)$$

$$D_{yP} = \left.\frac{dy}{dt}\right|_P = f_y(x^P, y^P, t_1)$$

$$x_1 = x_o + \left(\frac{D_x + D_{xP}}{2}\right)h$$

$$y_1 = y_o + \left(\frac{D_y + D_{yP}}{2}\right)h$$

x 1 and y1 are used in the next iteration. To solve the swing equation by Modified Euler"s method, it is written as two first order differential equations:

$$\frac{d\delta}{dt} = \omega$$

$$\frac{d\omega}{dt} = \frac{P_a}{M} = \frac{P_m - P_{\max}\sin\delta}{M}$$

Starting from an initial value _o, _o at the beginning of any time step, and choosing a step size _t s, the equations to be solved in modified Euler‟s are as follows:

$$\left.\frac{d\delta}{dt}\right|_0 = D_1 = \omega_o$$

$$\left.\frac{d\omega}{dt}\right|_0 = D_2 = \frac{P_m - P_{\max}\sin\delta_0}{M}$$

$$\delta^P = \delta_0 + D_1\,\Delta t$$

$$\omega^P = \omega_0 + D_2\,\Delta t$$

$$\left.\frac{d\delta}{dt}\right|_P = D_{1P} = \omega^P$$

$$\left.\frac{d\omega}{dt}\right|_P = D_{2P} = \frac{P_m - P_{\max}\sin\delta^P}{M}$$

$$\delta_1 = \delta_0 + \left(\frac{D_1 + D_{1P}}{2}\right)\Delta t$$

$$\omega_1 = \omega_0 + \left(\frac{D_2 + D_{2P}}{2}\right)\Delta t$$

δ_1 and ω_1 are used as initial values for the successive time step. Numerical errors are introduced because of discarding higher–order terms in Taylor's expansion. Errors can be decreased by choosing smaller values of step size. Too small a step size, will increase computation, which can lead to large errors due to rounding off. The Runge- Kutta method which uses higher–order terms is more popular.

Example :A 50 Hz, synchronous generator having inertia constant H = 5.2 MJ/MVA and x_d' = 0.3 pu is connected to an infinite bus through a double circuit line as shown in Fig. 9.21. The reactance of the connecting HT transformer is 0.2 pu and reactance of each line is 0.4 pu. $|E_g|$ = 1.2 pu and $|V|$ = 1.0 pu and P_e = 0.8 pu. Obtain the swing curve using modified Eulers method for a three phase fault occurs at the middle of one of the transmission lines and is cleared by isolating the faulted line.

Solution:

Before fault transfer reactance between generator and infinite bus

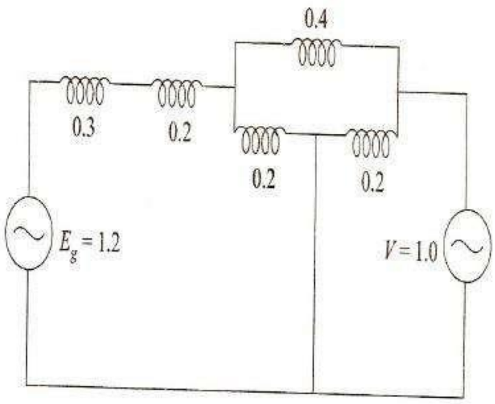
$$X_I = 0.3 + 0.2 + \frac{0.4}{2} = 0.7 \text{ pu}$$

$$P_{max\ I} = \frac{1.2 \times 1.0}{0.7} = 1.714 \text{ pu.}$$

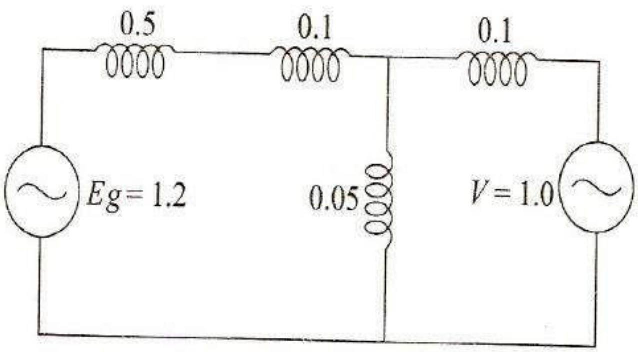
Initial $P_e = 0.8$ pu $= P_m$

Initial operating angle $\delta_o = \sin^{-1} \dfrac{0.8}{1.714} = 27.82^o = 0.485$ rad.

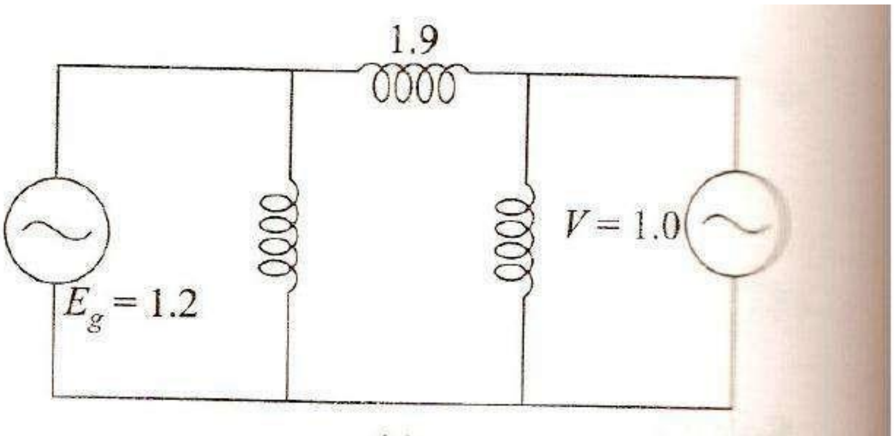
When fault occurs at middle of one of the transmission lines, the network and its reduction is as shown in Fig a to Fig c.



(a)



(b)



(c)

The transfer reactance is 1.9 pu.

$$P_{max\ II} = \frac{1.2 \times 1.0}{1.9} = 0.63 \text{ pu}$$

Since there is no outage, $P_{max\ III} = P_{max\ I} = 1.714$

$$\delta_{max} = \pi - \sin^{-1}\left(\frac{P_m}{P_{max\ III}}\right) = \pi - \sin^{-1}\left(\frac{0.8}{1.714}\right) = 2.656 \text{ rad}$$

$$\cos \delta_{cr} = \frac{P_m\,(\delta_{max} - \delta_o) - P_{max\ II}\cos\delta_o + P_{max\ III}\cos\delta_{max}}{P_{max\ III} - P_{max\ II}}$$

$$= \frac{0.8(2.656 - 0.485) - 0.63\cos(0.485) + 1.714\cos(2.656)}{1.714 - 0.63}$$

$$= \frac{1.7368 - 0.5573 - 1.5158}{1.084} = -0.3102$$

$$\delta_{cr} = \cos^{-1}(-0.3102) = 1.886 \text{ rad} = 108.07^o$$

with line outage

$$X_{III} = 0.3 + 0.2 + 0.4 = 0.9 \text{ pu}$$

$$P_{max\ III} = \frac{1.2 \times 1.0}{0.9} = 1.333 \text{ pu}$$

$$\delta_{max} = \pi - \sin^{-1}\frac{0.8}{1.333} = 2.498 \text{ rad}$$

7.7

Modified Eulers method

$\delta_o = 27.82^o = 0.485$ rad

$\omega_o = 0.0$ rad / sec (at $t = 0^0$)

Choosing a step size of 0.05 s, the swing is computed. Table a gives the values of the derivatives and predicted values. Table b gives the initial values δ_o, ω_o and the values at the end of the interval δ_1, ω_1. Calculations are illustrated for the time step t = 0.2 s.

$\delta_o = 0.761$

$\omega_o = 2.072$

$P_m = 0.8$

$$M = \left[\frac{5.2}{\Pi \times 50}\right] = 0.0331 \ s^2 / rad$$

P_{max} (after fault clearance) = 1.333 pu

$D_1 = 2.072$

$$D_2 = \frac{0.8 - 1.333 \sin(0.761)}{0.0331} = -3.604$$

$\delta^P = 0.761 + (2.072 \times 0.05) = 0.865$

$\omega^P = 2.072 + (-3.604 \times 0.05) = 1.892$

$D_{1P} = 1.892$

$$D_{2P} = \frac{0.8 - 1.333 \sin(0.865)}{0.0331} = -6.482$$

$$\delta_1 = 0.761 + \left(\frac{2.072 + 1.892}{2}\right) 0.05 = 0.860$$

$$\omega_1 = 2.072 + \left(\frac{-3.604 - 6.482}{2}\right) 0.05 = 1.82$$

δ_1, ω_1 are used as initial values in next time step.

Table a : Calculation of derivatives in modified Euler's method

t	D_1	D_2	δ^P	ω^P	D_{1P}	D_{2P}
0^+	0.0	15.296	0.485	0.765	0.765	15.296
0.05	0.765	14.977	0.542	1.514	1.514	14.350
0.10	1.498	14.043	0.636	2.200	2.200	12.860
0.15	2.17	- 0.299	0.761	2.155	2.155	- 3.600
0.20	2.072	- 3.604	0.865	1.892	1.892	- 6.482
0.25	1.820	- 6.350	0.951	1.502	1.502	- 8.612
0.30	1.446	- 8.424	1.015	1.025	1.025	- 10.041
0.35	0.984	- 9.827	1.054	0.493	0.493	- 10.843
0.40	0.467	- 10.602	1.065	- 0.063	- 0.063	- 11.060
0.45	- 0.074	- 10.803	1.048	- 0.614	- 0.614	- 10.720
0.50	- 0.612	- 10.46	1.004	- 1.135	- 1.135	- 9.800

Table b : calculations of δ_o, ω_o and δ_1, ω_1 in modified Euler's method

T	P_{max} (pu)	δ_o rad	ω_o rad / sec	δ_1 rad	ω_1 rad / sec	δ_1 deg
0^-	1.714	0.485	0.0	–	–	-
0^+	0.630	0.485	0.0	0.504	0.765	28.87
0.05	0.630	0.504	0.765	0.561	1.498	32.14
0.10	0.630	0.561	1.498	0.653	2.170	37.41
0.15	1.333	0.653	2.170	0.761	2.072	43.60
0.20	1.333	0.761	2.072	0.860	1.820	49.27
0.25	1.333	0.860	1.820	0.943	1.446	54.03
0.30	1.333	0.943	1.446	1.005	0.984	57.58
0.35	1.333	1.005	0.984	1.042	0.467	59.70
0.40	1.333	1.042	0.467	1.052	- 0.074	60.27
0.45	1.333	1.052	- 0.074	1.035	- 0.612	59.30
0.50	1.333	1.035	- 0.612	0.991	- 1.118	56.78

Runge - Kutta method

In Range - Kutta method, the changes in dependent variables are calculated from a given set of formulae, derived by using an approximation, to replace a truncated Taylor's series expansion. The formulae for the Runge - Kutta fourth order approximation, for solution of two simultaneous differential equations are given below;

Given $\frac{dx}{dt} = f_x(x, y, t)$

$$\frac{dy}{dt} = f_y(x, y, t)$$

Starting from initial values x_0, y_0, t_0 and step size h, the updated values are

$$x_1 = x_0 + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$

$$y_1 = y_0 + \frac{1}{6}(l_1 + 2l_2 + 2l_3 + l_4)$$

where $k_1 = f_x(x_0, y_0, t_0)\, h$

$$k_2 = f_x\left(x_0 + \frac{k_1}{2}, y_0 + \frac{l_1}{2}, t_0 + \frac{h}{2}\right) h$$

$$k_3 = f_x\left(x_0 + \frac{k_2}{2}, y_0 + \frac{l_2}{2}, t_0 + \frac{h}{2}\right) h$$

$$k_4 = f_x(x_0 + k_3, y_0 + l_3, t_0 + h)\, h$$

$$l_1 = f_y(x_0, y_0, t_0)\, h$$

$$l_2 = f_y\left(x_0 + \frac{k_1}{2}, y_0 + \frac{l_1}{2}, t_0 + \frac{h}{2}\right) h$$

$$l_3 = f_y\left(x_0 + \frac{k_2}{2}, y_0 + \frac{l_2}{2}, t_0 + \frac{h}{2}\right) h$$

$$l_4 = f_y(x_0 + k_3, y_0 + l_3, t_0 + h)\, h$$

The two first order differential equations to be solved to obtain solution for the swing equation are:

$$\frac{d\delta}{dt} = \omega$$

$$\frac{d\omega}{dt} = \frac{P_a}{M} = \frac{P_m - P_{max} \sin\delta}{M}$$

Starting from initial value δ_0, ω_0, t_0 and a step size of Δt the formulae are as follows

$$k_1 = \omega_0 \; \Delta t$$

$$l_1 = \left[\frac{P_m - P_{max} \sin\delta_0}{M}\right] \Delta t$$

$$k_2 = \left(\omega_0 + \frac{l_1}{2}\right) \Delta t$$

$$l_2 = \left[\frac{P_m - P_{max} \sin\left(\delta_0 + \frac{k_1}{2}\right)}{M}\right] \Delta t$$

$$k_3 = \left(\omega_0 + \frac{l_2}{2}\right) \Delta t$$

$$l_3 = \left[\frac{P_m - P_{max} \sin\left(\delta_0 + \frac{k_2}{2}\right)}{M}\right] \Delta t$$

$$k_4 = (\omega_0 + l_3) \; \Delta t$$

$$l_4 = \left[\frac{P_m - P_{max} \sin(\delta_0 + k_3)}{M}\right] \Delta t$$

$$\delta_1 = \delta_0 + \frac{1}{6} \; [k_1 + 2k_2 + 2k_3 + k_4]$$

$$\omega_1 = \omega_0 + \frac{1}{6} \; [l_1 + 2l_2 + 2l_3 + l_4]$$

Example

Obtain the swing curve for previous example using Runge - Kutta method.

Solution:

$\delta_0 = 27.82^0 = 0.485$ rad.

$\omega_0 = 0.0$ rad / sec. (at $t = 0^-$)

Choosing a step size of 0.05 s, the coefficient k_1, k_2, k_3, k_4 and l_1, l_2, l_3, and l_4 are calculated for each time step. The values of δ and ω are then updated. Table a gives the coefficient for different time steps. Table b gives the starting values δ_0, ω_0 for a time step and the updated values δ_1, ω_1 obtained by Runge - Kutta method. The updated values are used as initial values for the next time step and process continued. Calculations are illustrated for the time step t = 0.2 s.

$$\delta_0 = 0.756$$

$$M = 0.0331 \ s^2 / rad$$

$$\omega_0 = 2.067$$

$$P_m = 0.8$$

$$P_{max} = 1.333 \text{ (after fault is cleared)}$$

$$k_1 = 2.067 \times 0.05 = 0.103$$

$$l_1 = \left[\frac{0.8 - 1.333 \sin(0.756)}{0.0331}\right] \times 0.05 = -0.173$$

$$k_2 = \left[2.067 - \frac{0.173}{2}\right] 0.05 = 0.099$$

$$l_2 = \left[\frac{0.8 - 1.333 \sin\left(0.756 + \frac{0.103}{2}\right)}{0.0331}\right] \times 0.05 = -0.246$$

$$k_3 = \left[2.067 - \frac{0.246}{2}\right] 0.05 = 0.097$$

$$l_3 = \left[\frac{0.8 - 1.333 \sin\left(0.756 + \frac{0.099}{2}\right)}{0.0331}\right] \times 0.05 = -0.244$$

$$k_4 = (2.067 - 0.244) \ 0.05 = 0.091$$

$$l_4 = \left[\frac{0.8 - 1.333 \sin(0.756 + 0.097)}{0.0331}\right] \times 0.05 = -0.308$$

$$\delta_1 = 0.756 + \frac{1}{6} [0.103 + 2 \times 0.099 + 2 \times 0.097 + 0.091] = 0.854$$

$$\omega_1 = 2.067 + \frac{1}{6}\ [-0.173 + 2 \times -0.246 + 2 \times -0.244 - 0.308] = 1.823$$

Now $\delta = 0.854$ and $\omega = 1.823$ are used as initial values for the next time step. The computations have been rounded off to three digits. Greater accuracy is obtained by reducing the step size.

Table a : Coefficients in Runge - Kutta method

T	k_1	l_1	k_2	l_2	k_3	l_3	K_4	l_4
0.0	0.0	0.764	0.019	0.764	0.019	0.757	0.038	0.749
0.05	0.031	0.749	0.056	0.736	0.056	0.736	0.075	0.703
0.10	0.075	0.704	0.092	0.674	0.091	0.667	0.108	0.632
0.15	0.108	– 0.010	0.108	– 0.094	0.106	– 0.095	0.103	– 0.173
0.20	0.103	– 0.173	0.099	– 0.246	0.097	– 0.244	0.091	– 0.308
8.25	0.091	– 0.309	0.083	– 0.368	0.082	– 0.363	0.073	– 0.413
0.30	0.073	– 0.413	0.063	– 0.455	0.061	– 0.450	0.050	– 0.480
0.35	0.050	– 0.483	0.038	– 0.510	0.037	– 0.504	0.025	– 0.523
0.40	0.025	– 0.523	0.012	– 0.536	0.011	– 0.529	– 0.001	– 0.534
0.45	– 0. 001	– 0.534	– 0.015	– 0.533	– 0.015	– 0.526	– 0.027	– 0.519
0.50	– 0.028	– 0.519	– 0.040	– 0.504	– 0.040	– 0.498	– 0.053	– 0.476

Table b: δ, ω computations by Runge - Kutta method

t (sec)	P_{max} (pu)	δ_0 (rad)	ω_0 rad/sec	δ_1 rad	ω_1 rad/sec	δ_1 deg
0^-	1.714	0.485	0.0			
0^+	0.630	0.485	0.0	0.504	0.759	28.87
0.05	0.630	0.504	0.756	0.559	1.492	32.03
0.10	0.630	0.559	1.492	0.650	2.161	37.24
0.15	1.333	0.650	2.161	0.756	2.067	43.32
0.20	1.333	0.756	2.067	0.854	1.823	48.93

0.25	1.333	0.854	1.823	0.936	1.459	53.63
0.30	1.333	0.936	1.459	0.998	1.008	57.18
0.35	1.333	0.998	1.008	1.035	0.502	59.30
0.40	1.333	1.035	0.502	1.046	– 0.029	59.93
0.45	1.333	1.046	– 0.029	1.031	– 0.557	59.07
0.50	1.333	1.031	– 0.557	0.990	– 1.057	56.72

Note: δ_0, ω_0 indicate values at beginning of interval and δ_1, ω_1 at end of interval. The fault is cleared at 0.125 seconds. $\therefore$ $P_{max} = 0.63$ at t = 0.1 sec and $P_{max} = 1.333$ at t = 0.15 sec, since fault is already cleared at that time. The swing curves obtained from modified Euler's method and Runge - Kutta method are shown in Fig. It can be seen that the two methods yield very close results.

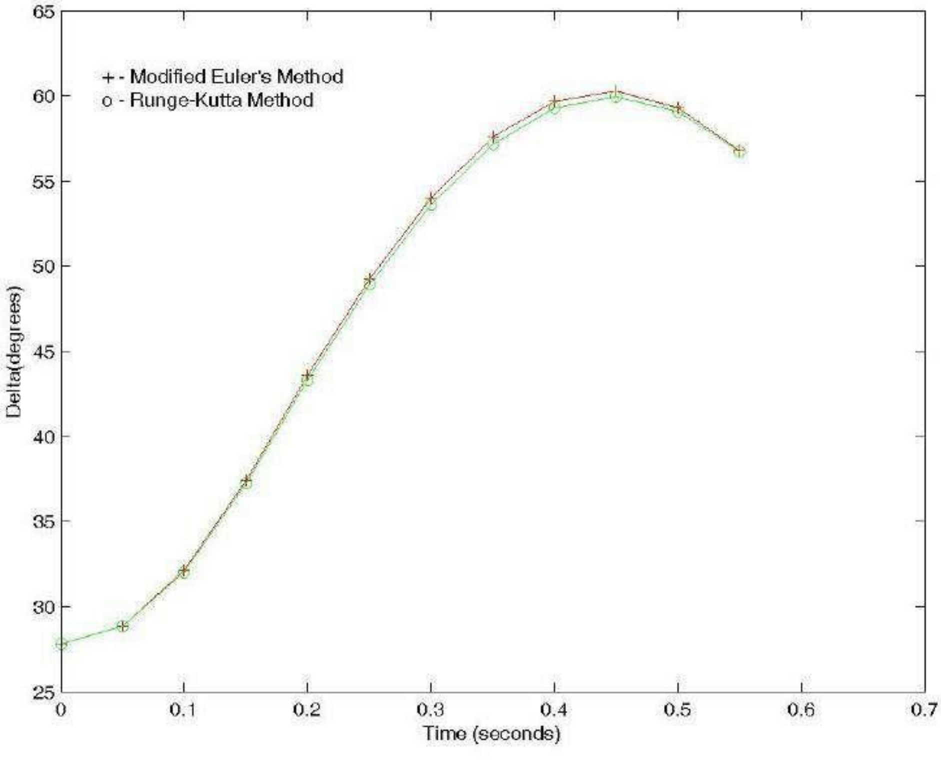


Fig: : Swing curves with Modified Euler' and Runge-Kutta methods

Milne's Predictor Corrector method:

The Milne's formulae for solving two simultaneous differential equations are given below.

Consider $\frac{dx}{dt} = f_x(x, y, t)$

$$\frac{dy}{dt} = f_y(x, y, t)$$

With values of x and y known for four consecutive previous times, the predicted value for n + 1[th] time step is given by

$$x^P_{n+1} = x_{n-3} + \frac{4h}{3}\left[2x'_{n-2} - x'_{n-1} + 2x'_n\right]$$

$$y^P_{n+1} = y_{n-3} + \frac{4h}{3}\left[2y'_{n-2} - y'_{n-1} + 2y'_n\right]$$

Where x' and y' are derivatives at the corresponding time step. The corrected values are

$$x_{n+1} = x_{n-1} + \frac{h}{3}\left[x'_{n-1} + 4x'_n + x'_{n+1}\right]$$

$$y_{n+1} = y_{n-1} + \frac{h}{3}\left[y'_{n-1} + 4y'_n + y'_{n+1}\right]$$

where $x'_{n+1} = f_x\left(x^P_{n+1}, y^P_{n+1}, t_{n+1}\right)$

$$y'_{n+1} = f_y\left(x^P_{n+1}, y^P_{n+1}, t_{n+1}\right)$$

To start the computations we need four initial values which may be obtained by modified Euler's method, Runge - Kutta method or any other numerical method which is self starting, before applying Milne's method. The method is applied to the solution of swing equation as follows:

Define $\delta'_n = \left.\frac{d\delta}{dt}\right|_n = \omega_n$

$$\omega'_n = \left.\frac{d\omega}{dt}\right|_n = \frac{P_m - P_{max}\sin\delta_n}{M}$$

$$\delta^P_{n+1} = \delta_{n-3} + \frac{4\Delta t}{3}\left[2\delta'_{n-2} - \delta'_{n-1} + 2\delta'_n\right]$$

$$\omega_{n+1}^{P} = \omega_{n-3} + \frac{4\Delta t}{3}\left[2\,\omega'_{n-2} - \omega'_{n-1} + 2\,\omega'_{n}\right]$$

$$\delta_{n+1} = \delta_{n-1} + \frac{\Delta t}{3}\left[\delta'_{n-1} + 4\,\delta'_{n} + \delta'_{n+1}\right]$$

$$\omega_{n+1} = \omega_{n-1} + \frac{\Delta t}{3}\left[\omega'_{n-1} + 4\,\omega'_{n} + \omega'_{n+1}\right]$$

where $\delta'_{n+1} = \omega_{n+1}^{P}$

$$\omega'_{n+1} = \frac{P_m - P_{\max}\sin\delta_{n+1}^{P}}{M}$$

Example

Solve example using Milne's method.

Solution:

To start the process, we take the first four computations from Range Kutta method

t = 0.0 s	$\delta_1 = 0.504$	$\omega_1 = 0.759$
t = 0.05 s	$\delta_2 = 0.559$	$\omega_2 = 1.492$
t = 0.10 s	$\delta_3 = 0.650$	$\omega_3 = 2.161$
t = 0.15 s	$\delta_4 = 0.756$	$\omega_4 = 2.067$

The corresponding derivatives are calculated using the formulae for δ'_n and ω'_n. We get

$\delta'_1 = 0.759$ $\quad \omega'_1 = 14.97$

$\delta'_2 = 1.492$ $\quad \omega'_2 = 14.075$

$\delta'_3 = 2.161$ $\quad \omega'_3 = 12.65$

$\delta'_4 = 2.067$ $\quad \omega'_4 = -3.46$

We now compute δ_5 and ω_5, at the next time step i.e t = 0.2 s.

$$\delta_5^{P} = \delta_1 + \frac{4\,\Delta t}{3}\left[2\,\delta'_2 - \delta'_3 + 2\,\delta'_4\right]$$

$$= 0.504 + \frac{4 \times 0.05}{3}\left[2 \times 1.492 - 2.161 + 2 \times 2.067\right] = 0.834$$

$$\omega_5^{P} = \omega_1 + \frac{4\,\Delta t}{3}\left[2\,\omega'_2 - \omega'_3 + 2\,\omega'_4\right]$$

$$= 0.759 + \frac{4 \times 0.05}{3}\left[2 \times 14.075 - 12.65 + 2 \times (-3.46)\right] = 1.331$$

$$\delta_5' = 1.331$$

$$\omega_5' = \frac{0.8 - 1.333 \sin (0.834)}{0.0331} = -5.657$$

$$\delta_5 = \delta_3 + \frac{\Delta t}{3}\left[\delta_3' + 4\delta_4' + \delta_5'\right]$$

$$= 0.65 + \frac{0.05}{3}\left[2.161 + 4 \times 2.067 + 1.331\right] = 0.846$$

$$\omega_5 = \omega_3 + \frac{\Delta t}{3}\left[\omega_3' + 4\omega_4' + \omega_5'\right]$$

$$= 2.161 + \frac{0.05}{3}\left[12.65 - 4 \times 3.46 - 5.657\right] = 2.047$$

$$\delta_5' = \omega_5 = 2.047$$

$$\omega_5' = \frac{0.8 - 1.333 \sin (0.846)}{0.0331} = -5.98$$

The computations are continued for the next time step in a similar manner.

MULTI MACHINE TRANSIENT STABILITY ANALYSIS

A typical modern power system consists of a few thousands of nodes with heavy interconnections. Computation simplification and memory reduction have been two major issues in the development of mathematical models and algorithms for digital computation of transient stability. In its simplest form, the problem of a multi machine power system under going a disturbance can be mathematically stated as follows:

$$\dot{x}(t) = f_I(x(t)) \qquad -\infty \le t \le 0$$

$$\dot{x}(t) = f_{II}(x(t)) \qquad 0 < t \le t_{ce}$$

$$\dot{x}(t) = f_{III}(x(t)) \qquad t_{ce} < t < \infty$$

$x(t)$ is the vector of state variables to describe the differential equations governing the generator rotor dynamics, dynamics of flux decay and associated generator controller dynamics (like excitation control, PSS, governor control etc). The function f_I describes the dynamics prior to the fault. Since the system is assumed to be in steady state, all the state variable are constant. If the fault occurs at $t = 0$, f_{II} describes the dynamics during fault, till the fault is cleared at time t_{cl}. The post–fault dynamics is governed by f_{III}. The state of the system x_{cl} at the end of the fault-on period (at $t = t_{cl}$) provides the initial condition for the post–fault network described which determines whether a system is stable or not after the fault is cleared. Some methods are presented in the following sections to evaluate multi machine transient stability. However, a detailed exposition is beyond the scope of the present book.

REDUCED ORDER MODEL

This is the simplest model used in stability analysis and requires minimum data. The following assumptions are made:

- Mechanical power input to each synchronous machine is assumed to be constant.
- Damping is neglected.
- Synchronous machines are modeled as constant voltage sources behind transient reactance.
- Loads are represented as constant impedances.

With these assumptions, the multi machine system is represented as in Fig. 9.26.

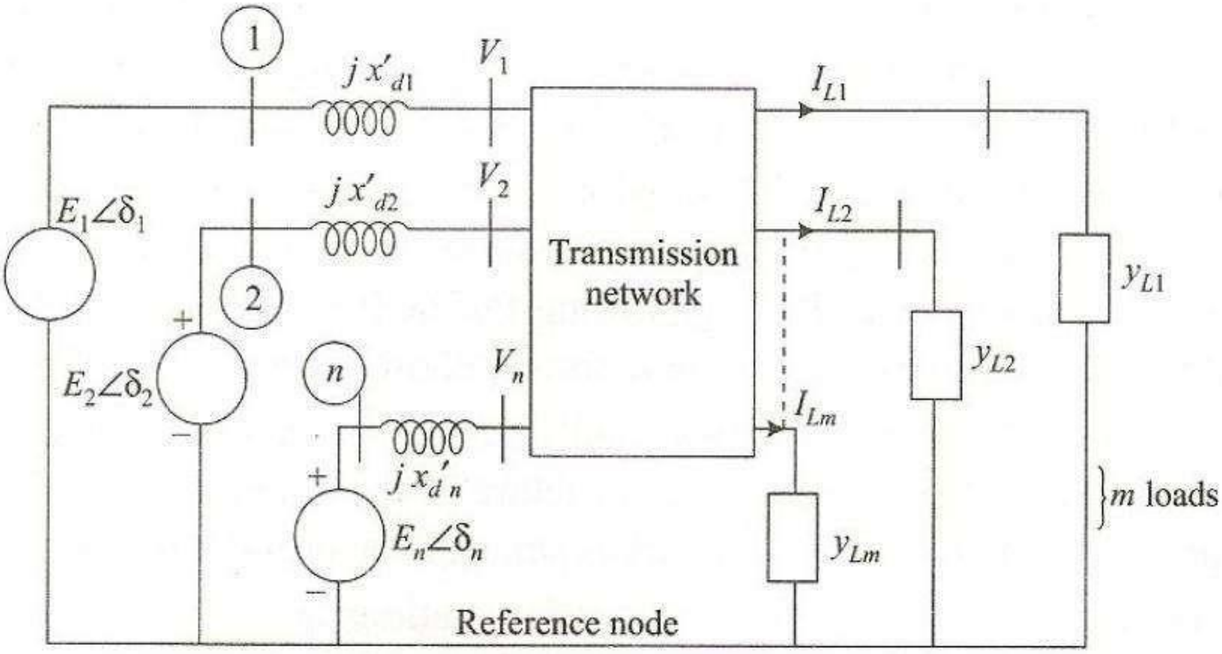


Fig 9.26 Multi machine system

Nodes 1, 2 …… n are introduced in the model and are called internal nodes (the terminal node is the external node connected to the transmission network). The swing equations are formed for the various generators using the following steps:

Step 1: All system data is converted to a common base.

Step 2: A prefault load flow is performed, to determine the prefault steady state voltages, at all the external buses. Using the prefault voltages, the loads are converted into equivalent shunt admittance, connected between the respective bus and the reference node. If the complex load at bus i is given by

$$S_i = P_{Li} + Q_{Li}$$

the equivalent admittance is given by

$$Y_{Li} = \frac{S^*_{Li}}{|V_{Li}|^2} = \frac{P_{Li} - jQ_{Li}}{|V_{Li}|^2}$$

Step 3: The internal voltages are calculated from the terminal voltages, using

$$|E_i|\angle\delta_i' = |V_i| + j\,x_{di}'\,I_i$$

$$= |V_i| + jx_{di}'\frac{S_{Gi}^*}{|V_i|}$$

$$= |V_i| + j\,x_{di}'\frac{(P_{Gi} - j\,Q_{Gi})}{|V_i|}$$

δ_i' is the angle of E_i with respect to V_i. If the angle of V_i is β_i, then the angle of E_i, with respect to common reference is given by $\delta_i = \delta_i' + \beta_i$. P_{Gi} and Q_{Gi} are obtained from load flow solution.

Step4: The bus admittance matrix Y_{bus} formed to run the load flow is modified to include the following.

(i) The equivalent shunt load admittance given by, connected between the respective load bus and the reference node.

(ii) Additional nodes are introduced to represent the generator internal nodes. Appropriate values of admittances corresponding to x_d', connected between the internal nodes and terminal nodes are used to update the Y_{bus}.

(iii) Y_{bus} corresponding to the faulted network is formed. Generally transient stability analysis is performed, considering three phase faults, since they are the most severe. The Y_{bus} during the fault is obtained by setting the elements of the row and column corresponding to the faulted bus to zero.

(iv) Y_{bus} corresponding to the post–fault network is obtained, taking into account line outages if any. If the structure of the network does not change, the Y_{bus} of the post-fault network is same as the prefault network.

Step 5: The admittance form of the network equations is

$$I = Y_{bus}\,V$$

Since loads are all converted into passive admittances, current injections are present only at the *n* generator internal nodes. The injections at all other nodes are zero. Therefore, the current vector I can be partitioned as

$$I = \left[\frac{I_n}{0}\right]$$

where I_n is the vector of current injections corresponding to the *n* generator internal nodes. Y_{bus} and V are also partitioned appropriately, so that

$$\left[\frac{I_n}{0}\right] = \left[\begin{array}{c|c} \frac{Y_1}{Y_3} & \frac{Y_2}{Y_4} \end{array}\right]\left[\frac{E_n}{V_t}\right]$$

where E_n is the vector of internal emfs of the generators and V_t is the vector of external bus voltages. From (9.91) we can write

$$I_n = Y_1 E_n + Y_2 V_t$$

$$0 = Y_3 E_n + Y_4 V_t$$

we get

$$V_t = -Y_4^{-1} Y_3 E_n$$

$$I_n = \left(Y_1 - Y_2\, Y_4^{-1}\, Y_3\right) E_n = \hat{Y}\; E_n$$

where $\hat{Y} = Y_1 - Y_2\, Y_4^{-1}\, Y_3$ is called the reduced admittance matrix and has dimension $n \times n$. $\hat{Y}$ gives the relationship between the injected currents and the internal generator voltages. It is to be noted we have eliminated all nodes except the *n* internal nodes.

Step 6: The electric power output of the generators are given by

$$P_{Gi} = \Re\, [E_i\; I_i^* .]$$

Substituting for I_i from (9.94) we get

$$P_{Gi} = \left|E_i\right|^2 \hat{G}_{ii} + \sum_{j=1 \neq i}^{n} \left|E_i\right| \left|E_j\right| \left(\hat{B}_{ij} \sin\, (\delta_i - \delta_j) + \hat{G}_{ij} \cos\, (\delta_i - \delta_j)\right)$$

(This equation is derived in chapter on load flows)

Step 7: The rotor dynamics representing the swing is now given by

$$M_i \frac{d^2 \delta_i}{dt^2} = P_{Mi} - P_{Gi} \qquad i = 1 \ldots\ldots\ldots. n$$

The mechanical power P_{Mi} is equal to the pre-fault electrical power output, obtained from pre-fault load flow solution.

Step 8: The *n* second order differential equations can be decomposed into *2n* first order differential equations which can be solved by any numerical method .

Though reduced order models, also called classical models, require less computation and memory, their results are not reliable. Further, the interconnections of the physical network of the system is lost.

FACTORS AFFECTING TRANSIENT STABILITY:

The relative swing of a machine and the critical clearing time are a measure of the stability of a generating unit. From the swing equation, it is obvious that the generating units with smaller H, have larger angular swings at any time interval. The maximum power transfer $P_{max} = \frac{E_g V}{x'_d}$, where V is the terminal voltage of the generators. Therefore an increase in x'_d, would reduce P_{max}. Hence, to transfer a given power P_e, the angle δ would increase since $P_e = P_{max} \sin \delta$, for a machine with larger x'_d. This would reduce the critical clearing time, thus, increasing the probability of losing stability.

NIT-V
PF CONTROL

Introduction:

The main objective of power system operation and control is to maintain continuous supply of power with an acceptable quality, to all the consumers in the system. The system will be in equilibrium, when there is a balance between the power demand and the power generated. As the power in AC form has real and reactive components: the real power balance; as well as the reactive power balance is to be achieved.

There are two basic control mechanisms used to achieve reactive power balance (acceptable voltage profile) and real power balance (acceptable frequency values). The former is called the automatic voltage regulator (AVR) and the latter is called the automatic load frequency control (ALFC) or automatic generation control (AGC).

Automatic Load Frequency Control:

The ALFC is to control the frequency deviation by maintaining the real power balance in the system. The main functions of the ALFC are to i) to maintain the steady frequency; ii) control the tie-line flows; and iii) distribute the load among the participating generating units. The control (input) signals are the tie-line deviation ΔP_{tie} (measured from the tie line flows), and the frequency deviation Δf (obtained by measuring the angle deviation $\Delta\delta$). These error signals Δf and ΔP_{tie} are amplified, mixed and transformed to a real power signal, which then controls the valve position. Depending on the valve position, the turbine (prime mover) changes its output power to establish the real power balance. The complete control schematic is shown in Fig1.1.

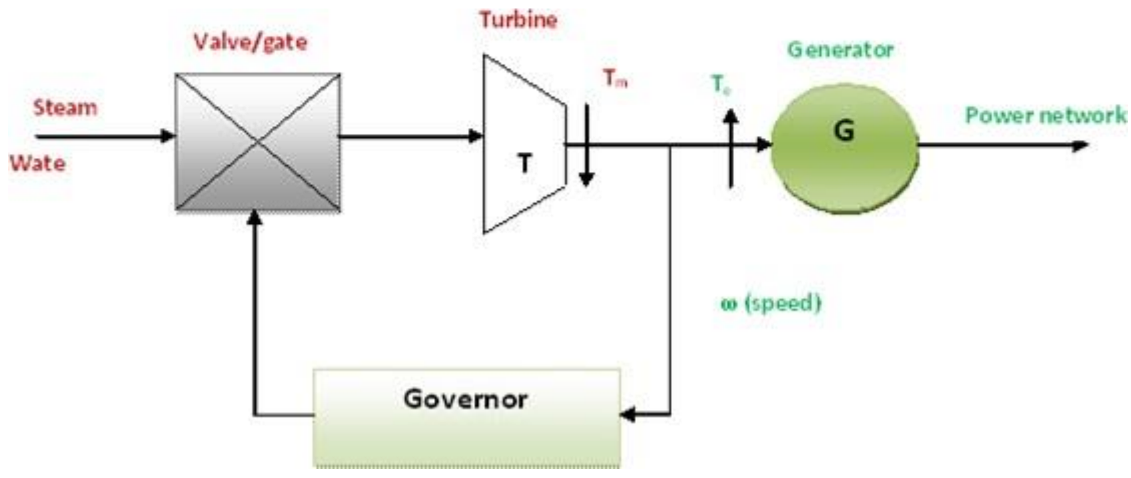


Fig.1.1: The Schematic representation of ALFC system

Speed Governing System:

Fig. 1.2 is the schematic representation of Turbine Speed Governing system. It has mainly four major components.

Speed governor: Speed governor senses the change in speed (or frequency) hence it can be regarded as heart of the system. The standard model of speed governor operates by fly-ball mechanism. Fly-balls moves outward when

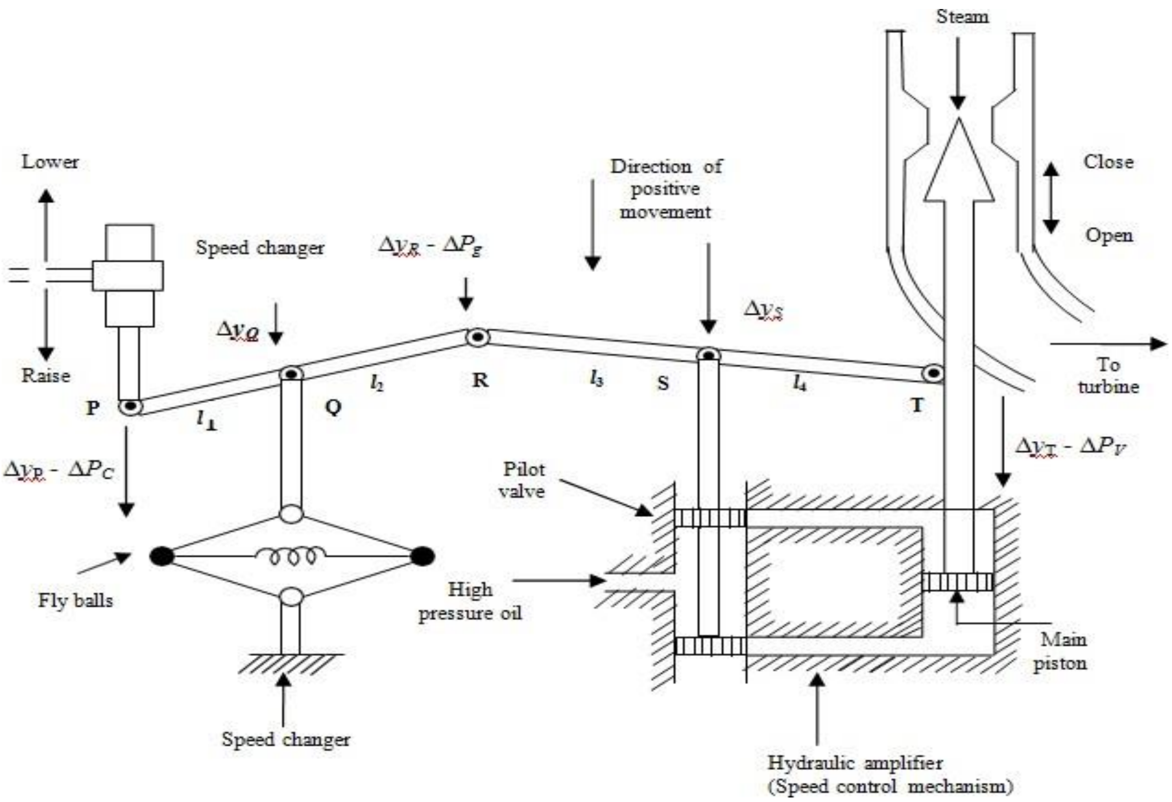
speed increases and the point Q on the linkage mechanism moves downwards. the reverse happens when the speed decreases. The movement of point Q is proportional to change in shaft speed.

Linage mechanism: PQR is a rigid link pivoted at Q and RST is another rigid link pivoted at S. This link mechanism provides a movement to the control valve in proportion to change in speed. It also provides a feedback from the steam valve movement.

Hydraulic amplifier: It comprises a pilot valve and main piston arrangement. It converts low power level pilot valve movement into high power level piston valve movement. This is necessary in order to open or close the steam valve against high pressure steam.

Speed changer: It provides a steady state power output setting for the turbine. Its downwards movement opens the upper pilot valve so that more steam is admitted to the turbine under steady conditions the reverse happens for upward movement of speed changer. By adjusting the linkage position of point P the scheduled speed/frequency can be obtained at the given loading condition.

Fig.1.2: Schematic of Speed Governing System



For the analysis, the models for each of the blocks in Fig1.1 are required. The generator and the electrical load constitute the power system. The valve and the hydraulic amplifier represent the speed governing system. Using the swing equation, the generator can be modeled by

$$\frac{2Hd^2\Delta\delta}{\omega_s dt^2} = \Delta P_m - \Delta P_e$$

Expressing the speed deviation in pu,

$$\frac{d\Delta\omega}{dt} = \frac{1}{2H}(\Delta P_m - \Delta P_e)$$

This relation can be represented as shown in Fig.1.3.

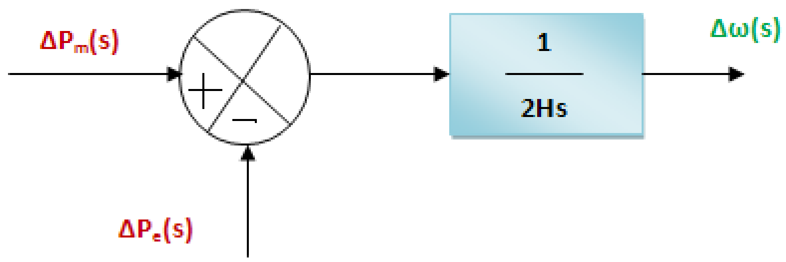


Fig.1.3:The block diagram representation of the Generator

The load on the system is composite consisting of a frequency independent component and a frequency dependent component. The load can be written as $\Delta P_e = \Delta P_0 + \Delta P_f$ where, ΔP_e is the change in the load; ΔP_0 is the frequency independent load component; ΔP_f is the frequency dependent load component. $\Delta P_f = D\Delta\omega$ where, D is called frequency characteristic of the load (also called as damping constant) expressed in percent change in load for 1% change in frequency. If D=1.5%, then a 1% change in frequency causes 1.5% change in load. The combined generator and the load (constituting the power system) can then be represented as shown in Fig 1.4.

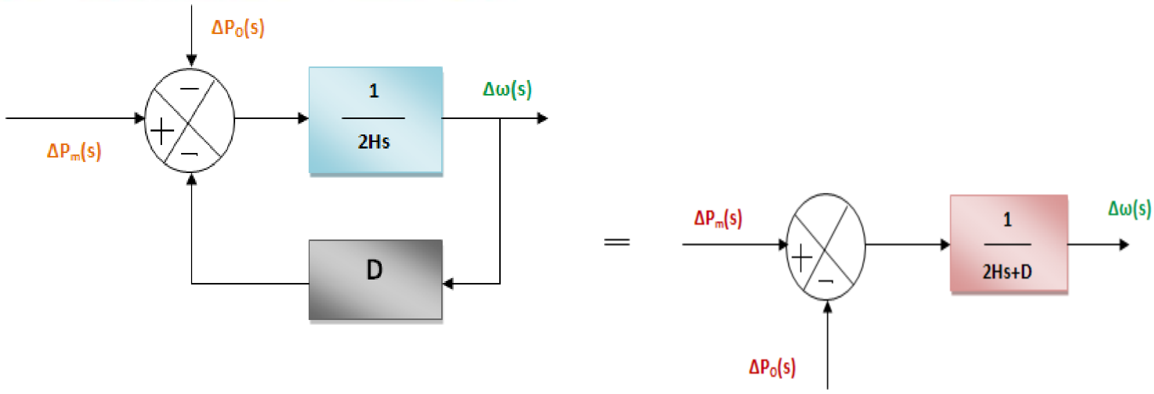
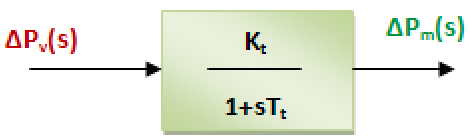


Fig1.4:The block diagram representation of the Generator and load

The turbine can be modeled as a first order lag as shown in the Fig1.5



$$G_t(s) = \frac{\Delta P_m(s)}{\Delta P_V(s)} = \frac{K_t}{1+sT_t}$$

Fig1.5.The turbine model

Gt(s) is the TF of the turbine; ΔPV(s) is the change in valve output (due to action).
ΔPm(s) is the change in the turbine output the governor can similarly modeled as shown in Fig1.4.
The output of the governor is by

$\Delta P_g = \Delta P_{ref} - \frac{\Delta\omega}{R}$ where ΔP_{ref} is the reference set power, and $\Delta\omega/R$ is the power given by governor speed characteristic. The hydraulic amplifier transforms this signal ΔP_g into valve/gate position corresponding to a power ΔP_V. Thus $\Delta P_V(s) = (K_g/(1+sT_g))\Delta P_g(s)$.

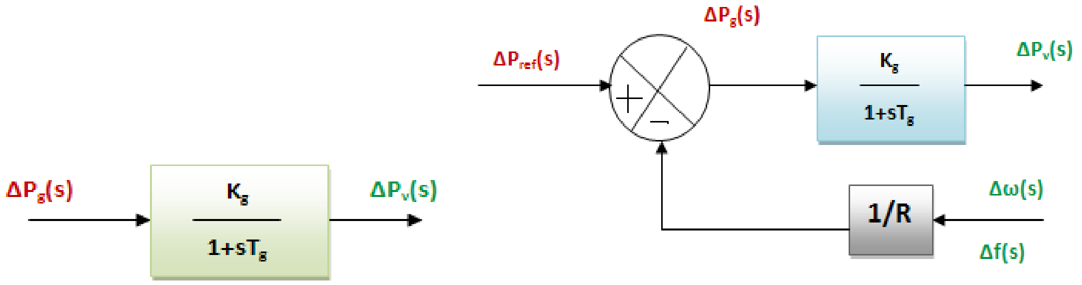


Fig1.6: The block diagram representation of the Governor

All the individual blocks can now be connected to represent the complete ALFC loop as Shown in Fig 1.7.

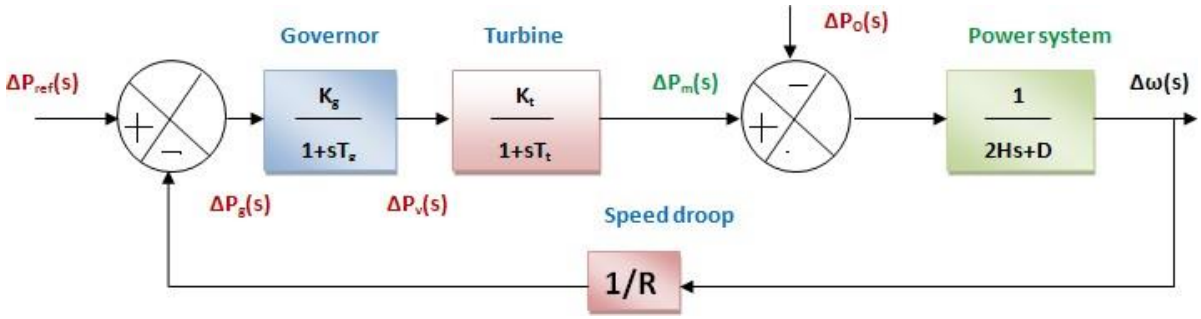


Fig1.7: The block diagram representation of the ALFC.

SINGLE AREA AND TWO AREA LOAD FREQUENCY CONTROL

Modern day power systems are divided into various areas. For example in India, there are five regional grids, e.g., Eastern Region, Western Region etc. Each of these areas is generally interconnected to its neighboring areas. The transmission lines that connect an area to its neighboring area are called **tie-lines.** Power sharing between two areas occurs through these tie-lines. Load frequency control, as the name signifies, regulates the power flow between different areas while holding the frequency constant.

As we have an Example 1 that the system frequency rises when the load decreases if $\Delta Pref$ is kept at zero. Similarly the frequency may drop if the load increases. However it is desirable to maintain the frequency constant such that $\Delta f=0$. The power flow through different tie-lines are scheduled - for example, area- i may export a pre-specified amount of power to area- j while importing another pre-specified amount of power from area- k . However it is expected that to fulfill this obligation, area- i absorbs its own load change, i.e., increase generation to supply extra load in the area or decrease generation when the load demand in the area has reduced. While doing this area- i must however maintain its obligation to areas j and k as far as importing and exporting power is concerned. A conceptual diagram of the interconnected areas is shown in Fig. 1.8.

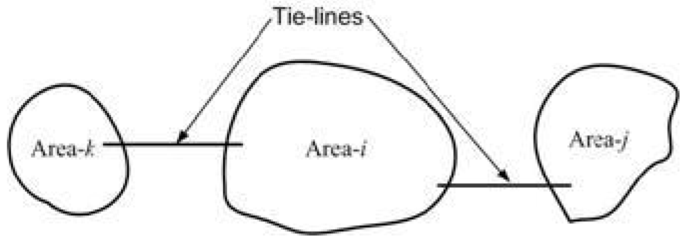


Fig. 1.8: Interconnected areas in a power system

We can therefore state that the load frequency control (LFC) has the following two objectives:

- Hold the frequency constant ($\Delta f = 0$) against any load change. Each area must contribute to absorb any load change such that frequency does not deviate.
- Each area must maintain the tie-line power flow to its pre-specified value.

$$\text{ACE} = (P_{tie} - P_{sch}) + B_f \Delta f = \Delta P_{tie} + B_f \Delta f$$

The first step in the LFC is to form the **area control error (ACE)**.

Where P_{tie} and P_{sch} are **tie-line power** and **scheduled power** through tie-line respectively and the constant B_f is called the **frequency bias constant.**

The change in the reference of the power setting $\Delta P_{ref,i}$, of the area- i is then obtained by

$$\Delta P_{ref,i} = -K_i \int \text{ACE}\, dt$$

The feedback of the ACE through an integral controller of the form where K_i is the integral gain. The ACE is negative if the net power flow out of an area is low or if the frequency has dropped or both. In this case the generation must be increased. This can be achieved by increasing $\Delta P_{ref,i}$. This negative sign accounts for this inverse relation between $\Delta P_{ref,i}$ and ACE. The tie-line power flow and frequency of each area are monitored in its control center. Once the ACE is computed and $\Delta P_{ref,i}$ is obtained , commands are given to various turbine-generator controls to adjust their reference power settings.

supplementary controls); ii) and to maintain the scheduled tie-line flows. A secondary objective of the AGC is to distribute the required change in generation among the connected generating units economically (to obtain least operating costs).

1.4 Steady State Performance of the ALFC Loop

In the steady state, the ALFC is in 'open' state, and the output is obtained by substituting $s \to 0$ in the TF.

With $s \to 0$, Gg(s) and Gt(s) become unity, then, (note that $\Delta P_m = \Delta P_T = \Delta P_G = \Delta P_e = \Delta P_D$;

That is turbine output = generator/electrical output = load demand)

$$\Delta P_m = \Delta P_{ref} - (1/R)\Delta\omega \quad \text{or} \quad \Delta P_m = \Delta P_{ref} - (1/R)\Delta f$$

When the generator is connected to infinite bus ($\Delta f = 0$, and $\Delta V = 0$), then $\Delta P_m = \Delta P_{ref}$.

If the network is finite, for a fixed speed changer setting ($\Delta Pref = 0$), then

If there are more than one generator present in the system, then

$$\Delta P_{m.\,eq} = \Delta P_{ref.eq} - (D + 1/R_{eq})\Delta f$$

where,

$$\Delta P_{m.\,eq} = \Delta P_{m1} + \Delta P_{m2} + \Delta P_{m3} + \ldots.$$

$$\Delta P_{ref.\,eq} = \Delta P_{ref1} + \Delta P_{ref2} + \Delta P_{ref3} + \ldots..$$

$$1/R_{eq} = (1/R_1 + 1/R_2 + 1/R_2 + \ldots.)$$

The quantity $\beta = (D + 1/R_{eq})$ is called the area frequency (bias) characteristic (response) or simply the stiffness of the system.

1.5 Concept of AGC (Supplementary ALFC Loop)

The ALFC loop shown in Fig1.7, is called the primary ALFC loop. It achieves the primary goal of real power balance by adjusting the turbine output ΔP_m to match the change in load demand ΔP_D. All the participating generating units contribute to the change in generation. But a change in load results in a steady state frequency deviation Δf. The restoration of the frequency to the nominal value requires an additional control loop called the supplementary loop. This objective is met by using integral controller which makes the frequency deviation zero. The ALFC with the supplementary loop is generally called the AGC. The block diagram of an AGC is shown in Fig1.8. The main objectives of AGC are i) to regulate the frequency (using both primary and

Derivation of Steady state error for single area load frequency control:

From the single area load frequency control block diagram we have,

$\Delta P_G(s) = k_g k_t$ / $(1+sT_g)(1+sT_t)[\Delta P_c(s)-1/R\Delta F(s)]$

The generator is synchronized to a network of very large size. So, the speed or frequency will be essentially independent of any changes in a power output of the generator

ie, $\Delta F(s) = 0$

Therefore $\Delta P_G(s) = k_g k_t$ / $(1+sT_g)(1+sT_t) * \Delta P_c(s)$

Steady state response:

(i) Controlled case:

To find the resulting steady change in the generator output:

Let us assume that we made a step change of the magnitude ΔP_C of the speed changer For step change, $\Delta P_C(s) = \Delta P_C/s$

$\Delta P_G(s) = [k_g k_t/(1+sT_g)(1+sT_t)]\Delta P_c(s)/s$ $s\Delta P_G(s) = [k_g k_t/(1+sT_g)(1+sT_t)]\Delta P_c(s)$

Applying final value theorem,

(ii)Uncontrolled case

Let us assume that the load suddenly increases by small amount ΔP_D.

Consider there is no external work and the generator is delivering a power to a single load.

$\Delta Pc = 0$

$Kg\ Kt = 1$

$\Delta P_G(s) = 1/(1+sT_g)(1+sT_t)$ [-

$\Delta F(s)/R]$ For a step change ,$\Delta F(s) =$

$\Delta f/s$ Therefore

$\Delta P_G(s) = 1/(1+sT_g)(1+sT_t)[-\Delta F/sR]$

$\Delta f/\Delta P_G$ (stat) =-R Hz/MW

Steady State Performance of the ALFC Loop

In the steady state, the ALFC is in open state, and the output is obtained by substituting s→0 in the TF.

With s→0, Gg(s) and Gt(s) become unity, then,(note that

$\mathbf{\Delta P_m = \Delta P_T = P_G = \Delta P_e = \Delta P_D;}$

That is turbine output = generator/electrical output = load demand)

$\mathbf{\Delta P_m = \Delta P_{ref} - (1/R)\ \Delta\omega}$ **or** $\mathbf{\Delta P_m = \Delta P_{ref} - (1/R)\Delta f}$

When the generator is connected to infinite bus ($\Delta f = 0$, and $\Delta V = 0$), then

$\Delta P_m = \Delta P_{ref}$.

If the network is finite, for a fixed speed changer setting ($\Delta Pref = 0$), then

$\Delta P_m = (1/R)\Delta f$ or $\Delta f = RP_m$.

DYNAMIC RESPONSE OF SINGLE AREA LOAD FREQUENCY CONTROL:

Now we are going to study the effect of a disturbance in the system derived above. Both loss of generation and loss of load can be simulated by imposing a positive or negative step input on the variable Pload. A change of the set value of the system frequency f0 is not considered as this is not meaningful in real power systems. From the block diagram in Figure.

ΔP_{load} and Δf ($\Delta P_{m0}^{set} = 0$):

$$\Delta f(s) = -\frac{1 + sT_t}{\frac{1}{S} + \frac{1}{D_l}(1 + sT_t) + (\frac{2W_0}{f_0} + \frac{2HS_B}{f_0})s(1 + sT_t)} \Delta P_{load}(s)$$

The step response for

$$\Delta P_{load}(s) = \frac{\Delta P_{load}}{s}$$

$$\Delta f_\infty = \lim_{s \to 0}(s \cdot \Delta f(s)) = \frac{-\Delta P_{load}}{\frac{1}{S} + \frac{1}{D_l}} = \frac{-\Delta P_{load}}{\frac{1}{D_R}} = -\Delta P_{load} \cdot D_R$$

with

$$\frac{1}{D_R} = \frac{1}{S} + \frac{1}{D_l}$$

In order to calculate an equivalent time constant Teq, Tt is put to 0. This can be done since for realistic systems the turbine controller time constant Tt is much smaller than the time constant.

$$\Delta f(s) = \frac{-\Delta P_{load}(s)}{\frac{1}{D_R} + T_M \frac{S_B}{f_0} s} = \frac{-1}{1 + T_M D_R \frac{S_B}{f_0} s} \frac{D_R \Delta P_{load}}{s}$$

or

$$\Delta f(s) = \frac{1}{1 + T_M D_R \frac{S_B}{f_0} s} \frac{\Delta f_\infty}{s}$$

with

$$T_{eq} = T_M D_R \frac{S_B}{f_0}$$

as the equivalent time constant.

1.6 AGC in a Single Area System

In a single area system, there is no tie-line schedule to be maintained. Thus the function of the AGC is only to bring the frequency to the nominal value. This will be achieved using the supplementary loop (as shown in Fig.1.7) which uses the integral controller to change the reference power setting so as to change the speed set point. The integral controller gain K_I needs to be adjusted for satisfactory response (in terms of overshoot, settling time) of the system. Although each generator will be having a separate speed governor, all the generators in the control area are replaced by a single equivalent generator, and the ALFC for the area corresponds to this equivalent generator.

1.7 AGC in a Multi Area System

In an interconnected (multi area) system, there will be one ALFC loop for each control area (located at the ECC of that area). They are combined as shown in Fig. 1.9 for the interconnected system operation. For a total change in load of ΔP_D, the steady state deviation in frequency in the two areas is given by $\Delta f = \Delta\omega_1 = \Delta\omega_2 = \dfrac{-\Delta P_D}{\beta_1 + \beta_2}$ where, $\beta_1 = (D_1+1/R_1)$; and $\beta_2 = (D_2+1/R_2)$.

1.8 Expression for tie-line flow in a two-area interconnected system

Consider a change in load ΔP_{D1} in area1. The steady state frequency deviation Δf is the same for both the areas. That is $\Delta f = \Delta f_1 = \Delta f_2$. Thus, for area1, we have

$$\Delta P_{m1} - \Delta P_{D1} - \Delta P_{12} = D_1 \Delta f$$

where, ΔP_{12} is the tie line power flow from Area1 to Area 2; and for Area 2

$$\Delta P_{m2} + \Delta P_{12} = D_2 \Delta f$$

The mechanical power depends on regulation. Hence

$$\Delta P_{m1} = -\frac{\Delta f}{R_1} \qquad \text{and} \qquad \Delta P_{m2} = -\frac{\Delta f}{R_2}$$

Substituting these equations, yields

$$(\frac{1}{R_1} + D_1)\Delta f = -\Delta P_{12} - \Delta P_{D1} \qquad \text{and} \qquad (\frac{1}{R_2} + D_2)\Delta f = \Delta P_{12}$$

Solving for Δf, we get

$$\Delta f = \frac{-\Delta P_{D1}}{(1/R_1 + D_1) + (1/R_2 + D_2)} = \frac{-\Delta P_{D1}}{\beta_1 + \beta_2}$$

and

$$\Delta P_{12} = \frac{-\Delta P_{D1}\beta_2}{\beta_1 + \beta_2}$$

where, β_1 and β_2 are the composite frequency response characteristic of Area1 and Area 2 respectively. An increase of load in area1 by ΔP_{D1} results in a frequency reduction in both areas and a tie-line flow of ΔP_{12}. A positive ΔP_{12} is indicative of flow from Area1 to Area 2 while a negative ΔP_{12} means flow from Area 2 to Area1. Similarly, for a change in Area 2 load by ΔP_{D2}, we have

$$\Delta f = \frac{-\Delta P_{D2}}{\beta_1 + \beta_2}$$

and

$$\Delta P_{12} = -\Delta P_{21} = \frac{-\Delta P_{D2}\beta_1}{\beta_1 + \beta_2}$$

Frequency bias tie line control

The tie line deviation reflects the contribution of regulation characteristic of one area to another. The basic objective of supplementary control is to restore balance between each area load generation. This objective is met when the control action maintains

- Frequency at the scheduled value
- Net interchange power (tie line flow) with neighboring areas at the scheduled values

The supplementary control should ideally correct only for changes in that area. In other words, if there is a change in Area1 load, there should be supplementary control only in Area1 and not in Area 2. For this purpose the area control error (ACE) is used (Fig1.9). The ACE of the two areas are given by

For area 1: $ACE_1 = \Delta P_{12} + \beta_1 \Delta f$

For area 2: $ACE_2 = \Delta P_{21} + \beta_2 \Delta f$

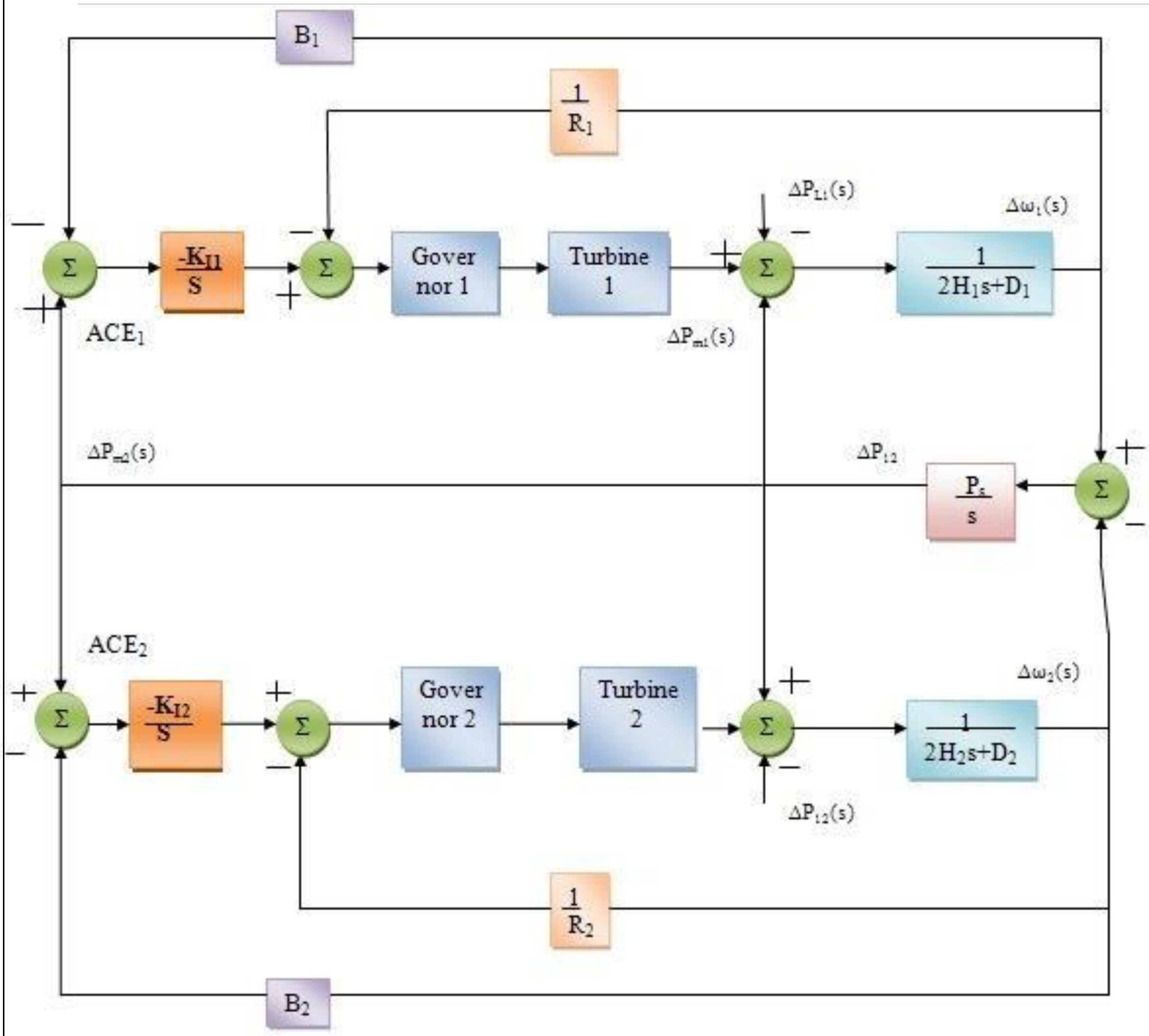


Fig.1.9:AGC for a multi-area operation

COMPUTER CONTROL OF POWER SYSTEMS

ENERGY CONTROL CENTRE:

The energy control center (ECC) has traditionally been the decision-center for the electric transmission and generation interconnected system. The ECC provides the functions necessary for monitoring and coordinating the minute-by-minute physical and economic operation of the power system. In the continental U.S., there are only three interconnected regions: Eastern, Western, and Texas, but there are many *control areas*, with each control area having its own ECC.

Maintaining integrity and economy of an inter-connected power system requires significant coordinated decision-making. So one of the primary functions of the ECC is to monitor and regulate the physical operation of the interconnected grid.

Most areas today have a two-level hierarchy of ECCs with the Independent System Operator (ISO) performing the high-level decision-making and the transmission owner ECC performing the lower-level decision-making.

A high-level view of the ECC is illustrated. Where we can identify the substation, the remote terminal unit (RTU), a communication link, and the ECC which contains the energy management system (EMS). The EMS provides the capability of converting the data received from the substations to the types of screens observed.

In these notes we will introduce the basic components and functionalities of the ECC. Note that there is no chapter in your text which provides this information.

Regional load control centre:

It decides generation allocation to various generating stations within the region on the basics of equal incremental operating cost considering line losses are equal and Frequency control in the region.

Plant load control room:

It decides the allocation of generation of various units in the plant on the basis of:

1. Equal incremented operating cost of various units
2. Minimize the reactive power flow through line so as to minimize line loss and maintain voltage levels and Frequency control in the plant

ECC Components:

The system control function traditionally used in electric utility operation consists of three main integrated subsystems: the energy management system (EMS), the supervisory control and data acquisition (SCADA), and the communications interconnecting the EMS and the SCADA (which is often thought of as part of the SCADA itself). Figure 3 provides a block diagram illustration of these three integrated subsystems. The SCADA and communications subsystems are indicated in

the dotted ovals at the top left hand corner of the figure. The res1of the figure indicates the EMS. We will describe each one in the following subsections.

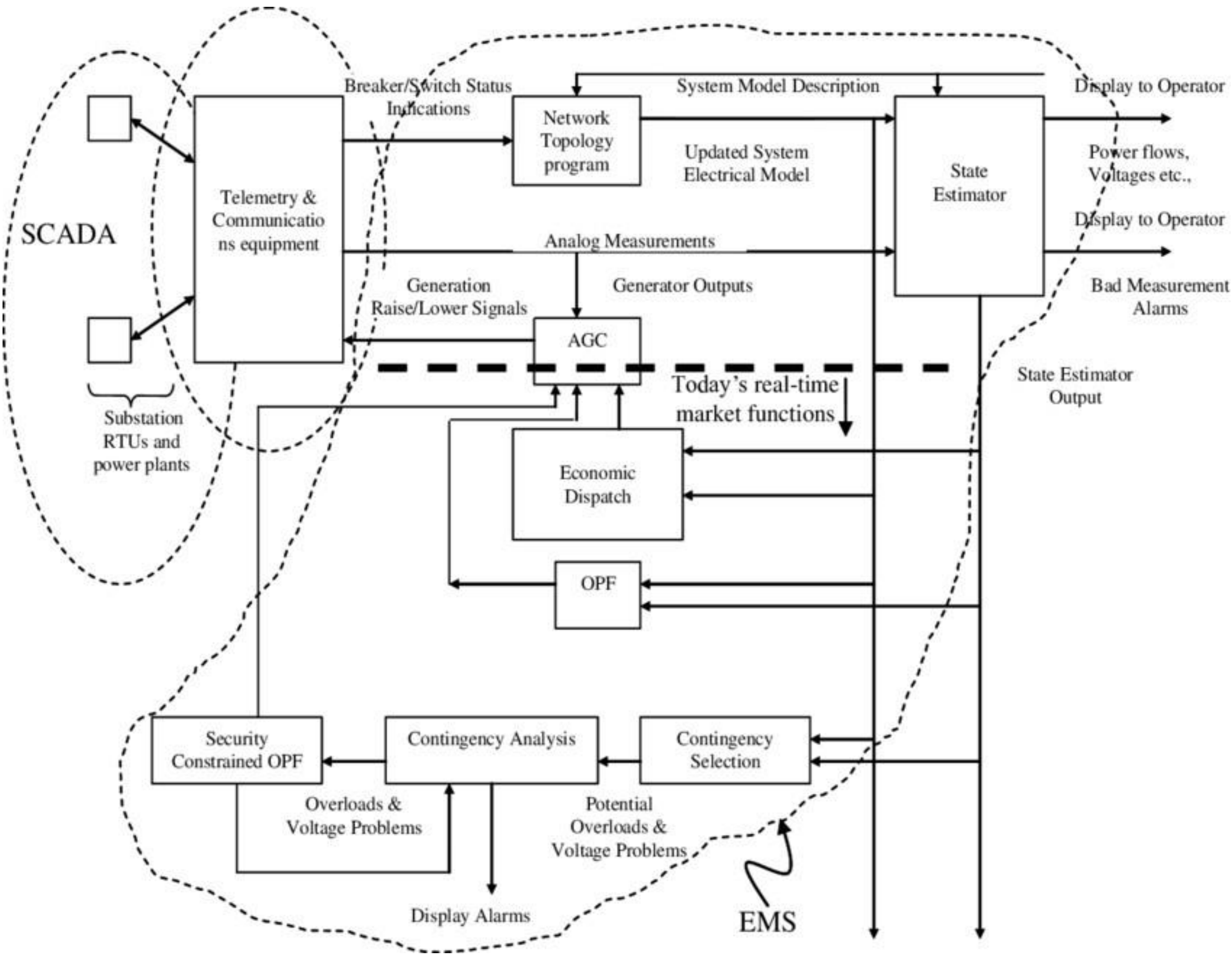


Fig.1:Block diagram of ECC

We distinguish EMS from distribution management systems (DMS). Both utilize their own SCADA, but for different functions. Whereas EMS/SCADA serves the high voltage bulk transmission system from the ECC, the DMS/SCADA serves the low voltage, distribution system from a distribution dispatch center. We are addressing in these notes the EMS/SCADA.

Operation of control centre:

- **Monitoring**
- **Data acquisition and Remote control level control**

1. Turbine – governor to adjust generation to balance changing load-instantaneous control.
2. AGC (called load frequency control (LFC)) maintains frequency and net power interchange.
3. Economic Dispatch Control (EDC) distributes the load among the units such that fuel cost is minimum.

B. Primary Voltage control

1. Excitation control
2. Transmission voltage control, SVC, Shunt capacitors, transformer taps.

2. SUPERVISORY CONTROL AND DATA ACQUISITION (SCADA)

There are two parts to the term SCADA *Supervisory control* indicates that the operator, residing in the energy control center (ECC), has the ability to control remote equipment. *Data acquisition* indicates that information is gathered characterizing the state of the remote equipment and sent to the ECC for monitoring purposes.

The monitoring equipment is normally located in the substations and is consolidated in what is known as the remote terminal unit (RTU). Generally, the RTUs are equipped with microprocessors having memory and logic capability. Older RTUs are equipped with modems to provide the communication link back to the ECC, whereas newer RTUs generally have intranet or internet capability.

Relays located within the RTU, on command from the ECC, open or close selected control circuits to perform a supervisory action. Such actions may include, for example, opening or closing of a circuit breaker or switch, modifying a transformer tap setting, raising or lowering generator MW output or terminal voltage, switching in or out a shunt capacitor or inductor, and the starting or stopping of a synchronous condenser.

Information gathered by the RTU and communicated to the ECC includes both analog information and status indicators. Analog information includes, for example, frequency, voltages, currents, and real and reactive power flows. Status indicators include alarm signals (over-temperature, low relay battery voltage, illegal entry) and whether switches and circuit breakers are open or closed. Such information is provided to the ECC through a periodic scan of all RTUs. A 2 second scan cycle is typical.

Functions of SCADA Systems

1. Data acquisition
2. Information display.
3. Supervisory Control (CBs: ON/OFF, Generator: stop/start, RAISE/LOWER command)
4. Information storage and result display.
5. Sequence of events acquisition.
6. Remote terminal unit processing.
7. General maintenance.
8. Runtime status verification.
9. Economic modeling.
10. Remote start/stop.
11. Load matching based on economics.
12. Load shedding.

Control Functions:

1.Control and monitoring of switching devices, tapped transformers, auxiliary devices etc..

2. Bay-and a station-wide interlocking Automatic functions such as load shedding, power restoration, and high speed bus bar transfer, Time synchronization by radio clock satellite signal.

Monitoring Functions:

1. Measurement and displaying of current, voltage, frequency, active and reactive power, energy, temperature, etc..

Alarm Functions:

1. Storage and evaluation of time stamped events.

Protection functions:

1. Substation protection functions includes the monitoring of events like start and trip.
2. Protection of bus bars. Line feeders, transformers, generators.

Communication technologies:

The form of communication required for SCADA is *telemetry*. Telemetry is the measurement of a quantity in such a way so as to allow interpretation of that measurement at a distance from the primary detector. The distinctive feature of telemetry is the nature of the translating means, which includes provision for converting the measure into a representative quantity of another kind that can be transmitted conveniently for measurement at a distance. The actual distance is irrelevant.

Telemetry may be analog or digital. In analog telemetry, a voltage, current, or frequency proportional to the quantity being measured is developed and transmitted on a communication channel to the receiving location, where the received signal is applied to a meter calibrated to indicate the quantity being measured, or it is applied directly to a control device such as a ECC computer.

Forms of analog telemetry include variable current, pulse-amplitude, pulse- length, and pulse-rate, with the latter two being the most common. In digital telemetry, the quantity being measured is converted to a code in which the sequence of pulses transmitted indicates the quantity. One of the advantages to digital telemetering is the fact that accuracy of data is not lost in transmitting the data from one location to another. Digital telemetry requires analog to digital (A/D) and

possible digital to analog (D/A) converters, as illustrated in the earliest form of signal circuit used for SCADA telemetry consisted of twisted pair wires; although simple and economic for short distances, it suffers from reliability problems due to breakage, water ingress, and ground potential risk during faults.

Improvements over twisted pair wires came in the form of what is now the most common, traditional type of telemetry mediums based on leased-wire, power-line carrier, or microwave. These are *voice grade* forms of telemetry, meaning they represent communication channels suitable for the transmission of speech, either digital or analog, generally with a frequency range of about 300 to 3000 Hz.

SCADA requires communication between Master control station and Remote control station:

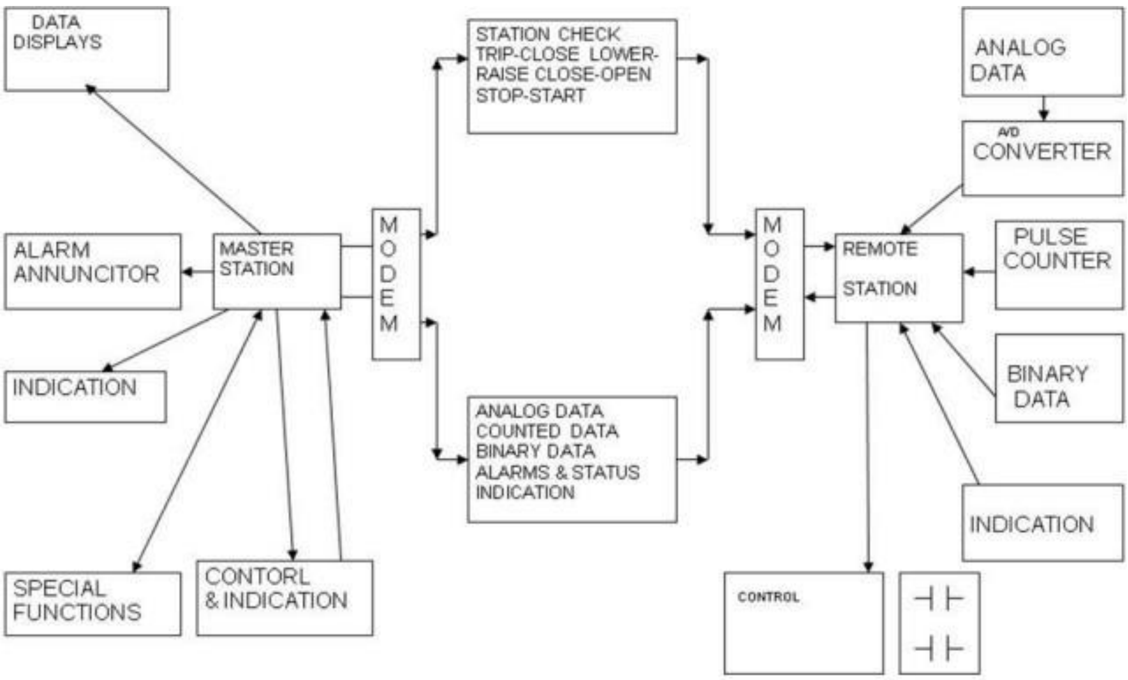


Fig.2:Communication between master and remote control station

Master and Remote station:

Leased-wire means use of a standard telephone circuit; this is a convenient and straightforward means of telemetry when it is available, although it can be unreliable, and it requires a continual outlay of leasing expenditures. In addition, it is not under user control and requires careful coordination between the user and the telephone company. Power-line carrier (PLC) offers an inexpensive and typically more reliable alternative to leased-wire. Here, the transmission circuit itself is used to modulate a communication signal at a frequency much greater than the 60 Hz power frequency. Most PLC occurs at frequencies in the range of 30-500 kHz. The security of PLC is very high since the communication equipment is located inside the substations through open disconnects, i.e., when the transmission line is outaged. Often, this is precisely the time when the communication signal is needed most. In addition, PLC is susceptible to line noise and requires careful signal-to-noise ratio analysis. Most PLC is strictly analog although digital PLC has become available from a few suppliers during the last few years.

Microwave radio refers to ultra-high-frequency (UHF) radio systems operating above 1 GHz. The earliest microwave telemetry was strictly analog, but digital microwave communication is now quite common for EMS/SCADA applications. This form of communication has obvious advantages over PLC and leased wire since it requires no physical conducting medium and therefore no right-of-way. However, line of sight clearance is required in order to ensure reliable communication, and therefore it is not applicable in some cases.

A more recent development has concerned the use of fiber optic cable, a technology capable of extremely fast communication speeds. Although cost was originally prohibitive, it has now decreased to the point where it is viable. Fiber optics may be either run inside underground power cables or they may be fastened to overhead transmission line towers just below the lines. They may also be run within the shield wire suspended above the transmission lines.

One easily sees that communication engineering is very important to power system control. Students specializing in power and energy systems should strongly consider taking communications courses to have this background. Students specializing in communication should consider taking power systems courses as an application area.

ENERGY MANAGEMENT SYSTEM (EMS):

The EMS is a software system. Most utility companies purchase their EMS from one or more EMS vendors. These EMS vendors are companies specializing in design, development, installation, and maintenance of EMS within ECCs. There are a number of EMS vendors in the U.S., and they hire many power system engineers with good software development capabilities during the time period of the 1970s through about 2000, almost all EMS software applications.

An attractive alternative today is, however, the application service provider, where the software resides on the vendor‟s computer and control center personnel access it from the Internet. Benefits from this arrangement include application flexibility and reliability in the software system and reduced installation cost.

One can observe from Figure 3 that the EMS consists of 4 major functions: network model building (including topology processing and state estimation), security assessment, automatic generation control, and dispatch. These functions are described in more detail in the following subsections.

Energy management is the process of monitoring, coordinating, and controlling the generation, transmission and distribution of electrical energy. The physical plant to be managed includes generating plants that produce energy fed

through transformers to the high-voltage transmission network (grid), interconnecting generating plants, and load centers. Transmission lines terminate at substations that perform switching, voltage transformation, measurement, and control. Substations at load centers transform to sub transmission and distribution levels. These lower-voltage circuits typically operate radially, i.e., no normally closed paths between substations through sub transmission or distribution circuits.(Underground cable networks in large cities are an exception.)

Since transmission systems provide negligible energy storage, supply and demand must be balanced by either generation or load. Production is controlled by turbine governors at generating plants, and automatic generation control is performed by control center computers remote from generating plants. Load management, sometimes called demand- Side management, extends remote supervision and control to sub-transmission and distribution circuits, including control of residential, commercial, and industrial loads.

Functionality Power EMS:

1. System Load Forecasting-Hourly energy, 1 to 7 days.
2. Unit commitment-1 to 7days.
3. Economic dispatch.
4. Hydro-thermal scheduling- up to 7 days.
5. MW interchange evaluation- with neighboring system.
6. Transmission loss minimization.
7. Security constrained dispatch.
8. Maintenance scheduling Production cost calculation.

Power System Data Acquisition and Control

A SCADA system consists of a master station that communicates with remote terminal units (RTUs) for the purpose of allowing operators to observe and control physical plants. Generating plants and transmission substations certainly justify RTUs, and their installation is becoming more common in distribution substations as costs decrease. RTUs transmit device status and measurements to, and receive control commands and setpoint data from, the master station. Communication is generally via dedicated circuits operating in the range of 600 to 4800 bits/s with the RTU responding to periodic requests initiated from the master station (polling) every 2 to 10 s, depending on the criticality of the data.

The traditional functions of SCADA systems are summarized:

• Data acquisition: Provides telemetered measurements and status information to operator.
• Supervisory control: Allows operator to remotely control devices, e.g., open and close circuit breakers. A “select before operate” procedure is used for greater safety.
• Tagging: Identifies a device as subject to specific operating restrictions and prevents

unauthorized operation.

- Alarms: Inform operator of unplanned events and undesirable operating conditions. Alarms

are sorted by criticality, area of responsibility, and chronology. Acknowledgment may be required

- Logging: Logs all operator entry, all alarms, and selected information.
- Load shed: Provides both automatic and operator-initiated tripping of load in response to system emergencies.
- Trending: Plots measurements on selected time scales.

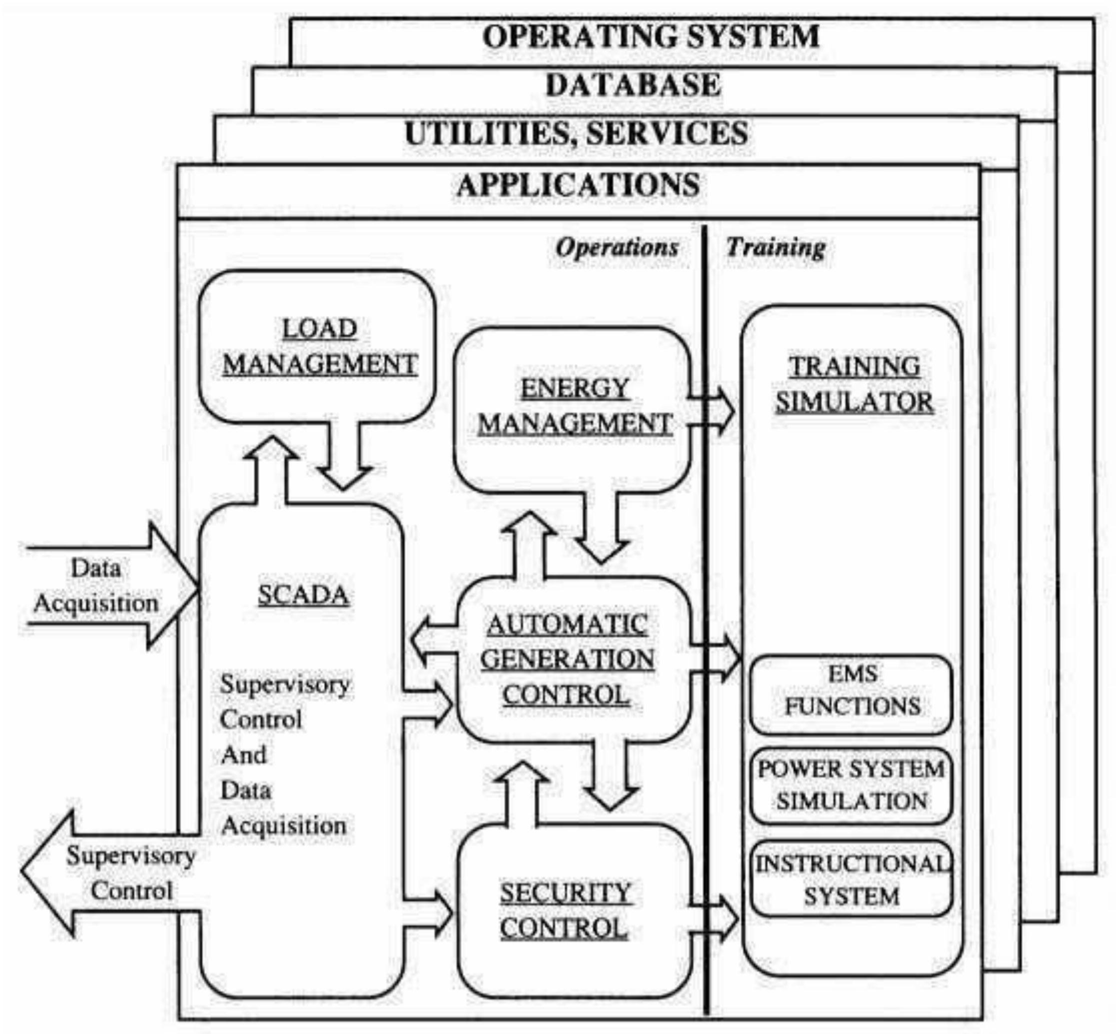


Fig.3.Layers of EMS

Layers of a modern EMS:

Since the master station is critical to power system operations, its functions are generally distributed among several computer systems depending on specific design. A dual computer system configured in primary and standby modes is most common. SCADA functions are listed below without stating which computer has specific responsibility.

- Manage communication circuit configuration
- Downline load RTU files
- Maintain scan tables and perform polling
- Check and correct message errors
- Convert to engineering units
- Detect status and measurement changes

• Monitor abnormal and out-of-limit conditions

• Log and time-tag sequence of events
• Detect and annunciate alarms
• Respond to operator requests to:
– Display information
– Enter data
– Execute control action
– Acknowledge alarms Transmit control action to RTUs
• Inhibit unauthorized actions
• Maintain historical files
• Log events and prepare reports
• Perform load shedding

Automatic Generation Control:

Automatic generation control (AGC) consists of two major and several minorfunctions that operate online in real time to adjust the generation against load at minimum cost. The major functions are load frequency control and economic dispatch, each of which is described below. The minor functions are reserve monitoring, which assures enough reserve on the system; interchange scheduling, which initiates and completes scheduled interchanges; and other similar monitoring and recording functions.

Load Frequency Control:

Load frequency control (LFC) has to achieve three primary objectives, which are stated below in priority order:

1. To maintain frequency at the scheduled value
2. To maintain net power interchanges with neighboring control areas at the scheduled values
3. To maintain power allocation among units at economically desired values.

The first and second objectives are met by monitoring an error signal, called *area control error* (ACE), which is a combination of net interchange error andfrequency error and represents the power imbalance between generation and load at any instant. This ACE must be filtered or smoothed such that excessive and random changes in ACE are not translated into control action. Since these excessive changes are different for different systems, the filter parameters have to be tuned specifically for each control area.

The filtered ACE is then used to obtain the proportional plus integral control signal. This control signal is modified by limiters, dead bands, and gain constants that are tuned to the particular system. This control signal is then divided among the generating units under control by using participation factors to obtain *unit control errors* (UCE).

These participation factors may be proportional to the inverse of the second derivative of the cost of unit generation so that the units would be loaded according to their costs, thus meeting the third objective. However, cost may not be the only consideration because the different units may have different response rates and it may be necessary to move the faster generators more to obtain an acceptable response. The UCEs are then sent to the various units under control and the generating units monitored to see that the corrections take place. This control action is repeated every 2 to 6 s. In spite of the integral control, errors in frequency and net interchange do tend to accumulate over time. These time errors and accumulated interchange errors have to be corrected by adjusting the controller settings according to procedures agreed upon by the whole interconnection. These

accumulated errors as well as ACE serve as performance measures for LFC.

The main philosophy in the design of LFC is that each system should follow its own load very closely during normal operation, while during emergencies; each system should contribute according to its relative size in the interconnection without regard to the locality of the emergency. Thus, the most important factor in obtaining good control of a system is its inherent capability of following its own load. This is guaranteed if the system has adequate regulation margin as well as adequate response capability. Systems that have mainly thermal generation often have difficulty in keeping up with the load because of the slow response of the units.

SECURITY ANALYSIS & CONTROL:

Security monitoring is the on line identification of the actual operating conditions of a power system. It requires system wide instrumentation to gather the system data as well as a means for the on line determination of network topology involving an open or closed position of circuit breakers. A state estimation has been developed to get the best estimate of the status and the state estimation provides the database for security analysis shown .

- **Data acquisition**:
  1. To process from RTU
  2. To check status values against normal value
  3. To send alarm conditions to alarm processor
  4. To check analog measurements against limits.
- **Alarm processor:**
  1. To send alarm messages
  2. To transmit messages according to priority
- **Status processor:**
  1. To determine status of each substation for proper connection.
- **Reserve monitor:**
  1. To check generator MW output on all units against unit limits
- **State estimator:**
  1. To determine system state variables
  2. To detect the presence of bad measures values.
  3. To identify the location of bad measurements
  4. To initialize the network model for other programs

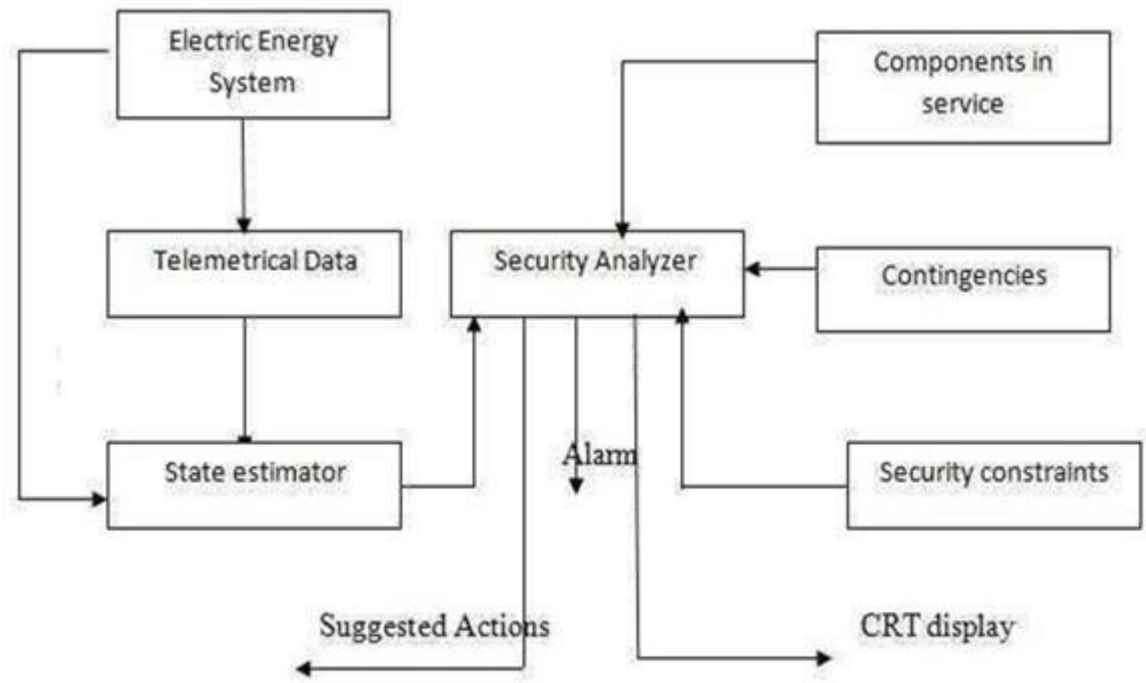


Fig.4: Practical Security Monitoring System

System Security:

1. System monitoring.
2. Contingency analysis.
3. Security constrained optimal power flow

Security Assessment:

Security assessment determines first, whether the system is currently residing in an acceptable state and second, whether the system would respond in an acceptable manner and reach an acceptable state following any one of a pre-defined contingency set. A *contingency* is the unexpected failure of a transmission line, transformer, or generator. Usually, contingencies result from occurrence of a *fault*, or short-circuit, to one of these components. When such a fault occurs, the protection systems sense the fault and remove the component, and therefore also the fault, from the system. Of course, with one less component, the overall system is weaker, and undesirable effects may occur. For example, some remaining circuit may overload, or some bus may experience an under voltage condition. These are called *static* security problems.

Dynamic security problems may also occur, including uncontrollable voltage decline, generator over speed (loss of synchronism), or undamped oscillatory behavior.

Security Control:

Power systems are designed to survive all probable contingencies. A contingency is defined as an event that causes one or more important components such as transmission lines, generators, and transformers to be unexpectedly removed from service. Survival means the system stabilizes and continues to operate at acceptable voltage and frequency levels without loss of load. Operations must deal with a vast number of possible conditions experienced by the system, many of which are not anticipated in planning. Instead of dealing with the impossible task of analyzing all possible system states, security control starts with a specific state: the current state if executing the real-time network sequence; a postulated state if executing a study sequence. Sequence means sequential execution of programs that perform the following steps:

1. Determine the state of the system based on either current or postulated conditions.
2. Process a list of contingencies to determine the consequences of each contingency on the system in its specified state.
3. Determine preventive or corrective action for those contingencies which represent unacceptable risk.

Security control requires topological processing to build network models and uses large-scale AC network analysis to determine system conditions. The required applications are grouped as a network subsystem that typically includes the following functions:

•**Topology processor**: Processes real-time status measurements to determine an electrical connectivity (bus) model of the power system network.

•**State estimator:** Uses real-time status and analog measurements to determine the„„best"""

estimate of the state of the power system. It uses a redundant set of measurements; calculates voltages, phase angles, and power flows for all components in the system; and reports overload conditions.

•**Power flow:** Determines the steady-state conditions of the power system network fora specified generation and load pattern. Calculates voltages, phase angles, and flows across the entire system.

•**Contingency analysis:** Assesses the impact of a set of contingencies on the state ofthe power system and identifies potentially harmful contingencies that cause operating limit violations.

•**Optimal power flow:** Recommends controller actions to optimize a specified objective function (such as system operating cost or losses) subject to a set of power system operating constraints.

•**Security enhancement:** Recommends corrective control actions to be taken to alleviate an existing or potential overload in the system while ensuring minimal operational cost.

•**Preventive action:** Recommends control actions to be taken in a "preventive" mode before a contingency occurs to preclude an overload situation if the contingency were to occur.

•**Bus load forecasting:** Uses real-time measurements to adaptively forecast loads for the electrical connectivity (bus) model of the power systemnetwork.

•**Transmission loss factors:** Determines incremental loss sensitivities for generating units;

calculates the impact on losses if the output of a unit were to be increased by 1 MW.

•**Short-circuit analysis:** Determines fault currents for single-phase and three-phase faults for fault locations across the entire power system network.

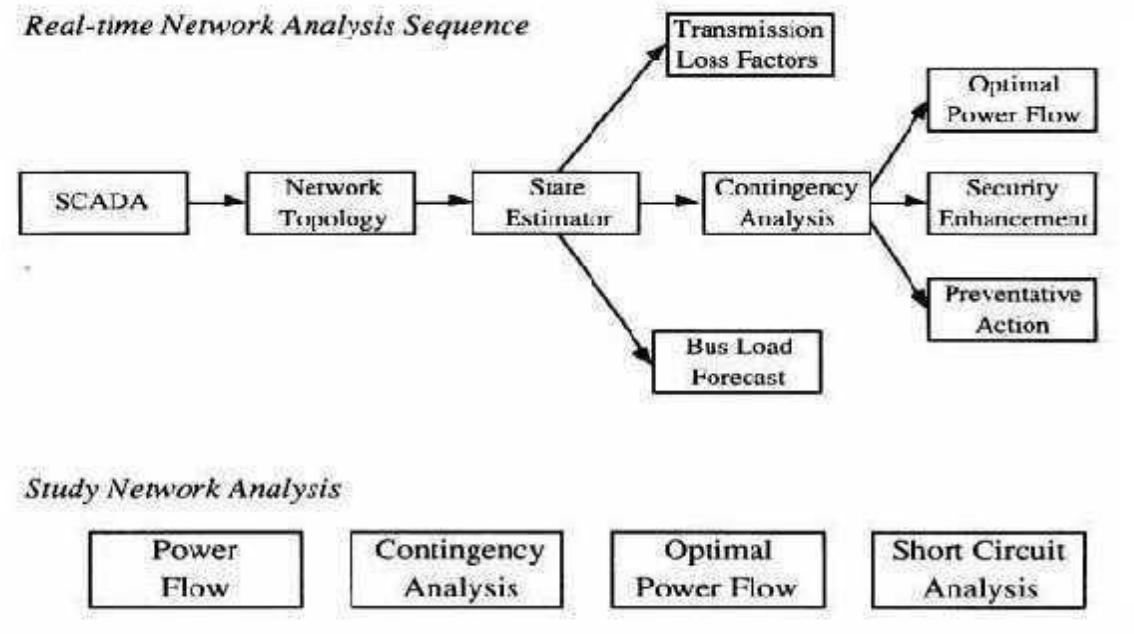


Fig.5:Real time network analysis sequence

VARIOUS OPERATING STATES:

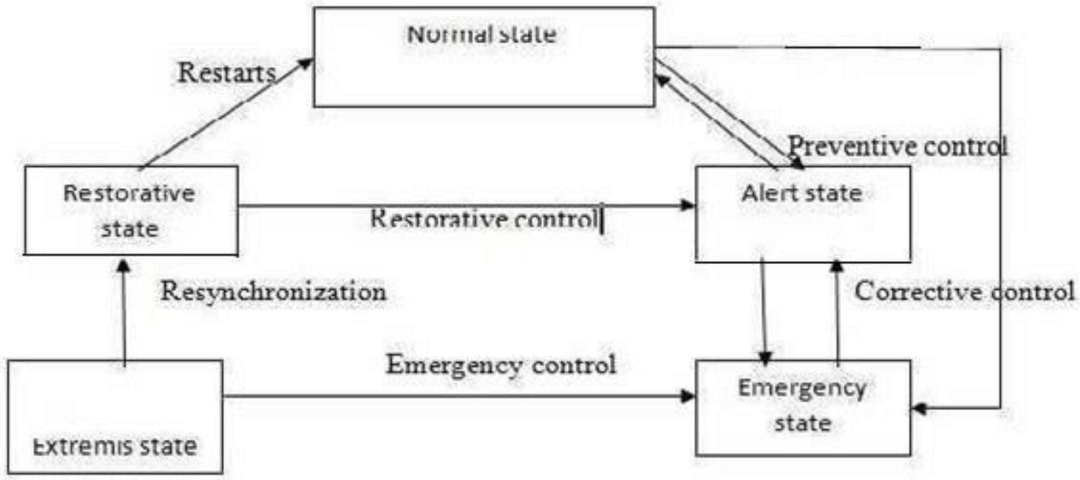


Fig.6:Various operating states

Operating states are:

1. Normal state
2. Alert state
3. Emergency state
4. Extremis state
5. Restorative state

- **Normal state:**

A system is said to be in normal if both load and operating constraints are satisfied .It is one in which the total demand on the system is met by satisfying all the operating constraints.

Alert state:

A normal state of the system said to be in alert state if one or more of the postulated contingency states, consists of the constraint limits violated. When the system security level falls below a certain level or the probability of disturbance increases, the system may be in alert state .All equalities and inequalities are satisfied, but on the event of a disturbance, the system may not have all the inequality constraints satisfied. If severe disturbance occurs, the system will push into emergency state. To bring back the system to secure state, preventive control action is carried out.

Emergency state:

The system is said to be in emergency state if one or more operating constraints are violated, but the load constraint is satisfied .In this state, the equality constraints are unchanged. The system will return to the normal or alert state by means of corrective actions, disconnection of faulted section or load sharing.

Extremis state:

When the system is in emergency, if no proper corrective action is taken in time, then it goes to either emergency state or extremis state. In this regard neither the load or nor the operating constraint is satisfied, this result is islanding. Also the generating units are strained beyond their capacity .So emergency control action is done to bring back the system state either to the emergency state or normal state.

Restorative state:

From this state, the system may be brought back either to alert state or secure state .The latter is a slow process. Hence, in certain cases, first the system is brought back to alert state and then to the secure state .This is done using restorative control action.