MALLA REDDY COLLEGE OF ENGINEERING & TECHNOLOGY

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DEPARTMENT OF MECHANICAL ENGINEERING

DIGITAL NOTES of MACHINE DESIGN-I

For

B.Tech – III YEAR – I

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UNIT - I

Machine Design is the creation of new and better machines and improving the existing ones. A new or better machine is one which is more economical in the overall cost of production and operation. The process of design is a long and time consuming one. From the study of existing ideas, a new idea has to be conceived. The idea is then studied keeping in mind its ~~economical~~ commercial success and given shape and form in the form of drawings. In the preparation of these drawings, care must be taken of the availability of resources in money, in ~~money~~ men and in materials required for the successful completion of the new idea into an actual reality. In designing a machine component, it is necessary to have a good knowledge of many subjects such as Mathematics, Engineering Mechanics, strength of materials, Theory of Machines, workshop Processes and Engineering Drawing.

Defination of machine design contains following important features:

- A designer uses principles of basic engineering sciences such as physics, mathematics, statics and Dynamics, thermodynamics and heat transfer, vibrations and fluid mechanics
- The designer has technical information of basic elements of machine. These elements include fastening devices, chain, belt and gear drives, bearings, oil seals and gaskets, springs, shafts, keys, couplings. and so on. A machine is combination of these elements and their suitability in different applications.

- The designer uses his skill and imagination to produce a configuration, which is a combination of these basic elements. However these combination is unique and different in different situations. The intellectual part of constructing a proper configuration is creative in nature.

- The final outcome of a design process consists of description of the machine. The design description is in the form of drawings of assembly and individual components.

- A design is created to satisfy the recognised need of a customer. The need may be to perform a specific function with maximum economy and efficiency.

"Machine Design is the creation of plans for machine to perform the desired functions. The machine may be entirely new in concept performing new type of work or it may perform more economically the work that can be done by an existing machine. It may be an improvement or enlargement of an existing for better economy and capability".

Classifications of Machine Design:

The machine design may be classified as follows:

1. Adaptive design:

In most cases, the designer's work is concerned with adaptation of existing designs. This type of design needs no special knowledge or skill and can be attempted by designers of ordinary technical training. The designer only makes minor alternation or modification in the existing designs of the product.

2. Development design:

This type of design needs considerable scientific training and design ability in order to modify the existing designs into a new idea by adopting a new material or different method of manufacture. In this case, though the designer starts from the existing design, but the final product may differ quite markedly from the original product.

3. New Design:

This type of design needs lot of research, technical ability and creative thinking. only those designers who have Personal Qualities of a sufficiently high order can take up the work of a new design.

The designs, depending upon the methods used, may be classified as follows:

(a) Rational design:

This type of design depends upon ~~the~~ mathematical ~~empirical~~ formulae ~~based on the~~ of Principle of ~~Practice~~ mechanics.

(b) Empirical design:

This type of design depends upon empirical formulae based on the practice and past experience.

(c) Industrial design:

This type of design depends upon the production aspects to manufacture any machine component in the industry.

(d) optimum design: It is the best design for the given objective function under the specified constraints. It may be achived by minimising the undesirable effects.

(e) System design. It is the design of any complex mechanical system like a motor car.

(f) Element design: It is the design of any element of the mechanical system like piston, crank shaft, connecting rod, etc.

(g) Computer aided design: This type of design depends upon the use of computer systems to assist in the creation, modification, analysis and optimisation of a design.

Basic Procedure of Machine Design,

The basic procedure of machine design consists of a step by step approach - from given specifications about the functional

Market survey

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Define specifications of product

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Study Alternative Mechanisms for product & Select proper Mechanism

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Prepare General Layout of Configuration and Select Joining Methods b/w Individual Components of product

↓

Design Individual Components

↓

Prepare Assembly & Detail Drawings & Modify drawings after Testing Prototype model

fig: Design Process

requirements of a product, to the complete description in the form of drawings of the final product. A logical sequence of steps, usualy common to all design projects, These steps are interrelated and interdependent, each reflecting and affecting all the other steps. Following steps are involved in the process of machine design.

Step 1: Product Specifications

The first step consists of preparing a complete list of the requirements of the product. The requirement include the output capacity of the machine, service life, cost & reliability. In some cases, the overall dimensions and weight of the product are specified. For example, while designing a scooter, the list of specification will be as follows:

(I) fuel consumption = 40 Km/lit

(II) maximum speed = 85 Km/hr

(III) Capacity for carrying = 2 persons with 10 kg luggage.

(IV) over all dimensions

width = 700 mm, length = 1750 mm, heigh = 1000 mm

(V) weight = 95 kg

(VI) cost = RS 15000 to 18000

In the consumer Products, external appearance, noiseless Performance and simplicity in operation of controls are important requirements. Depending upon the type of product, various requirements are given. weightages and a priority list of specifications is prepared.

Step 2: Selection of Mechanism

After a careful study of the requirements, the designer prepares rough sketches of different possible mechanism for the product. For Example, while designing a blanking or piercing Press, the following mechanisms are possible.

(I) Mechanism involving crank & connecting rod, converting the rotary motion of the electric motor into the reciprocating motion of the punch.

(II) Mechanism involving nut & screw, which is simple and cheap configuration but having Poor efficiency; and

(III) Mechanism consisting of hydraulic cylinders, piston and valves which is costly configuration but highly Efficient.

The alternative mechanisms are compared with each other and also with the mechanism of the products that are available in the market. An approximate estimation of the cost of each alternative Configuration is made and compared with the cost of existing products. This will reveal the Competitiveness of the product. While selecting the final configuration, the designer should consider whether the raw materials and standard Parts required for making the product are available in the market. He should also required to fabricate the non-standard Components are available in the factory. Depending upon the cost-Competitiveness, availability of raw materials and manufacturing facility, the best possible mechanism is selected for the product.

Step 3: Layout of Configuration

The next step in design procedure is to prepare a block diagram showing the general layout of the selected configuration. For example, the layout of an E.O.T (Electrically-Operated overhead Travelling) crane will consist of following Components:

(I) Electric motor for supply of power

(II) Flexible coupling to connect the motor shaft to the clutch shaft.

(III) Clutch to connect or disconnect the electric motor at the will of the operator:

(IV) Gear box to reduce the speed from 1440 r.p.m to about 15 r.p.m.

(V) Rope down to convert the rotary motion of the shaft to the linear motion of the wire rope:

(VI) Wire rope and pulley with the crane hook to attach the load.

(VII) Brake to stop the motion.

In this step, the designer specifies Joining methods, such as riveting, bolting or welding to connect individual components. Rough sketches of shapes of the individual Parts are prepared.

step 4: Design of Individual Components

The design of individual components or machine elements is an important step in the design process. It consists of following stages:

(I) Determine the forces acting on the component:

(II) Select proper material for the Component depending upon the functional requirements such as strength, rigidity, hardness and wear resistance.

(III) Determine the likely mode of failure for the component and depending upon it, select the criterion of failure, such as yield stength, ultimate tensile strength, endurance limit or permissible deflection;

(IV) Determine the geometric dimensions of the component using suitable factor of safety and modify the dimensions from assembly and manufacturing consideration:

This stage involves detailed stress and deflection analysis.

Step 5: Preparation of Drawing

The last stage in design process is to prepare drawings of the assembly and the individual components. on these drawings, the material of the component, its dimensions, tolerances, surface finish grades and machining symbols are specified. The designer prepares two separate lists of components- Standard components to be purchased directly from the market and special components to be machined in the factory. In many cases, a prototype model is prepared for the product and thoroughly tested before finishing the assembly drawings.

It is seen that process of machine design involves systematic approach from known specifications to unknown solutions. Quite often, problems arise on the shop floor, during the production stage and design may require modifi-cations. In such circumstances, the designer has to consult the manufacturing engineer and find out the suitable modification.

General Considerations in Designing a Machine Component

(I) Type of load and stresses caused by the load

The load or force is an external agent which, when applied on a machine part produce or tends to produce/destroy motion. Generally the machine members are subjected to various external forces due to

(I) Self weight of the machine

(II) Inertia due to reciprocating Parts

(III) Power transmission

(IV) Change of nature like temp. and other

(V) Frictional forces

The load is classified with respect to its nature of applications as

(a) steady or static or dead load: whose magnitude and direction will not change with respect to time.

(b) Live or variable or dynamic load: whose magnitude and direction change with time.

(c) Suddenly or shock load: Initial velocity is zero.

Ex: Designing a lathe bed require cast iron which is more hard and high compressive strength, for making dial gauge, glass may be employed. eg (. weight load of truck enter on weighting machine)

(d) Impact load: This is suddenly applied with some Velocity. (Load act at certain height)

Ex: Blows of hammer, rough road reactions to the whe
(to A hammer strike on a metal plate from cert height)

and axles of motor cars.

In general the components to be designed or dynamic and impact load should be stronger and bigger than that for steady load.

The load on a machine component may act in several ways due to which the internal stresses are set up: Tensile, Compr-essive, shear, Torsional, Bending, Combined stresses, Thermal stresses.

Load	Stresses
static or dead or steady load,	Tensile, compressive, shear
Live or variable load.	Torsion, Bending
suddenly applied or shock load,	Combination of any stresses
Impact load.	due to change in temp. (Thermal stresses)

2. ~~Motion of the Parts~~: Selection of materials

The selection of material for a Part depends upon the forces that are acting on that Part and stresses developed on that Part. The design engineer should have a through or complete Knowledge of the mechanical behavior of materials Under different loads.

~~3. Selection of materials: The selection of material for a part depen~~

3. Motion of the Parts: Depending upon the given specifications the suitable prescribed motion of the part is to be evaluated. The motion of the parts are

(I) Rectilinear motion (reciprocating)

(II) curvilinear motion (rotary)

(3) Constant speed

(4) Constant or variable acceleration.

4. Form and Size of the Parts: Based upon the ~~on~~ stresses acting on the Part; the size and form shape (appearance) of a Component is to be designed. In the process of reducing the size of the machine or existing component the design engineer should always check for the capability of that part to resist the stresses. The size is inversely proportional to material strength if the load is kept constant.

5. Lubrication: There is always a lot of heat is dissipated b/w movable parts (rotating, sliding or rolling bearings), a design engineer should always provide a better means of lubrication between the parts.

6. Operational features: The designer should always consider the operational features of the machine. For example the start button, controlling levers, stop button, should be designed based upon the convenient handling of the operator.

7. Use of Standard Parts: using the existing standard parts like bolts, nuts, washers, Gears and pulleys etc Reduces the cost of a machine and also it simplifies the manufacturing process. The designer should always go for selection of available parts of standard sizes: however, if the design requires a new part, the designer has to suggest a new manufacturing process.

8. Safety of operation: Some machines are ~~and~~ dangerous to operate at maximum speed. It is necessary that a designer

should always provide safety devices for the safety of the operator. The safety appliances should in no way interference with operation of the machin Ex: Electrical main-switch.

9. Work shop facilities: The designer has to always design the Part based upon his employer's workshop facilities available to him. Sometimes it is necessary to plan and supervice the work shop operations and to draft methods for casting, handling and machining special parts.

10. Number of Components to be manufactured

Based upon the number of Parts to be manufactured, the designer has flexibility in designing the Part. if the number of components to be manufactured are less, the designer should always look at using standard shapes and sizes of Parts available to him. However ~~the~~ if the number of components to be manufactured are more, he can go for a new product (design) of the part.

11. Cost of construction: The designer has to always try to minimize the cost of construction of a machine. use of standard parts and using the manufacturing process available to him can reduce the cost of construction.

12. Assembling: Based upon the local conditions at erection of the machine, the designer should design the different Components of a machine.

Materials:

Introduction: The knowledge of materials and their properties is of great significance for a design engineer. The machine elements should be made of such a material which has properties suitable for the conditions of operation. In addition to this, a design engineer must be familiar with the effects which the manufacturing processes and heat treatment have on the properties of the materials.

Classification of Engineering Materials:

The Engineering materials are mainly classified as:

1. Metals and their alloys, such as iron, steel, copper, aluminium etc.
2. Non-Metals, such as glass, rubber, plastic, etc.

The metals may be further classified as:

(a) Ferrous metals, and (b) Non-Ferrous metals.

The ferrous metals are those which have the iron as their main constituent, such as Cast-iron, wrough iron and steel.

The Non-ferrous metals are those which have a metal other than iron as their main constituent, such as copper, aluminium, brass, tin, zinc, etc.

Selection of materials for Engineering purposes.

The selection of a proper material, for engineering purposes, is one of the most difficult problem for the designer. The best material is one which serve the desired objective at the minimum cost. The following factors should be considered while selecting the material:

1. Availability of the materials
2. Suitability of the materials for the working conditions in service, and.
3. The cost of the materials.

The important properties, which determine the utility of the material and physical, chemical and mechanical. Properties. we shall now discuss physical and mechanical properties of the materials in

Physical properties of metals:

The physical properties of the metals include luster, colour, size and shape, density, electric and thermal conductivity and melting point.

Mechanical properties of metals:

The mechanical properties of the metals are those which are associated with ability of the material to resist mechanical forces. and load. These mechanical properties of the metals include strength, stiffness, elasticity, plasticity, ductility, brittileness, malleability, toughness, resilience, creep and hardness.

1. Strength: It is the ability of a material to resist the externally applied forces without breaking or yielding. The internal resistance offered by a part to an externally applied force is called Stress.

2. Stiffness: It is the ability of a material to resist deformation Under stress. The modulus of elasticity is the measure of stiffness.

3. Elasticity: It is the property of a material to regain its original shape after deformation when the external forces are removed. This property is desirable for materials used in tools and machines. It may be noted that steel is more elastic than rubber.

4. Plasticity: It is the property of a material which retains the deformation produced under load permanently. This property of the material is necessary for forgings, in stamping images on coins and in ornamental work.

5. Ductility: It is the property of a material enabling it to be drawn into wire with the application of a tensile force. A ductile material must be both strong and plastic. The ductility usually measured by the terms, percentage elongation and percentage reduction in area. The ductile material commonly used in engineering practice are mild steel, copper, aluminium, nickel, zinc, tin and lead.

6. Brittleness: It is the property of a material opposite to ductility. It is the property of braking of material with little permanent distortion. Brittle materials when subjected to tensile loads, snap off without giving any sensible elongation. Cast-iron is a brittle material.

7. Malleability: It is a special case of ductility which permits materials to be rolled or hammered into thin sheets. A malleable material should be plastic but it not essential to be so strong. The malleable materials commonly used in engineering practice are lead, soft steel, wrought iron, copper and aluminium.

8. Toughness: It is the property of a material to resist fracture due to high impact loads like hammer blows. The toughness of material decreases when it is heated. It is measured by the amount of energy that a unit volume of the material has absorbed after being stressed upto the point of fracture. This property is desirable in parts subjected to shock and impact loads.

9. Machinability: It is the property of material which refers to a relative case with which a material can be cut. The machinability of a material can be measured in a number of ways such as comparing the tool life for cutting different materials or thrust required to remove the material at some given rate or the energy required to remove a unit volume of the material. It may be noted that brass can be easily machined than steel.

10. Resilience: It is the property of a material to absorb energy and to resist shock and impact loads. It is measured by the amount of energy absorbed per unit volume within elastic limit. This property is essential for spring materials.

11. Creep: When a part is subjected to a constant stress at high temperature for a long period of time, it will undergo a slow and permanent deformation called creep. This property is considered in designing internal combustion engines, boilers and turbines.

12. Fatigue: When a material is subjected to repeated stresses, it fails at stresses below the yield point stresses. Such type of failure of a material is known as fatigue. The failure is caused by means of a progressive crack formation which are usually fine and of microscopic size. This property is considered in designing shafts, connecting rods, springs, gears, etc.

13. Hardness: It is very important property of the metals and has a wide variety of meanings. It embraces many different properties such as resistance to wear, scratching, deformation and machinability etc. It also means the ability of a metal to cut another metal. The hardness is usually expressed in numbers which are dependent on the method of making the test. The hardness of a metal may be determined by the following tests:

(I) Brinell hardness test
(II) Rockwell hardness test
(III) Vickers hardness test (also called Diamond Pyramid test, and
(IV) Shore scleroscope.

Steel : It is an alloy of iron and carbon, with carbon content upto a maximum of 1.5%. The carbon occurs in the form of iron carbide, because of its ability to increase the hardness and strength of the steel. other elements e.g. silicon, sulphur, phosphorus and manganese are also present to greater or lesser amount to impart

Certain desired properties to it. Most of the steel produced now-a-days is plain Carbon steel or simply Carbon steel. A Carbon steel is defined as a steel which has its properties mainly due to its carbon content and does not contain more than 0.5% of silicon and 1.5% of manganese.

The plain Carbon steels varying from 0.06% Carbon to 1.5% carbon are divided in the following types depending upon the carbon content.

1. Dead mild steel – up to 0.15% Carbon
2. Low Carbon or mild steel – 0.15% to 0.45% carbon
3. Medium Carbon steel – 0.45% to 0.8% carbon
4. High carbon steel – 0.8% to 1.5% Carbon

According to Indian Standard *[IS : 1762 (Part-1)-1974], a new system of designating the steel is recomended. According to this standard, steels are designated on the following two basis: (a) on the basis of mechanical properties, and (b) on the basis of chemical Composition

Steel Designated on the Basis of Mechanical properties

These steels are Carbon and low alloy steels where the main criterion in the selection and inspection of steel is the tensile strength or yield stress. According to Indian standard IS : 1570 (Part-1)-1978 (Reaffirmed 1993), these steels are designated by a symbol 'Fe' or 'Fe E' depending on whether the steel has been specified on the basis of minimum tensile strength or yield strength, followed by the figure indicating the minimum tensile strength or yield

Stress in N/mm^2. For example 'Fe 290' means a steel having minimum tensile strength of 290 N/mm^2 and 'Fe 220' means a steel having yield strength of 220 N/mm^2.

Steels Designated on the Basis of Chemical Composition:

According to Indian Standard, IS : 1570 (Part II/sec-I) -1979 (Reaffirmed 1991), the carbon steels are designated in the following order:

(a) Figure Indicating 100 times the average percentage of Carbon content,

(b) Letter 'C' and

(c) Figure indicating 10 times the average percentage of manganese content. The figure after multiplying shall be rounded off to the nearest integer.

For example 20C8 means a Carbon steel containing 0.15 to 0.25 per cent (0.2 per cent on average) Carbon and 0.60 to 0.90 percent (0.75 percent rounded off to 0.8 percent on an average) manganese.

Effect of Impurities on Steel:

The following are the effects of impurities like silicon, sulphur, manganese and phosphorous on steel.

Silicon: The amount of silicon in the finished steel usually ranges from 0.05 to 0.3 %. Silicon is added in low carbon steels to prevent them from becoming porous. It removes the gases and oxides, prevent blow holes and thereby makes the steel tougher and harder.

2. Sulphur: It occurs in steel either as iron sulphide or manganese sulphide. Iron sulphide because of its low melting point products red shortness, whereas manganese sulphide does not affect so much. Therefore, manganese sulphide is less objectionable in steel than iron sulphide.

3. Manganese: It serves as a valuable deoxidising and purifying agent in steel. Manganese also combines with sulphur and there by decreases the harmful effect of this element remaining in the steel. When used in ordinary low carbon steels, manganese makes the metal ductile and of good bending qualities. In high speed steels, it is used to toughen the metal and to increase its critical temperature.

4. Phosphorus: It makes the steel brittle. It also produces cold shortness in steel. In low carbon steels, it raises the yield point and improves the resistance to atmospheric corrosion. The sum of carbon and phosphorus usually does not exceed 0.25%

Manufacturing Consideration in Machine Design

Manufacturing Processes:

The knowledge of manufacturing processes is of great importance for a design engineer. The following are the various manufacturing processes used in Mechanical Engineering.

1. Primary shaping processes: The processes used for the preliminary shaping of the machine component are known

as Primary shaping Processes. The common operations used for this process are casting, forging, extruding, rolling, drawing, bending, shearing, spinning, power metal forming, squeezing, etc.

2. Machining Process: The processes used for giving final shape to the machine component, according to planned dimensions are known as machining processes. The common operations used for this process are turning, planning, shaping, drilling, boring, reaming, sawing, broaching, milling, hobbing, etc.

3. Surface finishing process: The processes used to provide a good surface finish for the machine component are known as surface finishing processes. The common operations used for this process are polishing, buffing, honing, lapping, abrasive belt grinding, barrel tumbling, electroplating, super finishing, sheradizing, etc.

4. Joining Process: The processes used for Joining machine Components are known as Joining processes. The common operations used for this process are welding, riveting, soldering, brazing, screw fastening, pressing, sintering etc.

5. Process effecting change in properties: These processes are used to impart certain specific properties to the machine Components so as to make them suitable for Particular operations or uses. Such processes are heat treatment, hot-working, cold-working and shot peening.

Other Considerations in Machine Design:

1. Work shop facilities.

(2) Number of machines to be manufactured.

(3) cost of construction

(4) Assembling.

Interchangeability:

The term interchangeability is normally employed for the mass production of identical items within the prescribed limits of sizes. A little consideration will show that in order to maintain the sizes of the part within a close degree of accuracy, a lot of time is required. But even then there will be small variations. if the variations are within certain limits, all parts of equivalent size will be equally fit for operating in machines mechanisms. Therefore, certain variations are recognized and allowed in the sizes of the mating parts to give the required fitting. This facilitates to select at random from a large number of parts for an assembly and results in a considerable saving in the cost of production.

In order to control the size of finished part, with due allowance for error, for interchangeable parts is called Limit system. It may be noted that when an assembly is made of two parts, the part which enters into the other, is known as enveloped surface (or ~~hole for~~ shaft for cylindrical part) and the other in which one enters is called enveloping surface (or hole for cylindrical part). The term shaft refers not only to the diameter of a circular shaft, but it is also used to determine any external dimension of a part. The term hole refers

" component selected randomly should assemble correctly with any other mating component. This is interchangeability" not only to the diameter of a circular hole, but it is also used to designate any internal dimension of a part.

Important Terms used in Limit system:

The following terms used in limit system (or inter-changeable system) are important from the subject point of view:

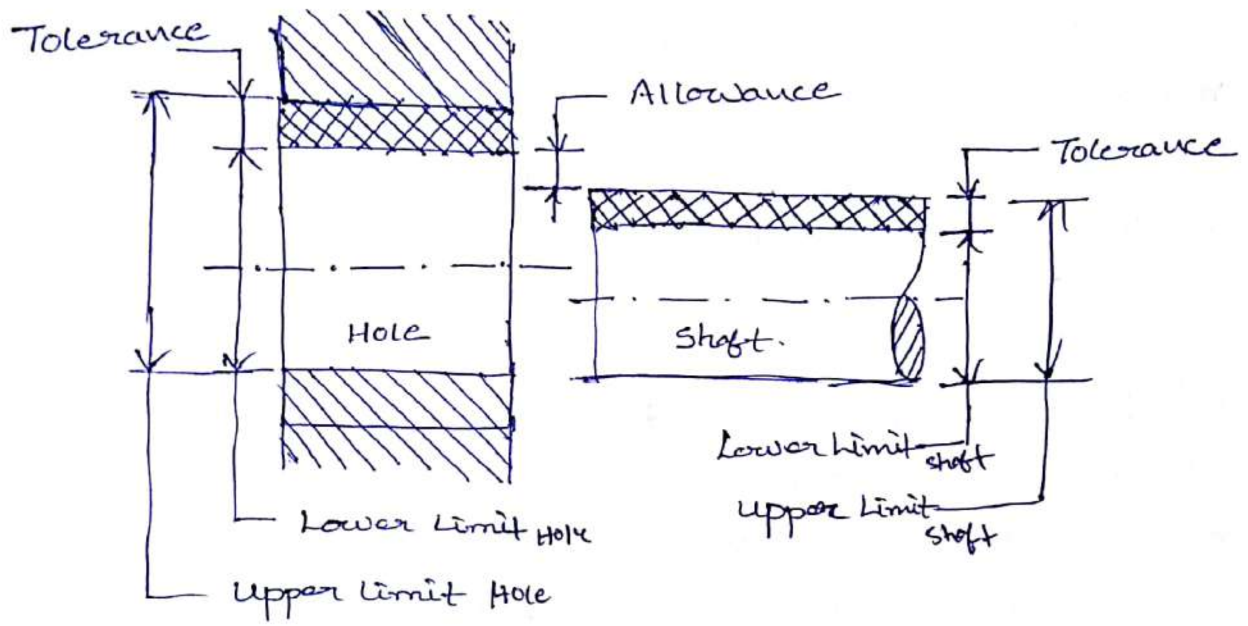


fig: Limits of sizes.

1. Nominal Size: It is the size of a Part specified in the drawing as a matter of convenience.

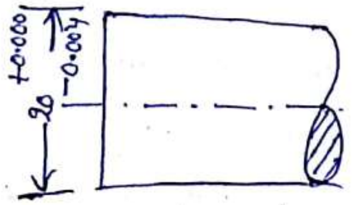
2. Base size: It is the size of a part to which all limits of variation (i.e tolerances) are applied to arrive at final dimensioning of the mating Parts. The nominal or basic size of a Part is often the same.

3. Actual Size: It is the actual measured dimension of the Part. The difference between the basic size and the actual size should not exceed a certain limit, otherwise it will interface with the interchangeability of the mating Parts.

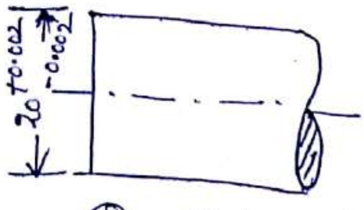
4. Limits of sizes: There are two extreme permissible for a dimension of the part as shown in fig. The largest permissible size for a dimension of the part is called upper or high or maximum limit, whereas the smallest size of the part is known as Lower or minimum limit.

5- Allowance:- It is the difference between the basic dimensions of the mating parts. The allowance may be positive or negative. when the shaft size is less than the hole size, the the allowance is positive and when the shaft size is greater than the hole size, then the allowance is negative.

6. Tolerance: It is the difference between the upper limit and lower limit of a dimension. In other words, it is the maximum permissible variation in a dimension. The tolerance may be Unilateral or bilateral. when all the tolerance is allowed on one side of the nominal size, e.g. $20^{+0.000}_{-0.004}$, then it is said to be Unilateral system of tolerance. The Unilateral system is mostly used in industries as it permits changing the tolerance value while still retaining the same allowance or type of fit. when the tolerances is allowed on both sides of the nominal size, e.g $20^{+0.002}_{-0.002}$, then it is said to be bilateral system of tolerance. In this case +0.002 is the upper limit and -0.002 is the lower limit.



(a) Unilateral tolerance



(b) Bilateral tolerance.

7. Tolerance zone: It is the zone between the maximum and minimum limit size.

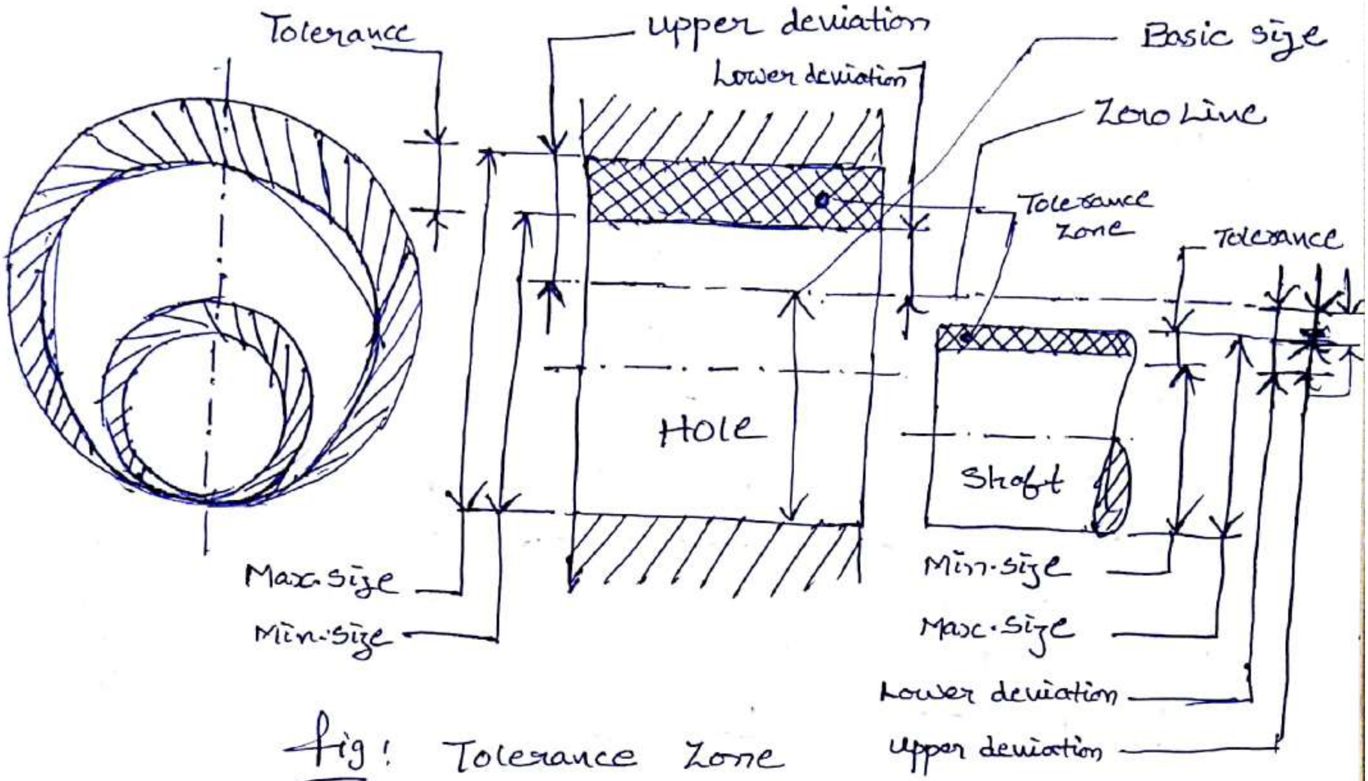


fig: Tolerance Zone

8. Zero Line: It is a straight line corresponding to the basic size. The deviations are measured from this line. The positive and negative deviations are shown above and below the zero respectively.

9. upper deviation: It is the algebraic difference between the maximum size and the basic size. The upper deviation of a hole is represented by a symbol ES (Ecart superior) and of a shaft, it is represented by ~~ai~~ es.

10. Lower deviation: It is the algebraic difference between the minimum size and the basic size. The lower deviation of a hole is represented by a symbol EI (Ecart Inferior) and of a shaft, it is represented by ei.

11. Actual deviation: It is the algebraic difference between an actual size and the corresponding basic size.

12. Mean deviation: It is the arithmetical mean between the upper and lower deviations.

13. Fundamental deviation: It is one of the two deviations which are conventionally chosen to define the position of the tolerance zone in relation to zero line, as shown below.

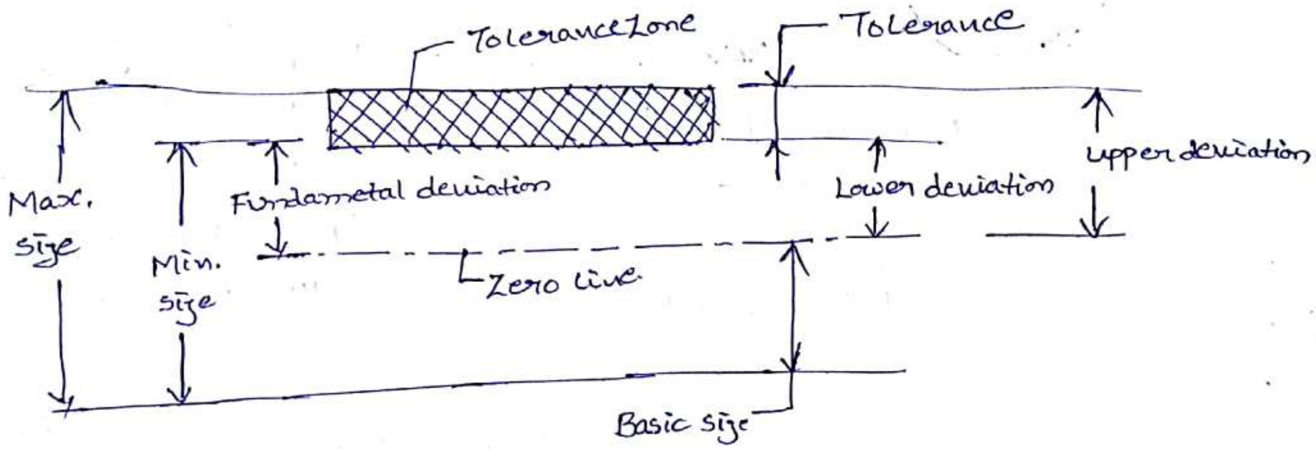
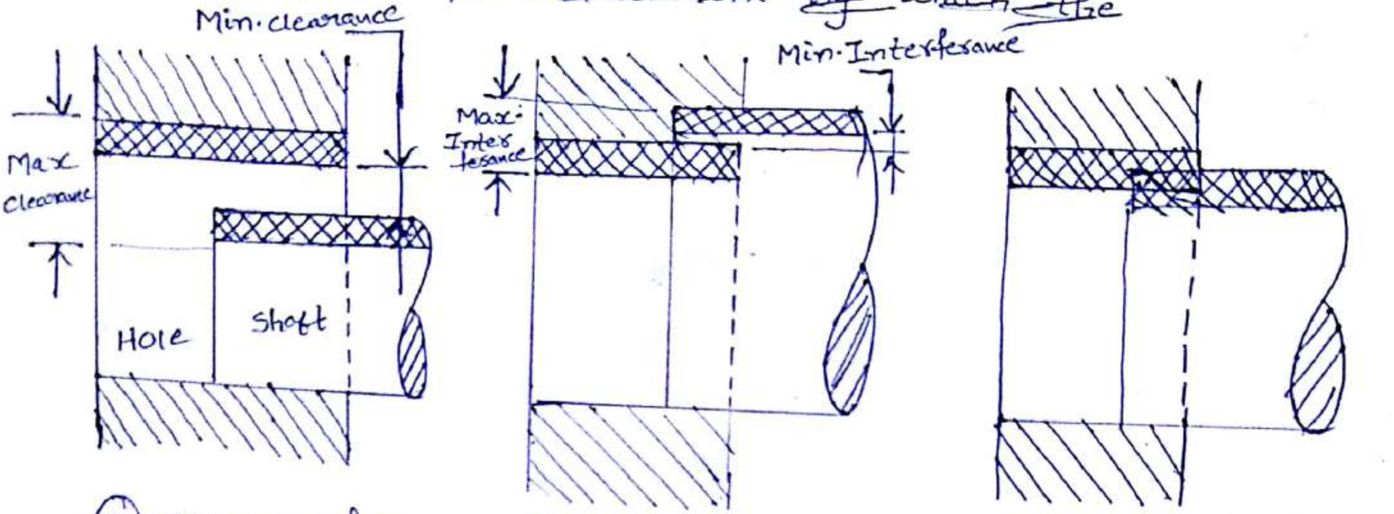


fig: Fundamental deviation

Fits: The degree of tightness or looseness between the two mating parts is known as a fit of the parts. The nature of fit is characterized by the presence and size of clearance and interference.

The clearance is the amount by which the actual size of the shaft is less than the actual size of the mating hole in an assembly as shown below. In other words, the clearance is the difference between the sizes of the hole and the shaft before assembly. The difference must be positive.

The clearance is the amount by which the



(a) Clearance fit (b) Interferance fit (c) Transition fit.

Fig: Types of fits.

The interference is the amount by which the actual size of a shaft is larger than the actual finished size of the mating hole in an assembly as shown above. In other words, the interference is the arithmetical difference between the sizes of the hole and the shaft, before assembly. The difference must be negative.

Types of fits:

According to Indian standards, the fits are classified into the following three groups:

1. Clearance fit: In this type of fit, the size limits for mating parts are so selected that clearance b/w them always occur, as shown above fig (a). It may be noted that in a clearance fit, the tolerance zone of the hole is entirely above the tolerance zone of the shaft. In a clearance fit, the difference b/w the minimum size of the hole and maximum size of the shaft is known as minimum clearance whereas the difference b/w the maximum size of the hole and minimum size of the shaft

is called maximum clearance as shown above fig (a). The clerance fits may be slide fit, easy sliding fit, running fit, slack running fit and loose running fit.

2. Interferance fit: In this type of fit, the size limits for the mating parts are so selected that interference b/w them always occur, as shown above fig (b). It may be noted that in an interference fit, the tolerance zone of the hole is entirely below the tolerance zone of the shaft. In an interferance fit, the difference b/w the maximum size of the hole and minimum size of the shaft is known as minimum interferance, whereas the difference b/w the minimum size of the hole and the maximum size of the shaft is called maximum interference as shown above fig (b). The interferance fits may be shrink fit, heavy drive fit and light drive fit.

3. Transition fit:- In this type of fit, the size limits for the mating parts are so selected that either a clearance or interferance may occur depending upon the actual size of the mating parts as shown above fig (c). It may be noted that in a transition fit, the tolerance zones of hole and shaft overlap. The transition fits may be force fit, tight fit and push fit.

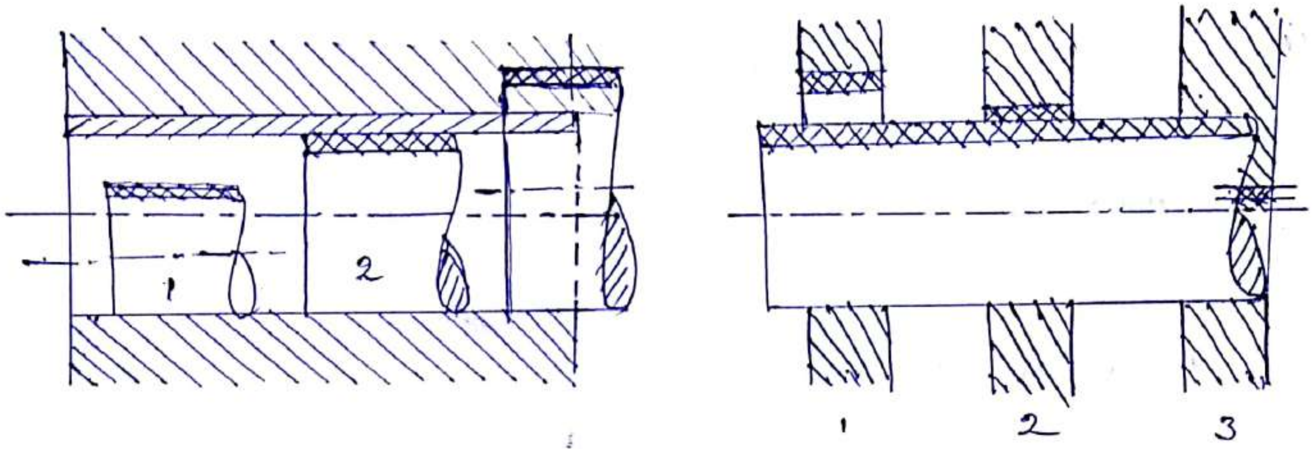
Basis of Limit system:

The following are two bases of limit system

1. Hole basis system: when the hole is kept as a constant member (i.e. when the lower deviation of the hole is

zero) and different fits are obtained by varying the shaft size, as shown in fig below, then the limit system is said to be on a hole basis.

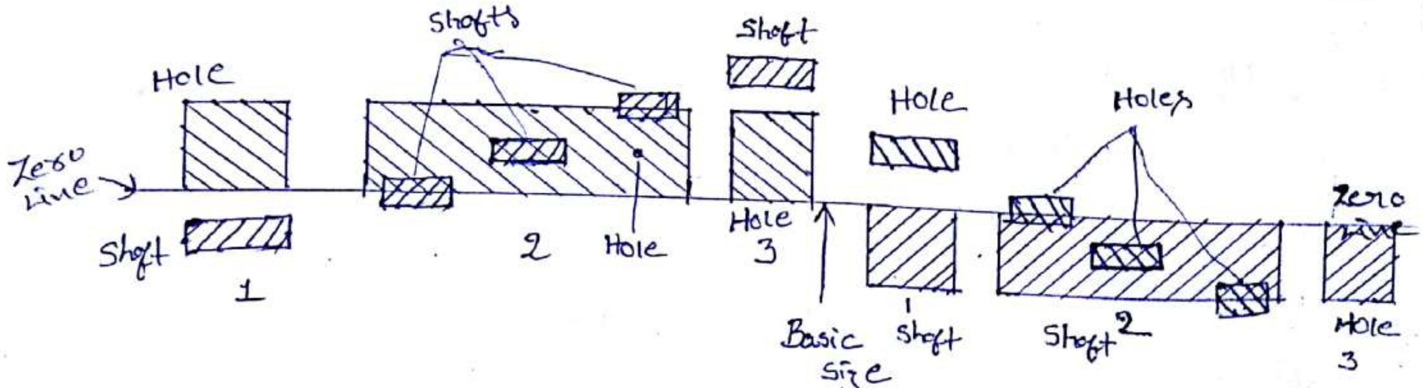
2- Shaft basis system: when the shaft is kept as a constant member (i.e. when the upper deviation of the shaft is zero) and different fits are obtained by varying the hole size, as shown in fig (b) below. Then the limit system is said to be on a shaft basis.



1- Clearance fit   2- Transition fit   3- Interference fit

fig. Hole Basis system   fig: Shaft Basis system

fig Basis of Limit system.



1. Clearance fit   2. Transition fit   3. Interference fit

Hole basis system   Shaft basis system.

fig. Basis of Limit system.

The hole basis and shaft basic system may also be shown above fig. with respect to the zero line. It may be noted that from the manufacturing point of view, a hole basis system is always preferred. This is because the holes are usually produced and finished by standard tooling like drill, reamers, etc., whose size is not adjustable easily. On the other hand, the size of the shaft (which is to go into the hole) can be easily adjusted and is obtained by turning or grinding operations.

(1) The dimensions of the mating parts, according to basic hole system, are given as follows:

Hole: 25.00 mm　　　Shaft: 24.97 mm

25.02 mm　　　24.95 mm

Find the hole tolerance, shaft tolerance and allowance.

Sol: Given that:

Lower limit of hole = 25 mm,

upper limit of hole = 25.02 mm

upper limit of shaft = 24.97 mm

Lower limit of shaft = 24.95 mm.

Hole tolerance

we know that hole tolerance

= upper limit of hole − Lower limit of hole

$= 25.02 - 25.00$

$= 0.02$ mm.

Shaft tolerance:

we know the shaft tolerance

= upper limit of shaft − Lower limit of shaft.

$= 24.97 - 24.95$

$= 0.02$ mm.

Allowance:

we know that allowance

= Lower limit of hole − upper limit of shaft

$= 25.00 - 24.97$

$= 0.03$ mm

(2) Calculate the tolerances, fundamental deviations and limits of sizes for the shaft designated as 40 H8/f7.

Sol Given: shaft designation = 40 H8/f7

The shaft designation 40 H8/f7 means that the basic size is 40 mm and the tolerance grade for the hole is 8. (i.e. IT8) and for the shaft is 7 (i.e IT7)

∵ (I.T = International Tolerance Grade.)

Tolerances

Since 40 mm lies in the diameter steps of 30 to 50 mm, therefore the geometric mean diameter,

$$D = \sqrt{30 \times 50} = 38.73 \text{ mm.}$$

we know that standard tolerance unit,

$$i = 0.45 \sqrt[3]{D} + 0.001 D$$

$$i = 0.45 \cdot \sqrt[3]{38.73} + 0.001 \times 38.73$$

$$i = 0.45 \times 3.38 + 0.03873$$

$$i = 1.55973 \text{ or } \underline{1.56} \text{ microns}$$

$$i = 1.56 \times 0.001 = 0.00156 \text{ mm}$$

$$\boxed{i = 0.00156} \qquad (\because 1 \text{ micron} = 0.001 \text{ mm})$$

We find that standard tolerance for the hole of grade $\underline{8}$ (IT 8)

$$= 25i = 25 \times 0.00156 = 0.039 \text{ mm}$$

and standard tolerance for the shaft of grade 7 (IT 7)

$$= 16i = 16 \times 0.00156 = 0.025 \text{ mm}$$

Fundametal deviation

we know that fundametal deviation (lower deviation) for Hole $\underline{H}$,

$$EI = 0$$

we find that fundametal deviation (upper deviation) for shaft $\underline{f}$

$$es = -5.5(D)^{0.41}$$

$$= -5.5(38.73)^{0.41} = -24.63 \text{ or } -\underline{25} \text{ microns.}$$

$$es = -25 \times 0.001 = -0.025 \text{ mm}$$

$\therefore$ Fundametal deviation (lower deviation) for shaft $\underline{f}$,

$$ei = es - IT = -0.025 - 0.025$$

$$= -0.050 \text{ mm.}$$

The $\underline{-ve}$ sign indicates that fundamental deviation lies below the $\underline{\text{zero}}$ line.

Limits of sizes:

we know that lower limit for hole

$= \text{Basic size} = 40\text{ mm}$

upper limit for hole = Lower limit for hole + Tolerance for hole

$= 40 + 0.039 = 40.039\text{ mm}$

upper limit for shaft = Lower limit for hole or Basic size − Fundametal deviation (~~up~~

= upper deviation) ...(∵ shaft f lies below the zero line)

$= 40 - 0.025 = 39.975\text{ mm}$

and lower limit for shaft = ~~Lower limit for hole or Basic size − Fundametal deviation~~

= upper limit for shaft − Tolerance for shaft

$= 39.975 - 0.025 = 39.95\text{ mm}.$

(3) A Journal of nominal or basic size of 75 mm runs in a bearing with close running fit. Find the limits of shaft and bearing. what is the maximum and minimum clearanc ?

Sol. we find that the close running fit is represented by H8/g7. i.e. a shaft g7 should be used H8 hole.

since 75 mm lies in the diameter step of 50 to 80 mm, therefore the geometric mean diameter,

$$D = \sqrt{50 \times 80} = 63\text{ mm}$$

we know that standard toleranc unit,

$$i = 0.45\sqrt[3]{D} + 0.001D = 0.45\sqrt[3]{63} + 0.001 \times 63$$

$i = 1.79 + 0.063 = 1.853$ micron

$= 1.853 \times 0.001 = 0.001853$ mm

$\therefore$ Standard tolerance for hole 'H' of grade 8 (IT 8)

$= 25i = 25 \times 0.001853$

$= 0.046$ mm.

and standard tolerance for shaft 'g' of grade 7 (IT7)

$= 16i = 16 \times 0.001853$

$= 0.03$ mm

we find that upper deviation for shaft g,

$es = -2.5(D)^{0.34} = -2.5(63)^{0.34} = -10$ micron.

$= -10 \times 0.001 = -0.01$ mm,

$\therefore$ Lower deviation for shaft g,

$ei = es - IT = -0.01 - 0.03 = -0.04$ mm

We know that lower limit for hole

$=$ Basic size $= 75$ mm

upper limit for hole $=$ Lower limit for hole $+$ Tolerance for hole

$= 75 + 0.046 = 75.046$ mm

upper limit for shaft $=$ Lower limit for hole $-$ upper deviation for shaft

($\because$ shaft g lies below zero line)

$= 75 - 0.01 = 74.99$ mm

and lower limit for shaft = upper limit for shaft − Tolerance for shaft

= 74.99 − 0.03 = 74.96 mm

we know that maximum clearance

= upper limit for hole − Lower limit for shaft

= 75.046 − 74.96 = 0.086 mm

and minimum clearance

= Lower limit for hole − upper limit for shaft

= 75 − 74.99

= 0.01 mm

Stress:

When some external system of forces or loads acts on a body, the internal forces (equal and opposite) are set up at various sections of the body, which resist the external forces. This internal force per unit area at any section of the body is known as unit stress or simply a stress. It is denoted by a Greek letter sigma (σ). Mathematically,

Stress, $\sigma = P/A$

where P = Force or load acting on a body, and

A = Cross-sectional area of the body.

In S.I units, the stress is usually expressed in pascal (Pa) such that 1 Pa = 1 N/m². In actual practice, we use bigger units of stress i.e megapascal (MPa) and gigapascal (GPa), Such that

$$1 \text{ MPa} = 1 \times 10^6 \text{ N/m}^2 = 1 \text{ N/mm}^2$$

and $$1 \text{ GPa} = 1 \times 10^9 \text{ N/m}^2 = 1 \text{ kN/mm}^2$$

Strain:

When a system of forces or loads act on a body, it undergoes some deformation. This deformation per unit length is known as unit strain or simply a strain. It is denoted by a Greek letter epsilion (ε). Mathematically,

$$\text{strain}, \ \varepsilon = \delta l / l \quad \text{or} \quad \boxed{\delta l = \varepsilon \cdot l}$$

where δl = change in length of the body, and

l = original length of the body.

Tensile stress and strain:

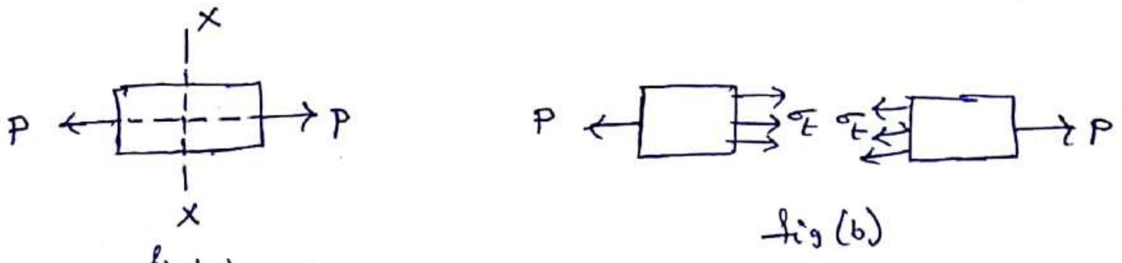


fig - Tensile stress and strain.

When a body is subjected to two equal and opposite axial pulls P (also called tensile load) as shown above fig(a), then the stress induced at any section of the body is known as tensile stress as shown in fig(b). A little consideration will show that due to the tensile load, there will be a decrease in cross-sectional area and an increase in length of the body. The ratio of the increase in length to the original length is known as tensile strain.

Let P = Axial tensile force acting on the body,

A = Cross-sectional area of the body,

l = original length, and

δl = Increase in length

Then Tensile stress, $\boxed{\sigma_t = P/A}$

and tensile strain $\boxed{\varepsilon_t = \dfrac{\delta l}{l}}$

Young's Modulus or Modulus of Elasticity

Hook's Law* states that when a material is loaded within elastic limit, the stress is directly proportional to strain

i.e. $\sigma \propto E$ (or) $\boxed{\sigma = E\varepsilon}$

$$E = \frac{\sigma}{\varepsilon} = \frac{P \times l}{A \times \delta l}$$

where E is a constant of proportionality known as Young's modulus or modulus of elasticity. In S.I Units, it is usually expressed in GPa i.e. GN/m^2 or KN/mm^2.

It may be noted that Hook's law holds good for tension as well as compression. The following values of (E) commonly used

For steel and Nickel	–	200 to 220 KN/mm^2
wrought iron	–	190 to 200 KN/mm^2
Cast-iron	–	100 to 160 KN/mm^2
copper	–	90 to 110 KN/mm^2

Brass — 80 to 90 KN/mm²

Aluminium — 60 to 80 KN/mm²

Timber — 10 KN/mm².

Shear stress and strain:

When a body is subjected to two equal and opposite forces acting tangentially across the resisting section, as a result of which the body tends to shear off the section, then the stress induced is called shear stress.

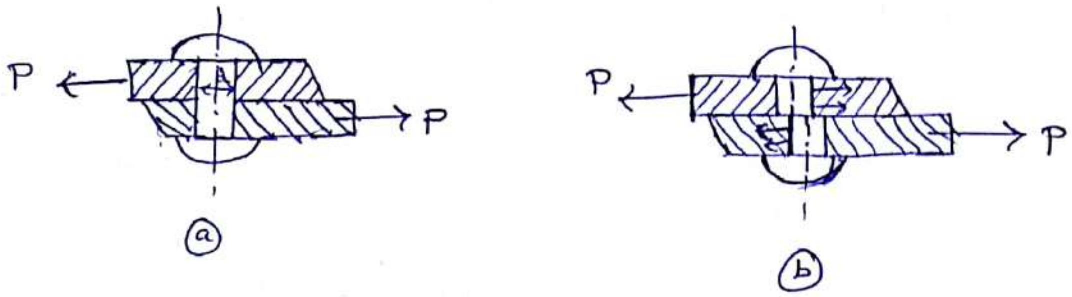


fig: single shearing of a riveted Joint.

The corresponding strain is known as shear strain and it is measured by the angular deformation accompanying the shear stress. The shear stress and shear strain are denoted by the Greek letters tau and phi (ψ) respectively,

Mathematically,

$$\text{Shear stress } \tau = \frac{\text{Tangential force}}{\text{Resisting area}}$$

Consider a body consisting of two plates connected by a rivet as shown above fig (a). In this case, the tangential force P tends to shear off the rivet at one cross-section as shown fig (b). It may be noted that when the tangential force is resisted by one cross-section of the rivet (or when shearing takes place at one cross-section of the rivet), then the rivets are said to be in single shear. In such a case, the area resisting the shear off the rivet,

$$A = \frac{\pi}{4} d^2$$

And shear stress on the rivet cross-section

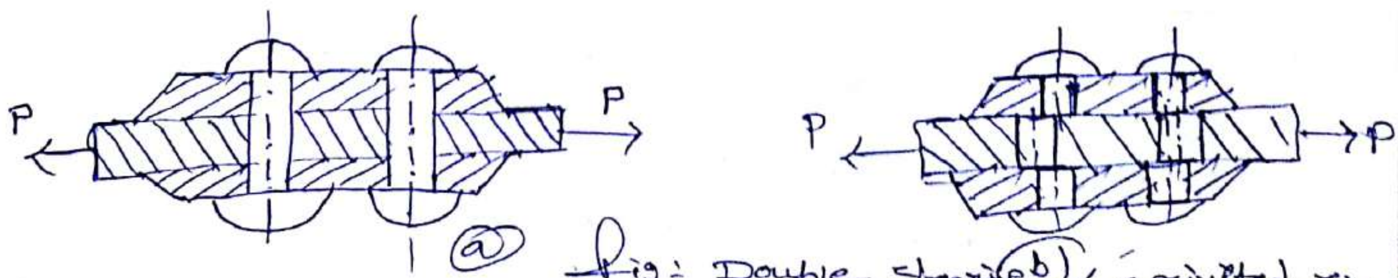
$$\tau = \frac{P}{A} = \frac{P}{\frac{\pi}{4} \times d^2} = \frac{4P}{\pi d^2}$$

Now let us consider two plates connected by the two cover plates as shown in fig (a). In this case, the tangential force P tends to shear off the rivet at two cross-sections as shown in fig (b). It may be noted that when the tangential force is resisted by two cross-sections of the rivet (or when the shearing takes place at two-cross-sections of the rivet), then the rivets are said to be in double shear. In such a case, the area resisting the shear off the rivet,

$$A = 2 \times \frac{\pi}{4} d^2 \quad \text{(for double shear)}$$

and shear stress on the rivet cross-section.

$$\tau = \frac{P}{A} = \frac{2P}{\pi d^2}$$



(a) Fig: Double Shearing (b) of a rivetted Joint.

Notes

1. All lap Joints and single cover butt Joints are in single shear while the butt Joints with double cover plates are in double shear.

2. In case of shear, the area involved is parallel to the external force applied.

3. When the holes are to be punched or drilled in the metal plates, then the tools used to perform the operations must overcome the ultimate shearing resistance of the material to be cut. If a hole of diameter 'd' is to be punched in a metal plate of thickness 't' then the area to be sheared,

$$\boxed{A = \pi d \times t}$$

And the maximum shear resistance of the tool or the force required to punch a hole,

$$P = A \times \tau_u = \pi d \times t \times \tau_u$$

where τ_u = ultimate shear strength of the material of the plate

Shear Modulus or modulus of Rigidity:

It has been found experimentally that within the elastic limit, the shear stress is directly proportional to shear strain. Mathematically,

$$\tau \propto \phi \quad \text{or} \quad \tau = C\phi \quad \text{(or)} \quad \frac{\tau}{\phi} = C$$

where τ = shear stress

ϕ = shear strain

C = Constant of proportionality. Known as shear modulus or modulus of rigidity. It is also denoted by N or G

Values of C for the commonly used materials

Steel	–	80 to 100 KN/mm^2
Wrought iron	–	80 to 90 KN/mm^2
Cast iron	–	40 to 50 KN/mm^2
Copper	–	30 to 50 KN/mm^2
Brass	–	30 to 50 KN/mm^2
Timber	–	10 KN/mm^2

Linear and Lateral Strain:

Consider a circular bar of diameter d and length L, subjected to a tensile force P as shown in fig (a)

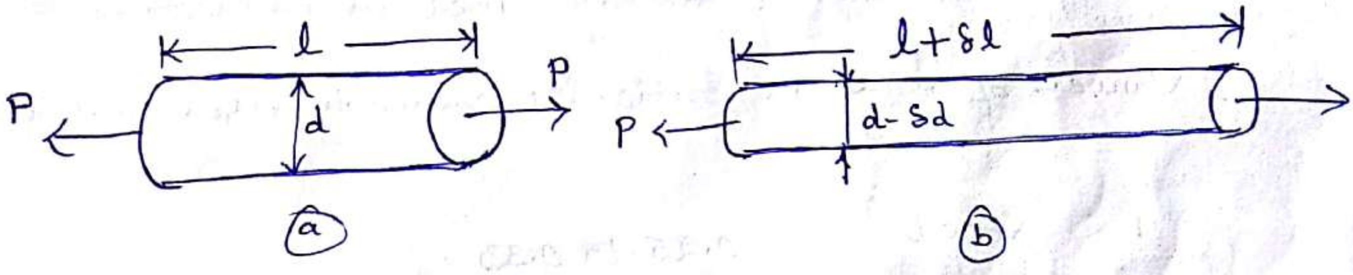


fig: Linear and lateral strain.

A little consideration will show that due to tensile force, the length of the bar increases by an amount δl and the diameter decreases by an amount δd, as shown in fig (b). Similarly if the bar is subjected to a compressive force the length of bar will decrease which will be followed by increase in diameter.

It is thus obvious, that every direct stress is accompained by a strain in its own direction which is known as Linear strain and an opposite kind of strain in every direction, at right angles to it, is known as lateral strain.

Poisson's Ratio :

It has been found experimentally that when a body is stressed within elastic limit, the lateral strain bears a constant ratio to the linear strain. Mathematically,

$$\frac{\text{Lateral strain}}{\text{Linear strain}} = \text{Constant.}$$

This constant is known as poisson's ratio and is denoted by $1/m$ or μ

Following are the values of poisson's ratio for some of the materials commonly used in engineering practice.

Values of poisson's ratio for commonly used materials

1.	Steel	0.25 to 0.33
2.	Cast iron	0.23 to 0.27
3.	Copper	0.31 to 0.34
4.	Brass	0.32 to 0.42
5.	Aluminium	0.32 to 0.36
6.	Concrete	0.08 to 0.18
7.	Rubber	0.45 to 0.50

Volumetric strain:-

When a body is subjected to a system of forces, it undergoes some changes in its dimensions. In other words, the volume of the body is changed. The ratio of the change in volume to the original volume is known as Volumetric strain.

Mathematically, Volumetric strain,

$$\boxed{\varepsilon_v = \delta v / v}$$

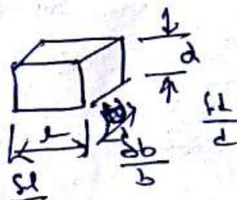
where δv = change in volume and $\underline{V}$ = original volume

Notes:

Total change in volume $= (L+\delta L)(d+\delta d)(b+\delta b)$

original $= L \cdot b \cdot d$

1. Volumetric strain of a rectangular body subjected to an axial force is given as



$$\varepsilon_v = \frac{\delta v}{V} = \varepsilon\left[1 - \frac{2}{m}\right], \text{ where } \frac{\delta l}{l}$$

ε = Linear strain.

Total change in ~~volume~~

2. Volumetric strain of a rectangular body subjected to three mutually perpendicular force is given by

$$\boxed{\varepsilon_v = \varepsilon_x + \varepsilon_y + \varepsilon_z}$$

where ε_x, ε_y and ε_z are the strains in the directions x-axis, y-axis and z-axis respectively.

Bulk Modulus:

when a body is subjected to three mutually perpendicular stresses, of equal intensity, then the ratio of the direct stress to the corresponding volumetric strain is known as bulk modulus. It is usually denoted by K. Mathematically, bulk modulus,

$$K = \frac{\text{Direct stress}}{\text{Volumetric strain}} = \frac{\sigma}{\delta V/V}$$

Relation Between Bulk Modulus and young's Modulus

The bulk modulus (K) and young's modulus (E) are related by the following relation,

$$K = \frac{m \cdot E}{3(m-2)} = \frac{E}{3(1-2\mu)}.$$

Relation between young's Modulus and Modulus of Rigidity

The young's modulus (E) and modulus of rigidity (G) are related by the following relation,

$$G = \frac{m \cdot E}{2(m+1)} = \frac{E}{2(1+\mu)}.$$

Factor of Safety:

It is defined, in general, as the ratio of the maximum stress to the working stress.

Mathematically,

$$F.S = \frac{\text{Maximum stress}}{\text{working or design stress}}$$

In case of ductile materials e.g. mild steel, where the yield point is clearly defined, the factor of safety is based upon the yield point stress. In such cases,

$$\text{Factor of safety} = \frac{\text{yield point stress}}{\text{working or design stress}}.$$

In case of brittle materials, e.g. cast iron, the yield point is not well defined as for ductile materials. Therefore, the factor of safety for brittle materials is based on ultimate stress.

$$\text{Factor of safety} = \frac{\text{ultimate stress}}{\text{working or design stress}}.$$

This relation may also be used for ductile materials.

The above relations for factor of safety are for static loading.

1. A steel bar 2.4 m long and 30 mm square is elongated by a load of 500 KN. if poisson's ratio is 0.25, find the increase in volume. Take $E = 0.2 \times 10^6$ N/mm^2.

sol. Given that $l = 2.4$ m $= 2400$ mm, $A = 30 \times 30 = 900$ mm^2

$P = 500$ KN $= 500 \times 10^3$ N, $\mu = 0.25$, $E = 0.2 \times 10^6$ N/mm^2.

Let $\delta V =$ Increase in volume.

we know that Volume of the rod,

$$V = \text{Area} \times \text{length} = 900 \times 2400$$
$$= 2160 \times 10^3 \text{ mm}^3.$$

and young's modulus.

$$E = \frac{stress}{Strain} = \frac{P/A}{\varepsilon}$$

$$\therefore \quad \varepsilon = \frac{P}{A \cdot E} = \frac{500 \times 10^{3}}{900 \times 0.2 \times 10^{6}} = 2.8 \times 10^{-3}$$

we know that volumetric strain,

$$\varepsilon_v = \frac{\delta V}{V} = \varepsilon\left(1 - \frac{2}{m}\right) = 2.8 \times 10^{-3}(1 - 2 \times 0.25)$$

$$= 1.4 \times 10^{-3}$$

$$\delta V = V \times 1.4 \times 10^{-3}$$

$$= 2160 \times 10^{3} \times 1.4 \times 10^{-3}$$

$$\boxed{\delta V = 3024 \text{ mm}^3}$$

Design strength and rigidity

~~At the~~

Design ~~for strength~~ Rigidity or stiffness.

It is the ability to resist deformations under the action of external load. Along with strength, rigidity is also very important operating property of many machine components.

Ex: Helical and leaf springs, elastic elements in various instruments. Shafts, bearings, toothed and worm gears and so on.

In many cases, this parameter of operating capacity proves to be most important and to ensure that the dimensions of the part have to be increased to such an extent that the actual induced stresses become much

Lower that the allowable ones. Rigidity also necessary to ensure that the mated parts and the machine as a whole operate effectively.

Forces subject the parts to elastic deformations:

Shafts are bent and twisted, bolts are streched etc..

1. when a shaft is deflected, its Journals are misaligned in the bearings there by causing the Uneven wear of the shells, heating and seizure in the sliding bearings.

2. Deflections and angles of turn of shafts at the plates where gears are fitted cause non-uniform load distribution over the length of the teeth.

3. With the deflection of an insufficiently rigid shaft, the operating conditions of antifriction bearings sharply deteriorate if the bearings cannot self-aligning.

4. Rigidity is particularly important for ensuring the adequate accuracy of items produced on machine tools.

Rigidity of machine elements is found with the help of formulae from the theory of strength of materials. The actual displacements like deflections, angles of turn, angles of twist should not be more that the allowable values. The most important design methods for increasing the rigidity of machine elements are as follows:

(a) The decrease in the arms of bending and twisting forces.

(b) The incorporation of additional supports.

(c) The application of cross sections which effectively resist torsion (closed tubular) and bending (in which the cross-section is removed as far as possible from the neutral axis).

(d) The decrease of the length of the parts in tension and the increase of their cross- section area.

from the above it's clear that the stiffness of a member depends not only on the shape and size of its cross section but also on elastic modulus of the material used.

Preferred Numbers

When a machine is to be made in several sizes with different powers or capacities, it is necessary to decide what capacities will cover a certain range efficiently with minimum number of sizes. It has been shown by experience that a certain range can be covered efficiently when it follows a geometrical progression with a constant ratio. The preffered numbers are the conventionally rounded off values derived from geometric series including the integral powers of 10 and having as common ratio of the following factors.

$$\sqrt[5]{10},\ \sqrt[10]{10},\ \sqrt[20]{10},\ \sqrt[40]{10}$$

The ratios are approximately equal to 1.58, 1.26, 1.12 and 1.06. The series of preferred numbers are designated as R5, R10, R20, ~~R80~~ and R40 respectively. These four series are called basic series. The other series called derived series may be obtained by simply multiplying or dividing the basic sizes by 10, 100 etc. The preferred numbers in the series R5 are 1, 1·6, 2·5, 4·0 and 6·3

Stresses due to change in Temperature - Thermal stresses

Whenever there is some increase or decrease in the temperature of a body, it causes the body to expand or contract. A little consideration will show that if the body is allowed to expand or contract freely, with the rise or fall of the temperature, no stresses are induced in the body. such stresses are known as thermal stresses.

Let l = original length of the body

t = Rise or fall of temperature, and

α = Coefficient of thermal expansion,

δl = Increase or decrease in length,

$$\boxed{\delta l = l \cdot \alpha \cdot t}$$

If the ends of the body are fixed to rigid supports, so that its expansion is prevented, then compressive strain induced in the body,

$$\boxed{\varepsilon_c = \frac{\delta l}{l} = \frac{l \cdot \alpha \cdot t}{l} = \alpha \cdot t}$$

Thermal stress,

$$\boxed{\sigma_{th} = \varepsilon_c \cdot E = \alpha \cdot t \cdot E}$$

1. When a body is composed of two or different materials having different coefficient of thermal expansion, then

due to the rise in temperature, the material with higher Coefficient of thermal expansion will be subjected to compressive stress whereas the material with low coefficient of expansion will be subjected to tensile stress.

2. when a thin tyre is shrunk on to a wheel of diameter D, its internal diameter d is a little less than the wheel diameter. When the tyre is heated, its circumference πd will increase to πD. In this condition, it is slipped on to the wheel. when it cools, it wants to return to its original circumference πd, but the wheel if it is assumed to be rigid, prevents it from doing so.

$$\text{Strain, } (\varepsilon) = \frac{\pi D - \pi d}{\pi d} = \frac{D-d}{d}$$

This strain is known as circumferential or hoop strain.

∴ Circumferential or hoop stress,

$$\sigma = E \cdot \varepsilon = \frac{E(D-d)}{d}.$$

Problem

① A composite bar made of aluminium and steel is held b/w the supports as shown in fig. The bars are stress free at a temperature of 37°c. what will be the stress in the two bars when the temperature is 20°c, if (a) the supports are unyielding, and (b) the supports yield and come nearer to each other by 0.10 mm?

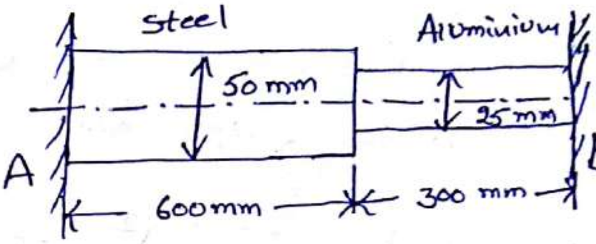
It can be assumed that the change of temperature is uniform all along the length of the bar.

Take $E_s = 210$ GPa, $E_a = 74$ GPa, $\alpha_s = 11.7 \times 10^{-6} /°c$ and

$\alpha_a = 23.4 \times 10^{-6} /°c$.

Sol

Given that:



$t_1 = 37°c$, $t_2 = 20°c$

$E_s = 210$ GPa $= 210 \times 10^9$ N/m^2, $E_a = 74$ GPa $= 74 \times 10^9$ N/m^2

$\alpha_s = 11.7 \times 10^{-6} /°c$, $\alpha_a = 23.4 \times 10^{-6} /°c$, $d_s = 50$ mm $= 0.05$ m,

$d_a = 25$ mm $= 0.025$ m, $l_s = 600$ mm $= 0.6$ m, $l_a = 300$ mm $= 0.3$ m.

Let us assume that the right support at B is removed and the bar is allowed to contract freely due the fall in temperature. we know that the fall in temperature,

$$t = t_1 - t_2 = 37 - 20 = 17°c$$

$\therefore$ Contraction in steel bar $= \alpha_s \cdot l_s \cdot t$

$$= 11.7 \times 10^{-6} \times 600 \times 17 = 0.12 \text{ mm}.$$

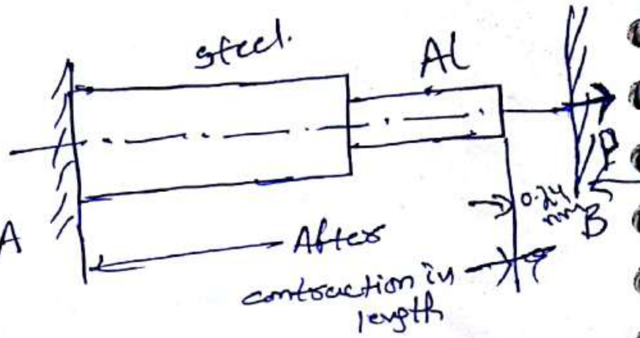
and contraction in aluminium bar

$$= \alpha_a \cdot l_a \cdot t = 23.4 \times 10^{-6} \times 300 \times 17 = 0.12 \text{ mm}.$$

Total Contraction $= 0.12 + 0.12 = 0.24$ mm $= 0.24 \times 10^{-3}$ m

It may be noted that even after this contraction (i.e 0.24 mm) in length, the bar is still stress free as the right hand end was assumed free.

Let an axial force **P** is applied to the right end till this end is brought with the right hand support at B, as shown in fig.



we know that cross-sectional area of the steel bar,

$$A_s = \frac{\pi}{4}(d_s)^2 = \frac{\pi}{4}(0.05)^2$$

$$= 1.964 \times 10^{-3}\ m^2$$

and cross-sectional area of the aluminium bar,

$$A_a = \frac{\pi}{4}(d_a)^2 = \frac{\pi}{4}(0.025)^2$$

$$= 0.491 \times 10^{-3}\ m^2$$

we know that elongation of the steel bar,

$$\delta l_s = \frac{P l_s}{A_s \times E_s} = \frac{P \times 0.6}{1.964 \times 10^{-3} \times 210 \times 10^9}$$

$$= \frac{0.6P}{412.44 \times 10^6}\ m = \boxed{\delta l_s = 1.455 \times 10^{-9}\ P}\ m$$

and elongation of the aluminium bar,

$$\delta l_a = \frac{P l_a}{A_a \times E_a} = \frac{P \times 0.3}{0.491 \times 10^{-3} \times 74 \times 10^9}$$

$$= \frac{0.3P}{36.334 \times 10^6}\ m$$

$$\boxed{\delta l_a = 8.257 \times 10^{-9}\ P}\ m$$

$\therefore$ Total elongation $\delta l = \delta l_s + \delta l_a$

$$= (1.455 \times 10^{-9})P + (8.257 \times 10^{-9})P$$

$$\boxed{\delta l = 9.712 \times 10^{-9} P} \text{ m}$$

Let σ_s = stress in the steel bar, and

σ_a = stress in the aluminium bar.

(a) When the supports are unyielding

when the supports are unyielding, the total contraction is equated to the total elongation. i.e

$$0.24 \times 10^{-3} = 9.712 \times 10^{-9} P$$

$$\boxed{P = 24712} \text{ N}$$

(b) when the supports yield by 0.1 mm

when the supports yield and come nearer to each other by 0.10 mm, the net contraction in length

$$= 0.24 - 0.1 = 0.14 \text{ mm} = 0.14 \times 10^{-3} \text{ m},$$

(2) A copper bar 50mm in diameter is placed within a steel tube 75 mm external diameter and 50mm internal diameter of exactly the same length. The two pieces are rigidly fixed together by two pins 18mm in diameter, one at each end passing through the bar and tube. Calculate stress

induced in the copper bar, steel tube and pins if the temperature of the combination is raised by 50°c. Take $E_s = 210\ GN/m^2$, $E_c = 105\ GN/m^2$, $\alpha_s = 11.5 \times 10^{-6}/°c$ and $\alpha_c = 17 \times 10^{-6}/°c$.

Sol Given that:

$d_c = 50$ mm, $d_{se} = 75$ mm, $d_{si} = 50$ mm, $d_{p_2} = 18$ mm $= 0.018$ m.

$t = 50°c$, $E_s = 210\ GN/m^2 = 210 \times 10^9\ N/m^2$,

$E_c = 105\ GN/m^2 = 105 \times 10^9\ N/m^2$

$\alpha_s = 11.5 \times 10^{-6}/°c$, $\alpha_c = 17 \times 10^{-6}/°c$

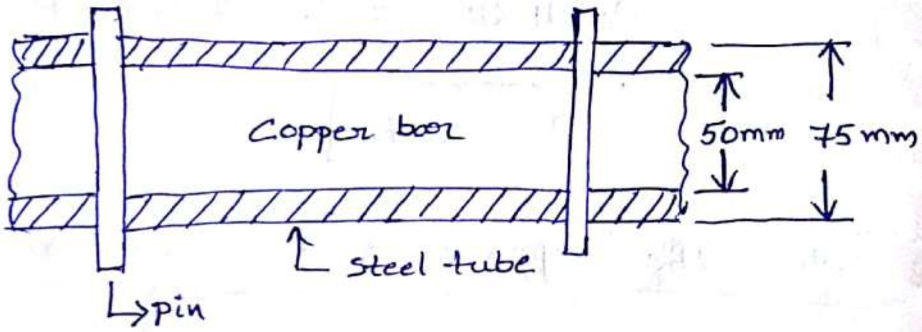


fig: copper bar in a steel tube

we know that cross-sectional area of the copper bar,

$$A_c = \frac{\pi}{4}(d_c)^2 = \frac{\pi}{4}(50)^2 = 1964\ mm^2 = 1964 \times 10^{-6}\ m^2$$

and cross-sectional area of the steel tube

$$A_s = \frac{\pi}{4}\left[(d_{se})^2 - (d_{si})^2\right]$$

$$= \frac{\pi}{4}\left[(75)^2 - (50)^2\right] = 2455\ mm^2$$

$$\boxed{A_s = 2455 \times 10^{-6}\ m^2}$$

Let l = Length of the copper bar and steel tube.

We know that free expansion of copper bar,

$$= \alpha_c \cdot l \cdot t = 17 \times 10^{-6} \times l \times 50$$

$$= \underline{850 \times 10^{-6}\, l}$$

and free expansion of steel tube

$$= \alpha_s \cdot l \cdot t = 11.5 \times 10^{-6} \times l \times 50$$

$$= 575 \times 10^{-6}\, l$$

$\therefore$ Difference in free expansion =

$$= 850 \times 10^{-6}\, l - 575 \times 10^{-6}\, l$$

$$= \underline{275 \times 10^{-6}\, l}$$

$\because$ The free expansion of the copper bar is more than the free expansion of the steel tube, therefore the copper bar is subjected to a compressive stress, while the steel tube is subjected to a tensile stress. Let a compressive force $\underline{P}$ newton on the copper bar opposes the extra expansion of the copper bar and an equal tensile force $\underline{P}$ on the steel tube pulls the steel tube so that the net effect of reduction in length of copper bar and the ~~the~~ increase in length of steel tube equalizes the difference in free expansion of the two.

$\therefore$ Reduction in length of copper bar due to force $\underline{P}$

$$\delta l = \frac{P \cdot l}{A_c \cdot E_c}$$

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$$= \frac{P.l}{1964\times10^{-6}\times105\times10^{9}} = \frac{P.l}{206.22\times10^{6}}\ m.$$

and increase in length of steel bar due to force P

$$= \frac{P.l}{A_S.E_S} = \frac{P.l}{2455\times10^{-6}\times210\times10^{9}} = \frac{P.l}{515.55\times10^{6}}\ m$$

$\therefore$ Net effect in length $= \frac{P.l}{206.22\times10^{6}} + \frac{P.l}{515.55\times10^{6}}$

$$= 4.85\times10^{-9}\ P.l + 1.94\times10^{-9}\ P.l$$

$$= 6.79\times10^{-9}\ P.l$$

Equating this net effect in length to the difference in free expansion, we have

$$6.79\times10^{-9}\ P.l = 275\times10^{-6}\ l$$

$$\boxed{P = 40500\ N}$$

Stress induced in the copper bar, steel tube and pins

we know that stress induced in the copper bar,

$$\sigma_c = \frac{P}{A_c} = 40500/(1964\times10^{-6}) = \underline{20.62\ MPa}$$

stress induced in the steel tube,

$$\sigma_s = P/A_s = 40500/(2455\times10^{-6}) = \underline{16.5\ MPa}$$

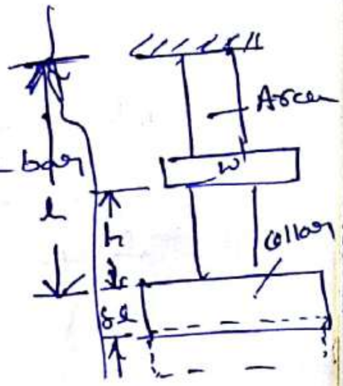
and shear stress induced in the pins,

$$\tau_p = \frac{P}{2A_p} = \frac{40500}{2\times\frac{\pi}{4}(0.018)^2} = 79.57\ MPa.$$

($\because$ The pin is double shear)

Impact stress:

Somtimes, machine mebers are subjected to the load with impact. (with some velocity). The stress produced in the member due to the falling load is know as impact stress. Consider a bar carrying a load W at a height h and falling on the collar provided at the lower end show in fig



Let A = Cross-sectional area of the bar

E = Young's modulus of the material of the bar

l = Length of the bar

δl = Deformation of the bar

P = Force at which the deflection δl is produced,

σ_i = stress induced in the bar due to the application of impact load, and

h = Height through which the load falls

we know that energy gained by the system in the form strain energy $= \frac{1}{2} \times P \times \delta l$

And potential energy lost by the weight $= W(h + \delta l)$

since the energy gained by the system is equal to the potential energy lost by the weight, therefore

$$\frac{1}{2} \times P \times \delta l = W(h + \delta l)$$

$$\frac{1}{2} \times \sigma_i \times A \times \frac{\sigma_i \times l}{E} = W\left(h + \frac{\sigma_i \times l}{E}\right)$$

$$\left[\because P = \sigma_i \times A, \quad \delta l = \frac{\sigma_i \times l}{E}\right]$$

$$\therefore \frac{Al}{2E}(\sigma_i)^2 - \frac{Wl}{E}(\sigma_i) - Wh = 0$$

from this quadratic eqn, we find that

$$\sigma_i = \frac{W}{A}\left[1 + \sqrt{1 + \frac{2hAE}{Wl}}\right] \quad \cdots \text{(Taking +ve sign for maximum Value)}$$

When $h=0$, then $\sigma_i = \frac{2W}{A}$. This means that the stress in the bar when the load in applied suddenly is double of the stress induced due to gradually applied load.

(1) An Unknown weight falls through 10 mm on a collar rigidly attached to the lower end of a Vertical bar 3 m long and 600 mm² in section. If the maximum instantaneous extension is known to be 2 mm, what is the corresponding stress and the value of unknown weight? Take $E = 200\ KN/mm^2$.

Sol. Given that:

$h = 10$ mm, $l = 3m = 3000$ mm, $A = 600\ mm^2$,

$\delta l = 2$ mm, $E = 200\ KN/mm^2 = 200 \times 10^3\ N/mm^2$

Stress in the bar

Let σ = stress in the bar

we know that young's modulus,

$$E = \frac{\text{Stress}}{\text{Strain}} = \frac{\sigma}{\varepsilon} = \frac{\sigma \cdot l}{\delta l}$$

$$\therefore \quad \sigma = \frac{E \cdot \delta l}{l} = \frac{200 \times 10^3 \times 2}{3000} = \frac{400}{3} = 133.3 \text{ N/mm}^2.$$

Value of the Unknown weight

Let W = value of the unknown weight.

we know that $\sigma = \frac{W}{A}\left[1 + \sqrt{1 + \frac{2hAE}{Wl}}\right]$

$$\frac{400}{3} = \frac{W}{600}\left[1 + \sqrt{1 + \frac{2 \times 10 \times 600 \times 200 \times 10^3}{W \times 3000}}\right]$$

$$\frac{400 \times 600}{3W} = \left[1 + \sqrt{1 + \frac{800\,000}{W}}\right]$$

$$\frac{80000}{W} - 1 = \sqrt{1 + \frac{800000}{W}}$$

Squaring both sides,

$$\frac{6400 \times 10^6}{W^2} + 1 - \frac{160000}{W} = 1 + \frac{800000}{W}$$

$$W = \frac{6400 \times 10^2}{96}$$

$$\boxed{W = 6666.7 \text{ N}}$$

Resilience:

When a body is loaded within the elastic limit, it changes its dimensions and on the removal of the load, it regains its original dimensions. So long as it remains loaded, it has stored energy in itself. On ~~removing~~ removing the load, the energy stored is given off as in the case of a spring. This energy, which is absorbed in a body when strained within elastic limit, is known as strain energy. The strain energy is always capable of doing some work.

The strain energy stored in a body due to external loading, within elastic limit, is known as resilience and the maximum energy which can be stored in a body up to the elastic limit is called proof resilience. The proof resilience per unit volume of a material is known as modulus of resilience. It is an important property of a material and gives capacity of the material to bear impact or shocks. Mathematically, strain energy ~~stor~~ stored in a body due to tensile or compressive load or resilience.

$$U = \frac{\sigma^2 \times V}{2E}$$

and modulus of resilience

$$= \frac{\sigma^2}{2E}$$

where σ = Tensile or compressive stress

V = volume of the body, and

E = young's modulus of the material of the body when a body is subjected to a shear load, then modulus of resilience (shear)

$$= \frac{\tau^2}{2C}$$

where τ = shear stress, and

C = Modulus of rigidity

when the body is subjected to torsion, the modulus of resilience

$$= \frac{\tau^2}{4C}$$

Problem

(1) A wrought iron bar 50 mm in diameter and 2.5 m long transmits shock energy of 100 N-m. Find the maximum instantaneous stress and the elongation. Take $E = 200\ GN/m^2$

Sol. Given that

$d = 50$ mm, $l = 2.5\ m = 2500$ mm, $U = 100$ N-m $= 100 \times 10^3$ N-mm

$E = 200\ GN/m^2 = 200 \times 10^3\ N/mm^2$

Maximum Instantaneous stress

Let σ = Maximum Instantaneous stress

we know that volume of the bar,

$$V = \frac{\pi}{4} \times d^2 \times l = \frac{\pi}{4}(50)^2 \times 2500$$

$$= 4.9 \times 10^6\ mm^3.$$

we also know that shock or strain energy stored in the body (U),

$$U = \frac{\sigma^2 \times V}{2E}$$

~~100~~

$$100 \times 10^3 = \frac{\sigma^2 \times 4.9 \times 10^6}{2 \times 200 \times 10^3} = 12.25\,\sigma^2$$

$$\sigma^2 = \frac{100 \times 10^3}{12.25}$$

$$\sigma^2 = 8163$$

(or)

$$\boxed{\sigma = 90.3\ N/mm^2}$$

Elongation produced

Let δl = Elongation produced,

we know that young's modulus,

$$E = \frac{\text{stress}}{\text{strain}} = \frac{\sigma}{\epsilon} = \frac{\sigma}{\frac{\delta l}{l}}$$

$$\delta l = \frac{\sigma \cdot l}{E} = \frac{90.3 \times 2500}{200 \times 10^3}$$

$$\boxed{\delta l = 1.13\ mm}$$

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Torsional shear stress:

when a machine member is subjected to the action of two equal and opposite couples acting in parallel planes (or torque or twisting moment), then the machine member is said to be subjected to torsion. The stress set up by torsion is known as torsional shear stress. It is zero at the centroidal axis and maximum at the outer surface consider a shaft fixed at one end and subjected to a torque (T) at the other end as shown below fig. As a result of this torque, every cross-section of the shaft is subjected to torsional shear stress. we have disscussed above that the torsional shear stress is zero at the centroidal axis and maximum at the outer surface. The maximum torsional shear stress at the outer surface of the shaft may be obtained from the following equation.

$$\frac{\tau}{r} = \frac{T}{J} = \frac{C \cdot \theta}{l} \longrightarrow (1)$$

where τ = Torsional shear stress induced at the outer surface of the shaft or maximum shear stress.

r = Radius of the shaft,

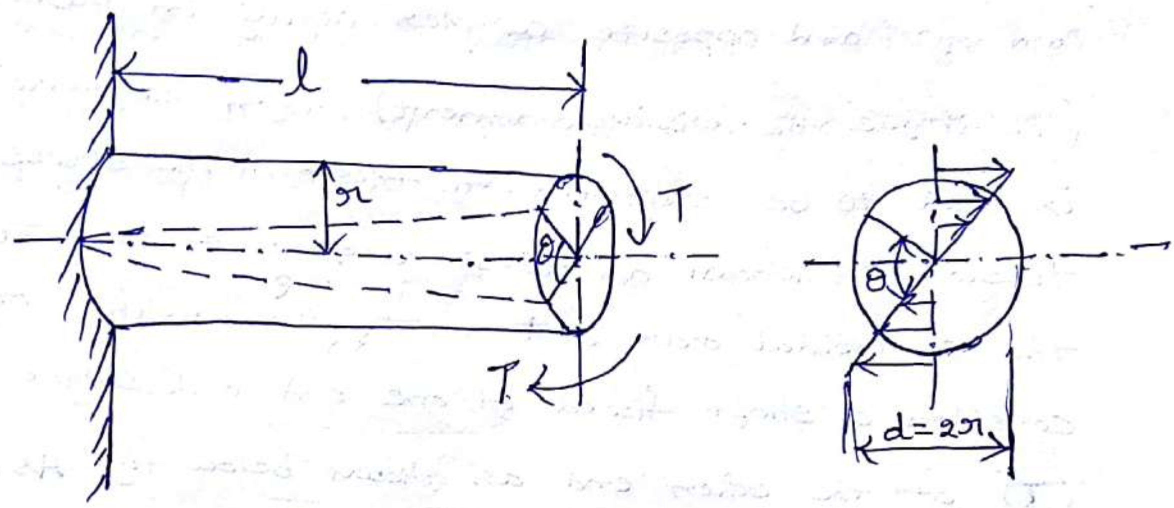
T = Torque or twisting moment,

J = Second moment of area of the section about its polar axis or polar moment of inertia,

C = Modulus of rigidity for the shaft material,

l = Length of the shaft, and

θ = Angle of twist in radians on a length l



The above equation is known as torsion equation. It is based on the following assumptions.

1. The material of the shaft is uniform throughout.

2. The twist along the length of the shaft uniform.

3. The normal cross-sections of the shaft, which were plane and circular before twist, remain plane and circular after twist.

4. All diameters of the normal cross-section which were straight before twist, remain straight with their magnitude unchanged, after twist.

5. The maximum shear stress induced in the shaft due to the twisting moment does not exceed its elastic limit value.

Note: 1. Since the torsional shear stress on any cross-section normal to the axis is directly proportional to the distance from the centre of the axis, therefore the torsional shear stress at a distance x from the centre of the shaft is given by

$$\boxed{\frac{\tau_x}{x} = \frac{\tau}{r}}$$

2) from eqn (1), we know that

$$\frac{T}{J} = \frac{\tau}{r} \quad \text{or} \quad \boxed{T = \tau \times \frac{J}{r}}$$

for a solid shaft of diameter (d), the polar moment of inertia,

$$J = I_{xx} + I_{yy} = \frac{\pi}{64} d^4 + \frac{\pi}{64} d^4 = \frac{\pi}{32} d^4$$

Therefore,

$$T = \tau \times \frac{\pi}{32} d^4 \times \frac{2}{d} = \frac{\pi}{16} \tau d^3$$

In case of a hollow shaft with external diameter (d_o) and internal diameter (d_i), the polar moment of inertia,

$$J = \frac{\pi}{32} \left[(d_o)^4 - (d_i)^4 \right] \quad \text{and} \quad r = \frac{d_o}{2}$$

$$T = \tau \times \frac{\pi}{32} \left[(d_o)^4 - (d_i)^4 \right] \times \frac{2}{d_o} = \frac{\pi}{16} \times \tau \left[\frac{(d_o)^4 - (d_i)^4}{d_o} \right]$$

$$= \frac{\pi}{16} \tau (d_o)^3 (1 - k^4) \quad \cdots \left(\text{substituting, } k = \frac{d_i}{d_o} \right)$$

3. The expression $(C \times J)$ is called torsional rigidity of the shaft.

4. The strength of the shaft means the maximum torque transmitted by it. Therefore, in order to design a shaft for strength, the above eqns are used. The power transmitted by the shaft (in watts) is given by

$$P = \frac{2\pi N T}{60} = T \cdot \omega$$

$$\boxed{P = T \cdot \omega} \qquad \left[\because \ \omega = \frac{2\pi N}{60}\right]$$

where T = Torque transmitted in N-m, and

ω = Angular speed in rad/s.

Problems

(1) A shaft is transmitting 100 KW at 160 r.p.m. Find a suitable diameter for the shaft, if the maximum torque transmitted exceeds the mean by 25%. Take maximum allowable shear stress as 70 MPa.

Sol Given that

$$P = 100 \text{ KW} = 100 \times 10^3 \text{ W}, \quad N = 160 \text{ r.p.m}$$

$$T_{max} = 1.25\, T_{mean}, \quad \tau = 70 \text{ MPa} = 70 \text{ N/mm}^2.$$

Let T_{mean} = Mean torque transmitted by the shaft in N-m, and

d = Diameter of the shaft in mm.

We know that the power transmitted (P),

$$100 \times 10^3 = \frac{2\pi N T_{mean}}{60} = \frac{\cancel{2\pi N}\ 2 \times \pi \times 160 \times T_{mean}}{60}$$

$$100 \times 10^3 = 16.76\ T_{mean}$$

$$\cancel{T_{mean} = 1.25 \times 5966.6}$$

$$\cancel{= 7458 \times 10^3\ N\text{-}mm}$$

$$T_{mean} = \frac{100 \times 10^3}{16.76} = 5966.6\ N\text{-}m.$$

$$\boxed{T_{mean} = 5966.6}\ N\text{-}m$$

and Maximum torque transmitted,

$$T_{max} = 1.25 \times 5966.6 = 7458\ N\text{-}m$$

$$\boxed{T_{max} = 7458 \times 10^3}\ N\text{-}mm$$

We know that maximum torque (T_{max}),

$$7458 \times 10^3 = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 70 \times d^3$$

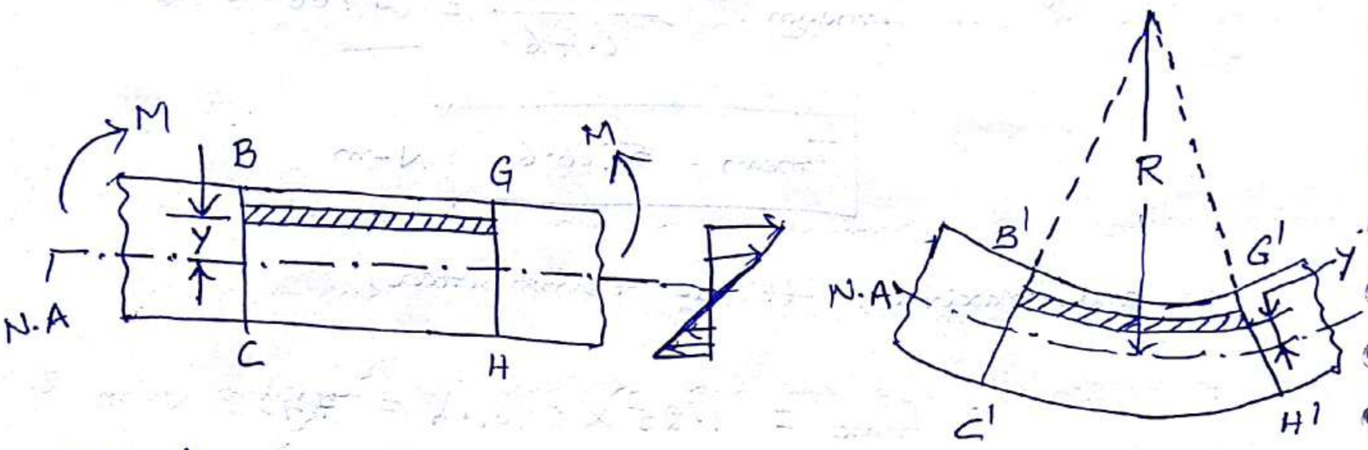
$$7458 \times 10^3 = 13.75\ d^3$$

$$d^3 = \frac{7458 \times 10^3}{13.75} = 542.4 \times 10^3$$

$$\boxed{d = 81.5}\ mm$$

Bending stress:

In Engineering Practice, the machine Parts of structural members may be subjected to static or dynamic loads which cause bending stress in the sections besides other types of Stresses such as tensile, compressive and shearing stresses. Consider a straight beam subjected to a bending moment M as shown in fig



The following assumptions are usually made while deriving the bending formula.

1. The material of the beam is ~~Perfec~~ Perfectly homogeneous (i.e. of the same material throughout) and isotropic (i.e. of equal elastic Properties in all directions).

2. The material of the beam obeys Hook's Law.

3. The transverse sections (i.e. BC or GH) which were plane before bending remain plane after bending also.

4. Each layer of the beam is free to expand or contact, independently, of the layer, above or below it

5. The Young's modulus (E) is the same in tension and Compression.

6. The loads are applied in the plane of bending.

A little consideration will show that when a beam is subjected to the bending moment, the fibres on the upper side of the beam will be shortened due to compression and those on the lower side will be elongated due to tension. It may be seen that somewhere between the top and bottom fibres there is a surface at which the fibres are neither shortened nor lengthened. Such a surface is called neutral surface. The intersection of the neutral surface with any normal cross-section of the beam is known as neutral axis. The stress distribution of a beam is shown above fig. The bending equation is given by

$$\boxed{\frac{M}{I} = \frac{\sigma}{y} = \frac{E}{R}}$$

where M = Bending moment acting at the given section,

σ = Bending stress,

I = Moment of inertia of the cross-section about the neutral axis.

y = Distance from the neutral axis to the extreme fibre,

E = Young's modulus of the material of the beam, and

R = Radius of curvature of the beam.

From the above eqn, the bending stress is given by

$$\boxed{\sigma = y \times \frac{E}{R}}$$

Since E and R are constant, therefore within elastic limit, the stress at any point is directly proportional to y. i.e the distance of the point from the neutral axis.

Also from the above eqn, the bending stress,

$$\sigma = \frac{M}{I} \times y = \frac{M}{I/y} = \frac{M}{Z}$$

The ratio I/y is known as section modulus and is denoted by Z.

Notes 1: The neutral axis of a section always passes through its centroid.

2. In case of symmetrical sections such as circular, square or rectangular, the neutral axis passes through its geometrical centre and the distance of extreme fibre from the neutral axis is $y = \frac{d}{2}$, where d is diameter in case of circular section or depth in case of square or rectangular section.

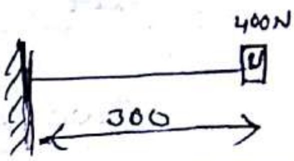
3. In case of unsymmetrical sections such as L-section or T-section, the neutral axis does not pass through its geometrical centre. In such cases, first of all the centroid of the section is calculated and then the distance of the extreme fibres for both lower and upper side of the section is obtained. Out of these two values, the bigger value is used in bending equation.

Problem

1. A beam of uniform rectangular cross-section is fixed at one end and carries an electric motor weighing 400 N at a distance of 300 mm from the fixed end. The maximum bending stress in the beam is 40 MPa. Find the width and depth of the beam, if depth is twice that of width.

Sol Given that: $W = 400\,N$, $L = 300\,mm$, $\sigma_b = 400\,MPa$
$= 400\,N/mm^2$

$h = 2b$



Let b = width of the beam in mm, and
h = Depth of the beam in mm.

$$\therefore\ Z = \frac{I}{y} = \frac{\frac{bh^3}{12}}{\frac{h}{2}} = \frac{bh^3}{12}\times\frac{2}{h} = \frac{bh^2}{6}$$

∴ Section modulus,

$$Z = \frac{b\cdot h^2}{6} = \frac{b(2b)^2}{6} = \frac{4b^3}{6} = \frac{2b^3}{3}\ mm^3$$

Maximum Bending moment (at the fixed end),

$$M = W\cdot L = 400\times 300 = 120000 = 120\times10^3\ N\text{-}mm$$

we know that bending stress (σ_b),

$$400 = \frac{M}{Z} = \frac{120\times10^3\times3}{2b^3} = \frac{180\times10^3}{b^3}$$

$$\frac{M}{I} = \frac{\sigma}{y} = \frac{E}{R}$$

$$\sigma = \frac{M}{I}\times y$$

$$b^3 = \frac{180\times10^3}{40} = 4.5\times10^3$$

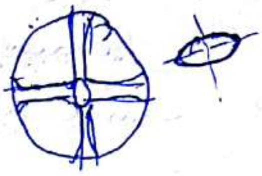
$$\sigma = \frac{M}{\frac{I}{y}}$$

$$\boxed{b = 16.5}\ mm$$

$$\boxed{\sigma = \frac{M}{Z}}$$

$$h = 2b = 2\times16.5 = 33\ mm.$$

(2) A cast-iron pulley transmits 10 kW at 400 r.p.m. The diameter of the pulley is 1.2 metre and it has four straight arms of elliptical cross-section, in which the major axis is twice the minor axis. Determine the dimensions of the arm if the allowable bending stress is 15 MPa.



Sol Given that $P = 10\,kW = 10 \times 10^3\,W$, $N = 400\,r.p.m$

$D = 1.2\,m = 1200\,mm$ or $R = 600\,mm$, $\sigma_b = 15\,MPa = 15\,N/mm^2$.

Let T = Torque transmitted by the pulley.

we know that the power transmitted by the pulley (P),

$$10 \times 10^3 = \frac{2\pi N T}{60} = \frac{2\pi \times 400 \times T}{60} = 42\,T$$

$$T = 10 \times 10^3 / 42 = 238\,N\text{-}m$$

$$\boxed{T = 238 \times 10^3\ N\text{-}mm}$$

Since the torque transmitted is the product of the tangential load and the radius of the pulley,

$\therefore$ Tangential load acting on the pulley $= \frac{T}{R}$

$$= \frac{238 \times 10^3}{600} = 396.7\,N$$

Since the pulley has four arms, therefore tangential load on each arm,

$$W = \frac{396.7}{4} = 99.2\,N.$$

and maximum bending moment on the arm,

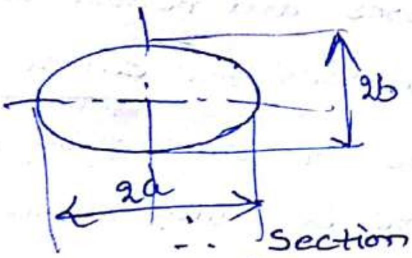
$$M = W \times R = 99.2 \times 600$$

$$\boxed{M = 59520} \text{ N-mm}$$

Let $2b$ = Minor axis in mm, and

$2a$ = Major axis in mm,

$= 2 \times 2b = \underline{4b}$. — (Given)



∴ Section modulus for an elliptical cross-section,

$$I_x = \frac{\pi}{4} ab^3 \qquad y = \frac{2a}{2} = a$$

$$Z = \frac{\pi}{4} \times a^2 b = \frac{\pi}{4}(2b)^2 \times b = \pi b^3 \text{ mm}^3$$

$$\frac{I}{y} = \frac{\pi}{4} a^3 b \qquad Z = \frac{I}{y} \qquad I_x = \frac{\pi}{4} ab^3$$

We know that bending stress (σ_b),

$$Z = \frac{I_y}{y} = \frac{\frac{\pi}{4} a^3 b}{a} = \frac{\pi}{4} a^2 b$$

$$15 = \frac{M}{Z} = \frac{59520}{\pi b^3} = \frac{18943}{b^3}$$

$$\boxed{b^3 = 1263}$$

or $\boxed{b = 10.8}$ mm.

∴ Minor axis $2b = 2 \times 10.8 = \underline{21.6 \text{ mm}}$

and Major axis $2a = 2 \times 2b = 2 \times 21.6 = \underline{43.2 \text{ mm}}$

Principal stresses and principal planes

When a system of forces act on a body, all the particles of that body are distributed and their dimensions and locations are varied due to straining action by the forces. At any strained particle, there are three planes mutually $\perp$er to each other which carry only normal stresses and no shear stresses. These planes which are having only normal stresses are called principal planes and these normal stresses are called principal stresses. Among these principal stresses one is having maximum value and other is having minimum value. There are some planes which are 45^0 to principal planes and carry only maximum shear stresses and hence known as shear stress.

Determination of principal stress for a Member subjected to Biaxial stress.

When a member is subjected to bi-axial stress (i.e direct stress in two mutually perpendicular planes accompained by a simple shear stress), then the normal and shear stresses are obtained as disscussed below.

Consider a rectangular body ABCD of uniform cross-sectional area and unit thickness subjected to normal stresses σ_1 and σ_2 as shown in fig(a). In addition to these normal stresses, a shear stress (τ) also acts. It has been shown in fig. The normal stress across any oblique section such as EF inclined at an angle θ with the direction of σ_2, as shown fig(a). is given by

$$\sigma_t = \frac{\sigma_1+\sigma_2}{2} + \frac{\sigma_1+\sigma_2}{2}\cos 2\theta + \tau \sin 2\theta \longrightarrow (i)$$

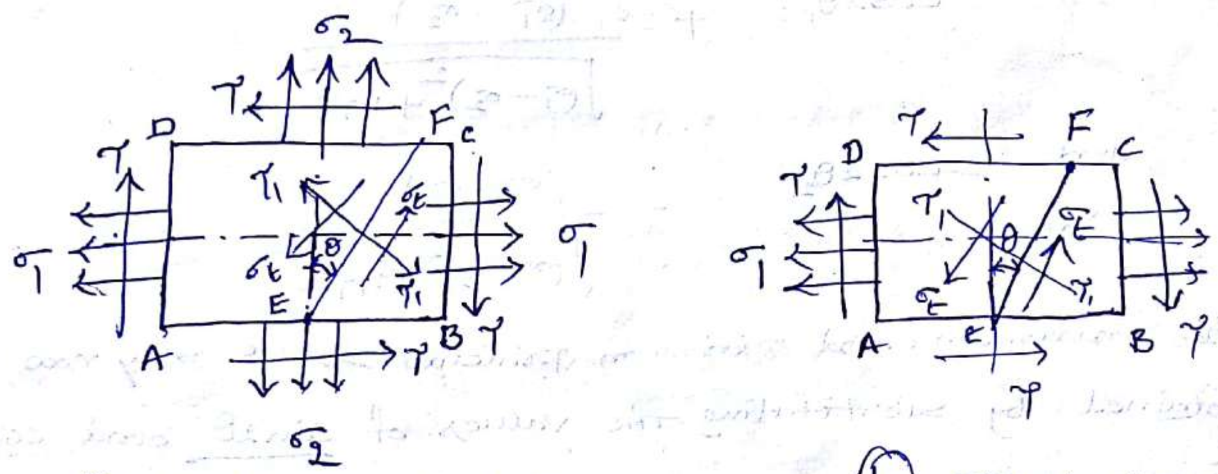
And tangential stress (i.e shear stress) across the section EF,

$$\tau_1 = \frac{1}{2}(\sigma_1 - \sigma_2)\sin 2\theta - \tau \cos 2\theta \longrightarrow (II)$$

since the planes of maximum and minimum normal stress (i.e. principal planes) have no shear stress, therefore the inclination of principal planes is obtained by equating $\tau_1 = 0$ in the above eqn (II) i.e

$$\frac{1}{2}(\sigma_1-\sigma_2)\sin 2\theta - \tau\cos 2\theta = 0.$$

$$\boxed{\tan 2\theta = \frac{2\tau}{\sigma_1-\sigma_2}} \longrightarrow (III)$$

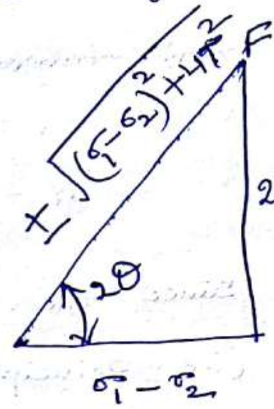


(a) Direct stress in two mutually $\perp^{er}$ planes accompained by a simple shear stress

(b) Direct stress in one plane accompained by a simple shear stress.

fig: Principal stresses for a member subjected to bi-axial stress.

we know that there are two principal planes at right angles to each other. Let θ_1 and θ_2 be inclinations of these planes with the normal cross-section. From the following fig. we find that



$$\sin 2\theta = \pm \frac{2\tau}{\sqrt{(\sigma_1-\sigma_2)^2+4\tau^2}}$$

$$\therefore \quad \sin 2\theta_1 = + \frac{2\tau}{\sqrt{(\sigma_1-\sigma_2)^2+4\tau^2}}$$

and $$\sin 2\theta_2 = - \frac{2\tau}{\sqrt{(\sigma_1-\sigma_2)^2+4\tau^2}}$$

Also $$\cos 2\theta = \pm \frac{(\sigma_1-\sigma_2)}{\sqrt{(\sigma_1-\sigma_2)^2+4\tau^2}}$$

$$\therefore \quad \cos 2\theta_1 = + \frac{(\sigma_1-\sigma_2)}{\sqrt{(\sigma_1-\sigma_2)^2+4\tau^2}}$$

and $$\cos 2\theta_2 = - \frac{(\sigma_1-\sigma_2)}{\sqrt{(\sigma_1-\sigma_2)^2+4\tau^2}}$$

The maximum and minimum principal stresses may now be obtained by substituting the values of $\sin 2\theta$ and $\cos 2\theta$ in eqn (I)

So, Maximum principal (or normal) stress,

$$\sigma_{t_1} = \frac{\sigma_1+\sigma_2}{2} + \frac{1}{2}\sqrt{(\sigma_1-\sigma_2)^2+4\tau^2} \longrightarrow \text{(IV)}$$

And minimum principal (or normal) stress,

$$\sigma_{t_2} = \frac{\sigma_1 + \sigma_2}{2} - \frac{1}{2}\sqrt{(\sigma_1-\sigma_2)^2 + 4\tau^2} \longrightarrow (V)$$

The planes of maximum shear stress are at right angles to each other and are inclined at $\underline{45^\circ}$ to the principal planes. The maximum shear stress is given by one-half the algebraic difference b/w the principal stresses, i.e.

$$\tau_{max} = \frac{\sigma_1 - \sigma_2}{2} = \frac{1}{2}\sqrt{(\sigma_1-\sigma_2)^2 + 4\tau^2} \longrightarrow (VI)$$

<u>Notes</u> 1. When a member is subjected to direct stress in one plane accompained by a simple shear stress, then the principal stresses are obtained by substituting $\underline{\sigma_2 = 0}$ in eqn (IV), (V) and (VI).

$$\sigma_{t_1} = \frac{\sigma_1}{2} + \frac{1}{2}\left[\sqrt{(\sigma_1)^2 + 4\tau^2}\right]$$

$$\sigma_{t_2} = \frac{\sigma_1}{2} - \frac{1}{2}\left[\sqrt{(\sigma_1)^2 + 4\tau^2}\right]$$

$$\tau_{max} = \frac{1}{2}\left[\sqrt{(\sigma_1)^2 + 4\tau^2}\right].$$

2. In the above expression of $\underline{\sigma_{t_2}}$, the value of $\frac{1}{2}\left[\sqrt{(\sigma_1)^2 + 4\tau^2}\right]$ is more than $\sigma_1/2$. Therefore the nature of $\underline{\sigma_{t_2}}$ will be opposite to that of $\underline{\sigma_{t_1}}$, i.e. if $\underline{\sigma_{t_1}}$ is tensile then $\underline{\sigma_{t_2}}$ will be compressive and vice-versa.

Applications of principal stresses in Designing Machine Members.

There are many cases in practice, in which machine members are subjected to combined stresses due to simultaneous action of either tensile or compressive stresses combined with shear stresses. In many shafts such as propeller shafts, C-frames etc., there are direct tensile or compressive stresses due to the external force and shear stress due to torsion, which acts normal to direct tensile or compressive stresses. The shafts like crank shafts, are subjected simultaneously to torsion and bending. In such cases, the maximum principal stresses, due to the combination of tensile or compressive stresses with shear stresses may be obtained. The results obtained in the previous article may be written as follows.

1. Maximum tensile stress,

$$\sigma_{t(max)} = \frac{\sigma_t}{2} + \frac{1}{2}\left[\sqrt{(\sigma_t)^2 + 4\tau^2}\right]$$

2. Maximum Compressive stress,

$$\sigma_{c(max)} = \frac{\sigma_c}{2} + \frac{1}{2}\left[\sqrt{(\sigma_c)^2 + 4\tau^2}\right]$$

3. Maximum shear stress,

$$\tau_{max} = \frac{1}{2}\left[\sqrt{(\sigma_t)^2 + 4\tau^2}\right]$$

where σ_t = Tensile stress due to direct load and bending.

σ_c = Compressive stress, and

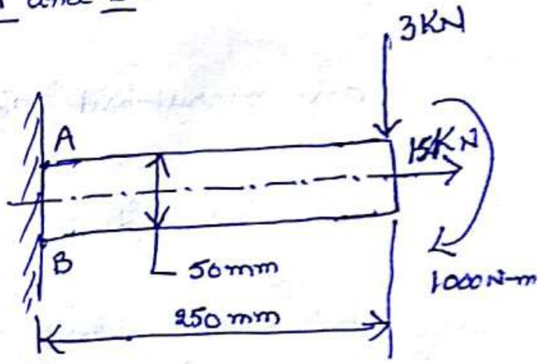
τ = Shear stress due to torsion.

Notes: 1. when $\tau = 0$ as in the case of thin cylindrical shell subjected in internal fluid pressure, then $\sigma_{max} = \sigma_t$.

2. when the shaft is subjected to an axial load (P) in addition to bending and twisting moments as in the propeller shafts of ship and shafts for driving worm gears, then the stress due to axial load must be added to the bending stress (σ_b). This will give the resultant tensile stress or compressive stress (σ_t or σ_c) depending upon the type of axial load (i.e pull or push).

Problem

① A shaft, as show in fig., is subjected to a bending load of 3KN, pure torque of 1000 N-m and an axial pulling force of 15KN. Calculate the stresses at A and B.



sol Given that

$W = 3\ KN = 3000\ N$,

$T = 1000\ N\text{-}m = 1 \times 10^6\ N\text{-}mm$

$P = 15\ KN = 15 \times 10^3\ N$

$d = 50\ mm$, $x = 250\ mm$

We know that cross-sectional area of the shaft,

$$A = \frac{\pi}{4} \times d^2 = \frac{\pi}{4}(50)^2 = 1964\ mm^2.$$

∴ Tensile stress due to axial pulling at point A and B,

$$\sigma_a = \frac{P}{A} = \frac{15 \times 10^3}{1964} = 7.64\ N/mm^2 = 7.64\ MPa.$$

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(4)

Bending moment at points A and B

$$M = w \cdot x = 3000 \times 250 = 750 \times 10^3 \text{ N-mm.}$$

Section modulus for the shaft,

$$Z = \frac{\pi}{32} d^3 = \frac{\pi}{32}(50)^3 = 12.27 \times 10^3 \text{ mm}^3$$

$$\boxed{Z = \frac{I}{y} = \frac{\pi d^4 / 64}{d/2} = \frac{\pi d^3}{32}}$$

∴ Bending stress at points A and B,

$$\sigma_b = \frac{M}{Z} = \frac{750 \times 10^3}{12.27 \times 10^3} = 61.1 \text{ N/mm}^2 = 61.1 \text{ MPa.}$$

The bending stress is tensile at point A and compressive at point B.

∴ Resultant tensile stress at point A,

$$\sigma_A = \sigma_b + \sigma_a = 61.1 + 7.64$$

$$= 68.74 \text{ MPa}$$

and resultant compressive stress at point B,

$$\sigma_B = \sigma_b - \sigma_a = 61.1 - 7.64 = 53.46 \text{ MPa}$$

We know that the shear stress at points A and B due to the torque transmitted,

$$\tau = \frac{16T}{\pi d^3} = \frac{16 \times 1 \times 10^6}{\pi (50)^3}$$

$$\left(\because T = \frac{\pi}{16} \tau d^3 \right.$$

$$= 40.74 \text{ N/mm}^2$$

$$\boxed{\tau = 40.74 \text{ MPa}}$$

$$\frac{\tau}{r} = \frac{T}{J}$$

$$\tau = \frac{T}{J} r$$

$$\tau = \frac{16T}{\pi d^3} \leftarrow = \frac{T \cdot 32}{\pi d^4} \cdot \frac{d}{2}$$

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Stresses at point A

We know that maximum principal (or normal) stress at point A,

$$\sigma_{A(max)} = \frac{\sigma_A}{2} + \frac{1}{2}\left[\sqrt{(\sigma_A)^2 + 4\tau^2}\right]$$

$$= \frac{68.74}{2} + \frac{1}{2}\left[\sqrt{(68.74)^2 + 4(40.74)^2}\right]$$

$$= 34.37 + 53.3$$

$$\boxed{\sigma_{A(max)} = 87.67 \text{ MPa.}}$$ (tensile) +ve value

Minimum principal (or normal) stress at point A,

$$(\sigma_A)_{min} = \frac{\sigma_A}{2} - \frac{1}{2}\left[\sqrt{(\sigma_A)^2 + 4\tau^2}\right]$$

$$= 34.37 - 53.3$$

$$\boxed{(\sigma_A)_{min} = -18.93 \text{ MPa}}$$ Compressive (-ve) value

and maximum shear stress at point A.

$$\tau_{A(max)} = \frac{1}{2}\left[\sqrt{(\sigma_A)^2 + 4\tau^2}\right]$$

$$= \frac{1}{2}\left[\sqrt{(68.74)^2 + 4(40.74)^2}\right]$$

$$\boxed{\tau_{A(max)} = 53.3 \text{ MPa.}}$$

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Stresses at point B

we know that maximum principal (or normal) stresses at point B,

$$\sigma_{B(max)} = \frac{\sigma_B}{2} + \frac{1}{2}\left[\sqrt{(\sigma_B)^2 + 4\tau^2}\right]$$

$$= \frac{53.46}{2} + \frac{1}{2}\left[\sqrt{(53.46)^2 + 4(40.74)^2}\right]$$

$$= 26.73 + 48.73 = 75.46 \text{ MPa (compressive)}$$

Minimum principal (or normal) stress at point B,

$$\sigma_{B(min)} = \frac{\sigma_B}{2} - \frac{1}{2}\left[\sqrt{(\sigma_B)^2 + 4\tau^2}\right]$$

$$= 26.73 - 48.73 = -22 \text{ MPa}$$

$$= 22 \text{ MPa (tensile)}$$

and maximum shear stress at point B,

$$\tau_{B(max)} = \frac{1}{2}\sqrt{(\sigma_B)^2 + 4\tau^2}$$

$$= \frac{1}{2}\sqrt{\cancel{\frac{1}{2}}\left[(53.46)^2 + 4(40.74)^2\right]}$$

$$\boxed{\tau_{B(max)} = 48.73} \text{ MPa.}$$

Stress Concentration:

Whenever a machine Component changes the shape of its cross-section, the simple stress distribution no longer holds good and the neighborhood of the discontinuity is different. This irregularity in the stress distribution caused by abrupt changes of form is called stress concentration. It occurs for all kinds of stresses in the presence of fillets, notches, holes, keyways, splines, surface roughness or scratches etc. In order to understand fully the idea of stress concentration, consider a member with different cross-section under a tensile load as shown in figure.

A little consideration will show that the nominal stress in the right and left hand sides will be uniform but in the region where the cross-section is changing, a redistribution of the force within the member must take place. The material near the edges is stressed considerably higher than the average value. The maximum stress occurs at some point on the fillet and directed parallel to the boundary at that point.

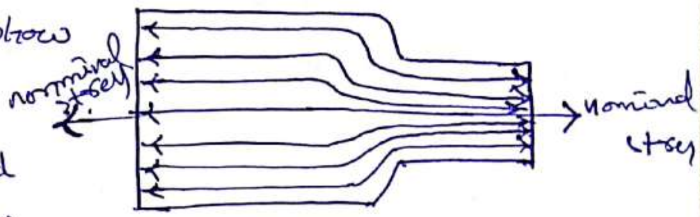


fig: stress concentration

Theoretical or Form stress concentration Factor

The theoretical or form stress concentration factor is defined as the ratio of the maximum stress in a member (at a notch or a fillet) to the nominal stress at the same section based upon net area. Mathematically theoretical or form stress concentration factor,

$$\boxed{K_t = \frac{\text{Maximum stress}}{\text{Nominal stress}}} = \frac{\sigma_{max}}{\sigma_0} = \frac{\tau_{max}}{\tau_0}$$

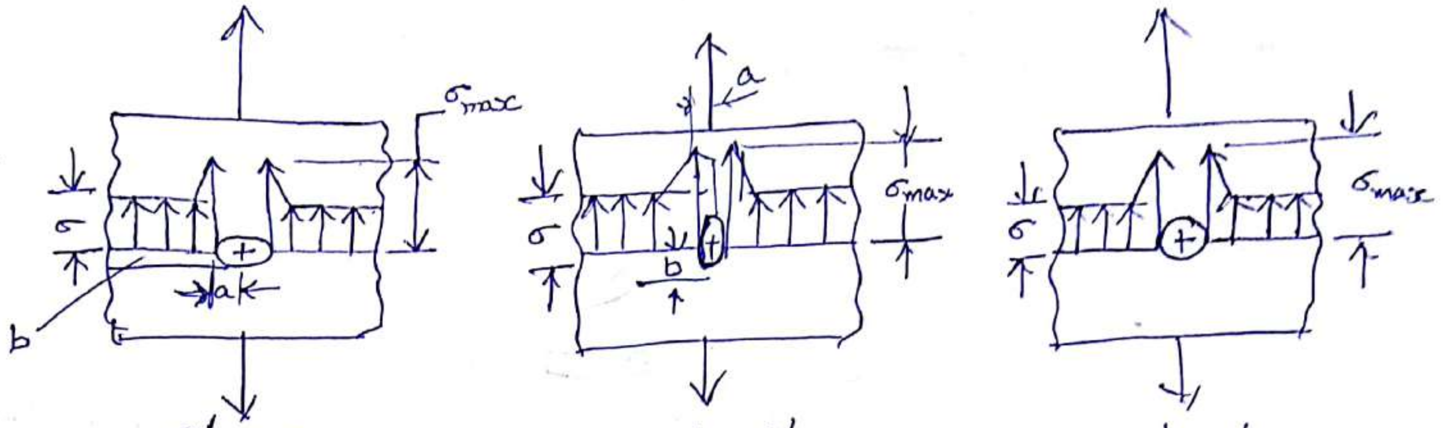
The value of K_t depends upon the material and geometry of the part. In static loading stress concentration in ductile materials is not so serious as in brittle materials, because in ductile materials local deformation or yielding takes place which reduces the concentration. In brittle materials cracks may appear at these local concentrations of stress which will increase the stress over the rest of the section. It is, therefore, necessary that in designing parts of brittle materials such as castings, care should be taken. In order to avoid failure due to stress concentration, fillets at the changes of section must be provided.

In cyclic loading, stress concentration in ductile materials is always serious because the ductility of the material is not effective in relieving the concentration of stress caused by cracks, flaws, surface roughness or any discontinuity in the geometrical form of the member. if the stress at any point in a member is above the endurance limit of the material, a crack may develop ~~ud~~ under the action of repeated load and the crack will lead to failure of the member.

In the above eqn σ_0 and τ_0 are stresses determined by elementary equation $\sigma_t = \frac{P}{A}$, $\sigma_b = \frac{M_b y}{I}$, $\tau = \frac{M_t R}{J}$ and τ_{max} & σ_{max} are localized stresses at the discontinuities

Stress concentration due to Holes and Notches

Consider a plate with transverse elliptical hole and subjected to an tensile load as shown in fig(a). we see from



$a/b = 2$

$\sigma_{max} = 5\sigma$

fig (a)

$a/b = 1/2$

$\sigma_{max} = 2\sigma$

fig (b)

$a/b = 1$

$\sigma_{max} = 3\sigma$

fig (c)

fig: stress concentration due to holes.

the stress-distribution that the stress at the point away from the hole is practically uniform and the maximum stress will be induced at the edge of the hole. The maximum stress is given by

$$\sigma_{max} = \sigma\left(1 + \frac{2a}{b}\right)$$

where a = half width of ellipse ⊥ler to the direction of load
b = half width of ellipse in the direction of load

And the theoretical stress concentration factor,

$$K_t = \frac{\sigma_{max}}{\sigma_o} = \left(1 + \frac{2a}{r}\right)$$

The stress concentration in the notched tension member, as shown below, is influenced by the depth of the notch and radius r at the bottom of the notch. The maximum stress, which applies to members having notches that are

Small in comparison with the width of the plate, may be obtained by the following equation,

$$\boxed{\sigma_{max} = \sigma\left(1 + \frac{2a}{r}\right)}$$

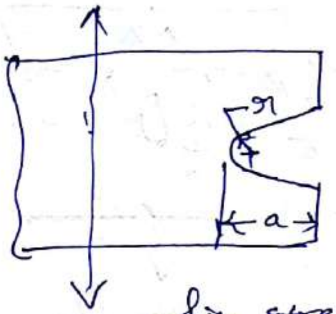
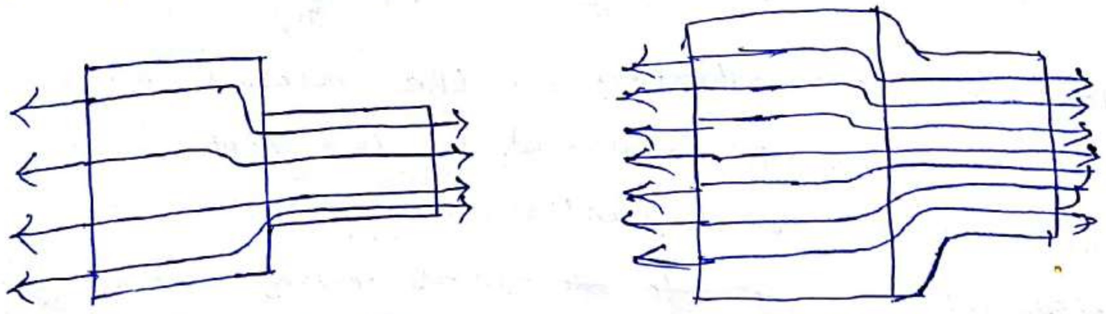


fig stress concentration due to notches

Methods of Reducting stress concentration

Whenever there is a change in cross-section, such as shoulders, holes, notches or keyways ~~and~~

Although, it is not possible to compleately eliminate the effect of stress concentration, there are methods to reduce stress concentration, ~~these are~~ This is achieved by providing specific geometric shape to the component.
In order to know what happens at the abrupt change of cross-section or at the discontinuity and reduce stress concentration, Understanding of flow analogy is useful.
There is a similarity between velocity distribution in fluid flow in a channel and the stress distribution in an axially loaded plate



Force flow Analogy
(a) Force flow around sharp corner

(b) Force flow around Rounded corner.

The equation of flow potential in fluid mechanics and stress potential in solid mechanics are same. Therefore, it is perfectly logical to use fluid analogy to understand the phenomena of stress concentration

When the cross-section of channel has uniform dimensions throughout, the velocities are uniform and the streamlines are equally spaced. The flow at any cross-section with in the channel is given by,

$$\boxed{\varphi = \int \sigma \, dA}$$

When the cross-section of plate has same dimensions throughout, the stresses are uniform and stress lines are equally spaced. The stress at any ~~d~~ section is given by,

$$P = \int \sigma \, dA.$$

When the cross-section of the channel is suddenly reduced, the velocity increases in order to maintain the same flow and the steamlines become narrower and narrower and crowd together. Similar phenomenon is observed in stressed plate. In order to transmit same force, the stress lines come closer and closer as the cross-section is reduced. At the change of cross-section, the stream lines as well as stress lines bend. When there is sudden change in cross-section, bending forces (stress) lines is very sharp and severe, resulting in stress-concentration. Therefore, stress concentration can be greatly reduced by reducing the bending by round the corners. Steam-lined shapes are used in channels to reduce turbulence and resistance to flow. Steamlining or rounding the counters of mechanical components has similar

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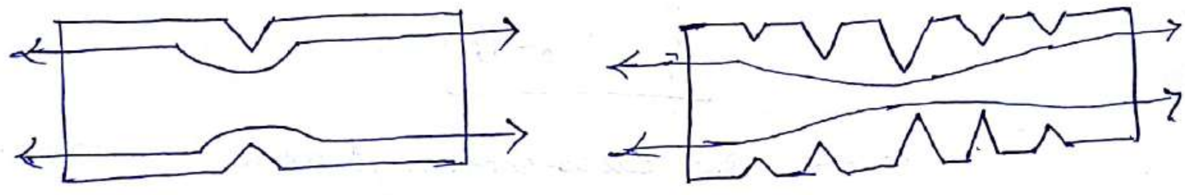
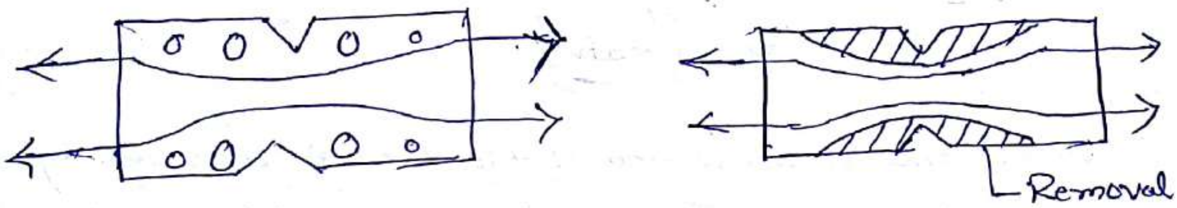
benafucial effects in reducing stress concentration. There are different methods to reduce the bending of the stress lines at the Jonction and reduce the stress concentration.

In practice reduction of stress concentration is achived by following methods:

(I) Additional Notches and Holes in Tension member



(a) original notch (b) Multiple notches



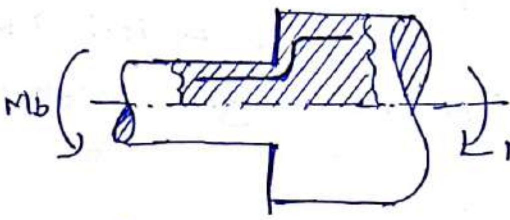
Removal of material.

(c) Drilled Holes (d) Removable of undesirable material.

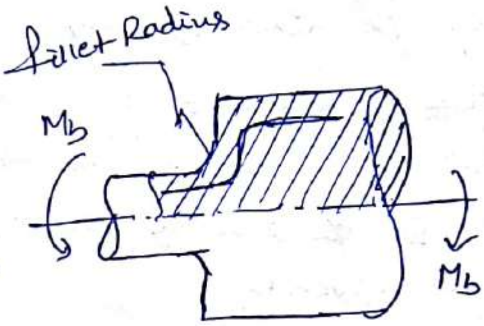
A flat plate with a V-notch subjected to tensile force is shown in fig (a). It is observed that a single notch results in a high degree of stress-concentration. The severity of stress concentration is reduced by three methods. (a) use of multiple notches, (b) drilling additional holes and (c) removal of undesirable material. The method of removing undesired material is called the principle of minimization of the material. In these three methods, the sharp bending of force flow line is reduced and it follows a smooth curve.

(I) Fillet Radius, Undercutting and Notch for Member in Bending

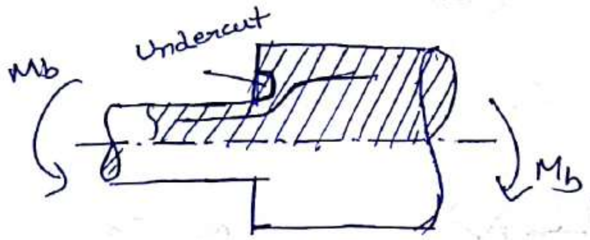
A bar of circular cross-section with a shoulder and subjected to bending



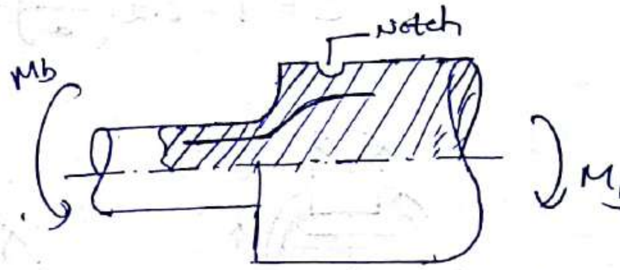
(a) Original cross-section



(b) Fillet Radius



(c) Undercutting



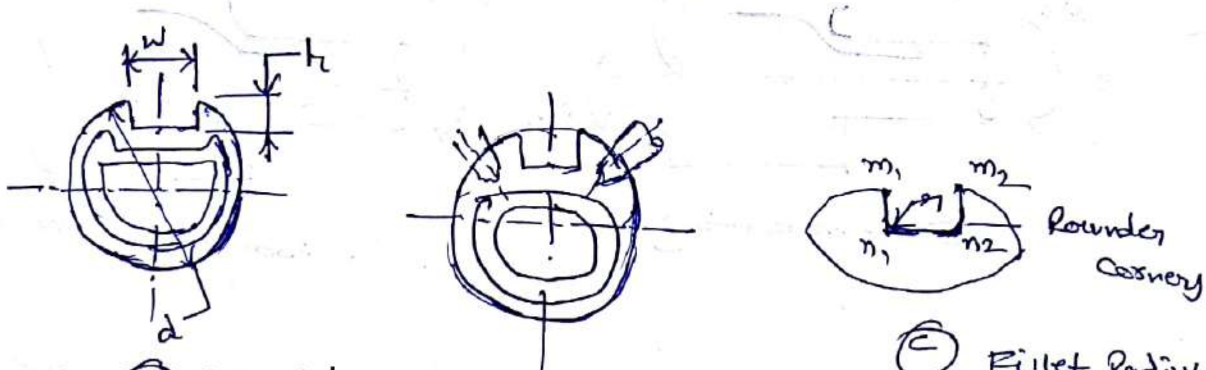
(d) Addition of notch

moment is shown above fig. (a) Ball bearings, gears or pulleys are seated against this shoulder. The shoulder creates change in cross-section of the shaft, that results in stress concentration. There are three methods to reduce stress concentration - (a) use of fillet radius (b) Undercutting (c) additional notch. fig (b) shows the shoulder with a fillet radius or. This results in gradual transition from a small diameter to a large diameter. The fillet radius should be as large as possible in order to reduce stress-concentration. In practice, fillet radius is limited by the design of mating components. The fillet radius can be increased by undercutting the shoulder as show in fig (c). A notch results in stress concentration. Surprisingly, cutting an additional notch is effective way to reduce stress concentration. Show the fig (d),

(III) Drilling Additional Holes for shaft:

A transmission shaft with a keyway is shown in fig below. Keyway is discontinuity and results in stress concentration at the corners of the keyway and reduces torsional shear strength. An empirical relationship developed by H.F. Moore for the ratio C of torsional strength of same sized shaft without keyway is given by

$$C = 1 - 0.2\left(\frac{w}{d}\right) - 1.1\left(\frac{h}{d}\right) \rightarrow (1)$$



(a) Original shaft (b) Drilled holes (c) Fillet Radius

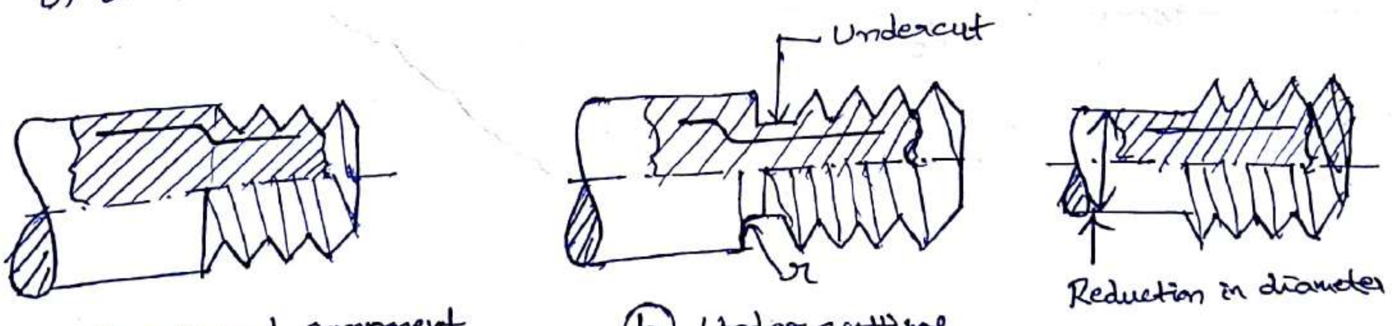
where w and h are width and height dimensions of the keyway respectively. and d is shaft diameter. The four corners of the key way. viz: m_1, m_2, n_1, n_2 are shown above fig (c).
It has been observed that torsional shear stresses at two points, viz. m_1 and m_2 are negligibly small in practice and theoretically equal to zero. on the other hand, the torsional shear stresses at two points, viz. n_1 and n_2 are excessive and theoritically infinite, that means even a small torque will produce permanent set at these points. Rounding corners at two points. viz- n_1 & n_2 by means of fillet radius can reduce the stress concentration. A stress concentration

factor $K_t = 3$ should be used when a shaft with key-way subjected to combined bending and torsional moments.

In addition to giving fillet radius at the inner corners of keyway, there is another method of drilling two symmetrical holes on the sides of keyway. These holes press the force flow lines and minimize their bending in the vicinity of the keyway

(IV) Reduction of stress concentration in Threaded members.

A threaded component is shown below fig(a). It is observed that the force flow line is bent as it passes from shank portion to threaded portion of the component. This results in stress concentration in the transition plane. in fig(b) a small under-cut is taken b/w the shank and the threaded portion of the component and fillet radius is provided for undercut. This reduces bending of the force flow line and consequently reduces stress concentration is in fig(c). where the shank diameter is reduced and made equal to the core diameter of the thread. In this case the force flow line is almost straight and there is no stress concentration.



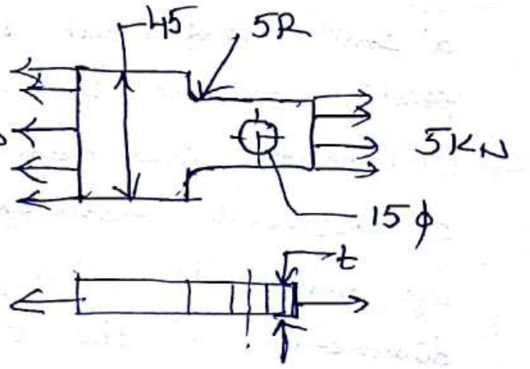
(a) original component (b) Undercutting

Many discontinuities found in machine components cannot be avoided. Therefore, stress concentration cannot be totally eliminated. However, it can be greatly reduced by selecting the correct geometric shape by the designer. Many difficult problems involving stress concentration have been solved by removing material instead of adding it. Additional notches, holes and undercuts are the simple means to achieve significant reduction in the stress concentration.

(1) A flat plate subjected to a tensile force of 5 KN is shown in fig below. The plate material is grey cast iron FG 200 and the factor of Safety is 2.5. Determine the thickness of the plate.

Sol: The stresses are critical at two sections the fillet section and hole section. At the fillet section,

$$\sigma_0 = \frac{P}{dt} = \left(\frac{5000}{30t}\right)$$



$$\frac{D}{d} = \frac{45}{30} = 1.5$$

and $\frac{r}{d} = \frac{5}{30} = 0.167$

from , K_t

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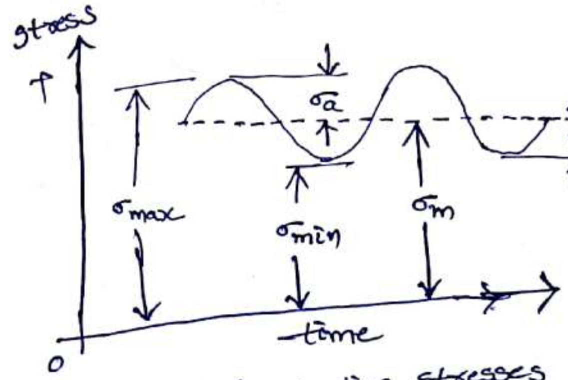
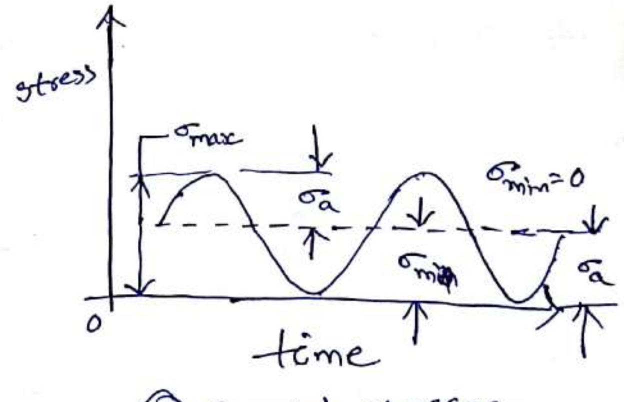
Fluctuating stress:

In the previous sections, (Generally assumed static force only) the external forces acting on a machine component were assumed to be static. In many applications, the components are subjected to forces, which are not static, but vary in magnitude with respect to time. The stresses induced due to such forces are called fluctuating stresses. It is observed that about 80% of failures of mechanical components are due to fatigue failure resulting from fluctuating stresses. In practice, the pattern of stress variation is irregular and unpredictable, as in case of stresses due to vibrations. For the purpose of design analysis, simple models for stress-time relationships are used. The most popular model for stress-time relationship is the sine curve.

There are three types of mathematical models for cyclic stresses - (I) fluctuating or alternating stresses; (II) Repeated stresses. and (III) reversed stresses. Stress-time relationships for these models are shown in fig. The fluctuating



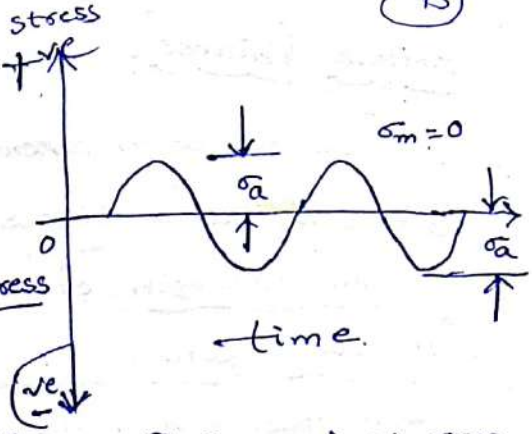
(a) fluctuating stresses



(b) Repeated stresses.

fig. Types of cyclic stresses.

(b) alternating stresses varies in a
Sinusoidal manner with respect to time.
It has some mean value as well as
amplitude value. It fluctuates b/w
two limits - maximum and minimum stress
The stress can be tensile or compressive
or partly tensile and partly compressive.



(c) Reversed stresses

The repeated stress varies in sinusoidal
manner with respect to time, but the variation is from
Zero to some maximum value. The minimum stress is zero
in this case and therefore, amplitude stress and mean stress
are equal. The reversed stress varies in a sinusoidal manner
with respect to time, but it has zero mean stress. In
this case, half portion of the cycle consists of tensile stress
~~and portion of the cycle consists of tensile stress~~ and
remaining half of compressive stress. There is complete
reversal from tension to compression b/w these two halves
and therefore, mean stress is zero. In fig, σ_{max} and σ_{min}
are maximum and minimum stresses, while σ_m and σ_a
are called mean stress and stress amplitude respectively,
It can be proove that,

$$\sigma_m = \frac{1}{2}\left(\sigma_{max} + \sigma_{min}\right) \longrightarrow (1)$$

$$\sigma_a = \frac{1}{2}\left(\sigma_{max} - \sigma_{min}\right) \longrightarrow (2)$$

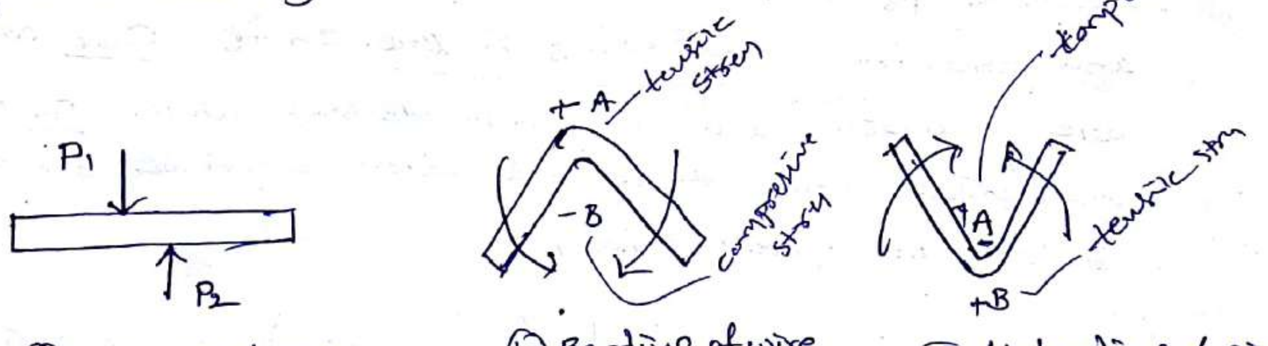
In analysis of fluctuating stresses, tensile stress is considered
as possitive, while compressive stress as negative. It can
be observed that repeated stress and reversed stress are special
cases of fluctuating stress with ($\sigma_{min} = 0$) and ($\sigma_m = 0$) respectively.

(a4)

Fatigue failure:

It has been observed that materials fail under fluctuating stresses, at a stress magnitude (extension size), that is lower than the ultimate tensile strength of the material. sometimes, the magnitude (extension) is even lower than the yield strength. Further, it has been found that the magnitude of the stress, causing fatigue failure decreases as the number of stress cycles increases. This phenomenon of decreased resistance of the materials to fluctuating stresses is the main characteristic of fatigue failure.

Let us examine the phenomenon that we have experienced in our childhood. suppose, there is a wire of 2 to 3 mm diameter and we want to cut it into two pieces without any device like hacksaw, one method is to shear the wire by applying equal and opposite forces P_1 and P_2 by left and right hands as shown. fig(a)



(a) shearing of wire (b) Bending of wire (c) Unbending of wire

fig: Shear and fatigue failure of wire.

It is difficult to cut the wire by this method. second method consists of alternatively bending and unbending the wire for few cycles. Let us consider two diametrically opposite points A and B on the surface of the wire. as shown above. fig(b). when the wire is bent, A is subjected to tensile stress while B to compressive stress. When the wire is unbent, there is compressive stress at A and tensile stress at B, as shown in fig(c). Therefore, there is a complete reversal of stress from tensile stress to compressive stress at point A due to alternate bending and Unbending. similarly, point B is subjected to reversal of stress during the same cycle. we have experienced that the wire can be cut very easily in few cycles of bending and Unbending. This is a fatigue failure and the magnitude of stress required to fracture is very low. In other words, there is decreased resistance of material to cyclic stresses. "Fatigu failure is defined as time delayed fracture Under cyclic loading". Examples of parts in which fatigue failures are common ~~are~~ in transmission shafts, connecting rods, gears, vehicle suspension springs and ball bearings.

There is a basic difference b/w failure due to static load and that due to fatigue. The failure due to static load is illustrated by the simple tension test. In this case the load is gradually applied and there is sufficient time for the elongation of fibres. In ductile materials, there is considerable plastic flow prior to fracture. This results in a silky fibrous structure due to the stretching of crystals at the fractured surface. on the other hand fatigue failure begins with a crack at some point in the material. The crack is more likely to occur in the following regions:

(I) Regions of discontinuity, such as oil holes, keyways, screw threads etc.

(II) Regions of irregularities in machining operations, such as scratches on the surface, stamp mark, inspection marks etc.

(III) Internal cracks due to to defects in materials like blow holes.

These regions are subjected to stress concentration due to the crack. The crack spreads due to fluctuating stresses until the cross-section of the component is so reduced that the remaining portion is subjected to sudden fracture. There are two distinct areas of fatigue failure – (I) Region Indicating slow growth of crack with a fine fibrous appearance; and (II) region of sudden fracture with a coarse grannular appearance.

In case of failure under static load, there is sufficient plastic deformation prior to failure, that gives warning well in advance. on the other hand, fatigue cracks are not visible till they reach the surface of the component and by that time the failure has already taken place. The fatigue failure is sudden and total. It is relatively easy to design a component for static load. The fatigue failure, however, depends upon a number of factors, such as number of cycles, mean stress, stress amplitude, stress concentration, residual stresses, corrosion and creep. This makes the design of components, subjected to fluctuating stresses, more complex.

Endurance Limit:(or) Limiting range of stress

The fatigue or endurance limit of a material is defined as the maximum amplitude (range) of completely reversed stress that the standard specimen can sustain for an unlimited number of cycles without fatigue failure." Since the fatigue test cannot be conducted for unlimited or infinite number of cycles, 10^6 cycles is considered as a sufficient number of cycles to define the endurance limit. There is another term called fatigue life, which is frequently used with endurance limit. The fatigue life is defined as number of stress cycles that the standard specimen can complete during the test before the appearance of the first fatigue crack. The dimensions of standard test specimen (in mm)

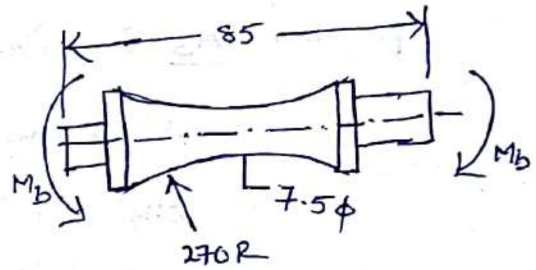
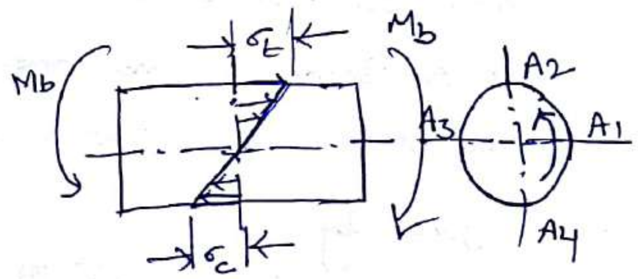
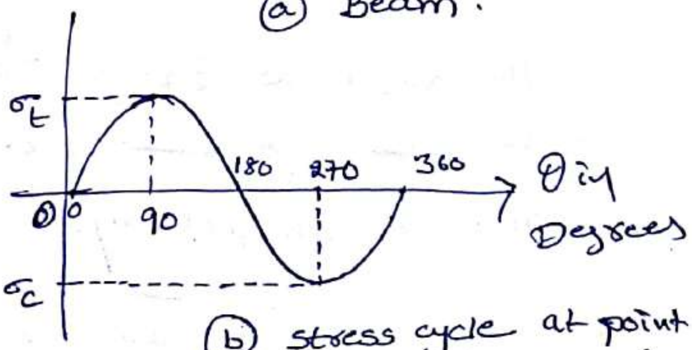


fig Specimen for fatigue test

The specimen is carefully machined and polished. The final polishing is done in axial direction in order to avoid circumferential scratches. In laboratory, the endurance limit is determined by means of a rotating beam machine developed by R.R. Moore. The principle of rotating beam is illustrated shown fig. below



(a) Beam.



(b) Stress cycle at point A.

A beam of circular cross-section is subjected to bending moment M_b. Under the action of B.M, tensile stresses are induced in the upper half of the beam and compressive stresses in the lower half. The maximum tensile stress σ_t in the upper most fibre is equal

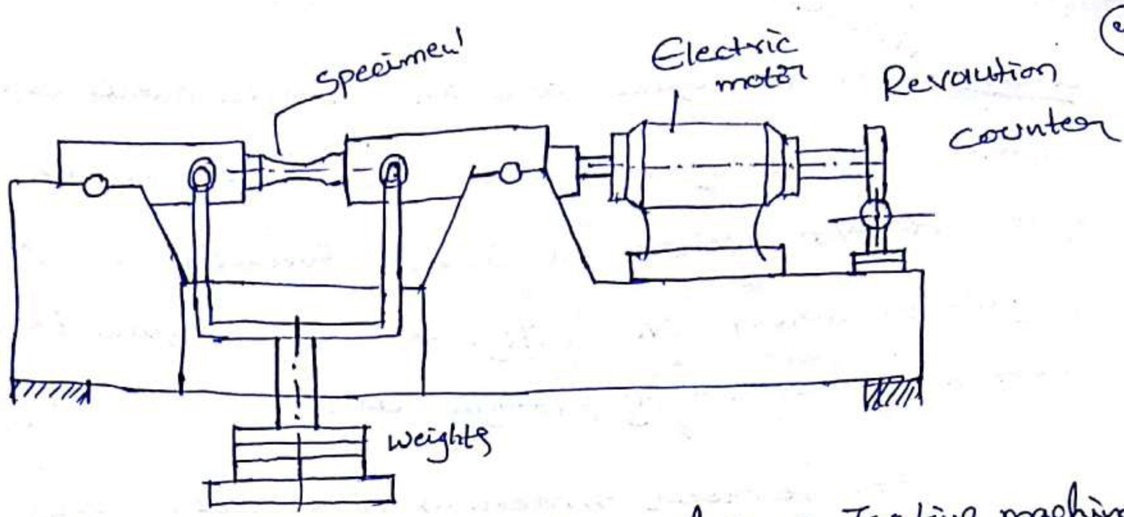
-to the maximum compressive stress $\underline{\sigma_c}$ in the lowermost fibre. There is zero stress at all fibres in the central horizontal plane passing through the axis of the beam. Let us consider a point $\underline{A}$ on the surface of the beam and let us try to find out stresses at this point when the beam rotated through one revolution. Initially point $\underline{A}$ occupies position $\underline{A_1}$ in the central horizontal plane with zero stress. When the beam is rotated through $\underline{90^\circ}$, it occupies position $\underline{A_2}$. It is subjected to maximum tensile stress $\underline{\sigma_t}$ in this position. When the beam is further rotated through $\underline{90^\circ}$, point $\underline{A}$ will occupy position $\underline{A_3}$ in the central horizontal plane with zero stress.

A further rotation of $\underline{90^\circ}$ will bring point $\underline{A}$ to position A_4. It is subjected to maximum compressive stress $\underline{\sigma_c}$ in this position. The variation of stresses at point A during one revolution of the beam as shown above fig (b). It is observed that the beam is subjected to completely reversed stresses with tensile stress in the first half and compressive stress in the second half. The distribution is sinusoidal and one stress cycle is completed in one revolution. The amplitude of this cycle (σ_t or σ_c) is given by

$$\sigma_t \text{ or } \sigma_c = \frac{M_b y}{I}.$$

The amplitude can be increased or decreased by ~~decreasing decreased by incre~~ increasing or decreasing bending moment respectively.

A schematic diagram of Rotating beam fatigue



~~fig~~: Rotating Beam ~~Engin~~ fatigue Testing machine

The specimen acts as rotating beam subjected to bending moment. Therefore, it is subjected to completely reversed stress cycle. changing the bending moment by addition or removal of weights can vary stress amplitude. The number of revolutions before apperance of the first fatigue crack is recorded on a revolution ~~meter~~. counter. In each test, two readings are taken, viz. stress amplitude (Sf) and number of stress cycles (N). These readings are used as two co-ordinates for plotting a point on S-N diagram. This point is called failure point. To determine the endurance limit of a material a number of tests are to be carried out.

The results of these tests are plotted by means of a S-N curve. The S-N curve is the graphical representation of stress amplitude (Sf) versus the number of stress cycles (N) before fatigue failure on a log-log graph paper. S-N curve for steels is represented in fig

Each test on fatigue testing machine gives one failure point on the S-N-diagram. In practice, the points are

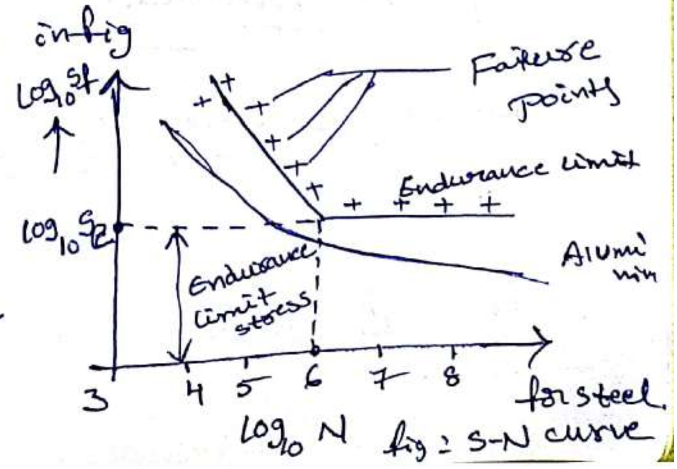


fig: S-N curve

Scattered (irregularity) in the figure and an average curve is drawn through them. The S-N diagram is also called wohler diagram, after August wohler, a German Engineer who published his fatigue research in 1870. The S-N diagram is a standard method of presenting fatigue data.

(asymptotic line that approaches curve)

For ferrous materials like steels, the S-N curve becomes asymptotic at 10^6 cycles, that indicates the stress amplitude corresponding to infinite number of stress cycles. The magnitude of this stress amplitude at 10^6 cycles represents the endurance limit of the material. The S-N curve shown above is valid only for ferrous metals. For non-ferrous metals like aluminium alloys, the S-N curve slopes gradually even after 10^6 cycles. These materials do not exhibit a distinct (definite) value of the endurance limit in a true-sense. For these materials, endurance limit stress is sometimes expressed as a function of the number of stress cycles. ** Number of tests are conducted by varying the loads from Higher to Lower value and corresponding failure will be noted for each load, if the load is going on reduced at a particular load, the specimen will not fail for Infinite cycle. The endurance limit, in a true sense, is ~~limit, in a~~ not exactly a property of material, like ultimate tensile strength. It is affected by factors such as the size of the component, shape of component, the surface finish, temperature and the notch sensitivity of the material.

Aluminium not having endurance
steel having enduran limit limit

Low cycle and High cycle Fatigue :

The S-N curve illustrated in fig ~~below~~ above is drawn from 10^3 cycles on log-log graph paper. The complete S-N curve from 10^0 cycle to 10^8 cycles is shown in fig below. There are two regions of this curve Low cycle fatigue & High cycle fatigue. The difference between these two fatigue failures is as follows:

(I) Any fatigue failure, when the number of stress cycles are less than 1000, is called Low cycle fatigue. Any fatigue failure when the number of stress cycles are more than 1000, is called high cycle fatigue.

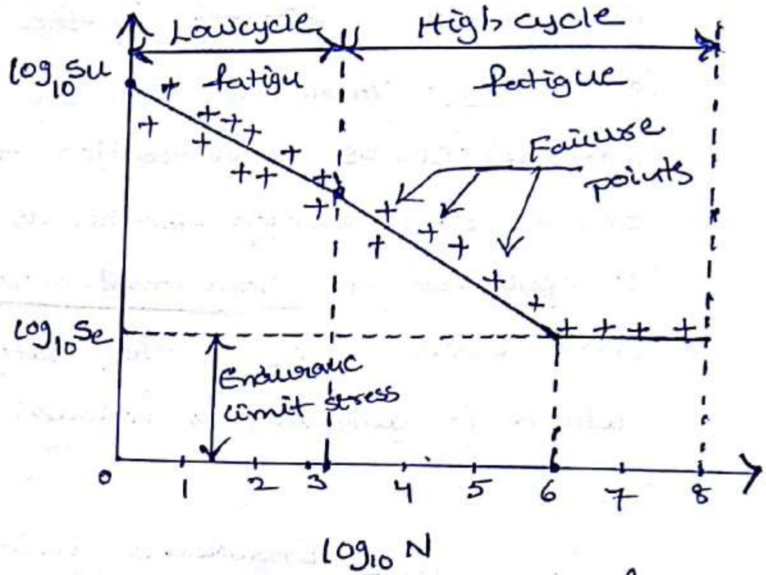


fig: Low and High cycle fatigue.

(II) Failure of studs on truck wheels, failure of setscrews for locating gears on shafts or failures of short-lived devices such as missiles are the examples of Low cycle fatigue. The failure of machine components such as springs, ball bearings or gears that are subjected to fluctuating stresses, are the examples of high cycle fatigue.

(III) The low cycle fatigue involves plastic yielding at localized areas of the components. There are some theories of Low cycle fatigue. However, in many applications the designers simply ignore the fatigue effect when the number of stress cycles are less than 1000. A greater factor of safety is used to account for this effect. Such components are designed on the basis of ultimate tensile strength or yield strength with suitable factor of safety.

Components subjected to high cycle fatigue are designated on the basis of endurance limit stress. S-N curve soderberg line, Gerber line or Goodman digram are used in design such components.

Notch Sensitivity:

It is observed that the actual (true or real) reduction in the endurance limit of a material due to stress concentration is less than the amount indicated by the theoretical stress concentration factor K_t. Therefore, two separate notations - K_t and K_f are used for stress concentration factors. K_t is the theoretical stress concentration factor, as defined in previous sections, which is applicable to ideal materials that are homogeneous, isotropic and elastic. K_f is the fatigue stress concentration factor, which is defined as follows:

$$K_f = \frac{\text{Endurance limit of the notch free specimen}}{\text{Endurance limit of the notched specimen}}$$

This factor K_f is applicable to actual materials and depends upon the grain size of the material. It is observed that due to stress concentration, there is a greater reduction in the endurance limit of fine grained materials as compared to coarse-grained materials.

Notch sensitivity is defined as the susceptibility of a material to succumb to the damaging effects of stress raising notches in fatigue loading. The notch sensitivity factor q is defined as follows:

$$q = \frac{\text{Increase of actual stress over nominal stress}}{\text{Increase of theoretical stress over nominal stress}}$$

since σ_0 = nominal stress as obtained by elementary eqns

$\therefore$ $\boxed{\text{actual stress} = K_f \sigma_0}$

$\because K_t = \frac{\sigma_{max}}{\sigma_0}$

$\sigma_{max(th)} = K_t \sigma_0$

$$\boxed{\text{theoretical stress} = K_t \sigma_o}$$

increase of actual stress over nominal stress

$$= (K_f \sigma_o - \sigma_o)$$

increase of theoritical stress over nominal stress

$$= (K_t \sigma_o - \sigma_o)$$

Therefore,

$$q = \frac{(K_f \sigma_o - \sigma_o)}{(K_t \sigma_o - \sigma_o)}$$

$$\boxed{q = \frac{(K_f - 1)}{(K_t - 1)}}$$

The above eqn can be rearranged in the following form:

$$K_f = 1 + q(K_t - 1)$$

following conclusions are drawn with the help of above eqn.

(I) when the material has no sensitivity to notches,

$$q = 0 \ \& \ K_f = 1$$

(II) when the material is fully sensitive to notches,

$$q = 1 \quad \text{and} \quad K_f = K_t$$

In general, the magnitude of notch sensitivity factor q varies from 0-1. The notch sensitivity factor for various materials for reversed bending or axial stresses ~~are~~

and reversed torsional shear stress are obtained from fig (1) and (2) below respectively. In case of doubt, the designer should use ($q=1$) or ($K_t = K_f$) and the design will be on the safe side.

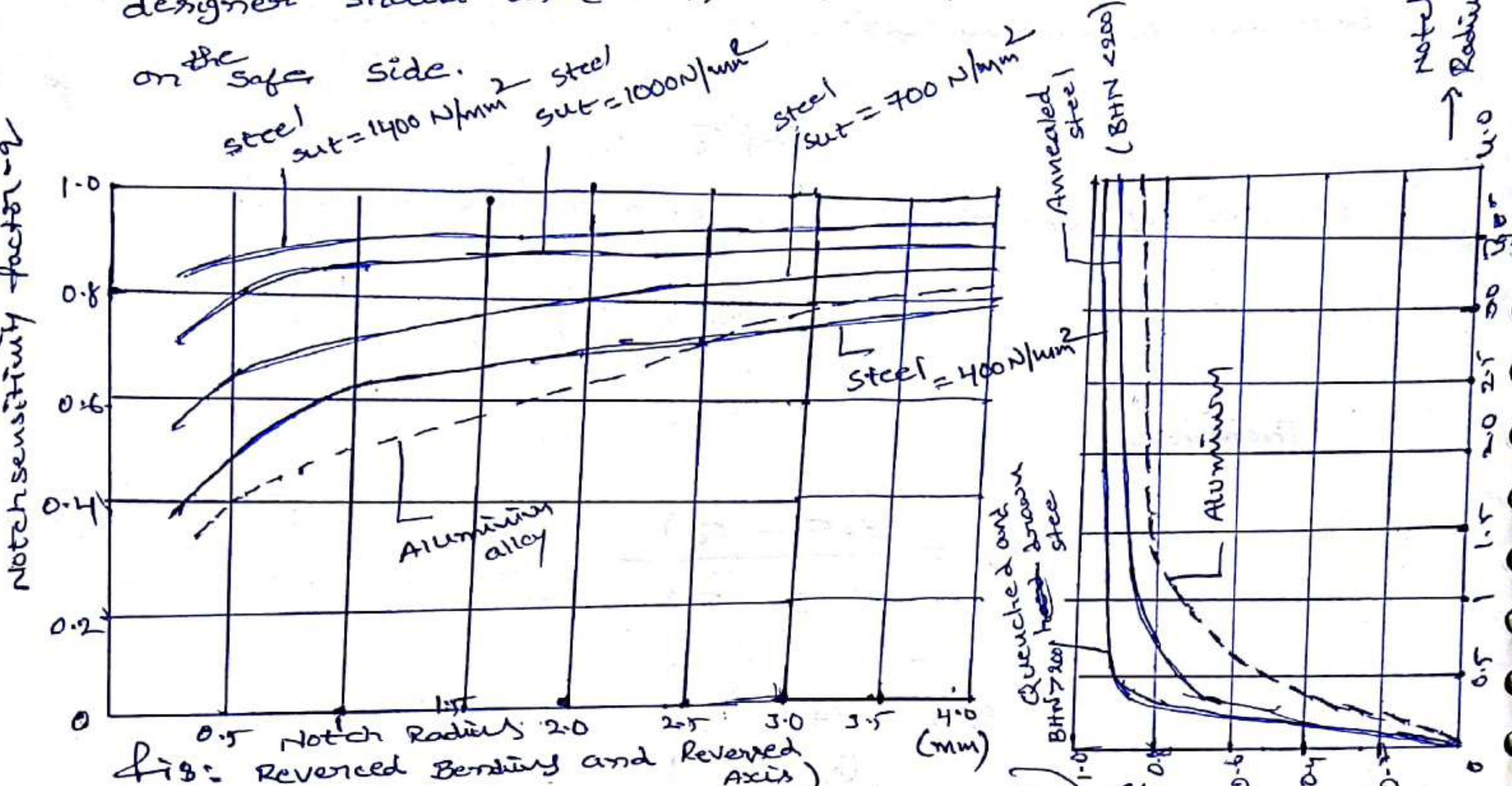


fig: Reverced Bending and Reversed Axis)

fig: Reversed Torsional stress

Endurance Limit Approximate Estimation:

The laboratory method for determining the endurance limit of materials, although more precise, is laborious and time consuming. Number of tests are prepare one S-N curve and each test takes considerable amount of time. It is, required to therefore, not possible to get the experimental data of each and every material. When the laboratory data regarding the endurance limit of the materials is not available, the procedure discussed in this section should be adopted.

Two seperate notations are used regarding endurance limit, (S_e') and (S_e) where,

S_e' = endurance limit stress of a rotating beam specimen subjected to reversed bending stress (N/mm^2)

S_e = endurance limit stress of a Particular mechanical Component subjected to reversed bending stress (N/mm^2)

There is an approximate relationship b/w the endurance limit and the ultimate tensile strength (S_{ut}) of the material.

For steels, $S_e' = 0.5\,S_{ut}$

For cast iron and cast steels,

$$S_e' = 0.4\,S_{ut}$$

For wrought alluminium alloys,

$$S_e' = 0.4\,S_{ut}$$

For cast alluminium alloys,

$$S_e' = 0.3\,S_{ut}$$

These relationships are based on 50% reliability.

The endurance limit of a mechanical Component is different from the endurance limit of a rotating beam specimen due to a number of factors. The difference arises due to the fact that there are Standard Specifications and working conditions for the rotating beam Specimen, while the actual components have different specifications and work under different conditions. Different modifying factors are used in practice to account for this difference. These factors are, sometimes, called derating factors. The purpose of derating factors is to derate or reduce the endurance limit of rotating beam specimen to suit the actual component. In this article, only four factors that normally require attention are discussed..

The relationship between (s_e) and (s_e^1) is as follows:

$$S_e = K_a \cdot K_b \cdot K_c \cdot K_d \, S_e^1$$

where

K_a = surface finish factor

K_b = size factor

K_c = reliability factor

K_d = modifying factor to account for stress concentration.

Surface Finish Factor, (K_a)

The surface of the rotating beam specimen is polished to mirror finish. The final polishing is carried out in the axial direction to smooth out any circumferential scratches. This makes the specimen almost free from surface scratches and imperfections. It is impractical to provide such an expensive surface finish for the actual component. The actual component may not even require such a surface finish. when the surface finish is poor, there are scratches and geometric irregularities on the surface. These surface scratches serve as stress raisers and result in stress concentration. The endurance limit is reduced due to introduction of stress concentration at these scratches. The surface finish factor takes into account the reduction in endurance limit due to the variation in surface finish b/w the specimen and the actual component. fig. below shows the surface finish factor for steel components. It should be noted that ultimate tensile strength is also

Note: (1) stress concentration is increases Endurance limit is decreases,
(2) ultimate tensile strength increases, surface face finish factor decreases;

a Parameter that affects the surface finish factor. High strength materials are more sensitive to stress concentration introduced by surface irregularities. Therefore, as the ultimate tensile strength increases, the surface finish factor decreases.

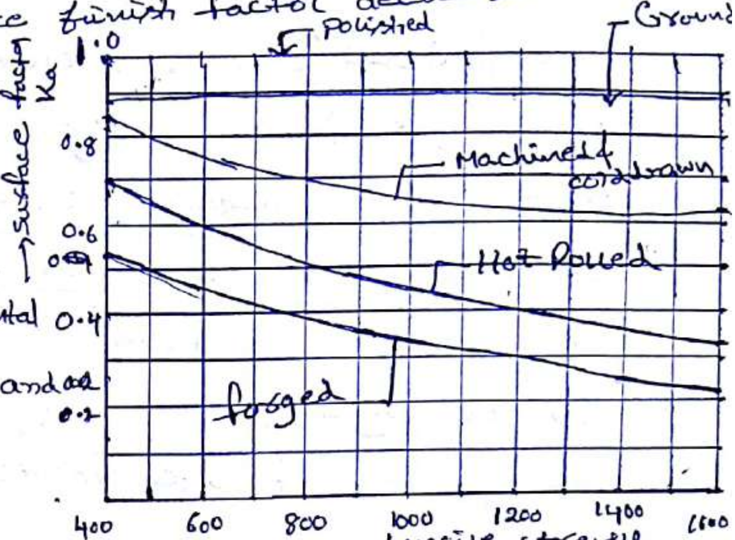


fig: surface finish factor

Shigley and Mischke have suggested an exponential eqn for surface finish factor. This eqn is based on experimental data points obtained by Noll and Lipson. This eqn is in the following form,

$$K_a' = a(S_{ut})^b \quad [\text{if } K_a > 1, \text{ set } K_a = 1]$$

The values of coefficients a & b are given in table.

The above-mentioned values of surface finish factors are developed only for steel components. They should not be applied to components made of other ductile materials like aluminium alloys.

The surface finish factor for ordinary grey cast-iron components is taken as one irrespective of their surface finish. It is observed that even mirror-finished samples of grey cast iron parts have surface discontinuities because of graphite flakes in the cast iron matrix. Adding some more surface scratches does not make any difference. Therefore, what ever is the machining method, the value of surface finish factor for cast iron parts is always taken as one.

Surface finish	a	b
Ground	1.58	-0.085
Machined or cold draw	4.51	-0.265
Hot rolled	57.7	-0.718
As forged	272	-0.995

Table values of coefficients a & b in surface finish factor

Size factor :- (K_b)

The rotating beam specimen is small with 7.5 mm diameter. Larger the machine part, greater is the probability that a flaw exist somewhere in larger volume. The chances of fatigue failure originating (statically) at any one of these flaws is more. The endurance limit, therefore, reduces with increasing the size of the component. The size factor K_b takes into account the reduction in endurance limit due to increase in the size of the component. For bending and torsion, the values of size factor (K_b) are given in table.

Table: Values of size factor

Dia (d) mm	K_b
$d \leq 7.5$	1.00
$7.5 < d \leq 50$	0.85
$d > 50$	0.75

Shigley and Mischke have suggested an exponential eqn for size factor. For bending and torsion, the eqn is in the following form,

For $2.79\text{ mm} \leq d < 51\text{ mm}$

$$\boxed{K_b = 1.24\, d^{-0.107}} \rightarrow (a)$$

For $51\text{ mm} < d \leq 254\text{ mm}$

$$K_b = 0.859 - 0.000873\, d$$

For axial loading,

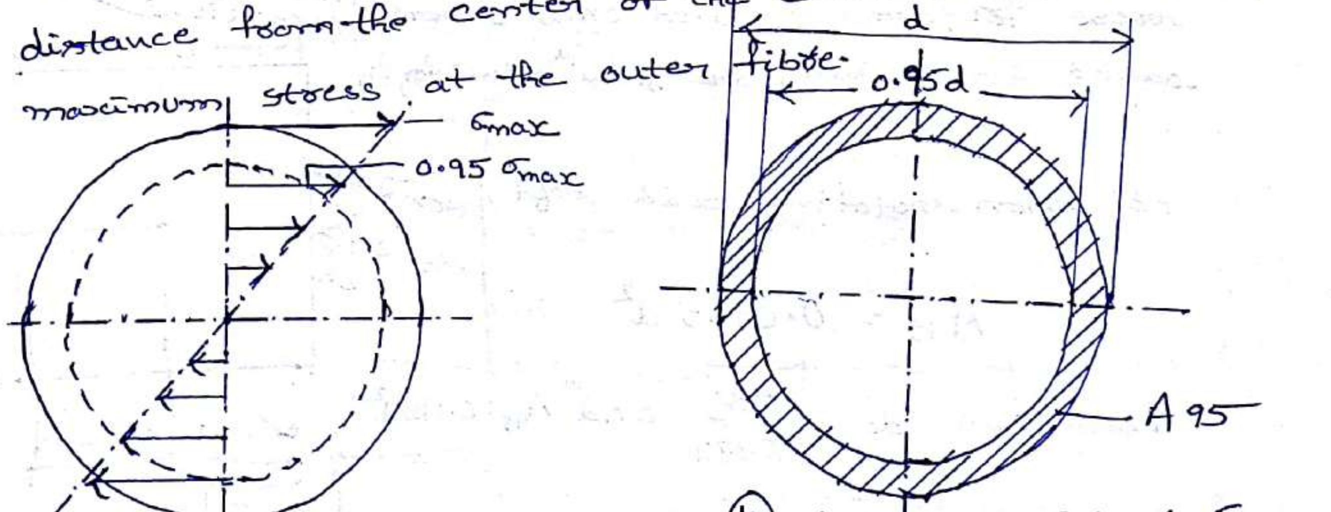
$$\boxed{K_b = 1} \rightarrow (b)$$

Instead of these two eqn (a & b) we use the above.

The above equations of Table can be used only for cylindrical components. It is difficult to determine the size factor for components having non-circular cross-section. However, since the endurance limit is reduced in such components,

[Note: Endurance limit decreases with increase of size of the component.]

it is necessary to define effective diameter based on an equivalent circular cross-section. In this case Kuguel's equality is widely used. This equality is based on the concept that fatigue failure is related to the probability of high stress interacting with a discontinuity. when the volume of material subjected to high stress is large, the probability of fatigue failure originating from any flow in that volume is more. Kuguel assumes a volume of material that is stressed to 95% of maximum stress or above, as high stress volume. According to Kuguel's equality, the effective diameter is obtained by equating the volume of the material stressed at and above 95% of the maximum stress to the equivalent volume in the rotating beam specimen. when these two volumes are equated, the lengths of component and specimen cancel out and only areas need be considered. this concept is illustrated fig below. The rotating beam specimen is subjected to bending stresses. The ~~distance~~ bending stress is linearly proportional to the distance from the center of the cross-section. There is maximum stress at the outer fibre.



(a) Stress distribution

(b) Area above 95% of σ_{max}

Therefore, the area (A_{95}) stressed above 95% of the maximum stress is the area of a ring, having an inside diameter of ($0.95d$) and outside diameter of ($1.0d$).

$$A_{95} = \pi\left[\frac{d^2 - (0.95d)^2}{4}\right] = 0.0766d^2$$

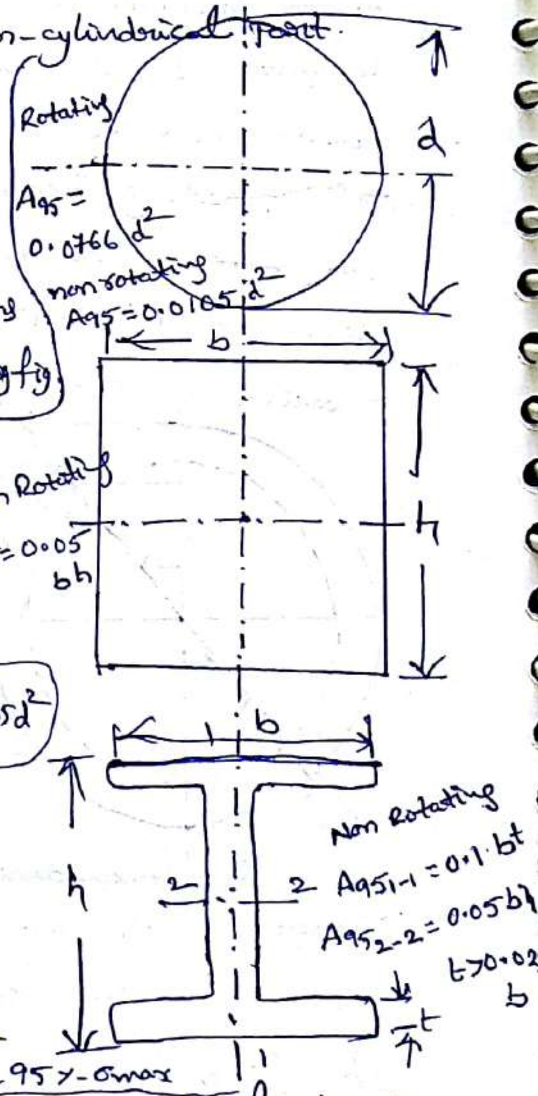
The above equation is also valid for hollow rotating shaft

The effective diameter of any non-circular cross-section is then given by,

$$d_e = \sqrt{\frac{A_{95}}{0.0766}}$$

where A_{95} = portion of cross-sectional area of the noncylin-
- drical part that is stressed b/w 95% and 100% of the maximum stress.

d_e = effective diameter of non-cylindrical part.



Formulae for areas that are stressed b/w 95% and 100% of maximum stress for commonly used cross-sections loaded in bending, are given in fig.

For non-rotating solid shaft,

$$A_{95} = 0.0105d^2$$

from eqn $d_e = \sqrt{\frac{A_{95}}{0.0766}}$ and $A_{95} = 0.0105d^2$

$$d_e = \sqrt{\frac{0.0105d^2}{0.0766}} = 0.37d$$

$$\boxed{d_e = 0.37d}$$

The above effective diameter $\underline{d_e}$ is used to find size

factor for non-rotating cylindrical component. For rectangular
cross-section having width $\underline{b}$ and depth $\underline{h}$,

$A_{95} = 0.05\,bh$ from eqn $d_e = \sqrt{\frac{A_{95}}{0.0766}}$ &

$A_{95} = 0.05\,bh$

$$d_e = \sqrt{\frac{0.05\,bh}{0.0766}}$$

$$\boxed{d_e = 0.808\sqrt{bh}}$$

The above effective diameter $\underline{d_e}$ is used to find outsize factor, table, similar procedure is followed for I-section beam.

Reliability factor:

The laboratory values of endurance limit are usually mean values. There is considerable dispersion of the data when number of tests are conducted even when using the Same material and Same conditions. The standard deviation of endurance limit tests is 8% of the mean value. The reliability factor K_c depends upon the reliability, that is used in the design of the component. The greater the likelihood that a part will survive, the more is the reliability & Lower is the reliability factor. The reliability factor is one for 50% reliability. This means that 50% of the components will survive in the given set of ~~tolerance~~ conditions. To insure that more than 50% of the parts will survive, the stress amplitude on the component should be lower than the tabulated value of endurance limit. The reliability factor is used to achieve this reduction. The reliability factors based on a

Standard deviation of 8% are given in Table.

Table : Reliability factor.

Reliability R(%)	K_c
50	1.000
90	0.897
95	0.868
99	0.814
99.9	0.753
99.99	0.702
99.999	0.659

Modifying factor to Account for stress Concentration

The endurance limit is reduced due to stress concentration. The stress concentration factor used for cyclic loading is less than the theoretical stress concentration factor due to the notch sensitivity of the material. To apply the effect of stress concentration, the designer can either reduce the endurance limit by K_d or increase the stress amplitude by K_f. we will use the first approach. The modifying factor K_d to account for the effect of stress concentration is defined as,

$$\boxed{K_d = \frac{1}{K_f}}$$

The above-mentioned four factors are used to find out the endurance limit of the actual component.

The endurance limit (S_{se}) of a component subjected to fluctuating torsional shear stresses is obtained from the endurance limit in reversed bending (S_e) using theories of failures.

According to the maximum shear-stress theory,

$$\boxed{S_{se} = 0.5\, S_e}$$

According to distortion-energy theory,

$$\boxed{S_{se} = 0.577\, S_e}$$

When the component is subjected to axial fluctuating load, the conditions are different. In axial loading, the entire cross-section is uniformly stressed to the maximum value. In rotating beam test, the specimen is subjected to bending stress. The bending stress is zero at the center of cross-section and negligible in vicinity of center. It is only outer region near the surface, that is subjected to maximum stress. There is more likelihood of a micro crack being present in much ~~higher~~ high stress field of axial loading than in the smaller volume outer region of rotating beam specimen. Therefore, endurance limit in axial loading is lower than the rotating beam test.

For axial loading,

$$\boxed{(S_e)_a = 0.8\, S_e}$$

Reversed Stresses Design for finite and infinite life

There are two types of problems in fatigue design —

(I) components subjected to completely reversed stresses, and

(II) components subjected to ~~completely~~ fluctuating stresses.

As shown figure previous chapters flutuating + completly reversed stress diagram, the mean ~~σ~~ stress is zero in case of completely reversed stresses. The stress distribution consists of tensile stresses for the first half cycle and compressive stresses for the remaining half cycle. and the stress cycle passes through zero. In case of fluctuating stresses, there is always a mean stress, and the stresses can be purely tensile, purely compressive or mixed depending upon the magnitude of the mean stress. Such problems are solved with the help of the modified Goodman diagram, that will be discussed in another section.

The design problems for completely reversed stresses are further divided into two groups - (I) Design for infinite life (II) design for finite life. When the component is to be designed for infinite life, the endurance limit becomes the criterion of failure. The amplitude stress induced in such components should be lower than the endurance limit in order to withstand the infinite number of cycles. Such components are designed with the help of the following equations:

$$\sigma_a = \frac{S_e}{(f.s)}$$

$$\tau_a = \left(\frac{S_{se}}{f.s}\right)$$

where σ_a and (τ_a) are stress amplitudes in the components and S_e and S_{se} are corrected endurance limits in reversed bending and torsion respectively.

When the component is to be designed for finite life, the S-N curve shown below can be used. The curve is valid for steels. It consists of a straight line AB drawn from ($0.9 S_{ut}$) at 10^3 cycles to (S_e) at 10^6 cycles on a log-log paper. The design procedure for such problems is as follows:

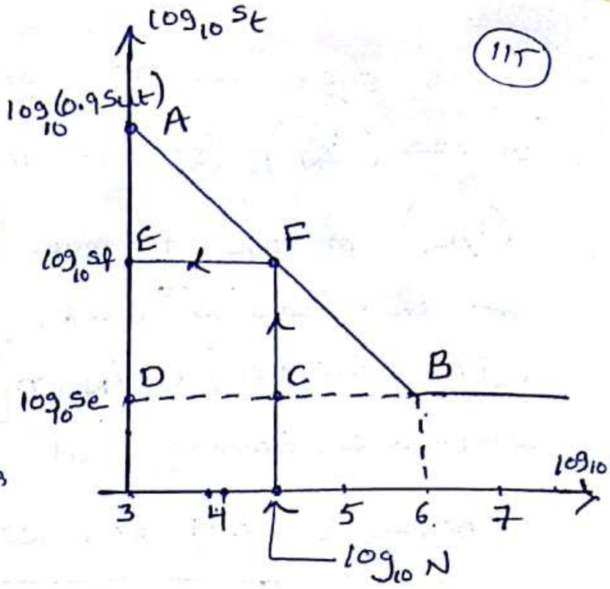


fig: S-N curve

(I) Locate point A with coordinates $[3, \log_{10}(0.9\, S_{ut})]$

since $\log_{10}(10^3) = 3$.

(II) Locate point B with coordinates $[6, \log_{10}(S_e)]$

since $\log_{10}(10^6) = 6$.

(III) Join $\overline{AB}$, which is used as a criterion of failure for finite-life problems.

(IV) Depending upon the life $\underline{N}$ of the component, draw a vertical line passing through $\log_{10}(N)$ on the abscissa. This line intersects $\overline{AB}$ at point F.

(V) Draw a line $\overline{FE}$ parallel to the abscissa. The ordinate at point E, i.e. $\log_{10}(S_f)$, gives the fatigue strength corresponding to $\underline{N}$ cycles.

The value of fatigue strength (S_f) obtained by the above procedure is used for the design calculations.

Cumulative Damage in fatigue:

In certain applications, the mechanical component is subjected to different stress levels for different parts of the work cycle. The life of such a component is determined by Miner's equation. Suppose that a component is subjected to completely reversed stresses (σ_1) for (n_1) cycles, (σ_2) for (n_2) cycles and so on. Let N_1 be the number of stress cycles before fatigue failure, if only the alternating stress (σ_1) is acting. one stress cycle will consume $(1/N_1)$ of the fatigue life and since there are n_1 such cycles at this stress level, the proportionate damage of fatigue life will be $[(1/N_1) n_1]$ or $[n_1/N_1]$. similarly, the proportionate damage at stress level (σ_2) will be (n_2/N_2). Adding these quantities, we get

$$\boxed{\frac{n_1}{N_1} + \frac{n_2}{N_2} + \cdots\cdots + \frac{n_x}{N_x} = 1.}$$

The above eqn is known as the Miner's equation. Sometimes, the number of cycles $n_1, n_2 \cdots$ at stress levels $\sigma_1, \sigma_2 \cdots$ are unknown. suppose that $\alpha_1, \alpha_2, \cdots$ are proportions of the total life that will be consumed by the stress levels $\sigma_1, \sigma_2,$ $\sigma_3, \cdots$ etc. Let $\underline{N}$ be the total life of the component.

Then,

$$n_1 = \alpha_1 N \qquad n_2 = \alpha_2 N$$

Substituting these values in Miner's equation,

$$\frac{\alpha_1}{N_1} + \frac{\alpha_2}{N_2} + \cdots\cdots + \frac{\alpha_x}{N_x} = \frac{1}{N}.$$

Also

$$\alpha_1 + \alpha_2 + \alpha_3 + \cdots\cdots + \alpha_x = 1$$

with the help of above equations, the life of the component subjected to different stress levels can be determined.

Note "where $\underline{\alpha_i}$ is the fraction of life consumed by exposure to the cycles at the different stress levels. In general

Soderberg and Goodman Lines

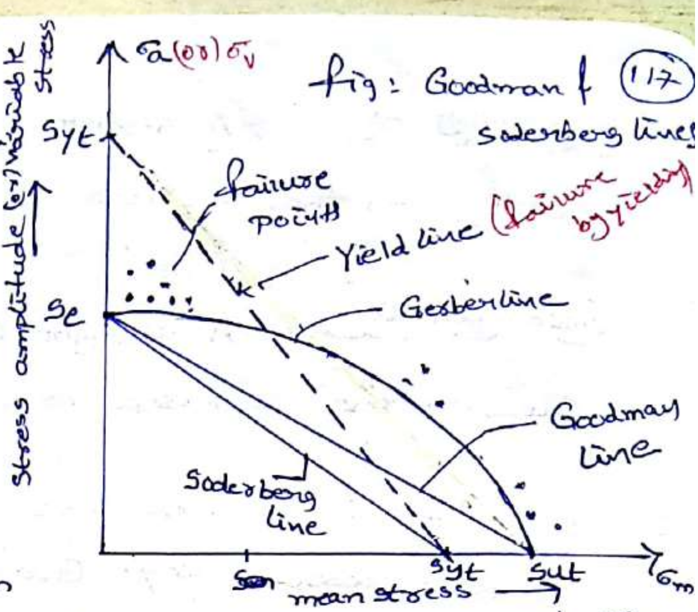


fig: Goodman & Soderberg lines

When a component is subjected to fluctuating stresses, as shown in figure, there is a mean stress (σ_m) as well as a stress amplitude (σ_a). It has been observed that mean stress component has effect on fatigue failure when it is present in combination with alternating component. The fatigue diagram for this general case is shown in above fig. In this diagram, the mean stress is plotted on the abscissa. The stress amplitude is plotted on the ordinate. The magnitudes of (σ_m) and (σ_a) depend upon the magnitudes of maximum and minimum force acting on the component. When stress amplitude (σ_a) zero, the load is purely static and the criterion of failure is Sut or Syt. These limits are plotted on the abscissa. When the mean stress (σ_m) is zero, the stress is completely reversing and the criterion of failure is endurance limit Se that is plotted on the ordinate. When the component is subjected to both components of stress, viz. (σ_m) and (σ_a), the actual failure occurs at different scattered points as shown above figure. There exists a border, that divides safe region from unsafe region for various combinations of (σ_m) and (σ_a). Different criterions are proposed to construct the borderline dividing safe zone and failure zone. They include Gerber Line, Soderberg Line and Goodman Line.

Gerber Line: A parabolic curve joining Se on the ordinate to Sut on the abscissa is called gerber line.

Soderberg Line: A straight line joining S_e on the ordinate to S_{yt} on the abscissa is called the soderberg line.

Goodman line: A straight line joining S_e on the ordinate to S_{ut} on the abscissa is called the Goodman line.

Gerber Parabola fits the failure points of test data in the best possible way. Goodman line fits beneath the scatter of this data. Both Gerber parabola and Goodman line intersect at (S_e) on the ordinate to (S_{ut}) on the abscissa. However, the goodman line is more safe from design considerations because it is completely inside the Gerber parabola and inside the failure points. The soderberg line is is more conservative failure criterion and there is no need to consider even yielding in this case. A yield line is constructed connecting (S_{yt}) on both axes. It is said ~~to be~~ as limit on first cycle of stress. This is because if a part yields, it has failed, regardless of its safety in fatigue.

We will apply following form for the eqn of straight line,

$$\frac{x}{a} + \frac{y}{b} = 1$$

where a and b are the intercepts of the line on X & Y axes respectively.

Applying the above formula, the eqn of soderberg line is given by

$$\boxed{\frac{\sigma_m}{S_{yt}} + \frac{\sigma_a}{S_e} = 1}$$

Similarly, the eqn of Goodman line is given by,

$$\boxed{\frac{\sigma_m}{S_{ut}} + \frac{\sigma_a}{S_e} = 1}$$

Goodman line is widely used as the criterion of fatigue failure when the component is subjected to mean stress as well as stress amplitude. It is because of the following reasons:

(I) Goodman line is safe from design considerations because it is completely inside the failure points of test data.

(II) The eqn of straight line is simple, compared with eqn of parabolic curve.

(III) It is not nessary to construct scale diagram. as a rough sketch is enough to construct fatigue diagram.

Modified Goodman Diagrams!

The components, which are subjected to fluctuating stresses, are designed by constructing the modified Goodman diagram. For the purpose of design, the problems are classified into two groups:

(I) Components subjected to fluctuating torsional shear stress

(II) Components subjected to fluctuating axial or bending stresses

Seperate diagrams are used for these two cases. The modified Goodman diagram for fluctuating axial or bending stresses is shown in below. fig. In this diagram, Goodman line is modified by combining fatigue failure with failure by yielding. In this diagram, the yield strength S_{yt} is plotted on both the axes - abscissa and ordinate, and a yield line $\overline{CD}$ is constructed to Join these two points to define failure by yielding. obviously, the line $\overline{CD}$ is inclined at 45° to the abscissa. similarly, a line $\overline{AF}$ is constructed ~~ab~~ to Join S_e on the ordinate with

Sut on the abscissa, that is the Goodman line discussed in the previous section. The point of intersection of these two lines is B. The area OABC represents the region of safety for components subjected to fluctuating stresses. The region OABC is called modified Goodman diagram should cause neither fatigue failure nor yielding. The modified Goodman diagram combines fatigue criteria as represented by the Goodman line and yield criteria as represented by yield line. Note that $\overline{AB}$ is the portion of Goodman line and $\overline{BC}$ is portion of yield line.

If the mean component of stress (σ_m) is very large and alternating component (σ_a) very small, their combination will define a point in the region BCF that would be safe within Goodman line but would yield on the first cycle.

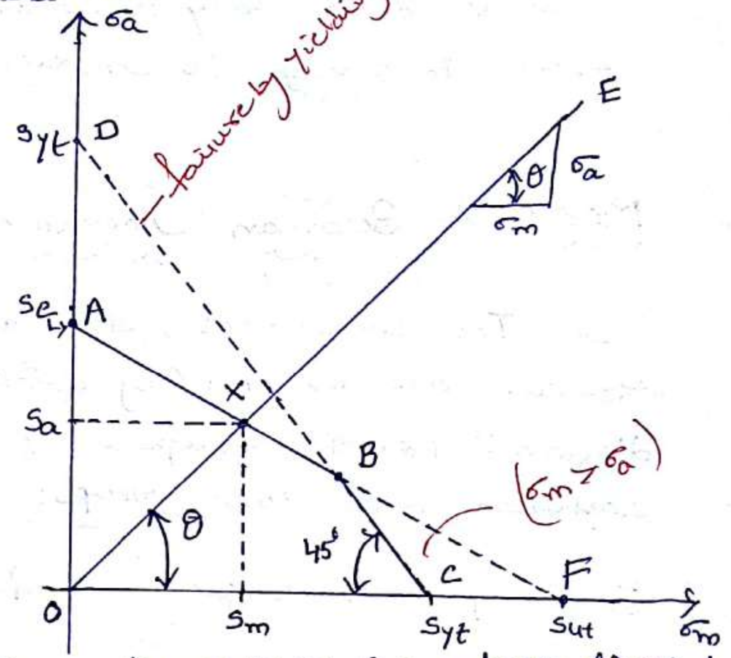


fig: Modified Goodman diagram for axial + Bending

This will result in failure irrespective of safety in fatigue failure. The portion $\overline{BF}$ of Goodman line is vulnerable portion and needs correction. This is the reason to modify the Goodman line

While solving a problem, a line $\overline{OE}$ with slope $\tan\theta$ is constructed in such a way that,

$$\tan\theta = \frac{\sigma_a}{\sigma_m}$$

$$\text{since} \quad \frac{\sigma_a}{\sigma_m} = \frac{(P_a/A)}{(P_m/A)} = \frac{P_a}{P_m}$$

$$\tan\theta = \frac{P_a}{P_m}$$

The magnitude of P_a and P_m can be determined from maximum and minimum force acting on the component.

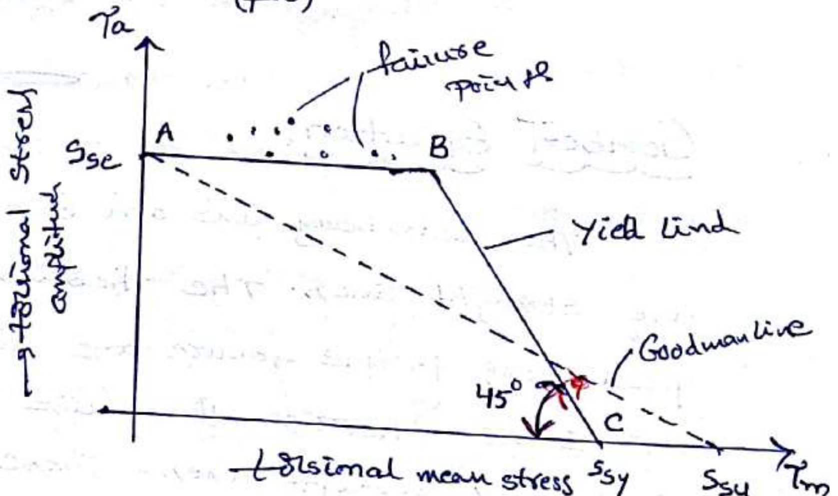
Similarly, it can be prooved that

$$\tan\theta = \frac{(M_b)_a}{(M_b)_m}$$

The magnitudes of $(M_b)_a$ and $(M_b)_m$ can be determined from maximum and minimum bending moment acting on the component.

The point of intersection of lines $\overline{AB}$ and $\overline{OE}$ is X. The point X indicates the dividing line b/w the safe region and the region of failure. The coordinates of point $X(S_m, S_a)$ represent the limiting values of stresses, that are used to calculate the dimensions of the component. The permissible stresses are as follows:

$$\sigma_a = \frac{S_a}{(f_s)} \quad \text{and} \quad \sigma_m = \frac{S_m}{(f_s)}$$



The modified Goodman diagram for fluctuating torsional shear stresses is shown in fig. In this diagram, torsional mean stress is plotted on abscissa while the torsional stress amplitude on ordinate. The torsional yield strength S_{sy} is plotted on abscissa and the yield line is constructed, that is inclined at 45° to the abscissa. It is interesting to note that,

fig: modified goodman diagram for torsional shear stress.

up to a certain point, torsional mean stress has no effect on torsional endurance limit. Therefore, a line is drawn through S_{se} on the ordinate and Parallel to the abscissa. The point of intersection of this line and yield line is $\underline{B}$. The area $\underline{OABC}$ represents the region of safety in this case. It is not necessary to construct fatigue diagram for fluctuating torsional shear stresses because $\overline{AB}$ is Parallel to X-axis. Instead a fatigue failure is indicated if

$$\tau_a = S_{se}$$

and a static failure is indicated if,

$$\tau_{max} = \tau_a + \tau_m = S_{sy}$$

The permissiable stresses are as follows:

$$\tau_a = \frac{S_{se}}{(f.s)}$$

and $$\tau_{max} = \frac{S_{sy}}{(f.s)}$$

Gerber Equation:

The soderberg line and Goodman line illustrated in fig are straight lines. The theories using such straight lines for predicting fatigue failure are called linear theories. There are some theories that use parabolic or elliptical curves instead of straight lines. These theories are called non-linear theories is Gerber theory that is based on parabolic curve. The Gerber curve is shown in fig. The equation for Gerber curve is as follows,

$$\frac{S_a}{S_e} + \left(\frac{S_m}{S_{ut}}\right)^2 = 1$$

The above eqn is called Gerber equation. It can be also written in the following form:

$$S_a = S_e\left[1 - \left(\frac{S_m}{S_{ut}}\right)^2\right]$$

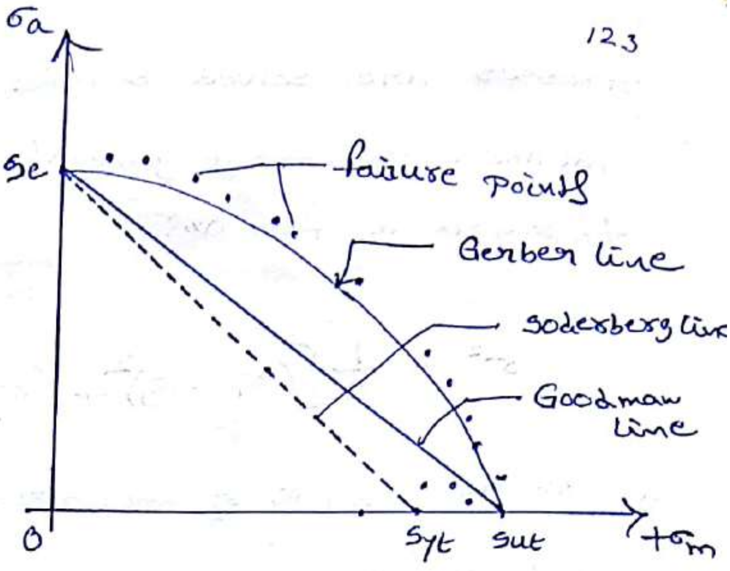


fig: Gerber line

Theories based on soderberg line or Goodman line, as failure criteria are conservative theories. This results in increased dimensions of the component. Gerber curve takes mean path through failure points. It is therefore more accurate in predicting fatigue failure.

Fatigue Design Under Combined stresses:

The problems discussed so far are based on the construction of the modified Goodman diagram for the component, that is subjected to either axial load or bending moment or torsional moment. Each type of loading is considered seperately. In practice, the problems are more complicated because the component may be subjected to two dimensional stresses, or to combined bending and torsional moments. In case of two-dimensional stresses, each of the two stresses may have two components - mean and alternating. similarly, the bending moment as well as torsional moment may have two components - mean and alternating. such problems involving combination of

Stresses are solved by the distortion-energy theory of failure. The most general equation of the distortion-energy theory is as follows:

$$\sigma^2 = \frac{1}{2}\left[(\sigma_x-\sigma_y)^2+(\sigma_y-\sigma_z)^2+(\sigma_z-\sigma_x)^2+6\left(\tau_{xy}^2+\tau_{yz}^2+\tau_{zx}^2\right)\right] \quad (a)$$

(a) where $\sigma_x, \sigma_y, \sigma_z$ are normal stresses in X, Y, and Z-directions and $\tau_{xy}, \tau_{yz}, \tau_{zx}$ are shear stresses in their respective planes. σ is a stress which is equivalent to these three dimensional stresses.

(b) In case of two-dimensional stresses, the component is subjected to stresses σ_x and σ_y in X and Y directions.

(c) substituting $\sigma_z = \tau_{xy} = \tau_{yz} = \tau_{zx} = 0$, in eqn (a)

$$\sigma = \sqrt{\left[(\sigma_x)^2-(\sigma_x\sigma_y)+(\sigma_y^2)\right]} \rightarrow (b)$$

The mean and alternating components of σ_x are σ_{xm} and σ_{xa} respectively. similarly, the mean and alternating components of σ_y are σ_{ym} and σ_{ya} respectively. In this analysis, the mean and alternating components are separately combined by eqn (b)

$$\sigma_m = \sqrt{\sigma_{xm}^2 - \sigma_{xm}\cdot\sigma_{ym} + \sigma_{ym}^2}$$

similarly,

$$\sigma_a = \sqrt{\sigma_{xa}^2 - \sigma_{xa}\cdot\sigma_{ya} + \sigma_{ya}^2}$$

The two stresses, σ_m and σ_a obtained by the above eqns are used in the modified Goodman diagram to design component.

In case of combined bending and torsional moments, there is a normal stress σ_x accompanid by the torsional shear stress τ_{xy}

Substituting $\sigma_y = \sigma_z = \tau_{yz} = \tau_{zx} = 0$ in eqn (a)

$$\sigma = \sqrt{\sigma_x^2 + 3\tau_{xy}^2} \longrightarrow (c)$$

The mean and alternating components of σ_x are σ_{xm} and σ_{xa} respectively. similarly, the mean and alternating components of τ_{xy} are τ_{xym} and τ_{xya} respectively. combining these components separately by eqn(c),

$$\sigma_m = \sqrt{\sigma_{xm}^2 + 3\tau_{xym}^2}$$

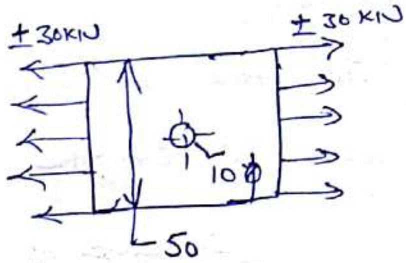
$$\sigma_a = \sqrt{\sigma_{xa}^2 + 3\tau_{xya}^2}$$

The two stresses σ_m ~~are~~ & σ_a obtained by the above equations are used in the modified Goodman diagram to design the component.

(1) A plate made of steel 20C8 ($S_{ut} = 440$ N/mm²) in hot rolled & normalized condition is shown in fig. It is subjected to a completely reversed axial load of 30 KN. The notch sensitivity factor q can be taken as 0.8 and the expected reliability is 90% The factor of safety is 2. The size factor can be taken as 0.85. Determine the plate thickness for infinite life.

Sol

S_e'



Relation-ship b/w endurance limit of specimen & Ultimate strength for steel.

$$S_e' = 0.5(S_{ut}) = 0.5(440) = 220\ N/mm^2$$

(Ex) A machine component is subjected to two-dimensional stresses. The tensile stress in the X direction varies from 40 to 100 N/mm² while the tensile stress in the Y direction varies from 10 to 80 N/mm². The frequency of variation of these stresses is equal. The corrected endurance limit of the component is 270 N/mm². S_{ut} = 660 N/mm², Determine the factor of safety used by the designer.

(2) The work cycle of a mechanical component subjected to completely reversed bending stresses consists of the following three elements:

(i) ± 350 N/mm² for 85% of time

(II) ± 400 N/mm² for 12% of time

(III) ± 500 N/mm² for 3% of time.

The material for the component is 50C4

($S_{ut} = 660$ N/mm²) and the corrected endurance limit of the component is 280 N/mm². Determine the life of the component.

Sol

$$0.9 S_{ut} = 0.9(660) = 594 \text{ N/mm}^2$$

$$\log_{10}(0.9 S_{ut}) = \log_{10}(594) = 2.7738$$

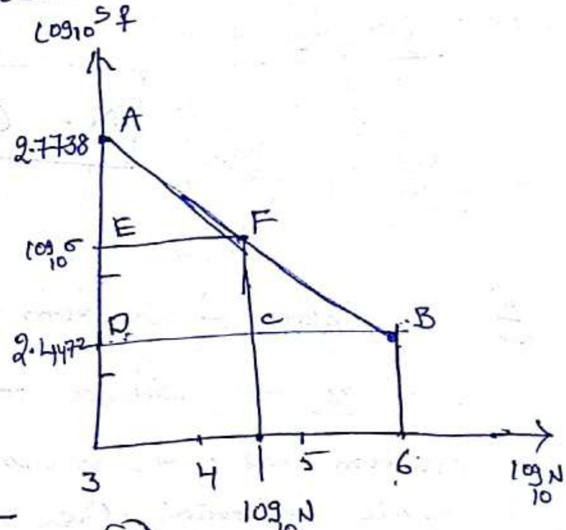
$$\log_{10}(S_e) = \log_{10}(280) = 2.4472$$

$$\log_{10}(\sigma_1) = \log_{10}(350) = 2.5441$$

$$\log_{10}(\sigma_2) = \log_{10}(400) = 2.6021$$

$$\log_{10}(\sigma_3) = \log_{10}(500) = 2.6990$$

The S-N curve for this problem



$$\frac{\overline{EF}}{\overline{DB}} = \frac{\overline{AE}}{\overline{AD}}$$

$$\overline{EF} = \frac{\overline{AE} \times \overline{DB}}{\overline{AD}}$$

$$\overline{EF} = \frac{(2.7738 - \log_{10}\sigma)(6-3)}{(2.7738 - 2.4472)} \rightarrow (a)$$

$$= 9.1855(2.7738 - \log_{10}\sigma)$$

and $\log_{10} N = 3 + \overline{EF} \rightarrow (b)$

from (a) & (b)

$$\log_{10} N = 3 + (2.7738 - \log_{10}\sigma)\, 9.1855$$

Therefore $\log_{10} N_1 = 3 + 9.1855(2.7738 - 2.5441)$

$$N_1 = 128798.$$

$$\log_{10} N_2 = 3 + 9.1855(2.7738 - 2.6021)$$

$$N_2 = 37770$$

$$\log_{10}(N_3) = 3 + 9.1855\,(2.7738 - 2.6990)$$

$$\boxed{N_3 = 4865}$$

<u>Miner's</u> <u>equation</u>

$$\frac{\alpha_1}{N_1} + \frac{\alpha_2}{N_2} + \frac{\alpha_3}{N_3} = \frac{1}{N}$$

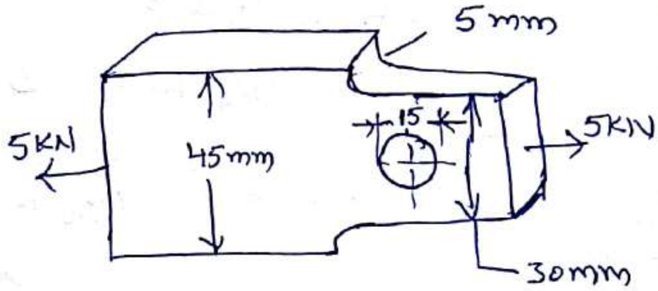
$$\frac{0.85}{128798} + \frac{0.12}{37770} + \frac{0.03}{4865} = \frac{1}{N}$$

$$\boxed{N = 62723} \text{ cycles}$$

(2) A plate of uniform thickness 't' has two widths of 45 mm and 30 mm with a fillet radius of 5 mm. The smaller width portion has a transverse hole of 15 mm diameter. for the plate material the ultimate tensile strength is 200 N/mm². considering stress concentration effect, and assuming a factor of safety of 2.5, find the thickness of plate for a maximum tensile load of 5 kN.

Sol Larger width of plate

$b_1 = 45$ mm



Smaller width of plate

$b_2 = 30$ mm

Thickness of plate $= t$ mm.

Fillet Radius, $r = 5$ mm

Dia of hole $d = 15$ mm.

ultimate strength $S_{ut} = 200\ N/mm^2$

factor of safety $f_s = 2.5$

Tensile load $P = 5\ KN = 5000\ N$

In general, more stress is induced in smaller width plate due to its smaller area of cross-section, especially across hole.

Nominal stress induced in the cross-section across hole,

$$\sigma_0 = \frac{P}{A} = \frac{5000}{(b-d)t} = \frac{5000}{(30-15)t} = \frac{333.3}{t}\ N/mm^2$$

Now $\frac{d}{b_2} = \frac{15}{30} = 0.5$ and $K_t = 2.17$, from J.BD page ~~304~~ 4.50

table ~~graph~~ no ~~304~~ 4.15

∴ Maximum stress induced in fillet section

$$\sigma_{max} = K_t \times \sigma_0 = 2.16 \times \frac{333.33}{t} = \frac{720}{t}\ N/mm^2$$

Now consider smaller width at fillet section:

Nominal stress induced in fillet section,

$$\sigma_0 = \frac{P}{b \cdot t} = \frac{5000}{30 \times t} = \frac{166.7}{t}\ N/mm^2$$

stress concentration factor K_t {for $\frac{b_1}{b_2} = \frac{45}{30} = 1.5$ &

$\frac{r}{b_2} = \frac{5}{30} = 0.17$) is

$K_t = 1.8$ from JBD page 4.40

graph no 4.11

Maximum stress induced $= K_t \times \sigma_0 = \frac{1.8 \times 166.7}{t} = \frac{300}{t}\ N/mm^2$

Hence more stress ~~concen~~ induced in the cross-section across hole which is given by

$$\sigma_{max} = \frac{720}{t}\ N/mm^2$$

This maximum stress should be less than permissible tensile strength for safe design.

$$\text{i.e} \quad \frac{720}{t} \leq \frac{S_u}{FS}$$

$$\text{i.e} \quad \frac{720}{t} < \frac{200}{2.5}$$

$$t > \frac{720 \times 2.5}{200} > 9\,mm$$

$$= 10\,mm \text{ (say)}$$

Thickness of the plate $= \underline{\underline{10\,mm}}$

(3) A plate made of steel 20 C8 ($S_{ut} = 440\ N/mm^2$) in hot-rolled and normalized condition is shown in fig. It is subjected to a completely reversed axial load of 30KN. The notch sensitivity factor q can be taken as 0.8 and the expected reliability is 90%. The factor of safety is 2. The size factor can be taken as 0.85. Determine the plate thickness for infinite life.

Sol

for steel. $S_e' = 0.5\, S_{ut} = 0.5(440) = 220\ N/mm^2$

from ~~graph~~ exponential ~~Ka = 0.67~~.

$$K_a = a(S_{ut})^b = 57.7\,(440)^{-0.718}$$

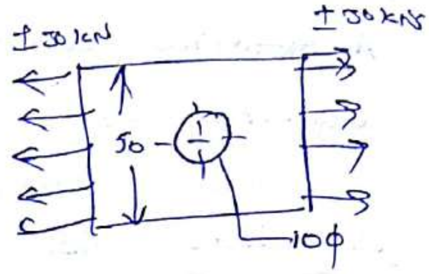
$$\boxed{K_a = 0.729}$$

size factor $K_b = 0.85$

for 90% Reliability. $K_c = 0.897$

factor of safety $^{K_e} = \underline{2}$

$\frac{d}{w} = \frac{10}{50} = 0.2$



$K_f = 1 + q(K_t - 1)$

from ~~fo~~ J.D.B, page 4.50, table 4.15

$\frac{d}{w} = 0.2, \quad K_t = 2.50$

~~K_f~~ $K_d = \frac{1}{K_f}$,

q given $= 0.8$

$K_f = 1 + 0.8(2.50 - 1) = 2.2$

$K_d = \frac{1}{2.2} = 0.4545$

$S_e = K_a K_b K_c K_d . S_e'$

$= 0.729 \times 0.85 \times 0.897 \times 0.4545 (220)$

$= 55.582 \; N/mm^2$

$\sigma_a = \frac{S_e}{fs} = \frac{55.582}{2} = 27.79 \; N/mm^2$

Also $\quad \sigma_a = \frac{P}{(w-d)t} = \frac{30 \times 10^3}{(50-10)t}$

$27.79 = \frac{30 \times 10^3}{40 t}$

$t = \frac{30000}{1111.65}$

$\boxed{t = 26.98 \; mm}$

(132)

(4) A rotating bar made of steel 45C8 ($S_{ut} = 630\ N/mm^2$) is subjected to a completely reversed bending stress. The corrected endurance limit of the bar is $315\ N/mm^2$. Calculate the fatigue strength of the bar for a life of 90,000 cycles.

Sol

$S_{ut} = 630\ N/mm^2$, $S_e = 315\ N/mm^2$

$N = 90{,}000$ cycles

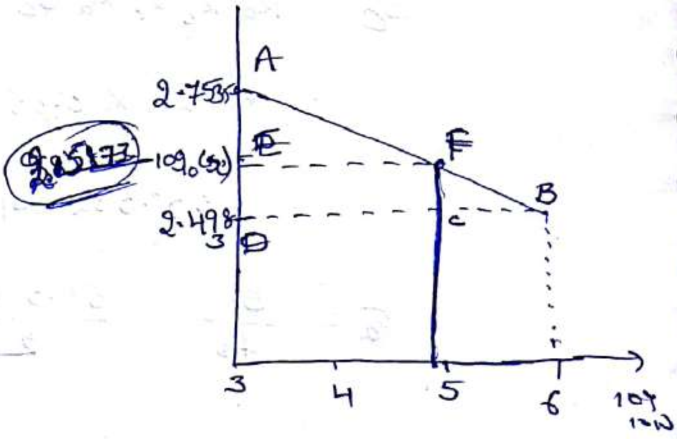
$S_e' = 0.5(S_{ut}) = 0.5(630) = 315\ N/mm^2$

$0.9\, S_{ut} = 0.9(630) = 567\ N/mm^2$

$\log_{10}(S_{ut}) = \log_{10}(567) = 2.7535$

$\log_{10}(S_e') = \log_{10}(315) = 2.4983$

$\log_{10}(90{,}000) = 4.9542$



$$\frac{EF}{DB} = \frac{AE}{AD}$$

$$\cancel{EF = \frac{AE \times DB}{AD}}$$

$$AE = \frac{EF \times AD}{DB}$$

$$AE = \frac{(4.9542-3)(2.7535-2.498)}{(6-3)}$$

$$(2.7535 - \log S_f) = \frac{(1.9542)(0.2555)}{3}$$

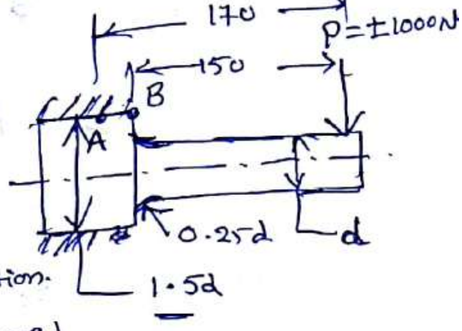
$$2.7535 - \log_{10} S_f = 0.1664327$$

$$\log_{10}(S_f) = 2.7535 - 0.166327$$

$$\log_{10}(S_f) = 2.5873$$

$$\boxed{S_f = 386.63 \ N/mm^2}$$

(5) A cantilever beam made of cold drawn steel. 20C8 ($S_{ut} = 540 \ N/mm^2$) is subjected to a completely reversed load of 1000 N as shown in fig. The notch sensitivity factor q at the fillet can be taken as 0.85 and the expected reliability is 90%. Determine the diameter d of the beam for a life of 10,000 cycles.



Sol: The failure will occur either at section A or at section B. At section A, although the bending moment is maximum, there is no stress concentration and the diameter is also more compared with that of section B. It is therefore, assumed that the failure will occur at section B.

$$S_e' = 0.5(S_{ut}) = 0.5(540) = 270 \ N/mm^2$$

$$K_a = a(S_{ut})^b = 4.51(540)^{-0.265}$$

$$= 0.85$$

Assuming $7.5 \le d < 50$

$$K_b = 0.85$$

for 90% Reliability $(K_c) = 0.897$

At section B, $\left(\frac{D}{d}\right) = \frac{1.5}{d} = 1.5$

$$\frac{r}{d} = \frac{0.25d}{d} = 0.25$$

from JDB page 4.47, table 4.10

$$\frac{D}{d} = 1.5, \quad \frac{r}{d} = 0.25$$

$$\boxed{K_t = 1.36}$$

$K_f = 1 + q(K_t - 1)$

$= 1 + 0.85(1.36 - 1)$

$\boxed{K_f = 1.306}$

$K_d = \frac{1}{K_f} = \frac{1}{1.306} = 0.7656$

$S_e = K_a K_b K_c K_d S_e'$

$= 0.85 \times 0.85 \times 0.897 \times 0.7656 (270)$

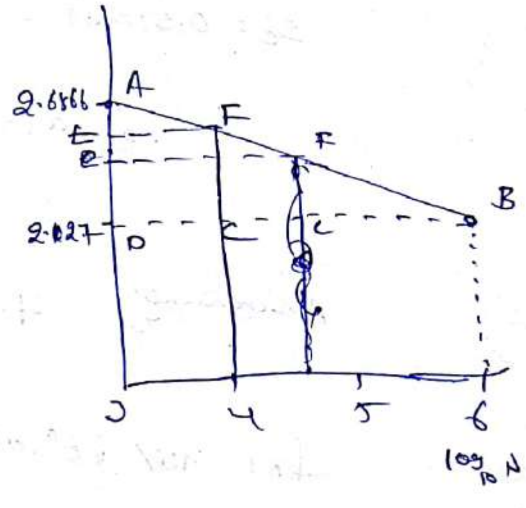
$S_e = 133.983 \ N/mm^2$

$\log_{10}(0.9 \, S_{ut}) = \log_{10}(0.9 \times 540) = 2.6866$

$\log_{10}(S_e) = \log_{10}(133.983) = 2.127$

$\log_{10}(10{,}000) = 4$

The S-N curve for this problem.



$\frac{AE}{AD} = \frac{EF}{DB}$

$AE = \frac{EF \times AD}{DB}$

$= \frac{(4-3)(2.6866 - 2.127)}{(6-3)}$

$AE = 0.1865$

~~$\log_{10}(S_f) + 3 = \log_{10} N$~~ $\quad 2.6866 - \log_{10}(S_f) = AE$

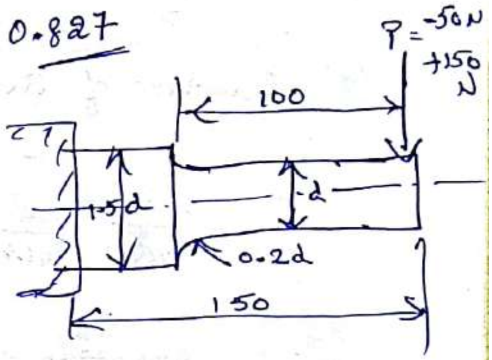
$\log_{10}(S_f) = 2.6866 - 0.1865 = 2.5000$

$S_f = 316.22 \ N/mm^2$

$S_f = \sigma_b = \frac{32 M_b}{\pi d^3} = \quad d^3 = \frac{32 M_b}{\pi \sigma_b} = \frac{32(1000 \times 150)}{\pi (316.22)}$

$d^3 = 1610, \quad d = 11.72$

(6) A Cantilever beam made of cold drawn steel 40C8 (S_{ut} = 600 N/mm² and S_{yt} = 380 N/mm²) is shown in fig. The force P acting at the free end varies from −50 N to +150 N. The expected reliability is 90% and the factor of safety is 2. The notch sensitivity factor at the fillet is 0.9. Determine the diameter d of the beam at the fillet cross-section.



sol

$$S_e' = 0.5\, S_{ut} = 0.5(600) = 300 \text{ N/mm}^2$$

$$K_a = a(S_{ut})^b = 4.51(600)^{-0.265} = 0.827$$

Assuming $7.5 < d < 50$ mm

$$K_b = 0.85$$

For 90% Reliability $K_c = 0.897$

$$\because \frac{r}{d} = \frac{0.2d}{d} = 0.2, \quad \frac{D}{d} = \frac{1.5d}{d} = 1.5$$

from Jad D.B, page no. 4-33, table 4.4

$$\frac{r}{d} = 0.2, \quad \frac{D}{d} = 1.5, \quad K_t = 1.32$$

$$K_f = 1 + q(K_t - 1) = 1 + 0.9(1.32 - 1) = 1.288$$

$$K_d = \frac{1}{K_f} = \frac{1}{1.288} = 0.776.$$

$$S_e = K_a \cdot K_b \cdot K_c \cdot K_d\, S_e'$$

$$= 0.827 \times 0.85 \times 0.897 \times 0.776 \times 300$$

$$= 146.86 \text{ N/mm}^2$$

At the fillet cross-section

$$(M_b)_{max} = 150 \times 100 = 15000 \text{ N-mm}$$

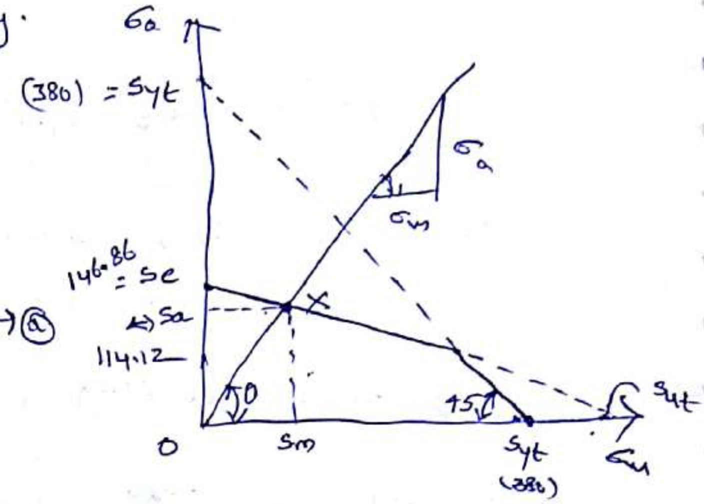
$$(M_b)_{min} = -50 \times 100 = -5000 \text{ N-mm}$$

$$(M_b)_m = \frac{1}{2}\left[(M_b)_{max} + (M_b)_{min}\right] = \frac{1}{2}\left[15000 - 5000\right] = 5000 \text{ N-mm}$$

$$(M_b)_a = \frac{1}{2}\left[(M_b)_{max} - (M_b)_{min}\right] = \frac{1}{2}\left[15000 + 5000\right] = 10{,}000 \text{ N-mm}$$

$$\tan\theta = \frac{(M_b)_a}{(M_b)_m} = \frac{10000}{5000} = 2, \quad \boxed{\theta = 63.435^\circ}$$

The modified Goodman diagram for this example is shown below. The co-ordinates of point X are determined by solving the following two equations simultaneously.



(I) Equation of line AB

$$\frac{Sa}{146.86} + \frac{Sm}{600} = 1 \rightarrow (a)$$

(II) Equation of line OX

$$= \frac{Sa}{Sm} = \tan\theta = 2 \Rightarrow \boxed{Sa = 2S_m}$$

solving the two equations,

$Sa = 114.12, \quad Sm = 57.06$

$$\sigma_a = \frac{Sa}{fs} \Rightarrow \frac{32(M_b)a}{\pi d^3} = \frac{Sa}{fs}$$

$$\frac{32(10{,}000)}{\pi d^3} = \frac{114.12}{2}$$

$$\boxed{d = 12.13 \text{ mm}}$$

(4) A machine component is subjected to fluctuating that varies from 40 to 100 N/mm². The corrected endurance limit stress for the machine component is 270 N/mm². The ultimate tensile strength & yield strength of material are 600 & 450 N/mm² respectively. Find the factor of safety using:

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(I) Gerber theory (II) Soderberg line (III) Goodman line; and
(IV) Also, find factor of safety against static failure.

Sol

$\sigma_{max} = 100\ N/mm^2, \quad \sigma_{min} = 40\ N/mm^2, \quad S_e = 270\ N/mm^2$

$\sigma_a = \frac{1}{2}(100-40) = 30\ N/mm^2, \quad \sigma_m = \frac{1}{2}(100+40) = 70\ N/mm^2$

$S_{ut} = 600\ N/mm^2$

$\sigma_a = \frac{S_a}{f_s} \Rightarrow S_a = \sigma_a \cdot f_s, \quad \sigma_m = \frac{S_m}{f_s} \Rightarrow S_m = \sigma_m \cdot f_s$

$S_a = 30 f_s, \quad S_m = 70 f_s$

(I) from Gerber equation

$$\frac{S_a}{S_e} + \left(\frac{S_m}{S_{ut}}\right)^2 = 1 \Rightarrow \frac{30 f_s}{270} + \left(\frac{70 f_s}{600}\right)^2 = 1$$

$$\frac{30 f_s}{270} + \frac{490 f_s^2}{360000} = 1. \qquad \cancel{734.693}$$

$$0.1111 f_s + 1.361111 \times 10^{-3} f_s^2 - 1 = 0$$

$$f_s^2 + 81.63 f_s - 756.944 = 0$$

solving the above quadratic equation

$$\boxed{f_s = 5.41}$$

(II) The eqn ~~of~~ soderberg line is as follows,

$$\frac{S_a}{S_e} + \frac{S_m}{S_{yt}} = 1 \Rightarrow \left(\frac{30n}{270}\right) + \left(\frac{70n}{450}\right) = 1$$

$$\boxed{n = 3.75}$$

(III) The eqn of Goodman line is as follows:

$$\frac{S_a}{S_e} + \frac{S_m}{S_{ut}} = 1.$$

$$\left(\frac{30n}{270}\right) + \left(\frac{70n}{600}\right) = 1 \qquad \boxed{n = 4.39}$$

(IV) The factor of safety against static failure is given by,

$$\sigma_{max} = \frac{S_{yt}}{f_s} \quad f_s = \frac{S_{yt}}{\sigma_{max}} = \frac{450}{100} = 4.5$$

(8) (130) A ~~machine~~ transmission shaft carries a pulley midway b/w the two bearings. The bending moment at the pulley varies from 200 N-m to 600 N-m, as the torsional moment in the shaft varies from 70 N-m to 200 N-m. The frequencies of variation of bending and torsional moments are equal to the shaft speed. The shaft is made of steel FeE 400 ($S_{ut} = 540\ N/mm^2$ and $S_{yt} = 400\ N/mm^2$). The corrected endurance limit of the shaft is $200\ N/mm^2$. Determine the diameter of the shaft using a factor of safety of 2.

sol

$$(M_b)_m = \frac{1}{2}\left[(M_b)_{max} + (M_b)_{min}\right] = \frac{1}{2}\left[600+200\right] = 400\ N\text{-}m.$$

$$(M_b)_a = \frac{1}{2}\left[(M_b)_{max} - (M_b)_{min}\right] = \frac{1}{2}\left[600-200\right] = 200\ N\text{-}m.$$

$$(M_t)_m = \frac{1}{2}\left[(M_t)_{max} + (M_t)_{min}\right] = \frac{1}{2}\left[200+70\right] = 135\ N\text{-}m$$

$$(M_t)_a = \frac{1}{2}\left[(M_t)_{max} - (M_t)_{min}\right] = \frac{1}{2}\left[200-70\right] = 65\ N\text{-}m.$$

$$\sigma_{xm} = \frac{32(M_b)_{max}}{\pi d^3} = \frac{32(400\times10^3)}{\pi d^3} = \left(\frac{4074.37\times10^3}{d^3}\right) N/mm^2$$

$$\sigma_{xa} = \frac{32(M_b)_a}{\pi d^3} = \frac{32(200\times10^3)}{\pi d^3} = \left(\frac{2037.18\times10^3}{d^3}\right) N/mm^2$$

$$\tau_{xym} = \frac{16(M_t)_m}{\pi d^3} = \frac{16(135\times10^3)}{\pi d^3} = \left(\frac{687.55\times10^3}{d^3}\right) N/mm^2$$

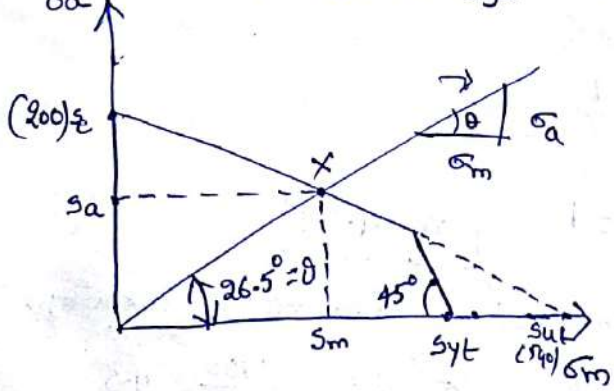
$$\tau_{xya} = \frac{16(M_t)_a}{\pi d^3} = \frac{16(65\times10^3)}{\pi d^3} = \left(\frac{331.04\times10^3}{d^3}\right) N/mm^2$$

$$\sigma_m = \sqrt{\sigma_{xm}^2 + 3\tau_{xym}^2} = \left(\frac{4244.84\times10^3}{d^3}\right) N/mm^2$$

$$\sigma_a = \sqrt{\sigma_{xa}^2 + 3\tau_{xya}^2} = \left(\frac{2116.33\times10^3}{d^3}\right) N/mm^2$$

$\tan\theta = \frac{\sigma_a}{\sigma_m} = 0.4986$

$\boxed{\theta = 26.5^\circ}$



$$\frac{S_a}{200} + \frac{S_m}{540} = 1, \quad \frac{S_a}{S_m} = 0.4988$$

$S_a = 114.76\ N/mm^2,\ S_m = 230.16\ N/mm^2$

$$\therefore \sigma_a = \frac{S_a}{f_s} = \frac{2116.33\times10^3}{d^3} = \frac{114.76}{2}$$

$\boxed{d = 33.29\ mm}$

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UNIT–II

Riveted Joints

Introduction :- A Rivet Joint is a short cylindrical metal pin (bar) with a head integral to it. The cylindrical portion of the rivet is called shank or body and lower portion of shank is known as tail. They are made of tough and ductile low carbon steel or nickel steel.

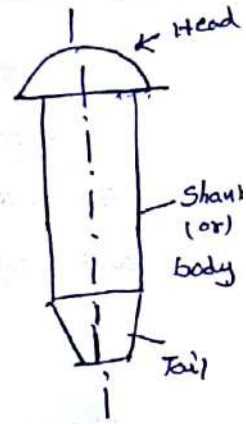


fig: Rivet

For light loads, sometimes, the rivets made of copper, aluminium or brass can be employed.

Although there are many ways of classifying riveted joints, the first distinction is b/w axially loaded & eccentrically loaded rivets. In axially loaded riveted joints, the line of action of applied force passes through the centre of Gravity of the rivet Group, whereas for the eccentrically loaded connection it does not.

Some of the applications of rivets are, such as, in boilers, buildings, bridges, ships and so on. Among the above applications, some are axially loaded like boilers, ships etc. and some other are eccentrically loaded like structures of buildings and bridges.

The riveted joints are widely used for joining light metals.

The fastenings (i.e Joints) may be classified into the following two groups:

1. Permanent fastenings, and
2. Temporary or detachable fastenings.

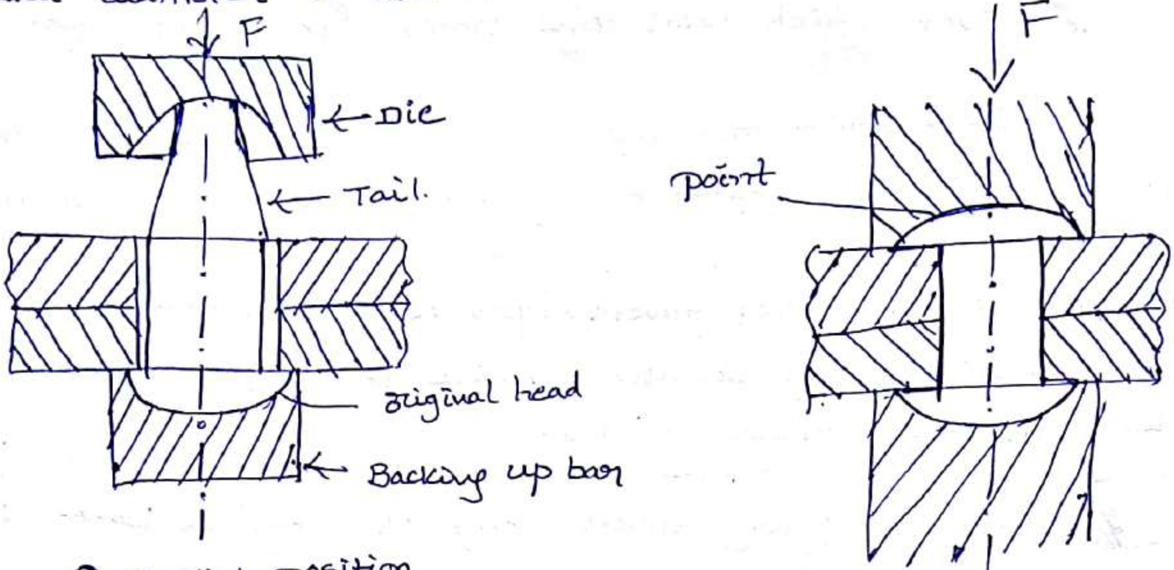
The Permanent fastenings are those fastenings which can not disassembled without destroying the connecting components. The examples of Permanent fastenings in order of strength are soldered, brazed, welded and riveted Joints.

The temporary or detachable fastenings are those fastenings which can be disassembled without distroy-ing the connecting components. The examples of temporary fastenings are screwed, keys, cotters, pins, and splined Joints.

Methods of Riveting (or) function of Riveting:

The function of rivets in a Joint is to make a connection that has strength and tightness. The strength is necessary to prevent failure of the Joint. The tightness is necessary in order to contribute to strength and to prevent leakage as in a boiler or in a ship hull.

When two plates are to be fastened (Joined) together by a rivet as shown below fig(a), the holes in the plates are punched and reamed or drilled. punching is the cheapest method and is used for relatively thin plates and in structural work. since punching injures the material around the hole, therefore drilling is used in most pressure vessel work. In structural and pressure vessel riveting, the diameter of the rivet hole is usually 1.5 mm larger than the nominal diameter of the rivet.



(a) Initial position

(b) final position.

fig: Methods of Riveting

The plates are drilled together and then separted to remove any burrs or chips so as to have a tight flush Joint b/w the plates. A cold rivet or a red hot rivet is introduced into the plates and the point (i.e. second head) is then formed. when a cold rivet is used, the process is known as cold riveting and when a hot rivet is used, the process is known as hot riveting. The cold riveting process is used for structural Joints while hot riveting is used to make leak proof Joints.

The riveting may be done by hand or by a riveting machine. In hand riveting, the original rivet head is backed up by a hammer or heavy bar and then the die or set, as shown above fig(a). is placed against the end to be headed and the blows are applied by a hammer. This causes the shank to expand thus filling the hole and the tail is converted into a point as shown above fig (b). As the rivet cools, it tends to contract. The lateral contraction will be slight, but there will be a longitudinal tension introduced in the rivet which holds the plates firmly together.

In machine riveting, the die is a part of the ~~ha~~ hammer which is operated by air, hydraulic or steam pressure.

Notes: 1. For steel rivets upto 12mm diameter, the cold riveting process may be used, while for larger diameter rivets, hot riveting process is used.

2. In case of long rivets, only the tail is heated and not the whole shank.

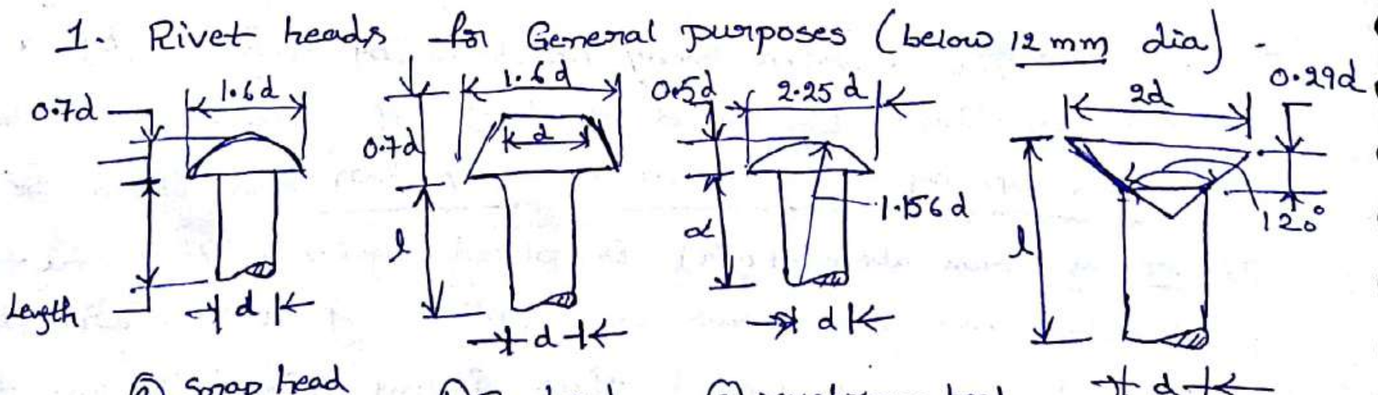
Essential Qualities of a Rivet

According to Indian standard, IS:2998-1982, the material of a rivet must have a tensile strength not less than 40 N/mm^2 and elongation not less than 26 percent

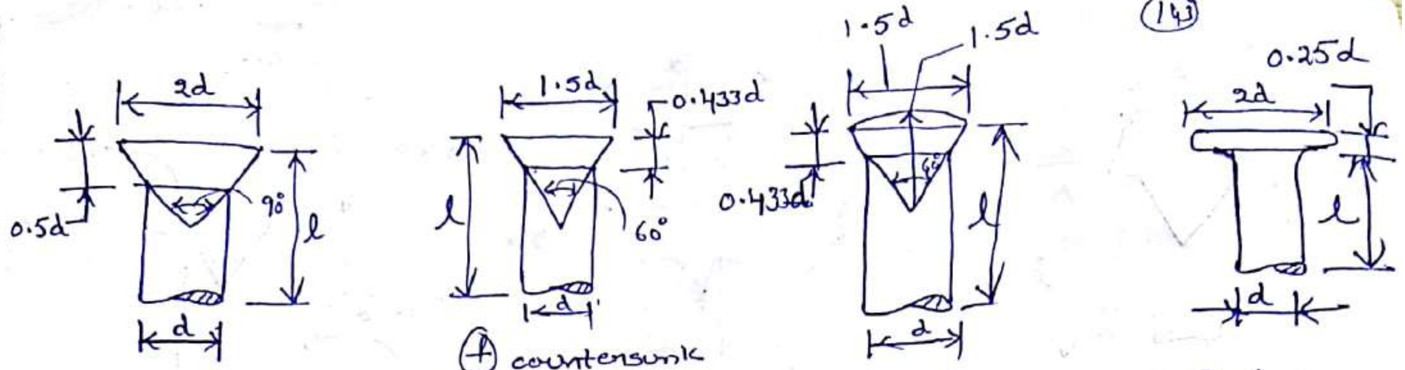
Types of Rivet Heads:

According to Indian standard specifications, the rivet heads are classified into the following three types:

1. Rivet heads for General purposes (below 12 mm dia).



(a) Snap head (b) Pan head (c) Mushroom head (d) countersunk head 120°



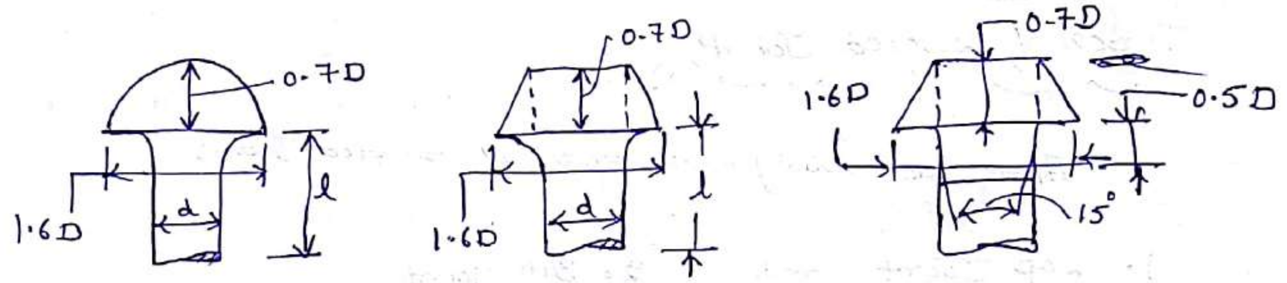
(e) countersunk head 90°

(f) countersunk head 60°

(g) Round counter sunk head 60°

(h) flat head

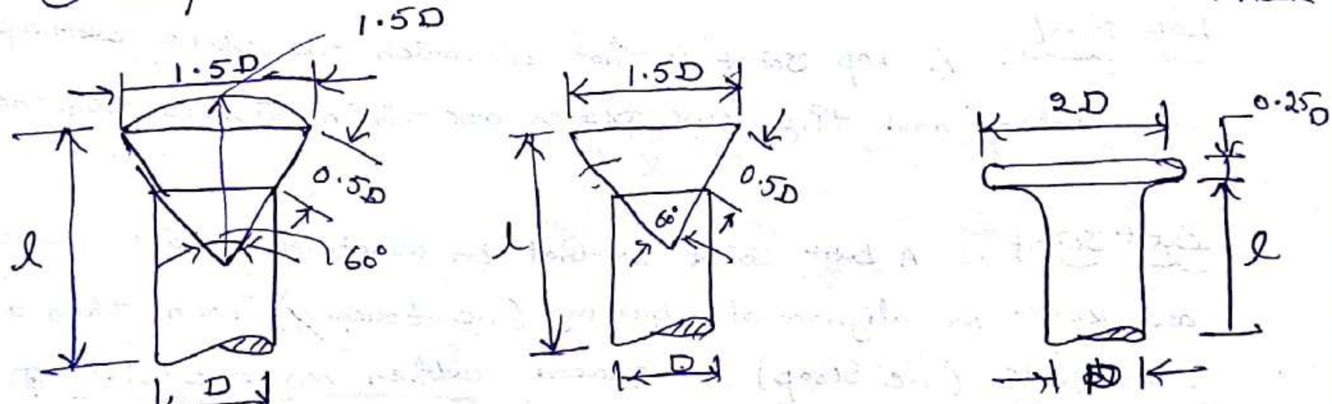
(2) Rivet head for general purposes (from 12mm to 48mm dia)



(a) Snap head

(b) Pan head

(c) Pan head with tapered neck

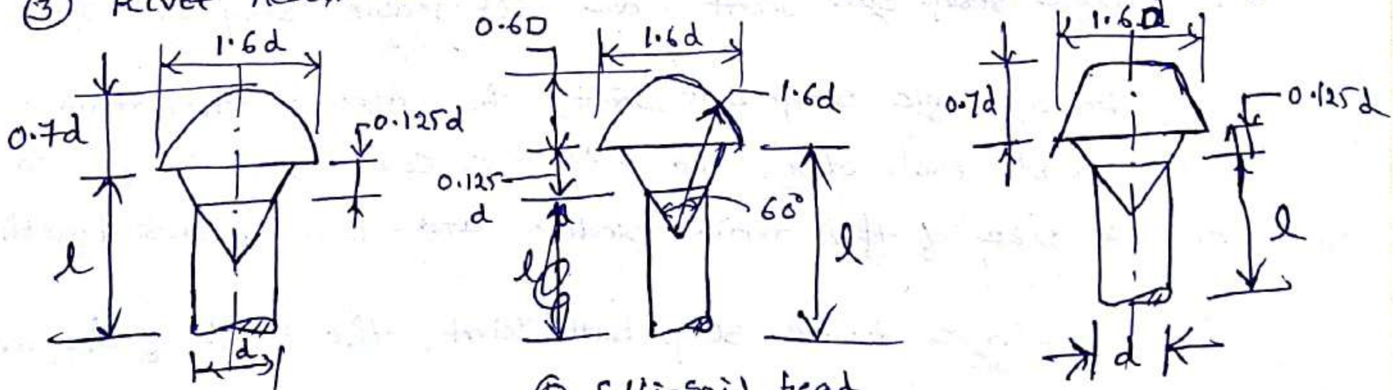


(d) Rounded countersunk head

(e) Flat countersunk head

Flat head.

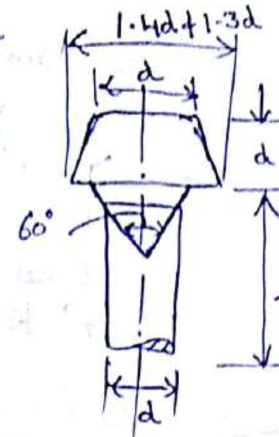
(3) Rivet heads for boiler work (from 12mm to 48mm dia)

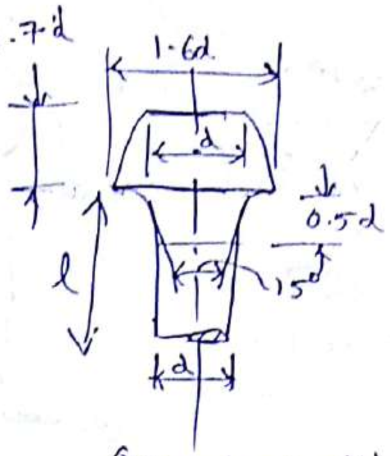


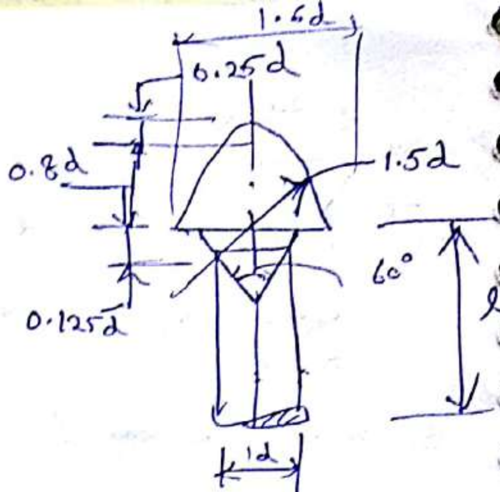
(a) Snap head

(b) Ellipsoid head

(c) Pan head (Type 1)







(d) 1.4 d for rivets under 24mm (e) Pan head with tapered neck. (f) steeple head

Types of Riveted Joints:

There are mainly two types of riveted Joints

1. Lap Joint, and 2. Butt Joint

Lap Joint: A lap Joint is that in which one plate overlaps the other and the two plates are then riveted together.

Butt Joint:- A butt joint is that in which the main plates are kept in alignment butting (i.e. touching) each other and a cover plate (i.e strap) is placed either on one side or on both sides of the main plates. The cover plate is then Riveted together with the main plates. Butt Joints are the following two types:

1. Single strap butt Joint, and (2) Double strap butt Joint

In a Single strap butt Joint, the edges of the main plates butt against each other and only one cover plate is placed on one side of the main plates and then riveted together.

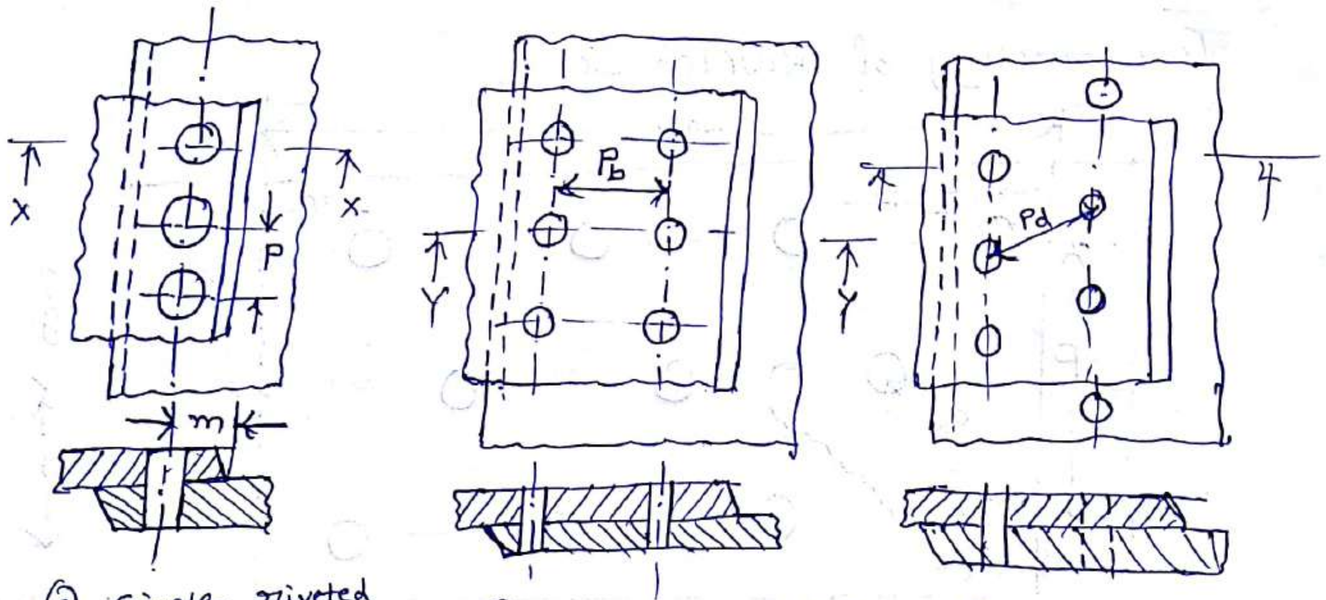
In a double strap butt Joint, the edges of the main plates butt against each other and two cover plates are placed on both sides of the main plates and then riveted together.

In addition to the above, following are the types of riveted Joints depending upon the number of rows of the rivets

1. Single reveted Joint, and (2) Double riveted Joint.

A single riveted Joint is that in which there is a single row of rivets in a lap Joint as shown in fig (a) below and there is a single row of rivets on each side in a butt Joint is shown in fig (d)

A double riveted Joint is that in which there are two rows of rivets in a lap Joint as shown below fig (b) & (c) and there are two rows of rivets on each side in a butt Joint is shown in fig (e)



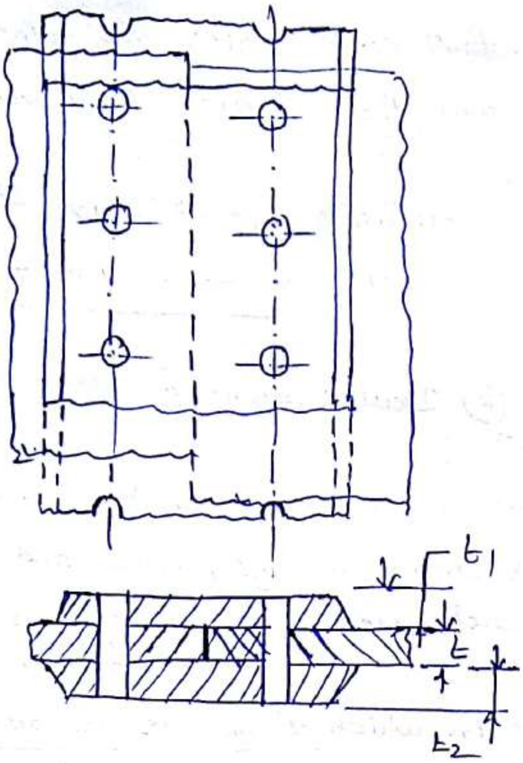
(a) Single riveted Lap Joint

(b) Double riveted Joint (chain riveting)

(c) Double riveted lap Joint (Zig-zag riveting)

fig: Single and double riveted lap Joint.

Similarly the Joints may be triple riveted or quadruple riveted.



(d) Sing riveted double strap Joint

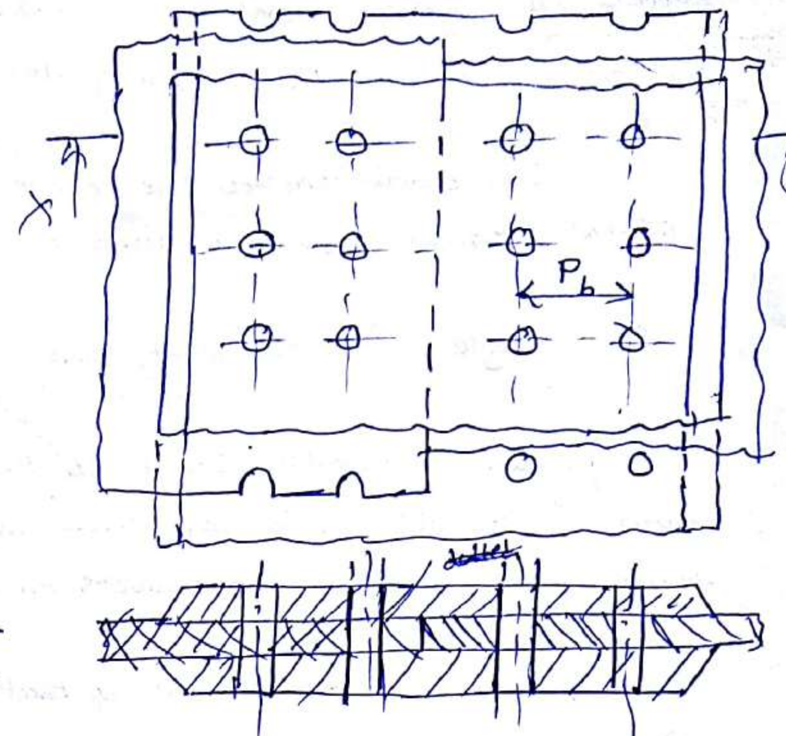


fig Double riveted double strap butt Joints

Terminology of Riveted Joint:

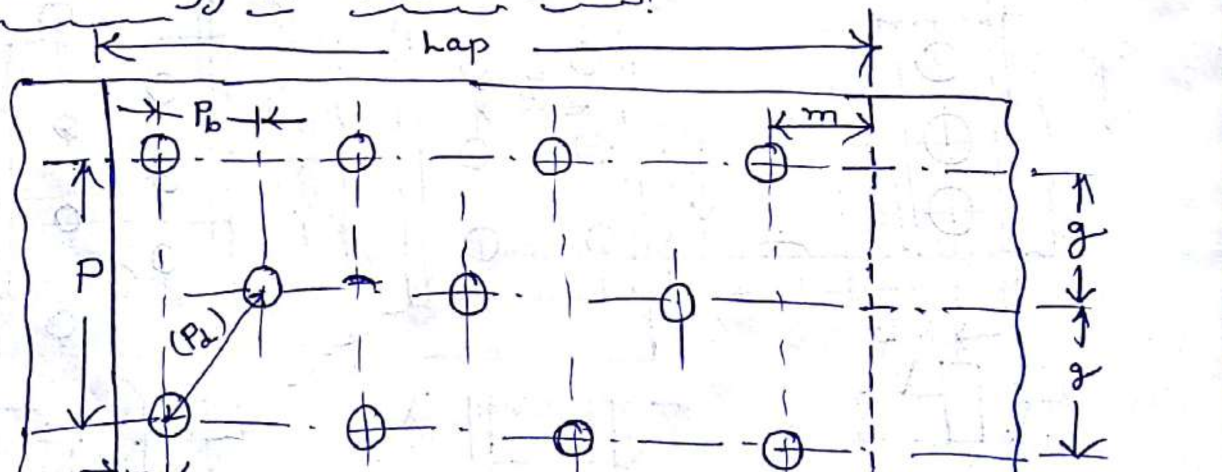


fig:- Terminology of a Riveted Joint

Pitch (P):- It is the distance between the centres of two consecutive rivets, arranged in the outer-most row, which is generally parallel to the seam.

Back pitch or transverse pitch :- (P_b)

It is the perpendicular distance between the succesive rows.

Diagonal pitch (P_d) :

It is the distance between the centres of two rivets arranged in successive rows of zig-zag riveted Joint.

Gauge line : It is the line of rivets which is parallel to the direction of stress.

Gauge distance (g) :- It is the perpendicular distance between two adjacent gauge lines. The maximum value of gauge distance is known as pitch (P), which is in the outermost row.

Margin, (m) It is the distance of the edge of member or cover plates from the nearest rivet hole.

Nominal diameter (d')

It is the standard diameter of the rivet. It is the diameter when the rivet is cold before use in the Joint.

Gross diameter (d) :

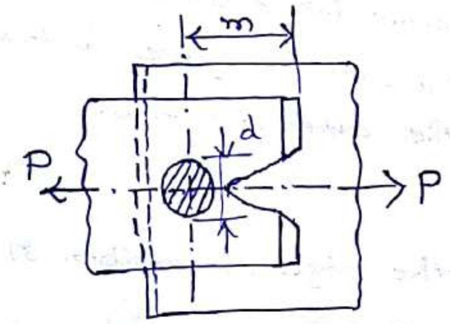
It is the diameter of the rivet in the hole after Joining and is, therefore, taken as diameter of hole itself.

Lap : It is the distance normal to the Joint b/w edges of the overlapping plates in a lap Joint or b/w the Joint and the end of cover plates in a butt Joint.

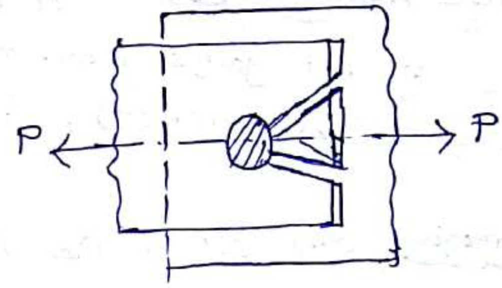
Modes of failure of a Riveted Joint and their ~~properties~~ Remedies

When the riveted Joint is subjected to axial load, the Joint may fail in the following modes.

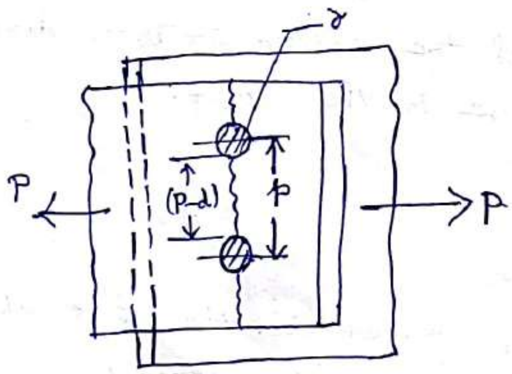
1. Tearing of the edge of plate near the rivet hole
2. Rupture or tearing of the plate at the section weakened by holes.
3. Shearing of rivets
4. Crushing of rivets.



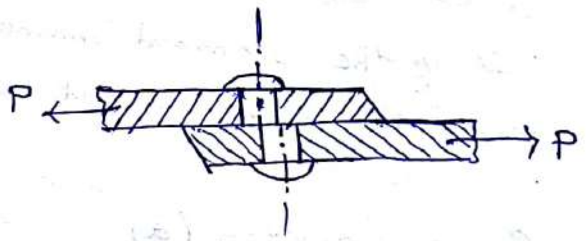
(a) cracking (or) Tearing of plate



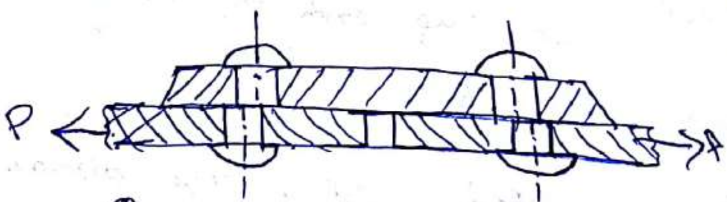
(b) shearing of plate at the edge



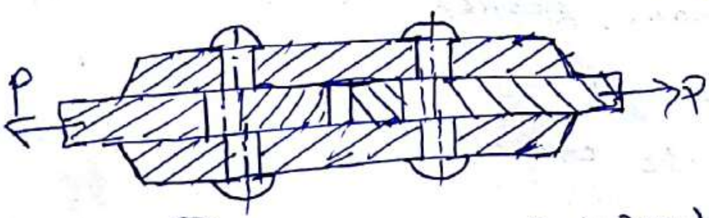
(c) Tearing of plates across the rows of rivets



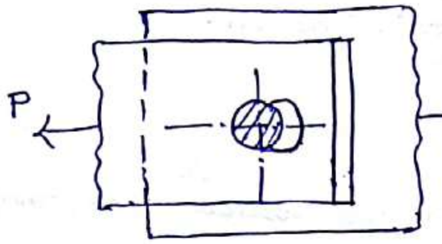
(d) shearing of rivets in a Lap Joint



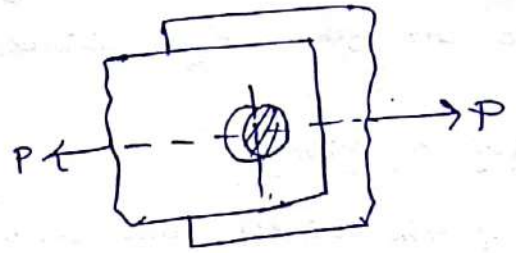
(e) Shearing of rivet in a single cover Butt Joint



(f) Shearing of rivets in a double cover butt Joint



(g) Crushing of plates



(h) crushing of rivet.

1. Tearing of the edge of plate

This type of failure may occur due to internal pressure of the overdriven rivet if sufficient edge distance (i.e margin) is not provided and hence by providing adequate margin (generally $m = 1.5d$); this failure may be avoided.

2. Rupture (i.e tearing) of the plate at the section weakened by holes.

When the riveted joint is subjected to an axial load, the plates should be strong enough to sustain the applied load. if not, at the section of riveting, perpendicular to the direction of applying load (i.e, in the rows, where the area of resistance is reduced by riveted holes) this type of failure will occur. For example, if p is the pitch length, d is the rivet hole diameter, t is the thickness of plate and S_t is the tensile strength of plate material, then for one pitch length, the net area of the plate resisting tension $= (p-d)\,t$ and the maximum ~~suitable~~ sustainable force

$$\boxed{F_t = (p-d)\,t \cdot S_t}$$

for overcoming this failure, this force F_t should be more than the applied load P. That is, by selecting high pitch value or choosing strong material thus by increasing its tensile strength, this failure may be eliminated.

3. Shearing of rivets:

If the rivets are unable to sustain the applied load, they can shear off at the plane between the plates where they are connected. A rivet can get sheared off at one plane or two planes according to the case whether the rivet is in single shear as in the case of lap joint and single cover butt joint or in double shear as in the case of double cover butt joint.

In the case of single shear, the resisting area for shear is $(\pi/4)d^2$ and in the case of double shear, this area will be double; i.e $2\left(\frac{\pi}{4}\right)d^2$. if s_s is the allowable shear stress of the rivet material and n is the number of of rivets per pitch length, then the resistance offered by the rivets as shearing strength of rivets is given by

$$\text{For single shear } (F_s) = n \times \frac{\pi}{4} d^2 \times s_s$$

$$\text{and double shear } (F_s) = n \times 2 \times \frac{\pi}{4} d^2 s_s .$$

According to Indian Boiler Regulations, abbreviated as I.B.R, the resisting area for double shear is considered as $1.875\left(\frac{\pi}{4}\right)d^2$ only instead of $2\left(\frac{\pi}{4}\right)d^2$ and hence the shear strength for double shear is,

$$\boxed{F_s = n \times 1.875 \times \frac{\pi}{4} d^2 \times s_s}$$

for overcoming this failure, this strength (F_s) should be more than the applied load (P). That is by increasing the number of rivets (n) or by choosing more strong rivet material thus by increasing its shear strength (S_s); this kind of failure may be avoided.

4. Crushing of rivets:

Some times the riveted joint may fail due to crushing of rivets by the plates or crushing of the plates by the rivets. When crushing of rivets or plates, the rivet's diameter or hole's diameter will become elliptical (i.e: oval shaped) from circular and the joint will become loose. In this case the bearing area or crushing area is taken as $d \times t$ which is also known as projected area and the resistance offered by the rivets, called as crushing strength of rivets, is given

by $F_c = d \cdot t \cdot S_c$ for one rivet

$\boxed{F_c = n \cdot d \cdot t \cdot S_c}$ for n rivets, where n is the no. of rivets per pitch length

For overcoming this failure; $F_c > P$ (applied load). That is by increasing the number of rivets or by selecting more strong rivets, this crushing failure may be eliminated.

Strength of a Riveted Joint:

The strength of a joint may be defined as the maximum force, which it can transmit, without causing it to fail. ~~as~~ and suppose ~~have~~ ~~se~~ when the load is applied on the joint and F_t, F_s, F_c are the tearing strength of plate, shearing and crushing strengths of rivets, then the joint will fail immediately when the least value of above three strengths reaches the applied load and other two strengths have no effect on the Joint. i.e: For existance, even the least of F_t, F_s and F_c should be more than the applied load, P.

Usually the strength of Joint may be calculated for one pitch length in the case of long Joint like boiler Joint and for the whole length of plate in the case of small Joint like tie bar used in roof, bridge work and so on.

If we consider F as the tensile strength of Unriveted or solid plate for the same one pitch length, then the efficiency (η) of the Joint may be defined as the ratio of strength of riveted Joint to the strength of Unriveted plate.

Strength of riveted Joint = Least of F_t, F_s & F_c

Strength of Unriveted plate $= F = p \cdot t \cdot s_t$

∴ Efficiency of the riveted Joint,

$$\boxed{\eta = \frac{\text{Least of } F_t, F_s \text{ and } F_c}{F}}$$

(1) A double riveted lap Joint is made b/w 15 mm thick plates. The rivet diameter and pitch are 25 mm and 75 mm, respectively. if the Ultimate stresses are 400 MPa in tension, 320 MPa in shear and 640 MPa in crushing, find the minimum force per pitch which will rupture the Joint. if the above

Joint is subjected to a load such that the factor of safety is 4, find out the actual stresses developed in the plates and the rivets.

sol Giventhat: $t = 15\,mm$; $d = 25\,mm$; $p = 75\,mm$, $\sigma_{tu} = 400\,MPa = 400\,N/mm^2$

$\tau_u = 320\,MPa = 320\,N/mm^2$, $\sigma_{cu} = 640\,MPa = 640\,N/mm^2$.

Minimum force per pitch which will rupture the Joint.

since the ultimate stresses are given, therefore we shall find the ultimate values of the resistances of the Joint. we know that ultimate tearing resistance of the plate per pitch.

$$P_{tu} = (p-d)\,t \times \sigma_{tu}$$

$$= (75-25)\,15 \times 400 = 300\,000\,N$$

ultimate shearing resistance of the rivets per pitch,

$$P_{su} = n \times \frac{\pi}{4} d^2 \times \tau_u = 2 \times \frac{\pi}{4}(25)^2\,320$$

$$= 314\,200\,N \qquad (\because n = 2)$$

and ultimate crushing resistance of the rivets per pitch

$$P_{cu} = n \times d \times t \times \sigma_{cu}$$

$$= 2 \times 25 \times 15 \times 640 = 480\,000\,N$$

From above we see that the minimum force per pitch which will rupture the Joint is 300 000 N or 300 KN

Actual stresses produced in the plates and rivets

Since the factor of safety is 4, therefore safe load per pitch length of the Joint.

$$= 300000/4 = 75000\,N$$

Let σ_{ta}, τ_a and σ_{ca} be the actual tearing, shearing & crushing stresses produced with a safe load of 75000 N in tearing, shearing & crushing.

We know that actual tearing resistance of the plates (P_{ta})

$$75000 = (p-d)\,t \times \sigma_{ta}$$

$$75000 = (75-25)\,15 \times \sigma_{ta}$$

$$75000 = 750\,\sigma_{ta}$$

$$\therefore \boxed{\sigma_{ta} = 100\text{ MPa}}$$

Actual shearing resistance of the rivets (P_{sa}),

$$75000 = n \times \frac{\pi}{4} d^2 \times \tau_a = 2 \times \frac{\pi}{4}(25)^2 \tau_a$$

$$\tau_a = 75000/982$$

$$\therefore \boxed{\tau_a = 76.4\text{ MPa}}$$

and actual crushing resistance of the rivets (P_{ca})

$$75000 = n \times d \times t \times \sigma_{ca}$$

$$75000 = 2 \times 25 \times 15 \times \sigma_{ca}$$

$$\therefore \boxed{\sigma_{ca} = 100\text{ N/mm}^2}$$

Problem

(2) Find the efficiency of the following riveted Joints:

(1) single riveted lap Joint of 6 mm plates with 20 mm diameter rivets having a pitch of 50 mm. (2) Double riveted lap Joint of 6 mm plates with 20mm diameter rivets having a pitch of 65 mm. Assume permissible tensile stress in plate = 120 MPa, Permissible shearing stress in rivets = 90 MPa, Permissible crushing stress in rivets = 180 MPa.

Sol Given that $t = 6mm$, $d = 20$ mm, $\sigma_t = 120 MPa = 120 N/mm^2$

$\tau = 90 MPa = 90 N/mm^2$, $\sigma_c = 180 MPa = 180 N/mm^2$.

(1) Efficiency of the first Joint

pitch $p = 50 mm$

First of all, Let us find the tearing resistance of the plate shearing & crushing resistances of the rivets.

(I) Tearing resistance of the plate:

we know that the tearing resistance of the plate per pitch length,

$$P_t = (p-d)\, t \times \sigma_t = (50-20)\, 6 \times 120 = 21600 N$$

(II) shearing resistance of the rivet

since the Joint is a single riveted lap Joint, therefore the strength of one rivet in single shear is taken. we know that shearing resistance of one rivet.

$$P_s = n \times \frac{\pi}{4} d^2 \times \tau = 1 \times \frac{\pi}{4} (20)^2 90$$

$$= 28278 N$$

(II) crushing resistance of the rivet

since the Joint is a single riveted, therefore ~~of~~ strength of one rivet is taken. we know that crushing resistance of one rivet,

$$P_c = n \times d \times t \times \sigma_c = 1 \times 20 \times 6 \times 180$$

$$= 21600\ N.$$

$\therefore$ strength of the Joint $=$ Least of P_t, P_s & P_c

$$= \underline{21600\ N}$$

we know that strength of the Unriveted or solid plate.

$$P = p \times t \times \sigma_t = 50 \times 6 \times 120 = \underline{36000\ N}$$

$\therefore$ Efficiency of the Joint,

$$\eta = \frac{\text{Least of } P_t, P_s \,\&\, P_c}{P} = \frac{21600}{36000}$$

$$= 0.60$$

$$\boxed{\eta = 60\%}$$

(2) Efficiency of the second Joint

pitch $\boxed{p = 65\ mm}$

(I) Tearing resistance of the plate,

we know that the tearing resistance of the plate per pitch length, $P_t = (p - d)\, t\, \sigma_t$

$$P_t = (65-20)6 \times 120 = \underline{32400\ N}$$

(ii) Shearing resistance of the rivets

Since the joint is double riveted lap joint, therefore strength of two rivets in single shear is taken. We know that shearing resistance of the rivets.

$$P_s = n \times \frac{\pi}{4} \times d^2 \times \tau = 2 \times \frac{\pi}{4}(20)^2 \times 90$$

$$= \underline{56556\ N}$$

(iii) Crushing resistance of the rivet:

Since the joint is double riveted, therefore strength of ~~two~~ two rivets is taken: we know that crushing resistance of rivets,

$$P_c = n \times d \times t \times \sigma_c = 2 \times 20 \times 6 \times 180$$

$$= \underline{43200\ N}$$

$\therefore$ Strength of the joint = Least of P_c, P_t & P_s

$$= \underline{32400\ N}$$

We know that the strength of the unriveted or solid plate,

$$P = p \times t \times \sigma_t = 65 \times 6 \times 120 = \underline{46800\ N}$$

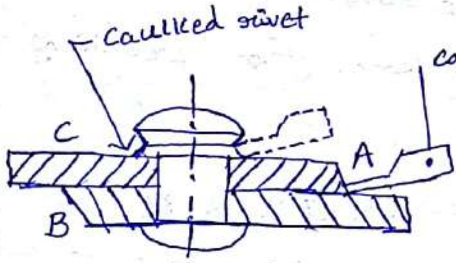
$\therefore$ Efficiency of the joint

$$\eta = \frac{\text{Least of } P_t, P_c \,\&\, P_s}{P} = \frac{32400}{46800} = 0.692$$

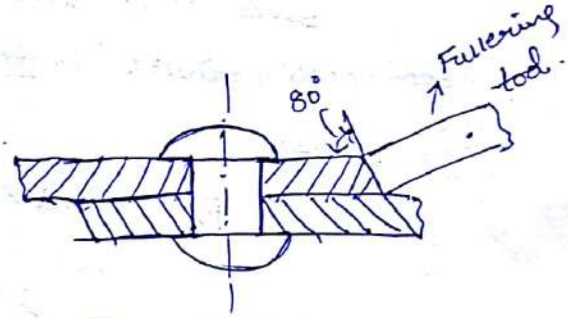
$$\boxed{\eta = 69.2\%}$$

Caulking and Fullering:

In order to make the Joints leak proof or fluid tight in pressure vessels like steam boilers, air receivers and tanks etc. a process known as caulking is employed. In this process, a narrow (unsharp) blunt tool called caulking tool, about 5 mm thick and 38 mm in breadth, is used. The edge of the tool is (down) ground (चिकना) to an angle of 080°. The tool is moved after each blow along the edge of the plate, which is planned to a a bevel of 75° to 80° to facilitate the forcing down of edge. It is seen that the tool (Rough edge) burrs down the plate at A in fig (a). forming a metal to metal Joint. In actual practice, both the edges at A and B are caulked.



(a) Caulking



(b) Fullering.

The head of the rivets as shown at C also turned down with a caulking tool to make a joint steam tight. A great care is taken to prevent injury to the plate below the tool.

A more satisfactory way of making the Joints (strong) staunch is known as fullering which has largely superseded (Replace b/c) caulking. In this case, a fullering tool with a thickness at the end equal to that of the plate is used in such a way that the greatest pressure due to the blows occur near the

Joint, giving a clean finish, with less risk of damaging the plate. A fullering process is shown above fig (b).

Design of Boiler Joints:

The boiler has a longitudinal Joint as well as circumferential Joint. The longitudinal Joint is used to Join the ends of the plate to get the required diameter of a boiler. for this purpose, a butt Joint with two cover plates is used. The circumferential Joint is used to get the required length of the boiler. for this purpose, a lap Joint with one ring overlapping the other alternately is used.

Since a boiler is made up of number of rings, therefore the longitudinal Joints are staggered for convenience of connecting rings at places where both longitudinal and circumferential Joints occur.

Design of Longitudinal Butt Joint for a Boiler:

According to Indian Boiler Regulation (I.B.R), the following procedure should be adopted for the design of longitudinal butt Joint for a boiler.

from Ja D.B, Page 10.4,

1 Thickness of boiler shell: First of all, the thickness of the boiler shell is determined by using the thin cylindrical formula, i.e.

$$t = \frac{PD}{2\sigma_t \eta_l} + 1 \text{ mm as corrosion allowance.}$$

where t = Thickness of the boiler shell,

P = Steam pressure in boiler,

D = Internal diameter of boiler shell,

σ_t = Permissible tensile stress, and

η_l = Efficiency of the longitudinal joint.

The following point may be noted.

(a) The thickness of the boiler shell should not be less than 7 mm

(b) The efficiency of the joint may be taken from the following table. 10.8, J.D.B. 10.21

Lap Joints	Efficiency (%)	Maximum efficiency
Single riveted	45-60	63.3
Double riveted	63-70	77.5
Triple riveted	72-80	86.6

Structural, Pressure vessel 10.22 table 10.9

Butt Joints (Double strap)	Efficiency (%)	Maximum efficiency
single riveted	55 to 60	63.3
Double riveted	70 to 83	86.6
Triple riveted (5 rivets per pitch with unequal width of straps)	80 to 90	95.0
Quadruple rivetel	85 to 94	98.1

* The η_{max} are valid for ideal equistrength joints with tensile stress = 77 MPa, τ = 62 MPa, σ_c = 133 MPa.

Indian Boiler Regulations (I.B.R) allow a maximum efficiency of 85% for the best Joint.

(c) According to I.B.R, the factor of safety (F.S) should not be less than 4. The following table shows the values of F.S for various kind of Joints in Boilers. from table ~~10.7~~ 10.20

Type of Joint	Factor of Safety	
	Hand riveting	Machine riveting
Lap Joint	4.75	4.5
Single strap butt Joint	4.75	4.5
Single riveted butt Joint with two equal cover straps	4.75	4.5
Double riveted butt Joint with two equal cover straps	4.25	4.0

9. Diameter of rivets: After finding out the thickness of the boiler shell (t), the diameter of the rivet hole (d) may be determined by using Unwin's empirical formula, i.e

Rivet hole dia $\boxed{d = 6\sqrt{t}}$ (when t is greater than 8 mm.)

But if the thickness of plate is less than 8mm, then the diameter of the rivet hole may be calculated by equating the shearing resistance. In this case, the diameter of rivet hole should not be less than the thickness of the plate, because there will be danger of punch crushing.

162 The following table gives the rivet diameter corresponding to the diameter of rivet hole.. from J.D.B page 10.19 table 10.5

Basic size (d^1)	12	14	16	18	20	22	24	27	30	33	36	39	42	48
Rivet hole dia (min) mm (standard size (d))	13	15	17	19	21	23	25	28.5	31.5	37.5	37.5	41	44	50

3. Pitch of rivets :- The pitch of the rivets is obtained by equating the tearing resistance of the plate to the shearing resistance of the rivets. It may noted that,

(a) The pitch of the rivets should not be less than 2d, $p \geq 2d$ which is necessary for the formation of head.

(b) The maximum value of the pitch of rivets for a longitudinal joint ~~of~~ of a boiler as per I.B.R. is

$$\boxed{P_{max} = C \times t + 41.28 \text{ mm}}$$

where t = Thickness of the shell plate in mm,

C = Constant.

The value of the constant $\underline{C}$ is given table 10.7, J.D.B page 10.24

Number of rivet/ pitch Length	Lap Joint	Butt Joint (single strap)	Butt Joint (Double strap)
1	1.31	1.53	1.75
2	2.62	3.06	3.50
3	3.47	4.05	4.63
4	4.17	—	5.52
5	—	—	6.00

Note: if the pitch of rivets as obtained by equating the tearing resistance to the shearing resistance is more than P_{max} and, then the value of P_{max} is taken.

4. Distance between the rows of rivets: The distance b/w the rows of rivets as specified by I.B.R is as follows:

(a) For equal number of rivets is more than one row for lap joint or butt joint, the distance b/w the rows of rivets (P_b) should not be less than

$0.33p + 0.67d$, for zig-zag riveting, and

$2d$, for chain riveting.

(b) For joint in which the number of rivets in outer rows is half the number of rivets in inner rows and if the inner rows are chain riveted, the distance b/w the outer rows and the next rows should not be less than

$0.33p + 0.67$ or $2d$, whichever is greater.

The distance b/w the rows in which there are full number of rivets shall not be less than $2d$.

(c) For joints in which the number of rivets in outer rows is half the number of rivets in inner rows and if the inner rows are zig-zag riveted, the distance b/w the outer rows and the next rows shall not be less than $0.2p + 1.15d$. The distance b/w the rows in which there are full number of rivets (zig-zag) shall not be less than

$0.165P + 0.67d$.

(5) Thickness of butt strap: According to I.B.R., the thickness for butt strap (t_1) are as given below:

(a) The thickness of butt strap, in no case, shall be less than 10 mm.

(b) $t_1 = 1.125t$, for ordinary (chain riveting) single butt strap.

$t_1 = 1.125t\left(\frac{p-d}{p-2d}\right)$, for single butt straps, every alternate rivet in outer rows being omitted.

$t_1 = 0.625\ t$, for double butt straps, ~~every alternate rivet in~~ of equal width having ordinary riveting (chain driving).

$t_1 = 0.625\ t\left(\frac{p-d}{p-2d}\right)$, for double butt straps of equal width having every alternate rivet in the outer rows being omitted.

(c) For unequal width of butt straps, the thicknesses of butt strap are

$t_1 = 0.75t$, for wide strap on the inside, and

$t_2 = 0.625t$, for narrow strap on the outside.

(6) Margin: The margin (m) is taken as $1.5\ d$.

Note: The above procedure may also be applied to ordinary riveted Joints.

Design of circumferential Lap Joint for a Boiler:

The following procedure is adopted for the design of circumferential lap joint for a boiler.

1. Thickness of the shell and diameter of rivets: The thickness of the boiler shell and the diameter of the rivet will be same as for longitudinal joint.

2. Number of rivets: Since it is a lap joint, therefore the rivets will be in single shear.

$\therefore$ Shearing resistance of the rivets

$$(P_s) = n \times \frac{\pi}{4} \times d^2 \times \tau \longrightarrow (I)$$

where $n =$ Total number of rivets.

Knowing the inner diameter of the boiler shell (D), and the pressure of steam (P), the total shearing load acting on the circumferential joint,

$$W_s = \frac{\pi}{4} \times D^2 \times P \longrightarrow (II)$$

from (I) & (II), we get

$$n \times \frac{\pi}{4} \times d^2 \times \tau = \frac{\pi}{4} \times D^2 \times P$$

$$n = \left(\frac{D}{d}\right)^2 \frac{P}{\tau}$$

3. Pitch of rivets: If the efficiency of the longitudinal joint is known, then the efficiency of the circumferential joint

Joint may be obtained. It is generally taken as 50% of tearing efficiency in longitudinal Joint, but if more than one circumferential Joints is used, then it is 62% for the intermediate Joints. Knowing the efficiency of the circumferential lap Joint (η_c), the pitch of the rivets for the lap Joint (p_1) may be obtained by using the relation :

$$\eta_c = \left(\frac{p_1 - d}{p_1}\right)$$

p_1 - pitch of lap Joint

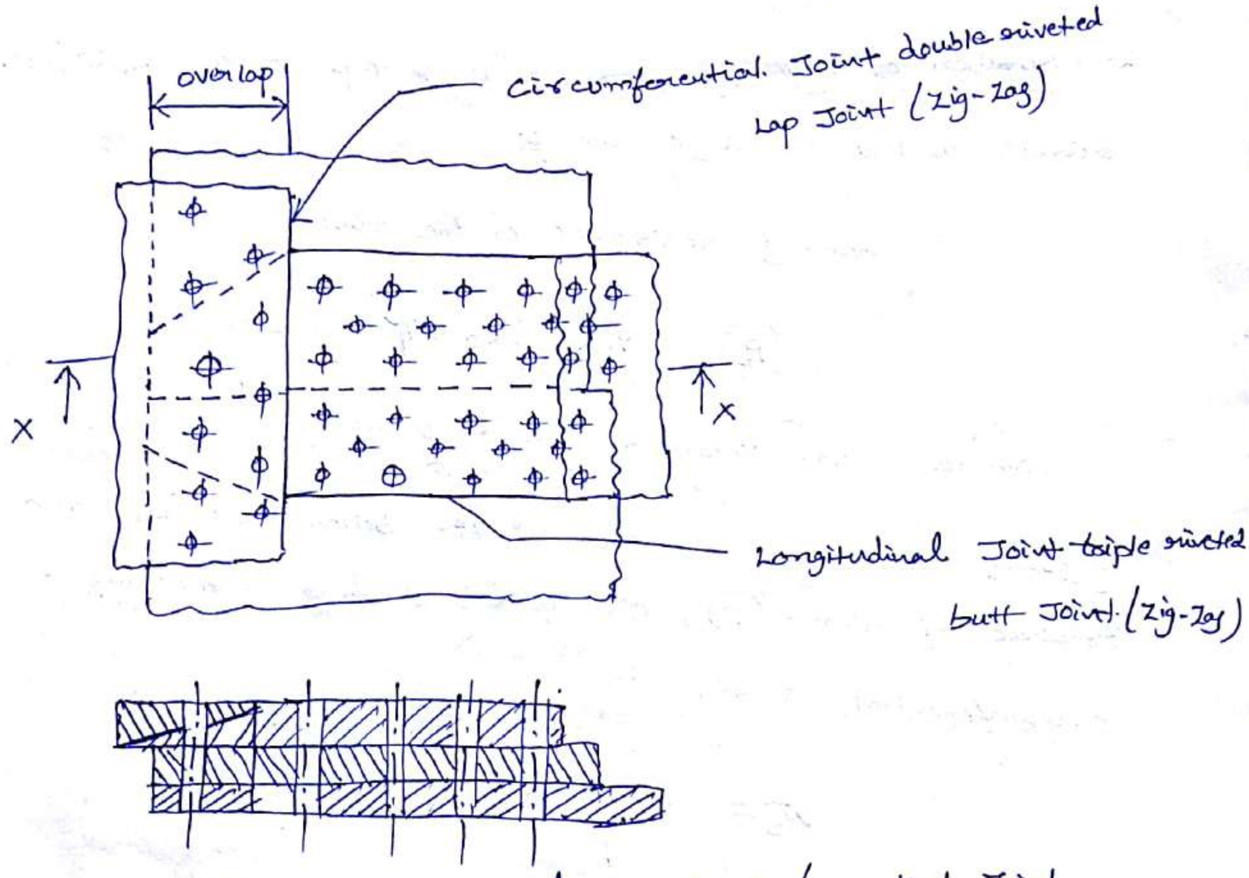


fig : Logitudinal & circumferential Joint

4. Number of rows :- The number of rows of rivets for the circumferential Joint may be obtained from the following relation:

$$\text{Number of rows} = \frac{\text{Total number of rivets}}{\text{Number of rivets in one row}}$$

and the number of rivets in one row

$$= \frac{\pi(D+t)}{p_1}$$

where D = Inner diameter of shell.

(5). After finding out the number of rows, the type of the Joint (i.e. single riveted or double riveted etc). may be decided. Then the number of rivets in a row and pitch may be re-adjusted. In order to have a leak-proof Joint, the pitch for the Joint should be checked from Indian Boiler Regulations.

(6) The distance b/w the rows of rivets (i.e back pitch) is calculated by using the relations as discussed in the previous article.

(7) After knowing the distance b/w the rows of rivet (p_b) the overlap of the plate may be fixed by using the relation,

$$\text{overlap} = (\text{No. of rows of rivets} - 1)\, p_b + m$$

where m = margin.

There are several ways of Joining the longitudinal Joint and the circumferential Joint. one of the methods of Joining the longitudinal and circumferential Joint show above fig.

table 10.6
Page 10.20

Tabl.: Recomended Joints for pressure vessels

Diameter of shell (meter)	Thickness of shell (mm)	Type of Joint
0.6 to 1.8	6 to 13	Double riveted
0.9 to 2.1	13 to 25	Triple riveted
1.5 to 2.7	19 to 40	Quadrupul riveted

(1) Design a double riveted butt Joint with two cover plates for the longitudinal steam of a boiler shell 1.5 m in diameter subjected to a steam pressure of 0.95 N/mm^2. Assume Joint efficiency as 75%, allowable tensile stress in the plate 90 MPa, Compressive stress 140 MPa, and shear stress in the rivet 56 MPa.

Sol. Given that: $D = 1.5 m = 1500\ mm$, $P = 0.95\ N/mm^2$, $\eta_l = 75\% = 0.75$

$\sigma_t = 90\ MPa = 90\ N/mm^2$, $\sigma_c = 140\ MPa = 140\ N/mm^2$

$\tau = 56\ MPa = 56\ N/mm^2$.

1. Thickness of boiler shell plate:

we know that thickness of boiler shell plate,

$$t = \frac{P.D}{2\sigma_t \times \eta_l} + 1\ mm$$

$$= \frac{0.95 \times 1500}{2 \times 90 \times 0.75} + 1 = 11.6 \text{ say } 12\ mm.$$

2. Diameter of rivet

Since the thickness of the plate is greater than 8mm, therefore the diameter of the rivet hole,

$$d = 6\sqrt{t} = 6\sqrt{12} = 20.8\ mm.$$

from table, Standard dia is 21 mm and the corresponding diameter of the rivet. 20 mm

(3) Pitch of rivets.

Let p = pitch of rivets

The pitch of the rivets is obtained by equating the tearing resistance of the plate to the shearing resistance of the rivets.

We know that tearing resistance of the plate,

$$P_t = (p-d)\, t \times \sigma_t = (p-21)\, 12 \times 90$$

$$= 1080\,(p-21)\ N \longrightarrow (I)$$

Since the joint is double riveted double strap butt joint, (i.e. $n=2$) therefore there are two rivets per pitch length. Assuming that the rivets are in double shear. and the rivets in double shear are 1.875 times stronger than rivets in single shear, we have

Shearing strength of the rivets,

$$P_s \text{ (or) } F_s = n \times 1.875 \times \frac{\pi}{4} d^2 \times \tau$$

$$= 2 \times 1.875 \times \frac{\pi}{4} (21)^2 \times 56$$

$$\boxed{F_s \text{ or } P_s = 72745\ N} \longrightarrow (II)$$

from eqns (I) & (II), we get

$$1080\,(p-21) = 72745$$

$$p = 63.35$$

$$\therefore\ p = 67.35 + 21 = 88.35 \text{ say } \underline{90\ mm}.$$

According to I.B.R., the maximum pitch of rivets for longitudinal Joint of a boiler is given by

$$P_{max} = C \times t + 41.28 \text{ mm}$$

from table, we find that for a double riveted double strap butt Joint and two rivets per pitch length, the value of C is 3.50

$$P_{max} = 3.5 \times 12 + 41.28$$

$$= 83.28 \text{ Say } 84 \text{ mm.}$$

Since the value of p is more than P_{max}, therefore we shall adopt pitch of the rivets,

$$\boxed{p = P_{max} = 84 \text{ mm}}$$

(4) Distance b/w rows of rivets:

Assuming Zig-Zag riveting, the distance b/w the rows of the rivets (according to I.B.R)

$$P_b = 0.33p + 0.67d$$

$$= (0.33 \times 84) + (0.67 \times 21)$$

$$= 41.8 \text{ Say } 42 \text{ mm.}$$

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(5) Thickness of cover plates:

According to I.B.R., the thickness of each cover plate of equal width is

$$t_1 = 0.625\,t = 0.625 \times 12 = \underline{7.5}\ \text{mm}$$

(6) Margin:- we know that the margin,

$$m = 1.5d = 1.5 \times 21 = 31.5 \text{ say } \underline{32\ \text{mm}}.$$

Let us now find the efficiency for the designed Joint.

Tearing resistance of the plate, $P_t = (p - d)\,t \times \sigma_t$

$$= (84 - 21)\,12 \times 90$$

$$= \underline{68040\ \text{N}}$$

Shearing resistance of the rivets,

$$P_s = n \times \frac{\pi}{4} d^2 \times 1.875 \times \tau$$

$$= 2 \times 1.875 \times \frac{\pi}{4} (21)^2 \times 56$$

$$P_s = \underline{72745\ \text{N}}$$

and crushing resistance of the rivets,

$$P_c = n \times d \times t \times \sigma_c = 2 \times 21 \times 12 \times 140 = \underline{70560\ \text{N}}$$

∵ The strength of riveted Joint is the least value of P_t, P_s, P_c,

therefore strength of the riveted Joint

$$\boxed{P_t = 68040\ \text{N}}$$

We know that strength of the Un-riveted plate,

$$P = p \times t \times \sigma_t = 84 \times 12 \times 90 = 90720\ N$$

$\therefore$ Efficiency of the designed Joint,

$$\eta = \frac{P_t}{P} = \frac{68040}{90720} = 0.75 \text{ or } 75\%$$

Since the efficiency of the designated Joint is equal to the given efficiency of 75%, therefore the design is satisfactory.

(2) Design the longitudinal Joint for a 1.25 m diameter steam boiler to carry a steam pressure of 2.5 N/mm². The ultimate strength of the boiler plate may be assumed as 420 MPa, crushing strength as 650 MPa and shear strength as 300 MPa. Take the Joint efficiency as 80%. Sketch the Joint with all the dimensions. Adopt the suitable factor of safety.

Sol Given that $D = 1.25\ m = 1250\ mm$, $p = 2.5\ N/mm^2$,

$\sigma_{tu} = 420\ MPa = 420\ N/mm^2$, $\sigma_{cu} = 650\ MPa = 650\ N/mm^2$,

$\tau_u = 300\ MPa = 300\ N/mm^2$, $\eta_l = 80\% = 0.8$

Assuming a factor of safety (F.S) at 5, the allowable stress are as follows:

$$\sigma_t = \frac{\sigma_{tu}}{F.S} = \frac{420}{5} = 84\ N/mm^2$$

$$\sigma_c = \frac{\sigma_{cu}}{F.S} = \frac{650}{5} = 130 \text{ N/mm}^2$$

$$\tau = \frac{\tau_u}{F.S} = \frac{300}{5} = 60 \text{ N/mm}^2.$$

1. Thickness of plate:

we know that thickness of plate,

$$t = \frac{P.D}{2\sigma_t \eta_l} + 1 \text{ mm} = \frac{2.5 \times 1250}{2 \times 84 \times 0.8} + 1 \text{ mm} = 24.3$$

$$\simeq 25 \text{ mm}$$

2. Diameter of rivet:

since the thickness of the plate is more than 8mm, therefore diameter of the rivet hole,

$$d = 6\sqrt{t} = 6\sqrt{25} = 30 \text{ mm}$$

From table, standard diameter of the rivet hole is 31.5 mm and the corresponding diameter of the rivet is 30 mm.

(3) Pitch of rivets:

Assume a triple riveted double strap butt Joint with unequal straps,

Let p = Pitch of the rivets in the outer most row

∴ Tearing strength of the plate per pitch length,

$$P_t = (p-d)\, t \times \sigma_t = (p - 31.5)\, 25 \times 84$$

$$P_t = 2100\,(p - 31.5) \text{ N} \longrightarrow (I)$$

since the Joint is triple riveted with two unequal cover straps, therefore there are 5 rivets per pitch length. out of these five rivets, four rivets are in double shear and one is in single shear. Assuming the strength of the rivets in double shear as 1.875 times that of single shear, therefore

Shearing resistance of the rivets per pitch length,

$$P_s = 4 \times 1.875 \times \frac{\pi}{4} d^2 \times \tau + \frac{\pi}{4} d^2 \times \tau$$

$$P_s = 8.5 \times \frac{\pi}{4} d^2 \times \tau \rightarrow \text{(II)}$$

From eqns (I) & (II)

$$2100 (p - 31.5) = 397500$$

$$p - 31.5 = 397500 / 2100 = 189.3$$

$$\boxed{p = 220.8 \text{ mm}}$$

$\therefore$ According to I.B.R, maximum pitch,

$$p_{max} = C \times t + 41.28 \text{ mm}$$

from table, we find that for double strap butt joint with 5 rivets per pitch length, the value of C is 6.

$$\therefore p_{max} = C \times t + 41.28$$

$$= 6 \times 25 + 41.28$$

$$= 191.28 \simeq 196 \text{ mm}$$

Since p_{max} is less than p, therefore we shall adopt

$$p = p_{max} = 196 \text{ mm.}$$

$\therefore$ pitch of rivets in the inner row,

$$p' = 196/2 = 98 \text{ mm.}$$

4. Distance b/w the rows of rivets:

According to I.B.R., the distance b/w the outer row and the next row,

$$= 0.2p + 1.15d = 0.2 \times 196 + 0.67 \times 31.5$$

$$= 75.4 \text{ say } \underline{76 \text{ mm}}$$

and the distance b/w the inner rows for zig-zag riveting

$$= 0.165p + 0.67d = 0.165 \times 196 + 0.67 \times 31.5$$

$$= 53.4 \text{ say } \underline{54 \text{ mm}}.$$

5. Thickness of butt straps

we know that for unequal width of butt straps, the thicknesses are as follows:

For wide butt strap, $t_1 = 0.75t = 0.75 \times 25 = 18.75$ say $\underline{20 \text{ mm}}$

and for narrow butt strap $t_2 = 0.625t = 0.625 \times 25$

$$= \underline{15.6} \text{ say } \underline{16 \text{ mm}}$$

It may be noted that wide and narrow butt straps are placed on the inside and outside of the shell respectively.

(6) Margin:

we know that the margin, $m = 1.5d = 1.5 \times 31.5$

$$= 47.25 \text{ say } \underline{47.5} \text{ mm}.$$

Let us now check the efficiency of the designed joint.

Tearing resistance of the plate in the outer row,

$$P_t = (p-d)t \times \sigma_t = (196-31.5)25 \times 84$$

$$= 345450 \text{ N}$$

Shearing resistance of the rivets,

$$P_s = 4 \times 1.875 \times \frac{\pi}{4} d^2 \times \tau + \frac{\pi}{4} d^2 \times \tau$$

$$= 8.5 \times \frac{\pi}{4} (31.5)^2 \times 60$$

$$P_s = 397500 \text{ N}$$

and crushing resistance of the rivets,

$$P_c = n \times d \times t \times \sigma_c = 5 \times 31.5 \times 25 \times 130$$

$$= 511875 \text{ N} \qquad (\because n = 5)$$

The Joint may also fail by tearing off the plate b/w the rivets in the second row. This is only possible if the rivets in the outermost row gives way (i.e shears), since there are two rivet holes per pitch length in the second row and one rivet is in the outer most row, therefore combined tearing & shearing resistance

$$= (p-2d)t \times \sigma_t + \frac{\pi}{4} d^2 \tau$$

$$= (196 - 2 \times 31.5)\, 25 \times 84 + \frac{\pi}{4} (31.5)^2 60$$

$$= 326065 \text{ N}$$

from above, we see that strength of the Joint $= 326065$ N

strength of the unriveted or solid plate $P = p \times t \times \sigma_t$

$$= 196 \times 25 \times 84$$

$$= 411600 \text{ N}$$

$\therefore$ Efficiency of the Joint

$$\eta = \frac{326065}{411600} = 0.792 \text{ or } 79.2\%$$

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Since the efficiency of the designed Joint is nearly equal to the given efficiency, therefore the design is satisfactory.

(3) A steam boiler is to be designed for a working pressure of 2.5 N/mm² with its inside diameter 1.6 m. Give the design Calculations for the longitudinal and circumferential Joints for the following working stresses for steel plates and rivets:

In tension = 75 MPa, In shear = 60 MPa, In crushing = 125 MPa

Draw the Joints to a suitable scale.

Sol Given that: $P = 2.5\ N/mm^2$, $D = 1.6\ m = 1600\ mm$,

$\sigma_t = 75\ MPa = 75\ N/mm^2$, $\tau = 60\ MPa = 60\ N/mm^2$, $\sigma_c = 125\ N/mm^2$

Design of longitudinal Joint:

The longitudinal Joint for a steam boiler may be designed as follows:

1. Thickness of boiler shell:

we know that the thickness of boiler shell,

$$t = \frac{PD}{2\sigma_t} + 1\ mm = \frac{2.5 \times 1600}{2 \times 75} + 1\ mm = 27.6 \simeq 28\ mm$$

2. Diameter of rivet:

since the thickness of the plate is more than 8 mm, therefore diameter of rivet hole, $d = 6\sqrt{t} = 6\sqrt{28} = 31.75\ mm$.

standard dia is 34.5 mm & corresponding dia 33 mm

3. Pitch of rivets:

Assume the Joint to be triple riveted double strap butt Joint with Unequal cover straps, as shown above.

Let p = pitch of the rivet in the outer most row.

∴ Tearing resistance of the plate per pitch length,

$$P_t = (p-d)\, t \times \sigma_t = (p-34.5)\, 28 \times 75$$

$$= 2100\,(p-34.5)\ \text{N} \longrightarrow \text{(I)}$$

Since the Joint is triple riveted with two Unequal Cover straps therefore there are 5 rivets per pitch length. out of these five rivets, four are in double shear and one is in single shear. Assuming the strength of rivets in double shear as 1.875 times that of single shear, therefore.

Shearing resistance of the rivets per pitch length,

$$P_s = 4 \times 1.875 \times \frac{\pi}{4} d^2 \tau + \frac{\pi}{4} d^2 \tau$$

$$= 8.5 \times \frac{\pi}{4} (34.5)^2\, 60 = 476\,820\ \text{N} \longrightarrow \text{(II)}$$

from (I) & (II), we get.

$$2100\,(p-34.5) = 476\,820$$

$$p = 227 + 34.5 = 261.5\ \text{mm}$$

∴ According to I.B.R, the maximum pitch,

$$p_{max} = C \cdot t + 41.28\ \text{mm}$$

5 rivets per pitch length, the value of $C = 6$

$$= 6 \times 28 + 41.28$$

$$= 220\ \text{mm}$$

Since P_{max} is less than p, therefore we shall adopt.

$$p = P_{max} = 220 \text{ mm}$$

∴ pitch of rivets in the inner row,

$$p' = \frac{220}{2} = 110 \text{ mm}$$

(4) Distance b/w the rows of rivets:

According to I.B.R, the distance b/w the outer row and the next row: $= 0.2p + 1.15d$

$= 83.7$ say 85 mm.

and the distance b/w the inner rows for zig-zag riveting.

$= 0.165p + 0.67d$

$= 59.4$ say 60 mm.

(5) Thickness of butt straps

we know that for unequal width of butt straps, the thickness are: For wide butt strap $t_1 = 0.75t = 21$ mm

for narrow " $t_2 = 0.625t = 18$ mm

(6) Margin: $m = 1.5d = 1.5 \times 34.5 = 52$ mm.

Tearing resistance of the plate in the outer row,

$$P_t = (p-d)t \times \sigma_t = 389550 \text{ N}$$

Shearing resistance of the rivets,

$$P_s = 4 \times 1.875 \times \frac{\pi}{4} d^2 \times \tau + \frac{\pi}{4} d^2 \times \tau$$

$$= 476820 \text{ N}$$

crushing resistance of the rivets

$$P_c = n \times d \times t \times \sigma_c = 603750 \text{ N}$$

The Joint may also fail by tearing off the plate b/w the rivets in the second row. This is only possible if the rivets in the outermost row gives way (i.e. shears). Since there are two rivet holes per pitch length in the second row and one rivet in the outermost row, therefore.

Combined Tearing & shearing resistance

$$= (p - 2d)\, t \times \sigma_t + \frac{\pi}{4} d^2 \tau$$

$$= 317100 + 56096 = 373196 \text{ N}$$

from above, we see that the strength of the Joint

$$= 373196 \text{ N}$$

strength of the Unriveted or solid plate,

$$P = p \times t \times \sigma_t = 220 \times 28 \times 75 = 462000 \text{N}$$

∴ Efficiency of the designed Joint,

$$\eta = \frac{373196}{462000} = 0.808$$

$$\boxed{\eta = 80.8\%}$$

Design of circumferential Joint:

The circumferential Joint for a steam boiler may be designed as follows:

1. The thickness of the boiler shell (t) and diameter of rivet hole (d) will be same as for longitudinal Joint, i.e

$$t = 28 \text{ mm} \ \& \ d = 34.5 \text{ mm.}$$

2. Number of rivets:

Let n = number of rivets

we know that shearing resistance of the rivets

$$= n \times \frac{\pi}{4} d^2 \times \tau \longrightarrow \text{(I)}$$

and total shearing load acting on the circumferential Joint

$$W_s = \frac{\pi}{4} D^2 \times P. \longrightarrow \text{(II)}$$

from (I) & (II)

$$n \times \frac{\pi}{4} d^2 \times \tau = \frac{\pi}{4} D^2 \times P$$

$$n = \frac{D^2 \times P}{d^2 \times \tau} = \frac{(1600)^2 \times 2.5}{(34.5)^2 \times 60} = 89.6 \qquad (\because P = \text{Pressure})$$

$$\therefore \boxed{n = 90}.$$

(3) Pitch of rivets:

Assuming the Joint to be double riveted lap Joint with Zig-Zag riveting, therefore number of rivet per row,

$$= \frac{90}{2} = 45$$

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we know that the pitch of the rivets,

$$P_1 = \frac{\pi(D+t)}{\text{Number of rivets per row}} = \frac{\pi(1600+28)}{45}$$

$$P_1 = 113.7 \text{ mm}$$

Let us take pitch of the rivets, $P_1 = 140$ mm.

(4) Efficiency of the Joint

we know that the efficiency of the circumferential Joint,

$$\eta_c = \frac{P_1 - d}{P_1} = \frac{140-34.5}{140} = 0.753 \text{ or } 75.3\%$$

(5) Distance b/w the rows of rivets

we know that the distance b/w the rows of rivets for

Zig-zag riveting, $= 0.33 P_1 + 0.67 d$

$= 69.3$ say 70 mm

(6) Margin :

we know that the ~~rag~~ margin,

$$m = 1.5\,d = 1.5 \times 34.5$$

$$m = 51.75$$

$$\simeq$$

$$\boxed{m = 52 \text{mm}}$$

Eccentric loaded Riveted Joint:

When the line of action of the load does not pass through the centroid of the rivet system and thus all rivets are not equally loaded, then the Joint is said to be an eccentric loaded riveted Joint, fig (a). The eccentric loading results in secondary shear caused by the tendency of force to twist the Joint about the centre of gravity in addition to direct shear or primary shear.

Let P = Eccentric load on the Joint, and

e = Eccentricity of the load i.e. the distance b/w the line of action of the load and the centroid of the rivet system i.e. G.

The following procedure is adopted for the design of an eccentrically loaded riveted Joint.

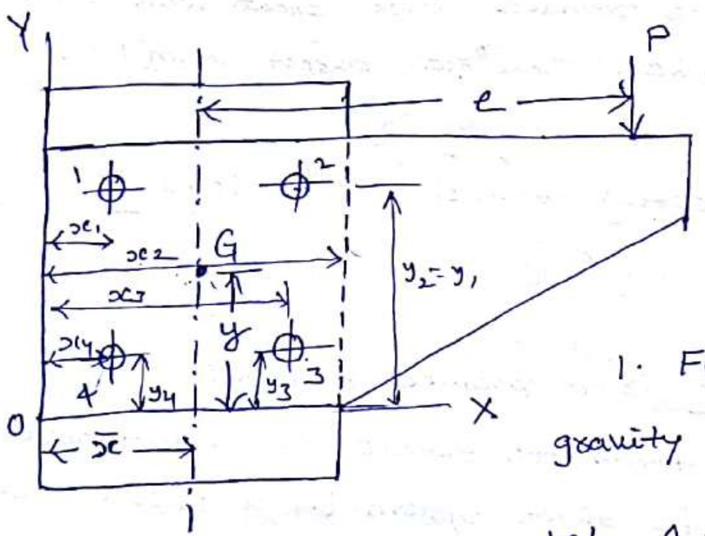


fig (a)

1. First of all, find the centre of gravity G of the rivet system.

Let A = Cross-sectional area of each rivet,

x_1, x_2, x_3 etc. = Distances of rivets from OY, and

y_1, y_2, y_3 etc = Distances of rivets from OX.

We know that $\bar{x} = \dfrac{A_1x_1 + A_2x_2 + A_3x_3 + \cdots}{A_1 + A_2 + A_3 + \cdots} = \dfrac{A_1x_1 + A_2x_2 + \cdots}{n \cdot A}$

$$\bar{x} = \frac{x_1 + x_2 + x_3 + \cdots}{n}$$

Similarly $\bar{y} = \dfrac{y_1 + y_2 + y_3 + \cdots}{n}$ (∵ where n = Number of rivets)

(2) Introduce two forces P_1 and P_2 at the centre of gravity 'G' of the rivet system. These forces are equal and opposite to P as shown fig (b)

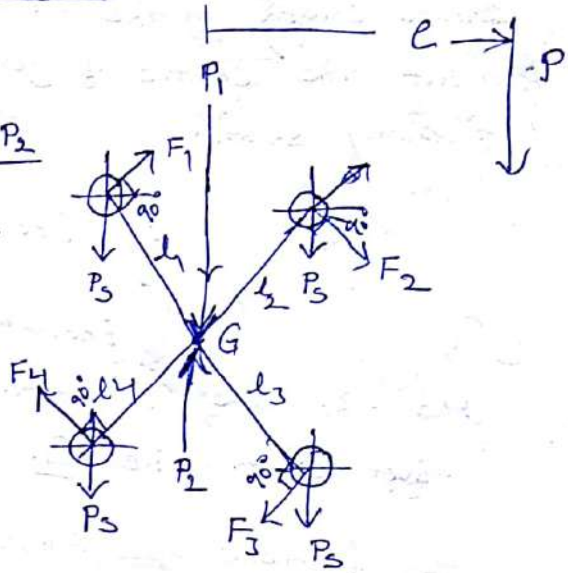


fig (b)

(3) Assuming that all the rivets are of the same size, the effect of $P_1 = P$ is to produce direct shear load on each rivet of equal magnitude. Therefore, direct shear load on each rivet,

$$P_s = \frac{P}{n}, \text{ acting parallel to the load } P$$

(4) The effect of $P_2 = P$ is to produce a turning moment of magnitude $P \times e$ which tends to rotate the joint about the centre of gravity 'G' of the rivet system in a clockwise direction. Due to the turning moment, secondary shear load on each rivet is produced. In order to find the secondary shear load, the following two assumptions are made:

(a) The secondary shear load is proportional to the radial distance of the rivet under consideration from the centre of gravity of the rivet system.

(b) The direction of secondary shear load is perpendicular to the line joining the centre of the rivet to the centre of Gravity of the rivet system.

Let $F_1, F_2, F_3 \ldots\ldots$ = Secondary shear loads on the rivets 1, 2, 3 ... etc

$l_1, l_2, l_3 \ldots$ = Radial distance of the rivets 1, 2, 3 ... etc. from the centre of gravity 'G' of the rivet system.

∴ From assumption (a),

$$F_1 \propto l_1 ; \quad F_2 \propto l_2 \quad \text{and so on}$$

or
$$\frac{F_1}{l_1} = \frac{F_2}{l_2} = \frac{F_3}{l_3} = \ldots$$

$$\therefore \quad F_2 = F_1 \times \frac{l_2}{l_1}, \quad \text{and} \quad F_3 = F_1 \frac{l_3}{l_1}$$

we know that the sum of the external turning moment due to the eccentric load and of internal resisting moment of the rivets must be equal to zero.

$$\left[\frac{F_1 l_1^2 + F_1 l_2}{l_1} \right.$$

$$P \cdot e = F_1 l_1 + F_2 l_2 + F_3 l_3 + \ldots\ldots$$

$$= F_1 l_1 + F_1 \frac{(l_2) l_2}{l_1} + F_1 \frac{l_3 (l_3)}{l_1} + \ldots\ldots$$

$$= \frac{F_1}{l_1} \left[(l_1)^2 + (l_2)^2 + (l_3)^2 + \ldots\ldots \right]$$

from the above expression, the value of F_1 may be calculated and hence F_2 and F_3 etc. are known. The direction of these forces are at right angles to the lines joining the centre of rivet to the centre of gravity of the rivet system, as shown above fig (b), and should produce the moment in the Same direction (i.e. clockwise or anticlockwise) about the centre of gravity, as the turning moment ($P \times e$).

(5.) The primary (or direct) and secondary shear load may be added vectorially to determine the resultant shear load (R) on each as shown below.
It may also be obtained by using the relation.

$$\boxed{R = \sqrt{(P_s)^2 + F^2 + 2F_s \times F \times \cos\theta}}$$

where θ = Angle b/w the primary or direct shear load (P_s) and Secondary shear load (F)

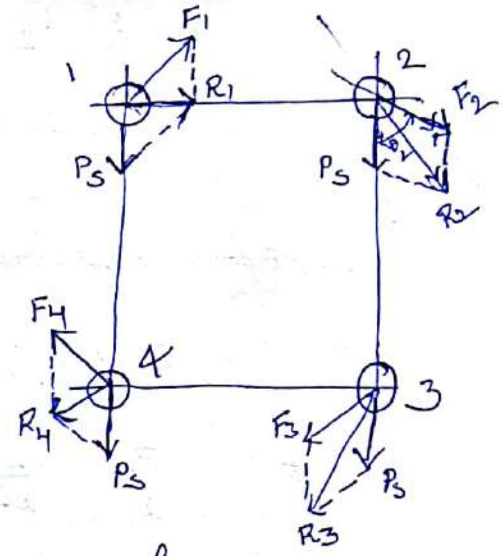


fig (x)

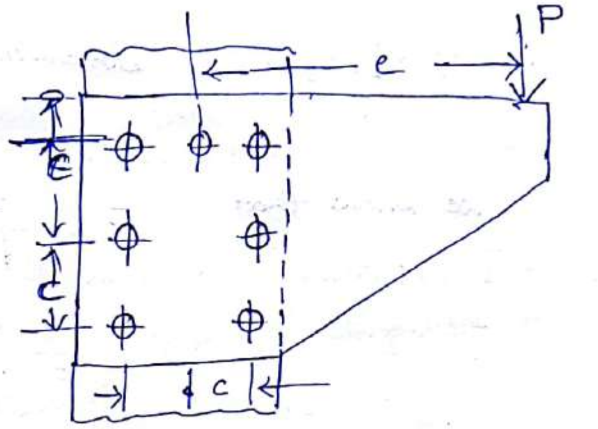
When the secondary shear load on each rivet is equal, then the heavily loaded rivet will be one in which the included angle b/w the direct shear load and secondary shear load is minimum. The maximum loaded rivet becomes the critical one for determining the strength of the riveted Joint. Knowing the permissible shear stress (τ), the diameter of the rivet hole may be obtained by using the relation,

$$\text{Maximum resultant Shear load } (R) = \frac{\pi}{4} \times d^2 \times \tau$$

$\underline{d}$ – standard diameter of the rivet hole.

1) An eccentrically loaded lap riveted Joint is to be designed for a steel brackets as shown.

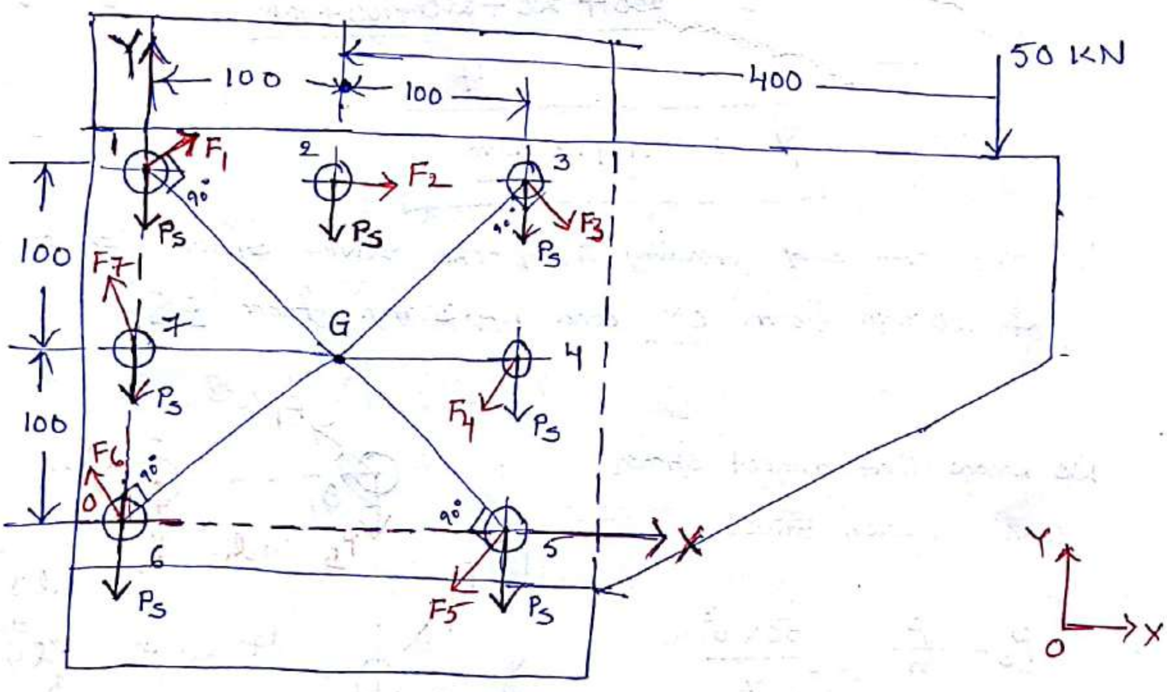
The bracket plate is 25 mm thick. All rivets are to be of the same size. Load on the bracket, P = 50 KN, rivet spacing, C = 100 mm, load arm e = 400 mm



Permissible shear stress is 65 MPa and crushing stress is 120 MPa. Determine the size of the rivets to be used for the Joint.

Sol Given that: $t = 25$ mm, $P = 50$ KN $= 50 \times 10^3$ N, $e = 400$ mm

$n = 7$, $\tau = 65$ MPa $= 65$ N/mm^2, $\sigma_c = 120$ MPa $= 120$ N/mm^2.



First of all, Let us find the centre of gravity (G) of the rivet system.

Let $\bar{x}$ = Distance of centre of Gravity from OY,

$\bar{y}$ = Distance of centre of Gravity from OX,

$x_1, x_2, x_3, \ldots$ = Distances of centre of gravity of each rivet from OY, and

$Y_1, Y_2, Y_3, \ldots$ = Distances of centre of gravity of each rivet from OX.

We know that.
$$\bar{x} = \frac{x_1+x_2+x_3+x_4+x_5+x_6+x_7}{n}$$

$$= \frac{100+200+200+200}{7}$$

$$\boxed{\bar{x} = 100 \text{ mm}}$$

$(\because x_1 = x_6 = x_7 = 0)$

$x_2 = 100, x_3 = 200$

$x_4 = x_5 = 200$

and
$$\bar{y} = \frac{y_1+y_2+y_3+y_4+y_5+y_6+y_7}{n}$$

$$= \frac{200+200+200+100+100}{7}$$

$[\because y_5 = y_6 = 0]$

$$\boxed{\bar{y} = 114.3 \text{ mm}}$$

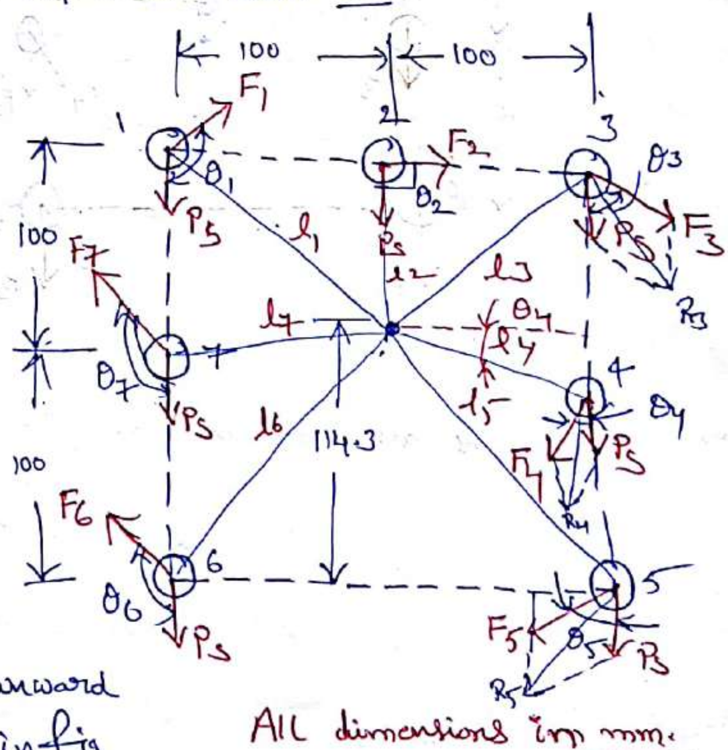
∴ The centre of Gravity (G) of the rivet system lies at a distance of 100 mm from OY and 114.3 mm from OX.

We know that direct shear load on each rivet,

$$P_s = \frac{P}{n} = \frac{50\times10^3}{7}$$

$$\boxed{P_s = 7143 \text{ N}}$$

The direct shear load acts Parallel to the direction of load P. i.e. vertically downward as shown in fig



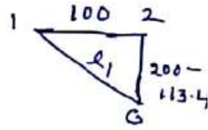
All dimensions in mm.

Turning moment produced by the load $\underline{P}$ due to eccentricity (e) $= P \times e = 50 \times 10^3 \times 400 = \underline{20 \times 10^6}$ N-mm.

This turning moment is resisted by seven rivets as shown above

Let $F_1, F_2, F_3, F_4, F_5, F_6$ and F_7 be the secondary shear load on the rivets 1, 2, 3, 4, 5, 6 and 7 placed at distances l_1, l_2, l_3, l_4, l_5, l_6 and l_7 respectively from the centre of gravity of the rivet system above

From the geometry of the figure, we find that



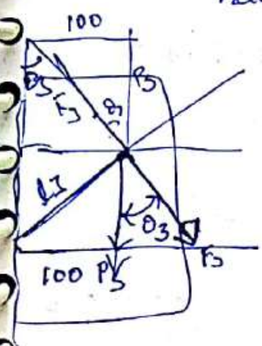
$$l_1 = l_3 = \sqrt{(100)^2 + (200 - 114.3)^2} = 131.7 \text{ mm}$$

$$l_2 = 200 - 114.3 = 85.7 \text{ mm}$$

$$l_4 = l_7 = \sqrt{(100)^2 + (114.3 - 100)^2} = 101 \text{ mm.}$$

and $$l_5 = l_6 = \sqrt{(100)^2 + (114.3)^2} = \underline{152 \text{ mm}}$$

Now equating the turning moment due to eccentricity of the load to the resisting moment of the rivets, we have



$$P \times e = \frac{F_1}{l_1}\left[(l_1)^2 + (l_2)^2 + (l_3)^2 + (l_4)^2 + (l_5)^2 + (l_6)^2 + (l_7)^2\right]$$

$$= \frac{F_1}{l_1}\left[2(l_1)^2 + (l_2)^2 + 2(l_4)^2 + 2(l_5)^2\right]$$

$$50 \times 10^3 \times 400 = \frac{F_1}{131.7}\left[2(131.7)^2 + (85.7)^2 + 2(101)^2 + 2(152)^2\right]$$ $(\because l_1 = l_3,\ l_4 = l_7$ and $l_5 = l_6)$

$$20 \times 10^6 \times 131.7 = F_1\,(34690 + 7345 + 20402 + 46208)$$

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$$20 \times 10^6 \times 131.7 = 108645\, F_1$$

$$\boxed{F_1 = 24244\, N}$$

Since the secondary shear loads are proportional to their radial distances from the centre of gravity, therefore.

$$F_2 = F_1 \times \frac{l_2}{l_1} = 24244 \times \frac{85.17}{131.7} = 15776\, N$$

$$F_3 = F_1 \times \frac{l_3}{l_1} = F_1 = 24244\, N$$

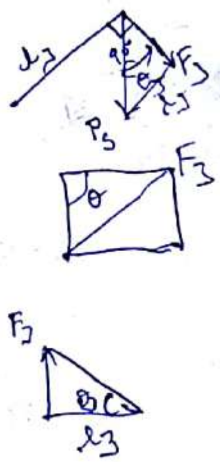
$$F_4 = F_1 \times \frac{l_4}{l_1} = 24244 \times \frac{101}{131.7} = \underline{18593\, N}$$

$$F_5 = F_1 \times \frac{l_5}{l_1} = 24244 \times \frac{152}{131.7} = 27981\, N$$

$$F_6 = F_1 \times \frac{l_6}{l_1} = F_5 = 27981\, N \qquad (\because l_6 = l_5)$$

$$F_7 = F_1 \times \frac{l_7}{l_1} = F_4 = \underline{18593\, N} \qquad \because (l_7 = l_4)$$

By drawing the direct and secondary shear loads on each rivet, we see that the rivets 3, 4 and 5 are heavily loaded. Let us now find the angles b/w the direct and secondary shear load for these three rivets. from the geometry, we find that (fig)



$$\cos\theta_3 = \frac{100}{l_3} = \frac{100}{131.7} = 0.76$$

$$\cos\theta_4 = \frac{100}{l_4} = \frac{100}{101} = 0.99$$

and $$\cos\theta_5 = \frac{100}{l_5} = \frac{100}{152} = \underline{0.658}$$

Scanned by CamScanner

Now resultant shear load on rivet 3,

$$R_3 = \sqrt{(P_3)^2 + (F_3)^2 + 2 P_3 \times F_3 \times \cos\theta_3}$$

$$= \sqrt{(7143)^2 + (24244)^2 + 2 \times 7143 \times 24244 \times 0.76}$$

$$\boxed{R_3 = 30.033 \text{ N}}$$

Resultant shear load on rivet 4,

$$R_4 = \sqrt{(P_3)^2 + (F_4)^2 + 2 P_3 \times F_4 \times \cos\theta_4}$$

$$= \sqrt{(7143)^2 + (18593)^2 + 2 \times 7143 \times 18593 \times 0.99}$$

$$\boxed{R_4 = 25684 \text{ N}}$$

and Resultant shear load on rivet 5,

$$R_5 = \sqrt{(P_3)^2 + (F_5)^2 + 2 P_3 \times F_5 \times \cos\theta_5}$$

$$= \sqrt{(7143)^2 + (27981)^2 + 2 \times 7143 \times 27981 \times 0.658}$$

$$\boxed{R_5 = 33121 \text{ N}}$$

The resultant shear load may be determined graphically, above fig.

From above we see that the maximum resultant shear load is on rivet 5. if d is the diameter of rivet hole, then maximum resultant shear load (R_5),

$$33121 = \frac{\pi}{4} \times d^2 \times \tau = \frac{\pi}{4} \times d^2 \times 65 = 51 d^2$$

$$d^2 = 649.4$$

$$\boxed{d = 25.5 \text{ mm}}$$

The standard diameter of the rivet hole (d) is 25.5 mm and the corresponding diameter of rivet is 24 mm.

Let us now check the Joint for crushing stress, we know that

$$\text{crushing stress} = \frac{\text{Max. load}}{\text{crushing area}} = \frac{R_5}{d \times t}$$

$$= \frac{33121}{25.5 \times 25} = \underline{51.95 \text{ N/mm}^2}.$$

Since this stress is well below the given crushing stress of 120 MPa, therefore the design is Satisfactory.

Welded Joints:

A welded Joint is a permanent Joint which is obtained by the fusion (melt) of the edges of the two parts to be Joined together, with or without the application of pressure and a filler material. The heat is required for the fusion of the material may be obtained by burning of gas (in case of gas welding) or by an electric arc (in case of electric arc welding). The latter method is exten-sively used because of greater speed of welding.

Welding is extensively used in fabrication as an alternative method for casting or forging and as a replacement for bolted and riveted Joints. It is also used as a repair medium e.g. to reunite metal at a crack, to buid up a small part that has broken off such as gear tooth or to repair a worn surface such as a bearing surface.

Advantages and Disadvantages of welded Joints over Riveted Joints

Advantages:

1. The welded structures are usually lighter than riveted structures. This is due to the reason that. in welding, gussets or other connecting components are not used.

2. The welded Joints provide maximum efficiency (may be 100%) which is not possible in case of riveted Joints.

3. Alterations and additions can be easily made in the existing structures.

4. As the welded structure is smooth in appearance, therefore it looks pleasing.

5. In welded connections, the tension members are not weakened as in the case of riveted Joints.

6. A welded Joint has a great strength. often a welded Joint has the strength of the parent metal itself.

7. Somtimes, the members are of such a shape (i.e circular steel pipes) that they afford difficulty for riveting. But they can be easily welded.

8. The welding provides very rigid Joint. This is in line with the modern trend of providing rigid frames.

9. It is possible to weld any part of a structure at any point. But riveting requires enough clearance.

10. The process of welding takes less time than the riveting.

Disadvantages

1. Since there is an uneven heating and cooling during fabrication, therefore the members may get distorted or additional stresses may develop.

2. It requires a highly skilled labour and supervision.

3. Since no provision is kept for expansion and contraction in the frame, therefore there is a possibility of cracks developing in it.

4. The inspection of welding work is more difficult than riveting work.

Types of welded Joints

Following two type of welded Joints are important from the subject point of view.

1. Lap Joint or fillet Joint

2. Butt Joint

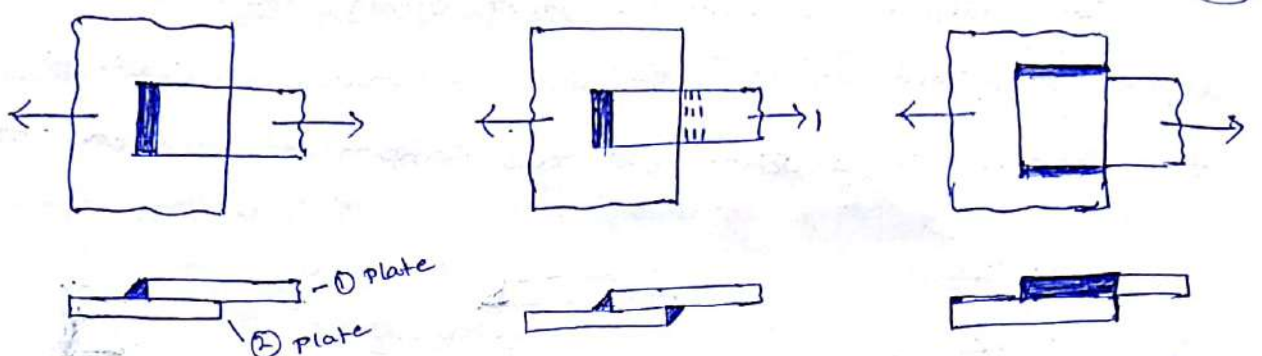
Lap Joint.

The Lap Joint or the fillet Joint is obtained by overlapping the plates and then welding the edges of the plates. The cross-section of the fillet is approximately triangular. The fillet Joints may be

1. Single transverse fillet

2. Double transverse fillet

3. Parallel fillet Joints



ⓐ single transverse ⓑ Double transverse ⓒ Parallel fillet

fig: Types of Lap and Butt Joints.
or Types of fillet Joints

A fillet weld is called transverse, if the direction of ~~the force~~ weld is perpendicular to the direction of the force acting on the Joint. single transverse fillet Joint is not Preferred because the edge of the plate, which is not welded, can ~~warp~~ warp out of shape. fig (a) The edge of the lower plate is free to deflect. Therefore, a double transverse fillet weld as shown fig (b). is preferred. A fillet weld is called Parallel or longitudinal, if the direction of weld is Parallel to the direction of the force acting on the Joint. in fig (c).

There are two types of cross-section for fillet weld - normal and convex as shown

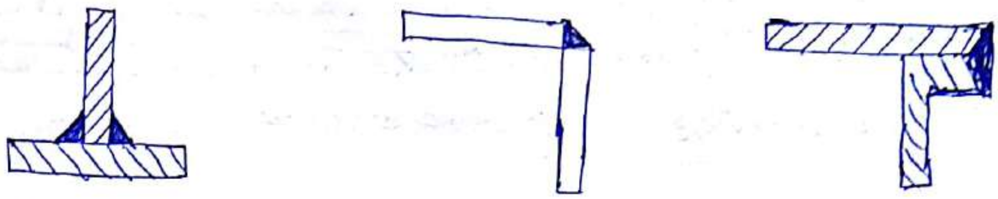
ⓐ Normal. ⓑ convex

The normal weld consists of an isosceles triangle. a triangle having two equal sides. shown in fig (a)

A convex weld is shown in fig (b).

convex weld requires more filler material and more labour. There is more stress concentration in convex weld compared ~~Com~~ with triangular weld. Therefore, normal weld is preferred over convex weld.

In addition to butt and fillet Joints, there are some other types of welded Joints, that are shown in fig. below. A tee-Joint is a Joint b/w two components located at right



ⓐ Tee Joint ⓑ corner Joint ⓒ Edge Joint

fig: Types of welded Joint

angles to each other in the form of a T. In this case, the end face of one component is welded to the side of the other component by means of a fillet weld as show above fig(a).

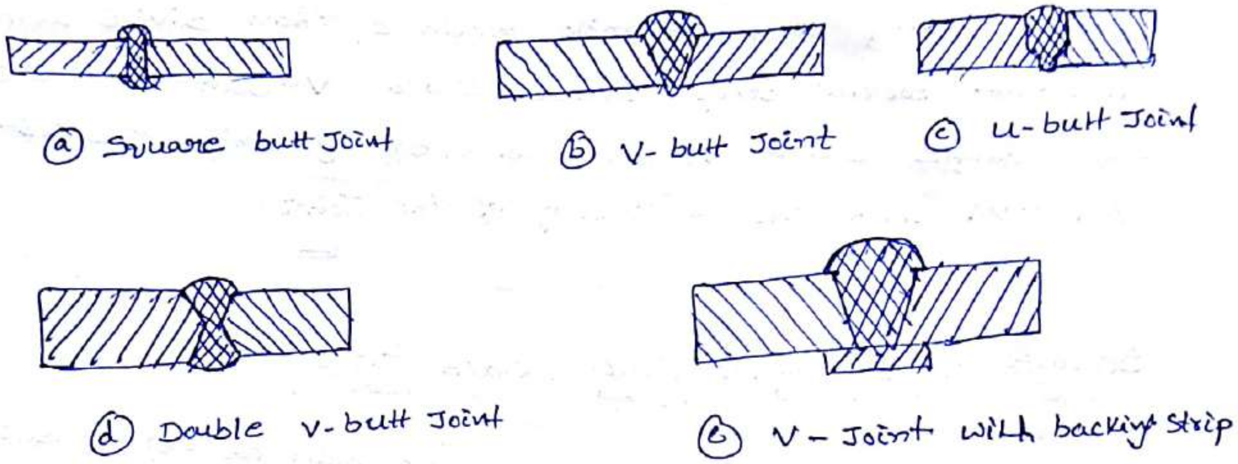
A corner Joint is a Joint b/w two components, that are at right angles to each other in the form of an angle. The adjacent edges are Joined by means of fillet weld as shown in fig(b).

A Edge Joint is a Joint b/w the edges of two or more parallel components as shown in fig(c). It is used for thin plates subjected to light loads.

Butt Joints:

A butt Joint can be defined as a Joint b/w two compo-nents lying approximately in the same plane. Butt Joint connects the ends of the two plates. The types of butt Joint are shown below. The selection of butt Joint depends upon the plate thickness and reliability.

(I) When the thickness of the plates is less than 5 mm, it is not necessary to bevel the edges of the plates. There is no preparation of the edges of the plates before welding. The edges are square with respect to the plates. Therefore, the Joint is called Square butt Joint. It is shown in fig (a).



(a) Square butt Joint (b) V- butt Joint (c) U- butt Joint

(d) Double V- butt Joint (e) V - Joint with backing strip

fig Types of Butt Joint

(II) when the thickness of the plates is b/w 5 to 25 mm, the edges are beveled before welding operation. The edges of two plates from V- shape. Therefore, the Joint is called V - Joint or single welded V - Joint. This Joint is welded only from one side. shown in fig (b)

(III) when the thickness of the plates is more than 20mm, the edges of the two plates are machined to form U - shape. The Joint is welded from one side. It is called single welded U - Joint. fig (c)

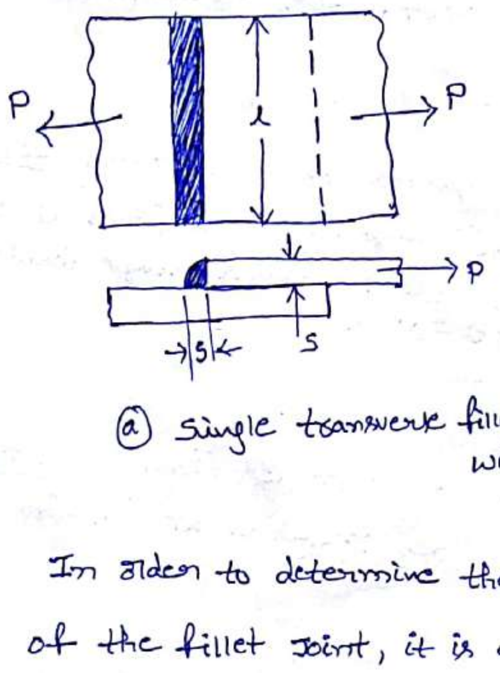
(IV) when the thickness of the plates is more than 30 mm, a double welded V - Joint is used. The Joint is welded from both sides of the plate in fig (d)

(V) when the welding is done by only ~~one~~ from one side, a Single welded V - Joint with backing strip is used to avoid the leakage of the molten metal on the other side. There are two types of the backing strip permanent steel backing, and removeable copper backing. fig (e).

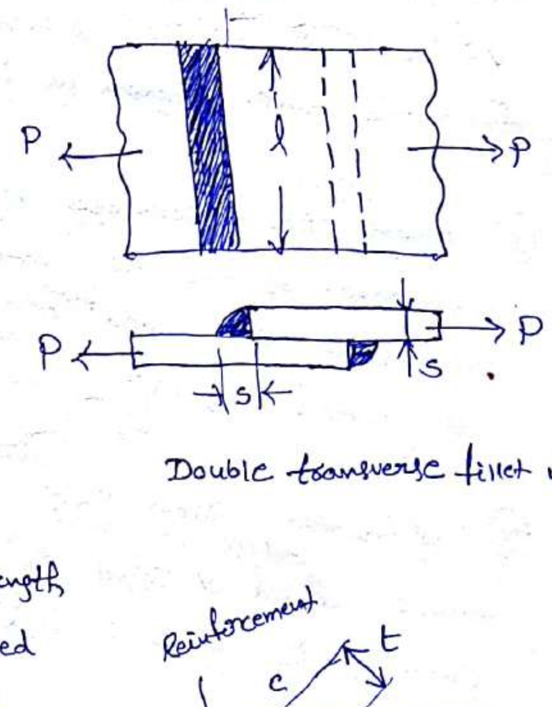
In applications like pressure vessels, the reliability of the Joint is an important consideration. single welded V-Joint is more reliable than square butt Joint. single welded V-Joint with backing strip is more reliable than single welded V-Joint without backing strip. Double welded V-Joint is more reliable than single welded V-Joint with backing strip. The cost also increases with the reliability of the Joint.

Strength of Transverse Fillet Welded Joints

The transverse fillet welds are designed for tensile strength. Transverse fillet welds are subjected to tensile stress. Let us consider a single and double transverse fillet welds as shown below fig (a) & fig (b) respectively.



(a) single transverse fillet weld



Double transverse fillet weld

In order to determine the strength of the fillet Joint, it is assumed that the section of fillet is a right angled triangle ABC with hypotenuse AC making equal angles with other two sides AB and BC.

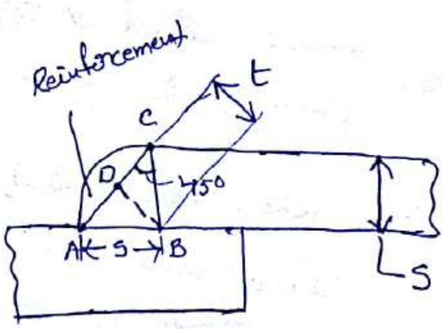


fig: Enlarged view of a fillet weld.

The enlarged view of the fillet is shown in fig. The length of each side is known as leg or size of the weld and the perpendicular distance of the hypotenuse from the intersection of legs (i.e. BD) is known as throat thickness. The minimum area of the weld is obtained at the throat BD, which is given by the product of the throat thickness and length of weld.

Let t = Throat thickness (BD)

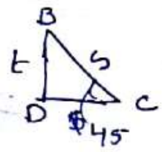
s = Leg or size of weld,

= Thickness of plate, and.

l = Length of weld.

from fig, we find that the throat thickness,

$$t = s \times \sin 45^\circ = 0.707\, s$$



$$\sin 45 = \frac{BD}{BC} = \frac{t}{s}$$

$$t = s \sin 45^\circ$$

∴ Minimum area of the weld or throat area,

A = Throat thickness × Length of weld

$$A = t \times l. = 0.707\, s\, l$$

If σ_t is the allowable tensile stress for the weld metal, then the tensile strength of the joint for single fillet weld,

P = Throat area × Allowable tensile stress

$$\boxed{P = 0.707\, s\, l \times \sigma_t} \qquad \because \sigma = \frac{P}{A}$$

and tensile strength of the joint for double fillet weld,

$$\boxed{P = 2 \times 0.707\, s\, l \times \sigma_t} = 1.414\, s\, l\, \sigma_t.$$

Note: Since the weld is weaker than the plate due to slag and blow holes, therefore the weld is given a reinforcement which may be taken as 10% of the plate thickness.

Strength of Parallel Fillet Welded Joints

The Parallel fillet welded Joints are designed for shear strength. Consider a double parallel fillet welded Joint as shown in fig (a). The minimum area of weld or the throat area,

$$\boxed{A = 0.707\, s l}$$

Note: The minimum area of the weld is taken because the stress is maximum at the minimum area.

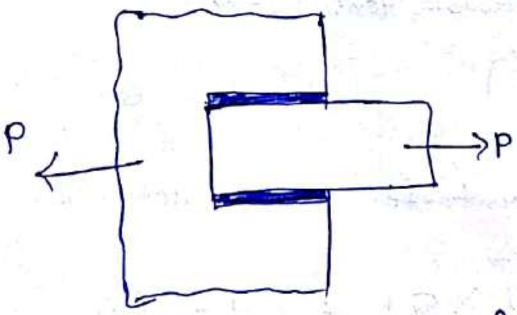
If τ is allowable shear stress for the weld metal, then the shear strength of the Joint for single parallel fillet weld,

$P =$ Throat area $\times$ Allowable shear stress

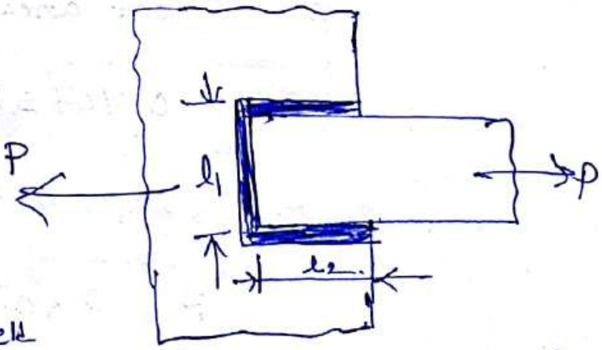
$$\boxed{P = 0.707\, s \times l \times \tau}$$

and shear strength of the Joint for double parallel fillet weld,

$$P = 2(0.707\, s l \tau) = 1.414\, s l \tau$$



(a) Double parallel fillet weld



(b) Combination of transverse & parallel fillet

Notes

1. If there is a combination of single transverse and double parallel fillet welds fig (b) then the strength of the joint is given by the sum of strengths of single transverse and double parallel fillet welds, Mathematically,

$$P = 0.707\, s \times l_1 \times \sigma_t + 1.414 \times l_2 \times \tau \times s$$

where l_1 is normally the width of the plate.

(2.) In order to allow for starting and stopping of the bead, 12.5 mm should be added to the length of each weld obtained by the above expression.

(3) For reinforced fillet welds, the throat dimension may be taken as 0.85t.

(1) A plate 100 mm wide and 10 mm thick is to be welded to another plate by means of double parallel fillets. The plates are subjected to a static load of 80 KN. Find the length of weld if the permissible shear stress in the weld does not exceed 55 MPa

Sol Given. width = 100 mm, ~~t = 10 mm~~, $P = 80\,KN = 80 \times 10^3\,N$

$\tau = 55\,MPa = 55\,N/mm^2$

Let l = Length of weld, and

s = size of weld = plate thickness = 10 mm

We know that maximum load which the plates can carry for double Parallel fillet weld. (P),

$$80 \times 10^3 = 1.414 \times 5 \times l \times \tau = 1.414 \times 10 \times l \times 55$$

$$80 \times 10^3 = 778\, l$$

$$\therefore \boxed{l = 103 \text{ mm}}$$

Adding 12.5 mm for ~~starting~~ starting and stopping of weld run, we have

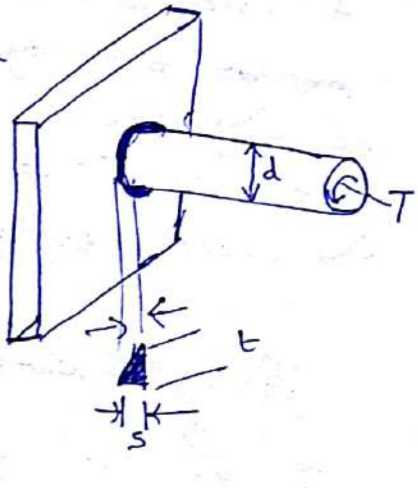
$$l = 103 + 12.5 = 115.5 \text{ mm}$$

Special cases of fillet welded Joints

The following cases of fillet ~~welded~~ welded Joints are important from the subject point of view.

1. Circular fillet weld subjected Torsion.

consider a circular rod connected to a rigid plate by a fillet weld as shown in fig



Let

d = Diameter of rod

r = Radius of rod,

T = Torque acting on the rod,

s = Size (or leg) of weld,

t = Throat thickness

J = Polar moment of Inertia of the weld Section $= \dfrac{\pi t d^3}{4}$

we know that shear stress for the material.

$$\tau = \frac{T \cdot r}{J} = \frac{T \times (d/2)}{J} \qquad \because \left[\frac{T}{J} = \frac{\tau}{r}\right]$$

$$= \frac{T \times (d/2) \times 4}{\pi t d^3} = \frac{2T}{\pi t d^2}$$

$$\boxed{\tau = \frac{2T}{\pi t d^2}}$$

~~we know that shear stress for the material,~~

$$\cancel{\tau = \frac{T \cdot r}{J} =}$$

This shear stress occurs in a horizontal plane along a leg of the fillet weld. The maximum shear occurs on the throat of weld which is inclined at 45° to the horizontal plane.

$\therefore$ Length of throat $t = s \sin 45° = 0.707s$

$$\boxed{t = 0.707s}$$

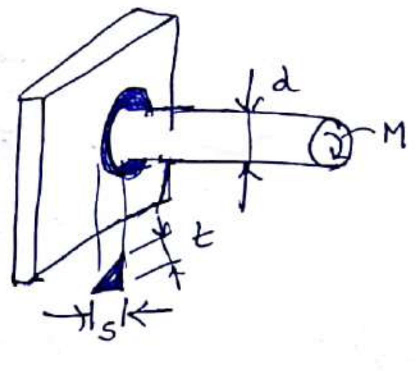
and maximum shear stress,

$$\tau_{max} = \frac{2T}{\pi \times 0.707 s \times d^2}$$

$$\boxed{\tau_{max} = \frac{2.83T}{\pi s d^2}}$$

2. circular fillet weld subjected to bending moment:

consider a circular rod connected to a rigid plate by a fillet weld as shown.



Let d = Diameter of rod,

M = Bending moment acting on the rod,

s = size (or leg) of weld,

t = Throat thickness,

Z = Section modulus of the weld section

$$= \frac{\pi t d^2}{4}.$$

we know that the bending stress,

$$\sigma_b = \frac{M}{Z} = \frac{M}{\frac{\pi t d^2}{4}} = \frac{4M}{\pi t d^2}$$

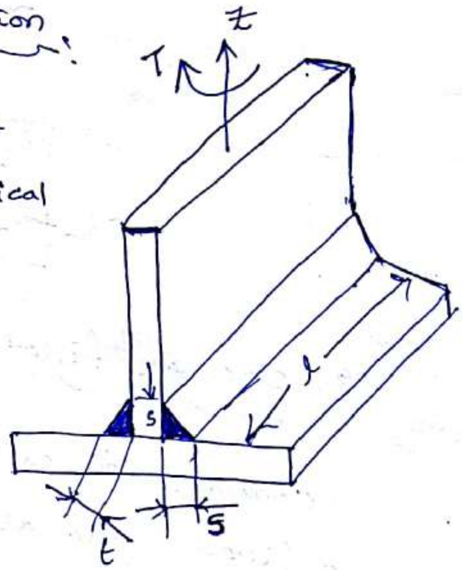
This bending stress occurs in a horizontal plane along a leg of the fillet weld. The maximum bending stress occurs on the throat of the weld which is inclined at 45° to the horizontal plane.

$\therefore$ Length of throat $t = s \sin 45^\circ = 0.707s$

and maximum bending stress,

$$(\sigma_b)_{max} = \frac{4M}{\pi \times 0.707 s \times d^2} = \frac{5.66M}{\pi s d^2}$$

(3) Long fillet weld subjected to torsion:



consider a vertical plate attached to a horizontal plate by two ~~vertical~~ indentical fillet welds as shown.

Let T = Torque acting on the vertical plate,

l = Length of weld,

s = Size (or leg) of weld

t = Throat thickness, and

J = Polar moment of inertia of the weld section

$$= 2 \times \frac{t \times l^3}{12} = \frac{t \times l^3}{6} \quad (\because \text{ of both sides weld})$$

It may be noted that the effect of the applied torque is to rotate the vertical plate about the Z-axis through its mid point. This rotation is resisted by shearing stresses developed b/w two fillet welds and the horizontal plate. It is assumed that these horizontal shearing stresses vary from zero at the Z-axis and maximum at the ends of the plate. This variation of shearing stresses is analogous to the variation of normal stress over the depth (l) of a beam subjected to pure bending.

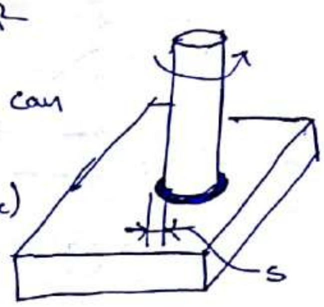
$$\therefore \text{ shear stress, } \tau = \frac{T \times l/2}{t \times l^3/6} = \frac{3T}{t \times l^2}$$

The maximum shear stress occurs at the throat and is given by

$$\tau_{max} = \frac{3T}{0.707 s \times l^2} = \frac{4.242 T}{s \times l^2}$$

① A 50 mm diameter solid shaft is welded to a flat plate by 10mm fillet weld as show in fig. find the maximum torque that the welded joint can sustain if the maximum shear stress intensity in the weld material is not to exceed 80 MPa.

sol Given that $d = 50mm$, $s = 10mm$, $\tau_{max} = 80 \ N/mm^2$



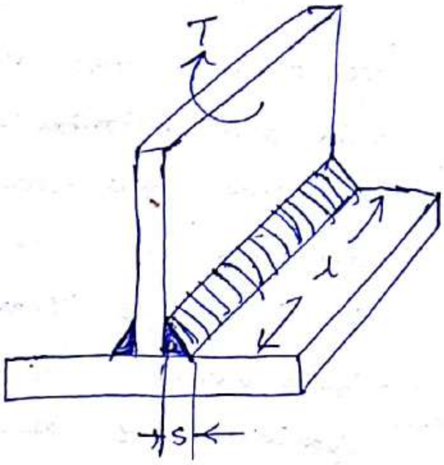
Let T = Maximum torque that the welded joint can sustain

We know that the maximum shear stress (τ_{max})

$$80 = \frac{2.83 T}{\pi s \ d^2}$$

$$\boxed{T = 2.22 \times 10^6 \ N\text{-}mm} = 2.22 \ kN\text{-}m.$$

(2) A Plate 1m long, 60 mm thick is welded to another plate at right angles to each other by 15 mm fillet weld, as shown. Find the maximum torque that the welded Joint can sustain if the permissible shear stress intensity in the weld material is not exceed 80 MPa.



Sol Given that

$l = 1m, = 1000 \text{ mm}$

$t = 60 \text{ mm}, \quad s = 15 \text{ mm},$

$\tau_{max} = 80 \text{ N/mm}^2$

Let T = Maximum torque that the welded Joint can sustain.

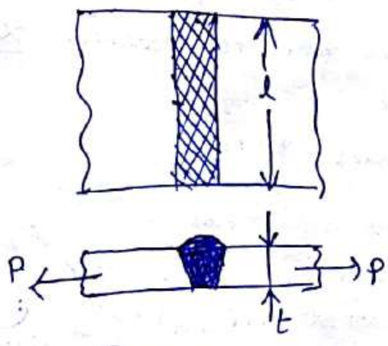
we Know that the maximum shear stress (τ_{max}),

$$80 = \frac{4.242\,T}{s \times l^2} = \frac{4.242\,T}{15(1000)^2} = \frac{0.283\,T}{10^6}$$

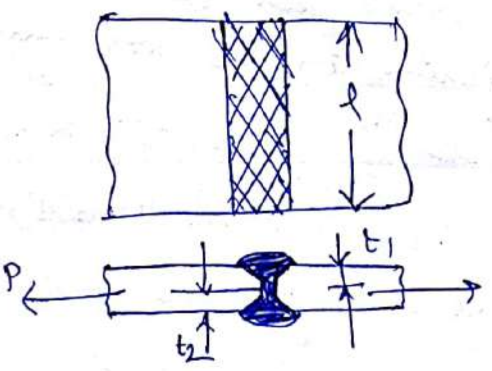
$$\therefore \boxed{T = 283 \text{ KN-m}}$$

Strength of Butt Joints:

The butt Joints are designed for tension or compression consider a single V-butt Joint as shown below.



(a) single v-butt Joint



(b) Double V-butt Joint

In case of butt Joint, the length of leg or size of welds is equal to the throat thickness which is equal to thickness of plates.

∴ Tensile strength of the butt Joint (single-V or square butt Joint)

$$\boxed{P = t \times l \times \sigma_t}$$

where l = Length of weld. It is generally equal to the width of plate.

and tensile strength for double V-butt Joint as shown above

$$\boxed{P = (t_1 + t_2)\, l \times \sigma_t}$$

where t_1 = Throat thickness at the top, and

t_2 = Throat thickness at the bottom.

It may be noted that size of the weld should be greater than the thickness of the plate, but it may be less. The following table shows recommended minimum size of the welds.

from J.D.B, Page 1145 & table 11.4

Thickness of plate (mm)	3-5	6-8	10-16	18-24	26-55	over 58
Minimum size of weld (mm)	3	5	6	10	14	20

Stresses for welded Joints!

The stresses in welded Joints are difficult to determine because of the variable and Unpredictable parameters like homogeneity of the weld metal, thermal stresses in the welds, changes of physical properties due to high rate of cooling etc. The stresses are obtained, on the following assumptions

1. The load is distributed uniformly along the entire length of the weld, and
2. The stress is spread uniformly over its effective section.

The following table shows the stresses for welded joints for joining ferrous metal with mild steel electrode under steady and fatigue or reversed load. from J.D.B 11.14 & table 11.2

Stresses for welded Joints

Type of weld	Bare electrode		Coated electrode	
	Steady load (MPa)	Fatigue load (MPa)	Steady load (MPa)	Fatigue load (MPa)
1. Fillet weld (All types)	80	21	98	35
2. Butt welds				
Tension	90	35	110	55
compression	100	35	125	55
Shear	55	21	70	35

Stress Concentration factor for welded Joints

The reinforcement provided to the weld produces stress concentration at the junction of the weld and the parent metal. When the parts are subjected to fatigue loading, the stress concentration factor as given

from J.D.B 11.15 table 11.3

Type of Joint	Stress Concentration factor
1. Reinforced butt welds	1.2
2. Toe of transverse fillet welds	1.5
3. End of parallel fillet weld	2.7
4. T-butt Joint with sharp corner	2.0

Note: For static loading and any type of Joint, stress concentration factor is 1.0.

① A plate 100 mm wide and 12.5 mm thick is to be welded to another plate by means of parallel fillet welds. The plates are subjected to a load of 50 KN. Find the length of the weld so that the maximum stress does not exceed 56 MPa. Consider the Joint first under static loading and then under fatigue loading.

Sol Given that width = 100 mm, $t = 12.5$ mm, $P = 50\times10^3$ N,

$\tau = 56$ MPa $= 56$ N/mm^2

Length of weld for static loading:

Let l = length of weld, and

s = size of weld = plate thickness

= 12.5 mm

We know that the maximum load which the plates can carry for double parallel fillet welds (P),

$$50\times10^3 = 1.414\, s \times l \times \tau$$

$$50\times10^3 = 1.414 \times 12.5 \times l \times 56 = 990\, l$$

$$\therefore \boxed{l = 50.5 \text{ mm}}$$

Adding 12.5 mm for starting and stopping of weld run, we have

$$l = 50.5 + 12.5 = 63 \text{ mm.}$$

Length of weld for fatigue loading:

from table we find that the stress concentration factor for Parallel fillet welding is 2.7

$\therefore$ Permissible shear stress,

$$\tau = \frac{56}{2.7} = 20.74 \ N/mm^2$$

We know that the maximum load which the plates can carry for double parallel fillet welds (P),

$$50 \times 10^3 = 1.414 \ s.l.\tau = 1.414 \times 12.5 \times l \times 20.74$$

$$\therefore \quad \boxed{l = 136.2 \ mm}$$

Adding 12.5 for starting and stopping of weld run, we have

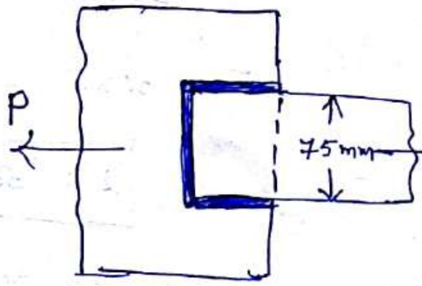
$$l = 136.2 + 12.5 = 148.7 \ mm$$

Q. A plate 75 mm wide and 12.5 mm thick is joined with another plate by a single transverse weld and a double parallel fillet weld as shown. The maximum tensile and shear stresses are 70 MPa and 56 MPa respectively. Find the length of each parallel fillet weld, if the joint is subjected to both static and fatigue loading.

Sol. Given: width = 75 mm,

thickness = 12.5 mm,

$\sigma_t = 70 \ N/mm^2$, $\tau = 56 \ N/mm^2$



The effective length of weld (l_1) for the transverse weld may be obtained by subtracting 12.5 mm from the width of the plate.

$$l_1 = 75 - 12.5 = 62.5 \ mm.$$

Length of each Parallel fillet for static loading

Let l_2 = length of each Parallel fillet

we know that the maximum load which the plate can carry is

$$P = \text{Area} \times \text{stress} = 75 \times 12.5 \times 70 = 65625\,N$$

Load carried by single transverse weld,

$$P_1 = 0.707\, s \times l_1 \times \sigma_t = 0.707 \times 12.5 \times 62.5 \times 70$$

$$\boxed{P_1 = 38664\,N}$$

and the load carried by double parallel fillet weld,

$$P_2 = 1.414\, s \times l_2 \times \tau = 1.414 \times 12.5 \times l_2 \times 56$$

$$\therefore \boxed{P_2 = 990\, l_2}\,N$$

∴ Load carried by the Joint (P),

$$65625 = P_1 + P_2 = 38664 + 990\, l_2$$

$$\boxed{l_2 = 27.2\text{ mm}}$$

Adding 12.5mm for starting and stopping of weld run, we have

$$l_2 = 27.2 + 12.5 = 39.7 \text{ say } 40\text{mm}$$

Length of each Parallel fillet for fatigue loading

from table, we find that the stress concentration factor for transverse weld is 1.5 and for Parallel fillet welds is 2.7.

∴ Permissible tensile stress,

$$\sigma_t = 70/1.5 = 46.7 \text{ N/mm}^2.$$

Load carried by single transverse weld,

$$P_1 = 0.707\, s \cdot l_1 \cdot \sigma_t = 0.707 \times 12.5 \times 62.5 \times 46.7$$

$$\boxed{P_1 = 25795 \text{ N}}$$

and load carried by double parallel fillet weld,

$$P_2 = 1.414\, s \cdot l_2 \cdot \tau = 1.414 \times 12.5\, l_2 \times 20.74$$

$$\boxed{P_2 = 366\, l_2} \text{ N}$$

∴ Load carried by the Joint (P),

$$65625 = P_1 + P_2 = 25795 + 366\, l_2$$

$$\boxed{l_2 = 108.8 \text{ mm}}$$

Adding 12.5 mm for starting and stopping of weld run, we have

$$l_2 = 108.8 + 12.5 = 121.3 \text{ mm},$$

(4) The fillet welds of equal legs are used to fabricate a 'T' as shown in fig. below (a) and (b), where s is the leg size and l is the length of weld.

Locate the plane of maximum shear stress in each of the following loading patterns:

1. Load parall to the weld (neglect eccentricity), and
2. Load at right angles to the weld (transverse load),

Find the ratio of these limiting loads.

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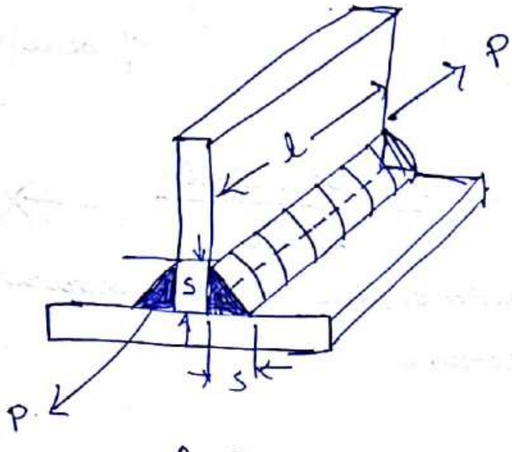


fig (a)

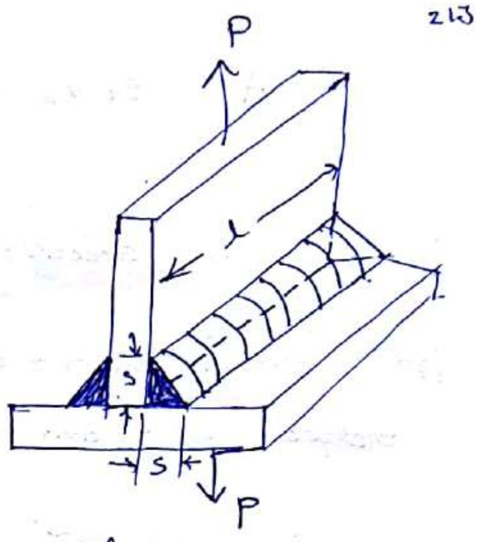


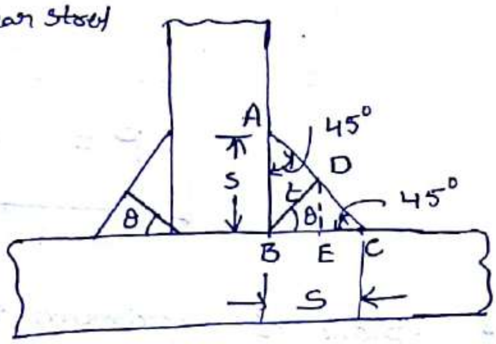
fig (b)

Sol Given: Leg size = s, Length of weld = l.

1. Plane of maximum shear stress when load acts parallel to the weld (neglecting eccentricity)

θ = Angle of plane of maximum shear stress

t = Throat thickness BD.



From figure, we find that

$$BC = BE + EC$$

$$= BE + DE \qquad (\because EC = DE)$$

or

$$s = BD\cos\theta + BD\sin\theta$$

$$= t\cos\theta + t\sin\theta$$

$$s = t(\cos\theta + \sin\theta)$$

$$\therefore \boxed{t = \frac{s}{\cos\theta + \sin\theta}}$$

We know that the minimum area of the weld or throat area,

$$A = 2t \times l = \frac{2s \times l}{(\cos\theta + \sin\theta)} \quad \ldots \left(\because \text{ of double fillet weld}\right)$$

and shear stress, $\tau = \frac{P}{A} = \frac{P(\cos\theta + \sin\theta)}{2s \times l} \rightarrow (I)$

for maximum shear stress, differentiate the above expression with respect to θ and equate to zero.

$$\frac{d\tau}{d\theta} = \frac{P}{2s \times l}\left(-\sin\theta + \cos\theta\right) = 0$$

$\sin\theta = \cos\theta$ or $\boxed{\theta = 45^\circ}$

substitue θ value of $\theta = 45^\circ$ in eqn (I), we have

$$\tau_{max} = \frac{P(\cos 45^\circ + \sin 45^\circ)}{2s \times l} = \frac{1.414 P}{2s \times l}$$

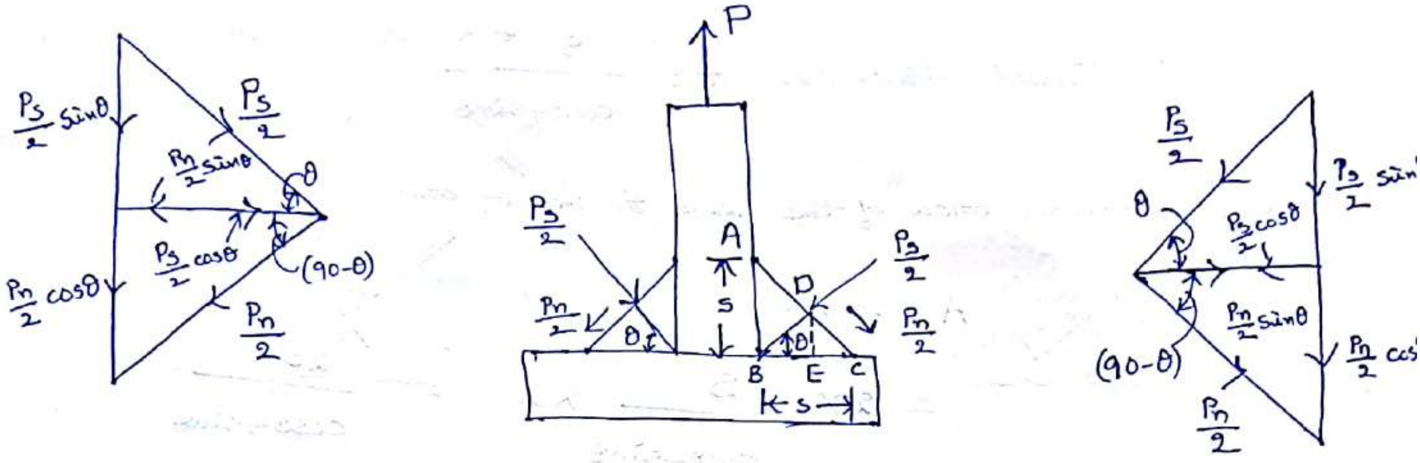
$$\boxed{P = 1.414 s \times l \times \tau_{max}}$$

(2.) Plane of maximum shear stress when load acts at right angles to the weld

When the load acts at right angles to the weld (transverse load), then the shear force and the normal force will act on each weld. Assuming that the two welds share the load equally, therefore summing up the vertical components, we have from fig below. $P = \frac{P_s}{2}\sin\theta + \frac{P_n}{2}\cos\theta + \frac{P_s}{2}\sin\theta + \frac{P_n}{2}\cos\theta$

$\boxed{P = P_s \sin\theta + P_n \cos\theta} \rightarrow (I)$

Assuming that the resultant of $\frac{P_s}{2}$ and $\frac{P_n}{2}$ is vertical, then the horizontal component are equal and opposite. we know



that, Horizontal component of $\frac{P_s}{2} = \frac{P_s}{2}\cos\theta$

and horizontal component of $\frac{P_n}{2} = \frac{P_n}{2}\sin\theta$

$$\frac{P_s}{2}\cos\theta = \frac{P_n}{2}\sin\theta$$

$$P_n = \frac{P_s\cos\theta}{\sin\theta}$$

Substituting the value of P_n in eqn (I)

$$P = P_s\sin\theta + \frac{P_s\cos\theta \times \cos\theta}{\sin\theta}$$

Multiplying throughout by $\sin\theta$, we have

$$P\sin\theta = P_s\sin^2\theta + P_s\cos^2\theta$$

$$= P_s(\sin^2\theta + \cos^2\theta)$$

$$\boxed{P\sin\theta = P_s} \longrightarrow (II)$$

from fig, we have

$$BC = BE + EC = BE + DE \qquad (\because EC = DE)$$

or $\quad s = t\cos\theta + t\sin\theta = t(\cos\theta + \sin\theta)$

$\therefore$ Throat thickness $t = \dfrac{S}{\cos\theta + \sin\theta}$

and minimum area of the weld or throat area,

$$A = 2t \times l.$$

$$= 2 \times \frac{S}{\cos\theta + \sin\theta} \times l = \frac{2 S l}{\cos\theta + \sin\theta}$$

$\therefore$ Shear stress, $\tau = \dfrac{P_s}{A} = \dfrac{P \sin\theta\,(\cos\theta + \sin\theta)}{2S \times l}$ [from (II)] $\rightarrow$ (III)

For maximum shear stress, differentiate the above expression with respect to θ and equate to zero.

$$\frac{d\tau}{d\theta} = \frac{P}{2Sl}\left[\sin\theta(-\sin\theta + \cos\theta) + (\cos\theta + \sin\theta)\cos\theta\right] = 0$$

or $-\sin^2\theta + \sin\theta\cdot\cos\theta + \cos^2\theta + \sin\theta\cdot\cos\theta = 0$ — $\left[\because \dfrac{d(u \cdot v)}{d\theta} = u\dfrac{dv}{d\theta} + v\dfrac{du}{d\theta}\right]$

$$\cos^2\theta - \sin^2\theta + 2\sin\theta\cdot\cos\theta = 0$$

Since $\cos^2\theta - \sin^2\theta = \cos 2\theta$ and $2\sin\theta\cdot\cos\theta = \sin 2\theta$, therefore

$$\cos 2\theta + \sin 2\theta = 0$$

$$\sin 2\theta = -\cos 2\theta$$

$$\frac{\sin 2\theta}{\cos 2\theta} = -1$$

$$\tan 2\theta = -1$$

$$\boxed{2\theta = 135^\circ}$$

$$\underline{\theta = 67.5^\circ}$$

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substituting the value of $\theta = 67.5^\circ$ in eqn (III), we have maximum shear stress,

$$\tau_{max} = \frac{P \sin(67.5^\circ)(\cos 67.5^\circ + \sin 67.5^\circ)}{2s \times l}$$

$$\tau_{max} = \frac{1.21\, P}{2s \times l}$$

$$P = \frac{2s \times l \times \tau_{max}}{1.21}$$

and $$\boxed{P = 1.65\, s \times l \times \tau_{max}}$$

Ratio of the limiting loads

we know that the ratio of the limiting (or maximum) loads

$$= \frac{1.414\, s \times l \times \tau_{max}}{1.65\, s \times l \times \tau_{max}}$$

$$= 0.857$$

Axially loaded Unsymmetrical welded sections

Sometimes unsymmetrical sections such as angles, channels, T-sections etc., welded on the flange edges are loaded axially as shown below. In such cases, the length of weld should be proportioned in such a way that the sum of resisting moments of the ~~wheels~~ welds about the gravity axis is zero. consider an angle section shown below.

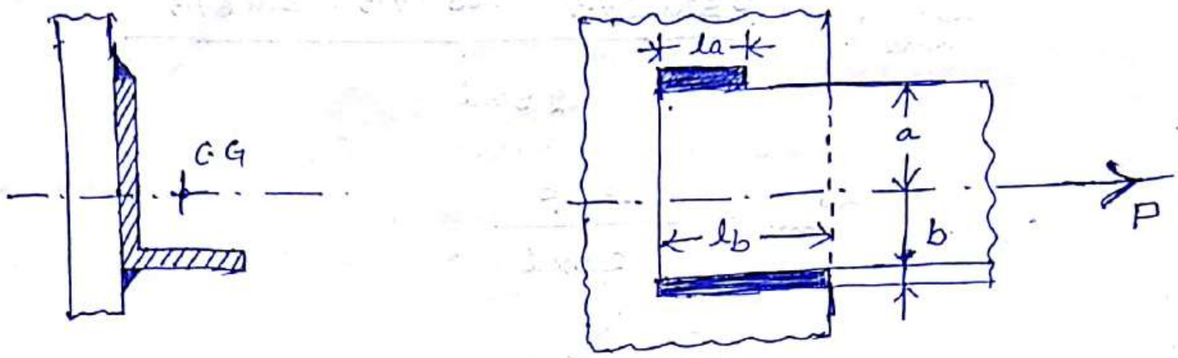


fig. Axially loaded unsymmetrical welded section

Let l_a = Length of weld at the top

l_b = Length of weld at the bottom

l = Total length of weld $= l_a + l_b$

P = Axial load,

a = Distance of top weld from gravity axis,

b = Distance of bottom weld from gravity axis, and

f = Resistance offered by the weld per unit length.

∴ Moment of the top weld about gravity axis

$$= l_a \times f \times a$$

and moment of the bottom weld about gravity axis

$$= l_b \times f \times b$$

Since the sum of the moments of the weld about the gravity axis must be zero, therefore,

$$l_a \times f \times a - l_b \times f \times b = 0$$

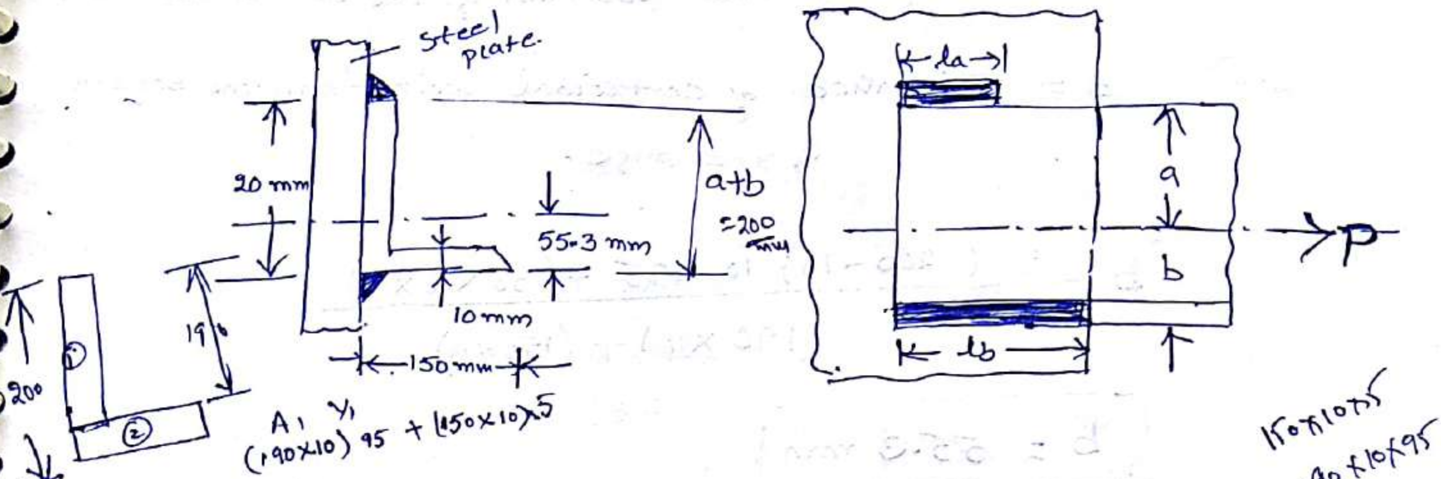
$$l_a \times a = l_b \times b \longrightarrow (I)$$

we know that $l = l_a + l_b \longrightarrow (II)$

$\therefore$ From eqns (I) & (II), we have

$$l_a = \frac{l \times b}{a+b}, \text{ and } l_b = \frac{l \times a}{a+b}.$$

(1) A 200 × 150 × 10 mm angle is to be welded to a steel plate by fillet welds as shown. If the angle is subjected to a static load of 200 KN, find the length of weld at the top and bottom. The allowable shear stress for static loading may be taken as 75 MPa.



Sol: Given: $a+b = 200$ mm, $P = 200$ KN $= 200 \times 10^3$ N,

95×2
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$\tau = 75$ MPa $= 75$ N/mm^2.

Let l_a = Length of weld at the top

l_b = Length of weld at the bottom, and

l = Total length of the weld $= l_a + l_b$

since, the thickness of the angle is 10 mm, therefore

Size of weld, $\boxed{s = 10 \text{ mm}}$

We know that for a single parallel fillet weld, the maximum load (P),

$$200 \times 10^3 = 0.707\ s \times l \times \tau$$

$$200 \times 10^3 = 0.707 \times 10 \times l \times 75$$

$$l = \frac{200 \times 10^3}{530.27}$$

$$\boxed{l = 377\ mm}$$

$$\boxed{l_a + l_b = 377\ mm} \qquad \because l = l_a + l_b$$

Now let us find out the position of the centroidal axis,

let b = Distance of centroidal axis from the bottom of the angle.

$$b = \frac{(200-10)\ 10 \times 95 + (150 \times 10 \times 5)}{(190 \times 10) + (150 \times 10)}$$

$$\boxed{b = 55.3\ mm}$$

and $a = 200 - 55.3 = 144.7\ mm.$

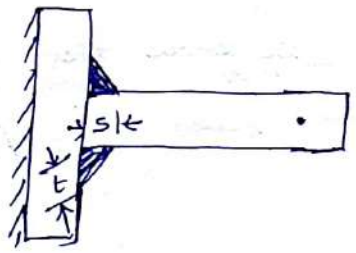
We know that $l_a = \frac{l \times b}{a+b} = \frac{377 \times 55.3}{200}$

$$\boxed{l_a = 104.2\ mm}$$

and $l_b = l - l_a = 377 - 104.2 = 272.8\ mm.$

Eccentrically loaded welded Joints:

An eccentric load may be imposed on welded Joints in many ways. The stresses induced on the Joint may be of different nature or of the same nature. The induced stresses are combined depending upon the nature of stresses. When the shear and bending stresses are simultaneously present in a Joint (case 1), the maximum stresses are as follows:

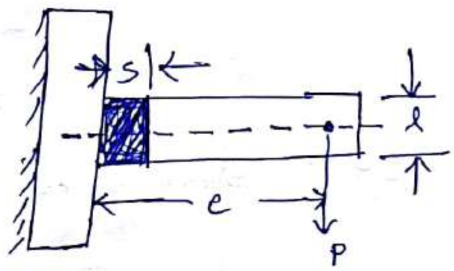


Maximum normal stress,

$$(\sigma_t)_{max} = \frac{\sigma_b}{2} + \frac{1}{2}\sqrt{(\sigma_b)^2 + 4\tau^2}$$

and Maximum shear stress

$$\tau_{max} = \frac{1}{2}\sqrt{(\sigma_b)^2 + 4\tau^2}$$



where σ_b = Bending stress, and

τ = Shear stress.

fig: Eccentrically loaded welded Joint

When the stresses are of the same nature, these may be combined vectorially (see case 2).

We shall now discuss the two cases of Eccentric loading as follows:

Case 1:-

Consider a T-Joint fixed at one end and subjected to an eccentric load P at a distance e as shown above.

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Let

s = Size of weld,

l = Length of weld, and

t = Throat thickness.

The Joint will be subjected to the following two types of Stresses:

1. Direct shear stress due to the shear force $\underline{P}$ acting at the welds, and
2. Bending stress due to the bending moment $\underline{P \times e}$.

We know that area at the moment,

$$A = \text{Throat thickness} \times \text{Length of weld}$$

$$= t \times l \times 2 = 2t \cdot l \quad \text{--- (for double fillet weld)}$$

$$= 2 \times 0.707 s \times l$$

$$= 1.414 s \times l \qquad (\because t = s\cos 45^\circ = 0.707 s)$$

$\therefore$ Shear stress in the weld (assuming uniformly distributed),

$$\tau = \frac{P}{A} = \frac{P}{1.414 \times s \times l}$$

Section modulus of the weld metal through the throat,

$$Z = \frac{t \times l^2}{6} \times 2 \quad \text{--- (for both sides weld)}$$

$$= \frac{0.707 s \times l^2}{6} \times 2 = \frac{s \times l^2}{4.242} \qquad \left[Z = \frac{I}{y} = \frac{\frac{tb^3}{12}}{b/2} \right]$$

Bending moment, $M = P \times e$

$\therefore$ Bending stress, $\sigma_b = \frac{M}{Z} = \frac{P \times e \times 4.242}{s \times l^2}$

$$\sigma_b = \frac{4.242\, P \times e}{s \times l^2}$$

we know that the maximum normal stress,

$$(\sigma_t)_{max} = \frac{1}{2}\sigma_b + \frac{1}{2}\sqrt{(\sigma_b)^2 + 4\tau^2}$$

and maximum shear stress,

$$\tau_{max} = \frac{1}{2}\sqrt{(\sigma_b)^2 + 4\tau^2}.$$

Case - 2. When a welded joint is ~~subjected~~ loaded eccentrically as shown below. the following two types of stresses are induced:

1. Direct or primary shear stress, and
2. Shear stress due to turning moment.

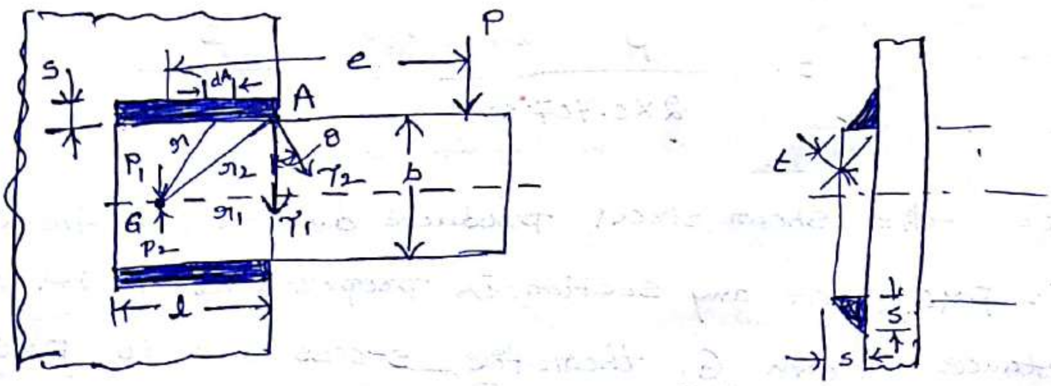


fig: Eccentrically loaded weld Joint.

Let P = Eccentric load,

e = Eccentricity i.e. perpendicular distance b/w the line of action of load and centre of gravity (G) of the throat section or fillets.

l = Length of single weld,

S = Size or leg of weld, and

t = Throat thickness

Let two loads P_1 and P_2 (each equal to P) are introduced at the centre of Gravity 'G' of the weld system. The effect of load $P_1 = P$ is to produce direct shear stress which is assumed to be uniform over the entire weld length. The effect of load $P_2 = P$ is to produce a turning moment of magnitude $P \times e$ which tends to rotate the Joint about the centre of Gravity 'G' of the weld system. Due to the turning moment, secondary shear stress is induced.

We know that the direct or primary shear stress,

$$\tau_1 = \frac{\text{Load}}{\text{Throat area}} = \frac{P}{A} = \frac{P}{2t \times l}$$

$$= \frac{P}{2 \times 0.707 s \times l} = \frac{P}{1.414 s \times l}$$

since the shear stress produced due to the turning moment ($T = P \times e$) at any section is proportional to its radial distance from G, therefore stress due to $P \times e$ at the point A is proportional to AG (r_2) and is in a direction at right angles to AG. In other words,

$$\frac{\tau_2}{r_2} = \frac{\tau}{r} = \text{constant}.$$

$$\boxed{\tau = \frac{\tau_2}{r_2} \times r} \longrightarrow (I)$$

where τ_2 is the shear stress at the maximum distance (r_2)

and τ is the shear stress at any distance r.

Consider a small section of the weld having area dA at a distance r from G.

$\therefore$ Shear force on this small section $= \tau \times dA$

and turning moment of this shear force about G,

$$dT = \tau \times dA \times r = \frac{\tau_2}{r_2} \times dA \times r^2 \quad \because (\text{from } \text{I})$$

$\therefore$ Total turning moment over the whole weld area,

$$T = P \times e = \int \frac{\tau_2}{r_2} \times dA \times r^2 = \frac{\tau_2}{r_2} \int dA \times r^2$$

$$= \frac{\tau_2}{r_2} \times J \qquad \left(\because J = \int dA \times r^2 \right)$$

where J = polar moment of inertia of the throat area about G.

$\therefore$ Shear stress due to the turning moment i.e secondary shear stress,

$$\tau_2 = \frac{T \times r_2}{J} = \frac{P \times e \times r_2}{J}$$

In order to find the resultant stress, the primary and secondary shear stresses are combined vectorially.

$\therefore$ Resultant shear stress at A,

$$\tau_A = \sqrt{(\tau_1)^2 + (\tau_2)^2 + 2\tau_1 \tau_2 \cos\theta}$$

where θ = Angle between τ_1 and τ_2, and

$$\boxed{\cos\theta = \frac{r_1}{r_2}}$$

Note: The polar moment of inertia of the throat area (A) about the centre of gravity (G) is obtained by the parallel axis theorem, i.e.

$$J = 2\left[I_{xx} + Ax^2\right] \quad \ldots \left[\because \text{of double fillet weld}\right]$$

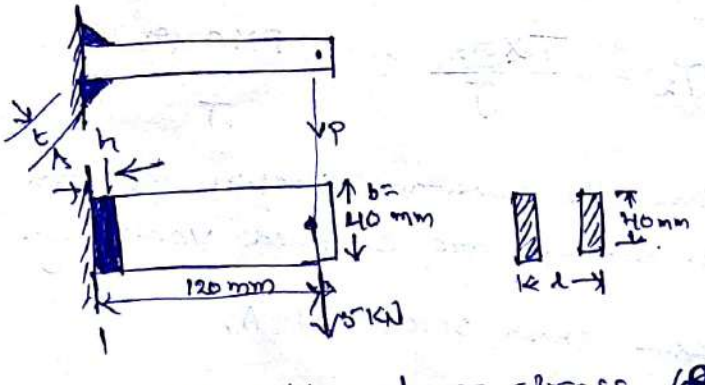
$$= 2\left[\frac{A \times l^2}{12} + Ax^2\right] = 2A\left(\frac{l^2}{12} + x^2\right)$$

where A = Throat area $= t \times l = 0.707\, s \times l$

l = Length of weld, and

x = perpendicular distance b/w the two parallel axes.

1) A welded Joint as shown in fig. is subjected to an eccentric load of 5KN. Find the size of weld if the maximum shear stress in the weld is 40 N/mm².



Load $P = 5000N$

Eccentricity $e = 120$ mm

Maximum allowable shear stress (τ_s) = 40 N/mm²

since the load is applied eccentrically, two types of stresses are induced in the weld, which are

1. Direct shear stress due to load, P

2. Bending stress due to bending moment, (P×e)

Let t be the throat thickness and h be the size of weld.

Now, Direct shear stress, $\tau = \dfrac{\text{Applied Load}}{\text{Total throat Area}} = \dfrac{P}{A}$

$$\tau = \frac{P}{2bt} = \frac{5000}{2\times 40\times t} = \frac{62.5}{t} \ N/mm^2.$$

Bending stress, $\sigma_b = \dfrac{M}{Z} = \dfrac{P.e}{\frac{tb^2}{3}}$ $\left(Z = \dfrac{tb^2}{3} \right)$

$$= \frac{5000\times 120}{\left(\frac{t\times 40^2}{3}\right)} = \frac{1125}{t} \ N/mm^2$$

The maximum shear stress induced in the weld is given by,

$$(\tau)_{max} = \frac{1}{2}\sqrt{(\sigma_b)^2 + 4\tau^2}$$

$$= \frac{1}{2}\sqrt{\left(\frac{1125}{t}\right)^2 + 4\times\left(\frac{62.5}{t}\right)^2}$$

$$= \frac{566}{t} \ N/mm^2$$

But the maximum allowable shear stress = 40 N/mm²

$$\frac{566}{t} = 40$$

$$t = \frac{566}{40} = 14.15 \text{ mm}$$

Size of the weld, $h = \sqrt{2}\times t = \sqrt{2}\times 14.15 = \underline{20 \text{ mm}}$

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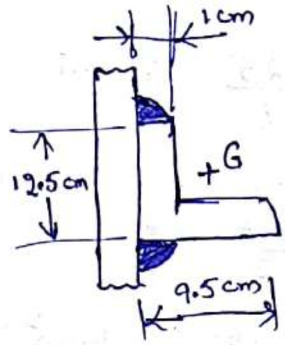
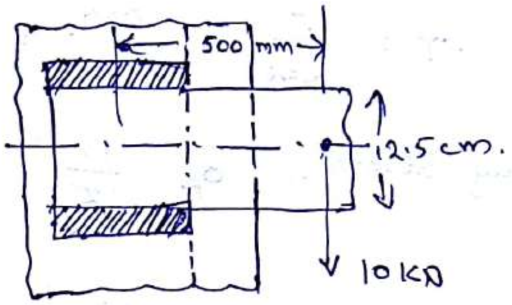
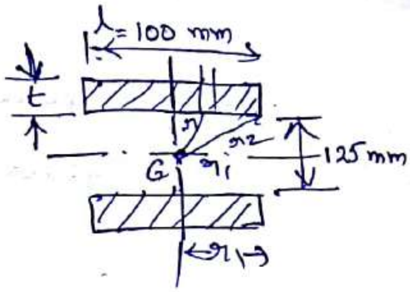
(2) A 12.5 × 9.5 × 1 cm angle is welded to a frame by two 1 cm fillet welds. A load of 10KN is applied normal to the centre of gravity axis at a distance of 500 mm from the centre of gravity of welds. Find the maximum shear stress in the welds, assuming each weld to be 100 mm long and parallel to the axis of the angle.





Sol Load $P = 10\ KN = 10{,}000\ N$.

Eccentricity, $e = 500\ mm$.



Size of weld, $h = 1\ cm = 10\ mm$

Length of each weld, $l = 100\ mm$.

Let the angle is welded in the frame in such a way that the height of angle may be 12.5 cm.

i.e $b = 12.5\ cm = 125\ mm$.

Since the weld is subjected to the eccentric load, two types of stresses are induced in the weld, which are,

1. primary (or direct) Shear stress due to load P.
2. secondary Shear stress due to twisting moment, T

Now, primary shear stress $s_{s_1} = \frac{P}{A} = \frac{P}{2lt}$

$$\tau_1 = s_{s_1} = \frac{P}{2l \times 0.707h} \quad \left(\because \text{throat thickness} = \frac{h}{\sqrt{2}} = 0.707h\right)$$

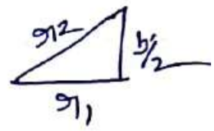
$$= \frac{10,000}{2 \times 100 \times 0.707 \times 10} = 7.1 \ N/mm^2$$

Secondary shear stress $S_{s_2} = \frac{T \cdot r_2}{J}$

where $T =$ Twisting moment $= P \cdot e$

$= 5000 \times 10^3$ N-mm.

$$J = \frac{tl(3b^2 + l^2)}{6} = \frac{0.707 \times 10 \times 100 (3 \times 125^2 + 100^2)}{6}$$



$$J = 6702 \times 10^3 \ mm^4$$

$$r_2 = \sqrt{r_1^2 + \left(\frac{b}{2}\right)^2} = \sqrt{(50)^2 + \left(\frac{125}{2}\right)^2} = 80 \ mm$$

$$\tau_2 = S_{s_2} = \frac{T \cdot r_2}{J} = \frac{5000 \times 10^3 \times 80}{6702 \times 10^3} = 59.7 \ N/mm^2.$$

Now, the resultant shear stress is given by

$$\tau_r = S_r = \sqrt{(\tau_1)^2 + (\tau_2)^2 + 2\tau_1 \tau_2 \cos\theta}$$

where $\cos\theta = \frac{r_1}{r_2} = \frac{50}{80} = 0.625$

$$\tau_r = \sqrt{(7.1)^2 + (59.7)^2 + 2 \times 7.1 \times 59.7 \times 0.625}$$

$$\boxed{\tau_r = 64.4 \ N/mm^2}$$

The maximum shear stress in the weld

$$\tau_r = 64.4 \ N/mm^2$$

UNIT - III

Shafts & Couplings

A rod of circular cross-section usually employed for transmitting power from one machine to another is called shaft. For transmitting power from one shaft to another, the elements like pulleys, gears, sprockets, coupling etc. are mounted on the shafts.

The shaft is always stepped with maximum diameter in the middle portion and minimum diameter at the two ends, where bearings are mounted. The steps on the shaft provide shoulders for positioning transmission elements like gears, pulleys and bearings. The rounded-off portion b/w two cross sections of different diameters is called a fillet. The fillet radius is provided to reduce the effect of stress concentration due to abrupt (sudden) change in the cross-section.

Types of shafts: Different types of shafts are employed for different types of applications. They are

(I) Line shaft: This shaft transmits power usually from single electric motor to many machines through belt drives as seen in rice mill.

(II) Propeller shaft: This is used to transmit power from the engine to the rear wheels of the automobile vehicle.

(III) Axle: It is also a shaft which may be stationary (Ex: front axle of automobile) or rotating (Ex: rear axle of automobile). The front axle simply supports the load where as the rear axle supports the load and also transmits power from propeller shaft to rear wheels through different gears.

(IV) Spindle: It is usually a hollow shaft used in machine-tools like lathe, drilling machine etc.

(V) Crank shaft: This is an integral part of the machine (Ex: I.C engine) and hence called as machine shaft. This shaft is not a straight one and is having a specific shape in order to fix with the connecting rod of the engine.

(VI) Counter shaft (or) Jack shaft:

This shaft connects the prime mover with a line shaft or a machine.

(VII) Flexible shaft: This shaft permits the transmission of motion when the axes of the shafts are not co-linear (i.e. at an angle to each other) for example the shaft of hand grinding machine as used in grill workshop.

Shafting Materials: usually the axles and shafts are made of mild steel of various grades. for heavy loaded shafts, alloy steels such as nickel,

nickel - chromium and chrome-vanadium steels are used.

Standard sizes of shafts:

The size (i.e diameter) of shafts may be decided by Preffered numbers, generally from R20 series. Some of the sizes are 10, 12, 14, 16, 18, 20, 22, 25, 28, 32, 36, 40, 45, 50, 56, 63, 71, 80, 90, 100, 110, 125, 140, 160, 180, 200, 220, 240, 260, 280, 300, 320, 340, 360, 380, 400, 420, 440, 450, 480 500 and so on. All diameters are mentioned in millimeters.

The standard length of shafts are 5m, 6m, and 7m.

step ratio(ϕ) R20 - $\sqrt[20]{10} = 1.12$.

R20 - R-stands for Charles Renard

Working stresses for shafts:

for tensile & compressive loads, the allowable working stress ranges from 60 N/mm² to 110 N/mm² and the optimum working stress is about 84 N/mm². similarly for torsional loads the allowable working stress ranges from 30 N/mm² to 55 N/mm² and optimum value is around 42 N/mm². All the above working stresses are based on the percentage of carbon content in the shafting materials which is mainly steel.

In general, the permissible tensile or compressive stress may be taken as 60 percent of yield strength and not more than 36 percent of the ultimate tensile strength. similarly, the permissible shear stress may be taken as 30 % of yield strength and not more than 18 % of ultimate tensile strength. i.e. the permissible

$S_t = \sigma_t = 0.6\, S_y (or)\, \sigma_y$, $S_t = 0.36 S_{ut}$

shear stress may be taken as 50% of permissible tensile or compressive stress.

Design Considerations:

Shafts are employed for power transmission and also load supporting. Hence they have to be designed to have sufficient strength in torsion and bending & also sufficient rigidity. wherever possible, the length of shaft may be reduced in order to increase the torsional stiffness.

In the case of long shafts, the pulleys & gears should be mounted close to the bearings to safeguard the shafts from deflection due to bending. Also the size of shaft should be designed in such a way that the critical speed of the shaft should not match with the natural frequency of the shaft. other-wise the shaft will fail due to lateral vibration.

usually the key slot, grooves etc. must be chamfered in order to reduce the stress concentration effect.

Design of Uniform cross sectional shafts:

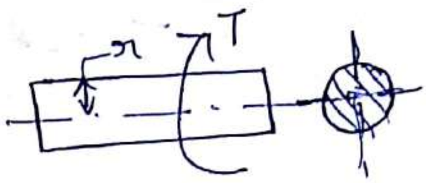
Shafts whose cross section is uniform throughout the length may be designed on the basis of two conditions.

(1) strength conditions and (2) stiffness or Rigidity conditions.

strength means whether the shaft is strong enough to transmit the required power, support the required load & so on. Depending upon the operating conditions, the shafts may be designed in four ways such that,

1. Design of shafts subjected to torque only.
2. Design of shafts subjected to bending moment only.
3. Design of shafts subjected to combined action of torque and bending moments, and
4. Design of shafts subjected to combined action of torque, bending moment and an axial load.

Design of shafts subjected to Torque (T) only:



when the power is transmitted from the motor to any machine, the shaft connecting the motor and machines by means of gears, couplings, pulleys or by other means may said to be subjected to twisting moment or torque.

from the known expression,

$$\frac{T}{J} = \frac{\tau}{r}$$

we may get the relationship for a circular shaft b/w torque, shear stress and diameter of the shaft which have already been derived ~~in th~~

$$T = \frac{\pi}{16} \tau d^3 \quad (\text{for solid shaft})$$

and $$T = \frac{\pi}{16} \tau d_o^3 (1 - k^4) \quad (\text{for hollow shaft})$$

where T = Twisting moment or torque

J = Polar moment of inertia

τ = Torsional shear stress

r = Radius of the shaft

d = Diameter of the shaft

k = Ratio of inner diameter to outer diameter

$$\boxed{\text{i.e } k = \frac{d_i}{d_o}}$$

The torque (T) used in the above relations is the maximum torque to be transmitted. But the torque calculated from the power is the mean torque.

i.e Torque developed due to power,

$$T = \frac{4500P}{2\pi N} \quad \text{where } P \text{ is HP, } T \text{ is kgf-m}$$

(or)

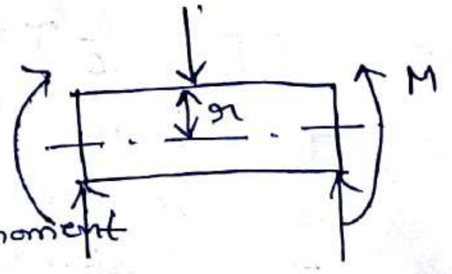
$$T = \frac{60P}{2\pi N} \quad \text{where } P \text{ in watts, } T \text{ in N-m}$$

i.e. The torque calculated in the "power-torque" relations is the mean torque.

Generally $T_{max} \geq T_{mean}$. When designing the shaft we can assume maximum torque equal to mean torque at some occations when the relationships b/w mean & maximum torque is not given.

Design of shafts subjected to bending moment (M) only

When a shaft is acted by a load, it will undergo bending & hence bending moment



we have already derived the relationship b/w Bending moment, bending stress & diameter of shaft for circular shaft from the known expression,

$$\frac{M}{I} = \frac{\sigma_b}{y}$$

$$\frac{M}{I} = \frac{\sigma}{y} = \frac{E}{R}$$

$$\boxed{y = \frac{d}{2}} = r$$

$$\boxed{\frac{M}{I} = \frac{\sigma}{r}}$$

$$M = \frac{\pi}{32} \sigma_b d^3 \quad \text{for solid shaft.}$$

$$M = \frac{\pi}{32} \sigma_b d_o^3 (1 - K^4) \quad \text{for hollow shaft.}$$

where M = Maximum bending moment

I = Area moment of Inertia

σ_b = Bending stress induced

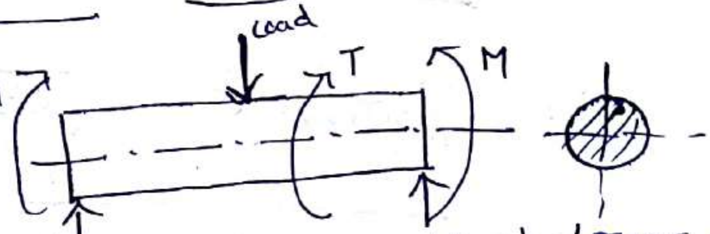
r = Radius of shaft

d = Dia of shaft.

K = Ratio of inner dia to outer dia.

$$K = \frac{d_i}{d_o}$$

Design of shafts subjected to combined Torque & Bending moment (T & M)



When a shaft, carrying heavy pulleys, gears or some loads, transmits power, it is said to be subjected to combined torque & bending moment. In this case, the shaft may be designed based on two theories.

1. Guest's theory (or) Maximum shear stress theory
2. Rankine's theory (or) Maximum normal stress theory.

Let τ = Torsional shear stress induced due to pure twisting moment, (T)

σ_b = Bending stress induced due to pure bending moment (M)

Guest's theory explain's that, the maximum shear stress due to combined action of twisting moment & bending moment is

$$\tau_{max} = \frac{1}{2}\sqrt{(\sigma_b)^2 + 4\tau^2}$$

where $\sigma_b = \frac{32M}{\pi d^3}$ and $\tau = \frac{16T}{\pi d^3}$ (for solid shaft)

By substituting, we get

$$\tau_{max} = \frac{1}{2}\sqrt{\left(\frac{32M}{\pi d^3}\right)^2 + 4\left(\frac{16T}{\pi d^3}\right)^2}$$

$$= \frac{1}{2} \times \frac{32}{\pi d^3}\sqrt{M^2 + T^2}$$

$$\boxed{\tau_{max} = \frac{16}{\pi d^3}\sqrt{M^2+T^2}}$$

i.e $\frac{\pi}{16}(\tau_{max})\, d^3 = \sqrt{M^2+T^2} = T_e$

$$\boxed{T_e = \frac{\pi}{16}\tau_{max}\, d^3}$$

where $T_e = \sqrt{M^2+T^2}$ is known as Equivalent torque.

By limiting maximum shear stress (τ_{max}) to allowable stress (τ), we can calculate the dia. of the shaft or any other required unknown data.

According to Rankine's theory, the maximum normal stress is given by

$$(\sigma_b)_{max} = \frac{1}{2}\left[\sigma_b + \sqrt{\sigma_b^2 + 4\tau^2}\right]$$

$$= \frac{1}{2}\left[\frac{32M}{\pi d^3} + \sqrt{\left(\frac{32M}{\pi d^3}\right)^2 + 4\cdot\left(\frac{16T}{\pi d^3}\right)^2}\right]$$

$$= \frac{32}{\pi d^3}\left[\frac{1}{2}\left(M + \sqrt{M^2+T^2}\right)\right]$$

or

$$\frac{\pi d^3}{32}(\sigma_b)_{max} = \frac{1}{2}\left(M + \sqrt{M^2+T^2}\right) = M_e$$

i.e $\boxed{M_e = \frac{\pi}{32}(\sigma_b)_{max}\, d^3}$

where $M_e = \frac{1}{2}\left(M + \sqrt{M^2+T^2}\right)$ is called equivalent bending moment.

for hollow shafts, the parameter d^3 in the above eqns may replaced by $d_o^3(1-k^4)$.

Generally, the Guest's theory will be used for ductile materials & Rankine's theory will be used for brittle materials.

for designing the shaft, the diameter is calculated based on both theories and the larger value will be chosen.

In actual practice, the torque & bending moment may not be constant because of change of power & loads due to voltage variations & surroundings nature like non-uniformity of roads as in the case of automobiles & soon. Hence for designing such shafts, subjected to this type of fluctuating loads, certain safety factors called shock & fatigue factors may be taken into account.

Let K_m = Combined shock & fatigue factor for bending.

K_t = Combined shock & fatigue factor for torsion.

By including the above factors, the equivalent torque, (T_e) may be changed as

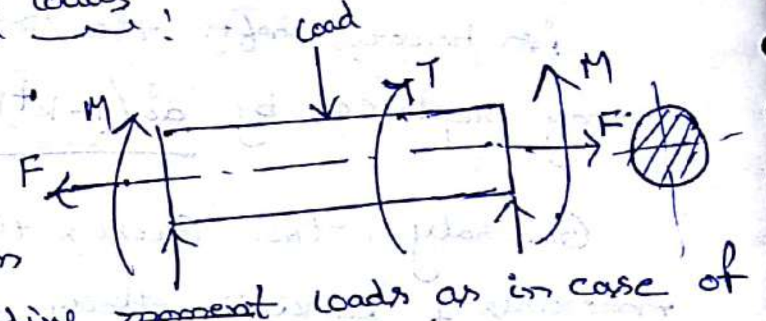
$$T_e = \sqrt{(K_m \cdot M)^2 + (K_t \cdot T)^2}$$

and the equivalent bending moment $\underline{M_e}$, may be changed as

$$M_e = \frac{1}{2}\left[K_m \cdot M + \sqrt{(K_m \cdot M)^2 + (K_t \cdot T)^2} \right]$$

Some recommended values of K_m & K_t

Design of shafts subjected to combined torque, bending moment and axial loads :



Sometimes the shafts may be subjected to axial loads in addition to torsional and bending ~~moment~~ loads as in case of propeller shafts of ships and shafts for driving worm gears and so on. In these cases, the equivalent torque (T_e) and equivalent bending moment (M_e) may be determined as follows.

Axial stress due to axial load,

$$\sigma_a = \frac{F}{\frac{\pi}{4}d^2} = \frac{4F}{\pi d^2} \text{ for solid circular shaft}$$

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and

$$\sigma_a = \frac{4F}{\pi d_o^2(1-k^2)}$$ for hollow circular shaft.

Bending stress due to bending moment.

$$\sigma_b = \frac{32M}{\pi d^3}$$ for solid circular shaft.

$$\sigma_b = \frac{32M}{\pi d_o^3(1-k^4)}$$ for hollow circular shaft

Resultant tensile stress for solid shaft.

$$\sigma_t = \sigma_b + \sigma_a$$

$$= \frac{32M}{\pi d^3} + \frac{4F}{\pi d^2} = \frac{32}{\pi d^3}\left(M + \frac{Fd}{8}\right)$$

$$= \frac{32M_1}{\pi d^3}$$ where $M_1 = \left(M + \frac{Fd}{8}\right)$

Similarly for hollow shaft.

$$\sigma_t = \frac{32M}{\pi d_o^3(1-k^4)} + \frac{4F}{\pi d_o^2(1-k^2)}$$

$$= \frac{32}{\pi d_o^3(1-k^4)}\left[M + \frac{Fd_o(1+k^2)}{8}\right]$$

$$= \frac{32M_2}{\pi d_o^3(1-k^4)}$$ where $M_2 = M + \frac{Fd_o(1+k^2)}{8}$

If the axial load is compressive and the shaft is very long, then a factor called column factor (α) may be introduced to take the column effect into account.

Now by combining all the conditions, the equivalent torque (T_e) for the hollow shaft which is subjected to fluctuating torsional & bending loads along with compressive axial load, is given by

$$T_e = \sqrt{\left[K_m M + \frac{\alpha F d_o (1+k^2)}{8}\right]^2 + (K_t T)^2}$$

$$\boxed{T_e = \frac{\pi}{16} \tau d_o^3 (1-k^4)}$$

And the equivalent bending moment (M_e) is given by

$$M_e = \frac{1}{2}\left[\left\{K_m M + \frac{\alpha F d_o (1+k^2)}{8}\right\} + \sqrt{\left\{K_m \cdot M + \frac{\alpha F d_o (1+k^2)}{8}\right\}^2 + (K_t T)^2}\right]$$

$$M_e = \frac{\pi}{32} \sigma_b d_o^3 (1-k^4)$$

where α = column factor for compressive loads.

$$= \frac{1}{1 - 0.0044\left(\frac{L}{K}\right)} \quad \left(\text{when } \frac{L}{K} < 115\right)$$

$$= \frac{\sigma_y \left(\frac{L}{K}\right)^2}{C \pi^2 E} \quad \left(\text{when } \frac{L}{K} > 115\right)$$

where L = Length of shaft b/w bearings

K = Least radius of gyration

σ_y = Compressive yield stress

E = Young's modulus, $\frac{L}{K}$ = slenderness ratio

C = Coefficient in Euler's formula depending upon end conditions.

= 1 for hinged ends

= 2.25 for fixed ends

= 1.6 for ends that are partly restrained as in bearings.

$K = \frac{d_i}{d_o}$ =

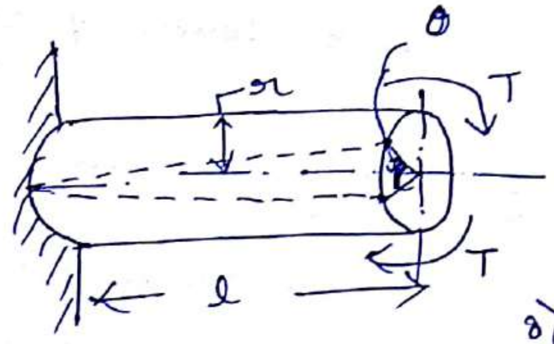
Depending upon various conditions, some changes should be made in the above eqns.

1. For solid shaft, $K=0$ & $d_o = d$

2. For no axial loading $F=0$

3. For axial tensile loading, $\alpha = 1$

Design of shafts Based on Torsional Stiffness:



Another important aspect to consider in shaft design is torsional stiffness or rigidity. Proper design requires that a shaft be able to transmit power uniformly and with a steady motion. Shaft that permit an excessive angular displacement may contribute to vibrations (both torsional & leteral), affect gear action and cause premature bearing wear or failure. In addition, the particular applications such as machine tools require that the spindles be especially rigid.

Although no standard torsional deflection has ever been established for different shaft applications, it has become standard practice to limit the torsional deflection for machinery shafting to 0.25° to 3° per meter length. and transmission shafting to 1° in a length of 20 times the shaft diameter. The deflection of camshafts particularly for internal combustion engines should be less than 0.5° regardless of the shaft length.

The equation established for torsional rigidity is

$$\boxed{\frac{T}{J} = \frac{G\theta}{L}}$$

where

T = Maximum torque to be transmitted

J = Polar moment of inertia

G = Modulus of rigidity of the shaft material

θ = Torsional deflection or angle of twist in radians.

L = Length of shaft.

Critical or whirling speeds of shafts:

Some-times the centre of gravity of a disk (such as gear, pulley, turbine rotor etc) mounted on the shaft may be slightly displaced from the axis of rotation of the shaft due to accidentally formed uneven masses of disk. This will produce deflection when the shaft rotates due to centrifugal force. Also the shaft itself may be deflected due to bending loads. This deflection is the function of speed and it will be maximum at certain speeds and the resulting vibration of the shaft can cause the damage to the shaft. This speed at which the deflection is maximum is called critical (or whirling) speed.

The critical speed depends on the magnitudes and locations of loads carried by the shaft, the length & stiffness of the shaft, the total mass of shaft and attached parts, the unbalance of the mass with respect to the axis of rotation, the amount of damping in the system and the type of end supports. The critical speed formulas are derived based on the supporting bearing conditions. Self-aligning or very short bearings are considered as simple supports while long bearings such as sleeve bearings are treated as fixed ends for the determination of critical speeds. The weight of shaft is either neglected or one-half to two-thirds of the weight of the shaft is added to the concentrated loads.

For any shaft there are many number of critical speeds, but only the lowest (first) and occasionally the second are generally considered for the design. The other critical speeds being so high from operating range of speeds, they are not usually taken into account.

The critical speeds for the shaft for different loading conditions are given as follows.

(1) For a shaft of negligible self-weight with single attached disk, the first critical speed is given by

$$\omega_c = \sqrt{\frac{g}{\delta}} \text{ rad/s}$$

where δ = static deflection of shaft in metre at the disk location by the load $\underline{W}$ newtons.

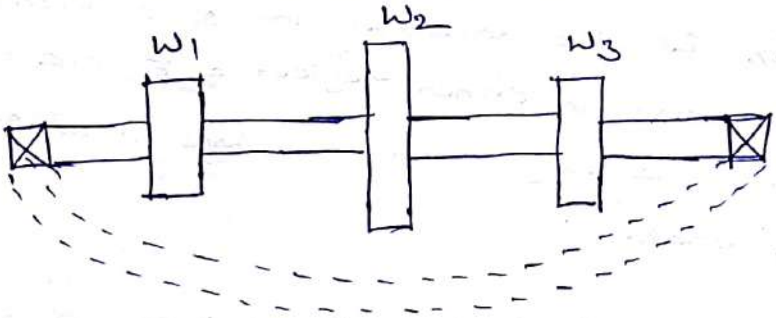
g = Gravitational constant (9.81 m/s^2)

(I) For a shaft of constant cross-section, simply supported at the ends with no weight involved other than that of the shaft itself, the first critical speed is given by,

$$\omega_c = \sqrt{\frac{5}{4}\left(\frac{g}{\delta_{max}}\right)} \quad rad/s$$

where δ_{max} is the maximum static deflection caused by a uniformly distributed load equal to the weight of the shaft.

(II) similarly for a shaft of some self-weight carrying several concentrated external weights (i.e., loads), the critical



speed may be found out using Dunkerley equation as

$$\frac{1}{\omega_c^2} = \frac{1}{\omega_{c_1}^2} + \frac{1}{\omega_{c_2}^2} + \cdots + \frac{1}{\omega_{cn}^2} + \frac{1}{\omega_{cs}^2}$$

where

ω_c = First critical speed of shaft with all loads acting together in rad/s

ω_{c_1} = First critical speed when the first load is acting seperately

ω_{c_2} = First critical speed when the second load is acting seperately

ω_{cs} = Critical speed of the shaft by its own weight and so on.

The above eqn can only be applied for the shaft of uniform diameter.

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if the speed is to be specified in terms of rpm, then ω_c, should be multiplied by $(60/2\pi)$

$$\text{i.e}\quad N_c = \left(\frac{60}{2\pi}\right)\omega_c = \left(\frac{30}{\pi}\right)\omega_c$$

Note: consider a shaft with two loads only: Now Dunkerley eqn for the critical speed is given by

$$\boxed{\frac{1}{\omega_c^2} = \frac{1}{\omega_{c1}^2} + \frac{1}{\omega_{c2}^2}} \quad \text{in rad/s}$$

or

$$\frac{1}{N_c^2} = \frac{1}{N_{c1}^2} + \frac{1}{N_{c2}^2} \quad \text{in rpm.}$$

$$\text{where}\quad N_c = \frac{30}{\pi}\omega_c = \frac{30}{\pi}\sqrt{\frac{g}{\delta}}$$

Assuming $g = 9810\ mm/s^2$ & δ is in mm, we get

$$N_c = \sqrt{\frac{30^2}{\pi^2}\left(\frac{g}{\delta}\right)} = \sqrt{\frac{900 \times 9810}{\pi^2 \times \delta}} = \frac{945.8}{\sqrt{\delta}}$$

$$\therefore\ N_{c1} = \frac{945.8}{\sqrt{\delta_1}} \quad \text{and} \quad N_{c2} = \frac{945.8}{\sqrt{\delta_2}}$$

Now the critical speed eqn implies that

$$\frac{1}{N_c^2} = \frac{1}{N_{c1}^2} + \frac{1}{N_{c2}^2} = \frac{\delta_1}{(945.8)^2} + \frac{\delta_2}{(945.8)^2}$$

$$= \frac{(\delta_1+\delta_2)}{(945.8)^2}$$

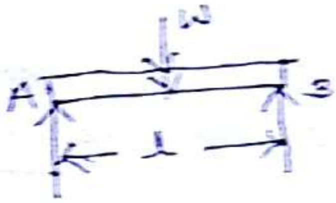
$$N_c^2 = \frac{(945.8)^2}{(\delta_1+\delta_2)} \Rightarrow N_c = \frac{945.8}{\sqrt{\delta_1+\delta_2}}\ \text{rpm.}$$

where δ_1 & δ_2 are in mm.

Since the critical speed is related with deflection we should know some standard formulas of deflections for the shaft subjected to different types of loading.

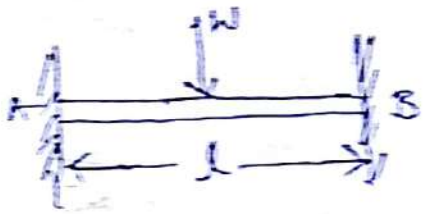
(a) For the shaft of negligible weight with central load in simple supports show

$$\boxed{\delta_c = \frac{Wl^3}{48EI}}$$



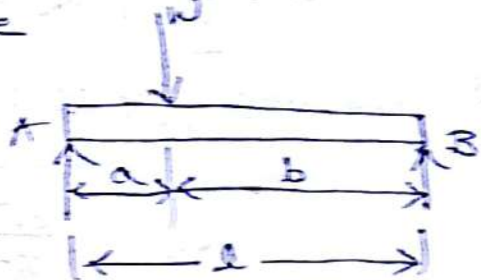
(b) For the shaft of negligible weight with central load in fixed ends

$$\boxed{\delta_c = \frac{Wl^3}{192EI}}$$



(c) For the shaft of negligible weight with a non-central load in simple supports

$$\boxed{\delta_c = \frac{Wa^2b^2}{3EI\,l}}$$



where a = Distance of load from left support

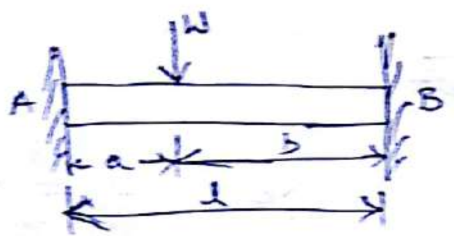
b = Distance " " " Right "

E = Young's modulus

I = Moment of inertia

l = Length of shaft.

(d)



$$\delta_c = \frac{Wa^3b^3}{3EI\,l^3}$$

(e)

$$\delta_c = \frac{5wl^4}{384EI}$$

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Steps followed in Shaft Design:

1. At first analyse the problem to know that the shaft is to be designed based on strength or rigidity.

2. If it is strength basis, use the eqn $\frac{T}{J} = \frac{\tau}{r}$ and if it is in rigidity basis. use the eqn $\frac{T}{J} = \frac{G\theta}{L}$.

3. From the power, calculate the mean torque using,

   (I) $P = \frac{2\pi NT}{4500}$ where P – in H.P, T – in kgf-m, N – rpm.

   (II) $P = \frac{2\pi NT}{60}$ where P – in Watts, T – in N-m., N – rpm.

4. Calculate the maximum torque using the given relationship b/w maximum torque & mean torque. if it is not given, then adopt mean torque for further design.

5. Determine the maximum bending moment based on the conditions of shafts like simply supported, or cantilever type and also types of loading such as vertical loading or horizontal loading or both and so on.

6. Select the suitable formula based on shear stress theory (Guest's theory) or normal stress theory (Rankine's theory) and determine the diameter of shaft by considering factor for fluctuating loads. column factor and also key-way factor if required.

① A shaft running at 500 rpm transmits a power of 10KW. Assume allowable shear stress as 40 N/mm², find the dia of the shaft.

Sol. Power $P = 10KW = 10{,}000$ watts

Speed $N = 500$ rpm

Allowable shear stress, $\tau = 40 N/mm^2$

We know that the torque to be transmitted by the shaft can be calculated using $P = \frac{2\pi NT}{60}$

$$T = \frac{60P}{2\pi N} = \frac{60 \times 10{,}000}{2 \times \pi \times 500}$$

∴ Torque $T = 191 . N\text{-}m = 191 \times 10^3 N\text{-}mm.$

Generally the torsional shear strength of shaft should be greater than the torque to be transmitted. otherwise the shaft will fail.

$$\text{i.e} \quad \frac{\pi}{16} \tau d^3 \geq T$$

$$d \geq \left\{ \frac{16T}{\pi \tau} \right\}^{1/3}$$

$$d = 28.9mm \simeq 30 mm$$

Take the diameter of shaft $= 30mm$.

② A hollow steel shaft transmits 500KW at 1000 rPm. The maximum shear stress is 50 N/mm². Find the outside & inside diameter of the shaft, if the outside diameter is twice the inside diameter, assuming that the maximum torque is 20% greater than the mean torque.

Sol Power $P = 500\ kW = 500 \times 10^3$ watts, speed $N = 1000$ rpm.

Allowable shear stress $\tau = 50\ N/mm^2$

$$P = \frac{2\pi N T}{60} \implies$$

Mean torque $T_{mean} = \frac{60P}{2\pi N} = \frac{60 \times 500 \times 10^3}{2\pi \times 1000} = 4775\ N\text{-}m$

Maximum torque $= \overset{20\%}{\cancel{102}}\ T_{mean}$

$$= \cancel{102}\ 4775 + \left(\frac{20}{100} \times 4775\right) = 5730\ N\text{-}m$$

$$= 5730 \times 10^3\ N\text{-}mm.$$

we know that torsional shear strength of shaft $\geq$ max. torque to be transmitted.

i.e. $\frac{\pi}{16} \tau d_o^3 (1 - K^4) \geq T_{max}$ where $K = \frac{d_i}{d_o} = \cancel{\frac{d_i}{d_o}}$

Condition given. $\boxed{d_o = 2d_i}$

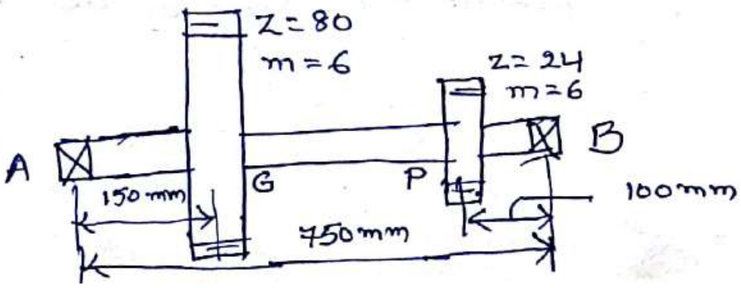
$$K = \frac{d_i}{2d_i} = 0.5$$

i.e $\frac{\pi}{16} \times 50 \times d_o^3 (1 - 0.5^4) \geq 5730 \times 10^3$

$d_o = 90\ mm$, $d_i = 45\ mm$

(3) A shaft is to transmit 20KW at 250 r.p.m. It is supported on two bearings 750 mm apart and has two gears keyed to it. The pinion having 24 teeth of 6mm module is located at 100 mm to the left of the right-hand bearing and delivers the power horizontally to the right. The gear having 80 teeth, 6mm module is located at 150 mm to the right of left-hand bearing and receives power in a vertical direction from below. selecting suitable material determine the required shaft size for a factor of safety of 2.

8



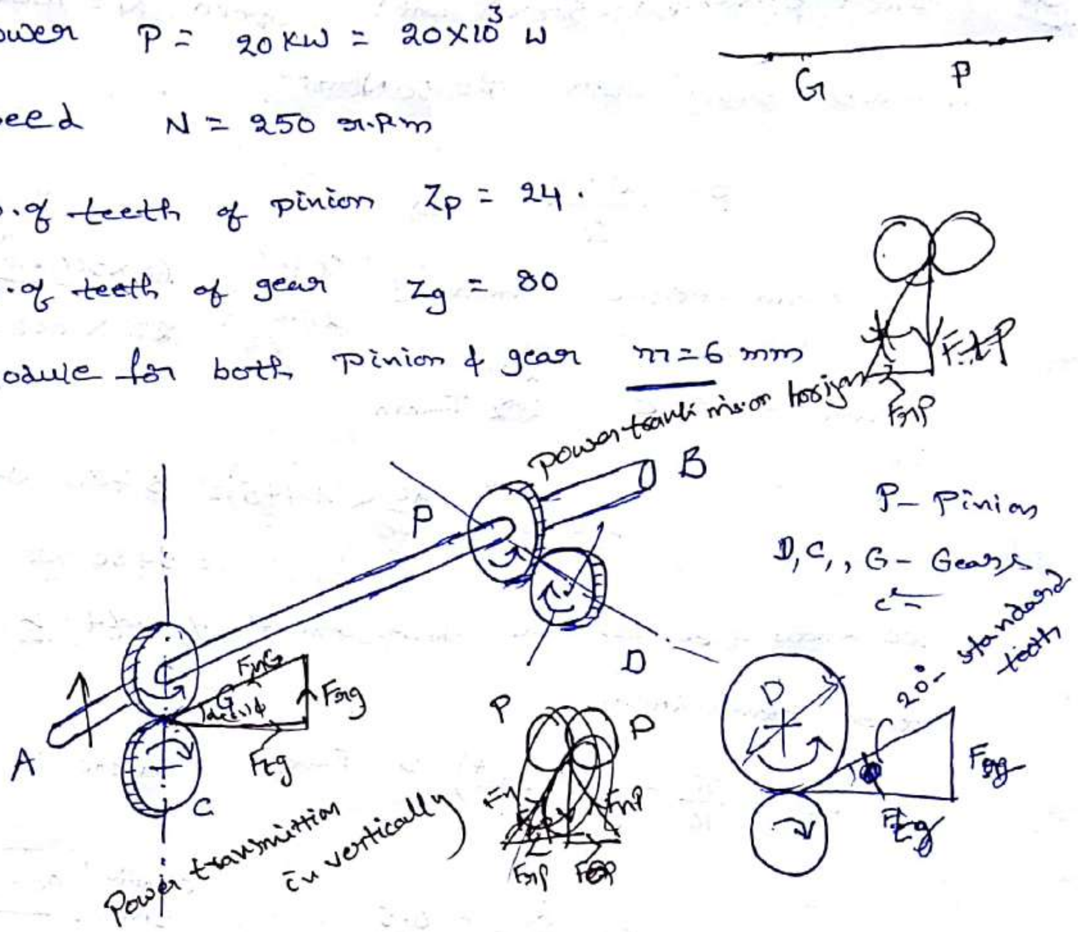
Sol Power $P = 20\,kW = 20\times10^3\,W$

Speed $N = 250$ r.p.m

No. of teeth of pinion $Z_p = 24$.

No. of teeth of gear $Z_g = 80$

Module for both pinion & gear $m = 6\,mm$



from fig. we can understand that power is received by the gear G in a vertical direction from the gear C, kept below and is transmitted to the pinion P and then transmitted to gear D in the horizontal direction to the right.

Torque transmitted by the gear G, is given by

$$T = \frac{60P}{2\pi N} = \frac{60\times20\times10^3}{2\times\pi\times250} = 764\ N\text{-}m = 764\times10^3\ N\text{-}mm.$$

usually, when the power (i.e. torque) is transmitted to a gear, two types of forces will be produced namely.

(I) Tangential force and (II) Radial force

Now for the Gear G, the produced tangential force,

$$F_{tg} = \frac{T}{\left(\frac{D_g}{2}\right)}$$

where D_g = Diameter of gear = module × No. of teeth

$= m \times z_g = 6 \times 80 = 480$ mm

standard value

$\phi = 20^\circ$, pressure angle

$$\therefore F_{tg} = \frac{764 \times 10^3}{\left(\frac{480}{2}\right)} = 3183 \text{ N}$$

Radial force, $F_{rg} = F_{tg} \times \tan 20^\circ = 3183 \times \tan 20^\circ = 1159$ N

Similarly for the pinion, the tangential force is

$\tan\phi = \frac{F_{rg}}{F_{tg}}$

$$F_{tp} = \frac{T}{\left(\frac{D_p}{2}\right)}$$

where $D_p = m \times z_p = 6 \times 24 = 144$ mm.

$$= \frac{764 \times 10^3}{\left(\frac{144}{2}\right)} = 10611 \text{ N.}$$

Radial force for the pinion $F_{rp} = F_{tp} \times \tan 20^\circ$

$= 10611 \times \tan 20^\circ = 3862$ N

Now the directions of loading are decided as follows. Assume the shaft rotation is viewed from left end A. Let the driver gear C rotates in clockwise direction. Then the driven gear G will rotate in anti-clockwise direction. The pinion P will rotate in anti-clockwise direction and driven gear D, in clock-wise direction.

Since the gear G, receives power from gear C, kept below, the tangential force F_{tg}, supplied by the gear C will act on the gear G, horizontally towards right and the radial force, F_{rg}, will act vertically upwards.

Similarly for the pinion P which is transmitting power to the gear D, the tangential force, F_{tp}, produced by the gear D will act on the pinion P, vertically downwards and the radial force, F_{rp}, will act horizontally toward left. The loading diagrams are shown in fig below.

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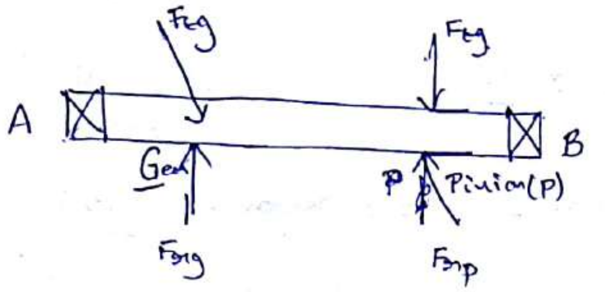
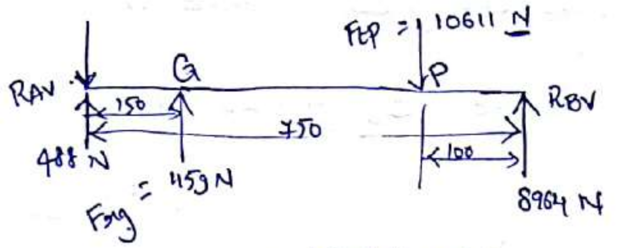
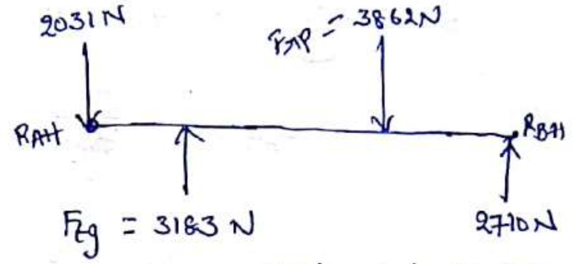
for finding the bending moments at various sections, the reactions at bearing ends should be determined.

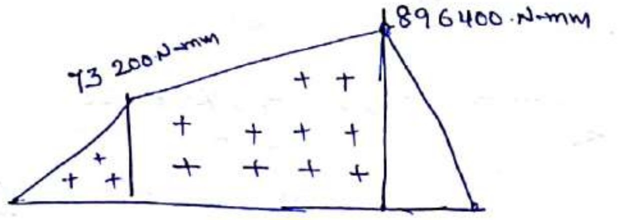




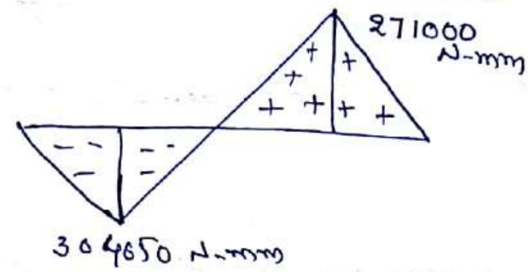
vertical loading



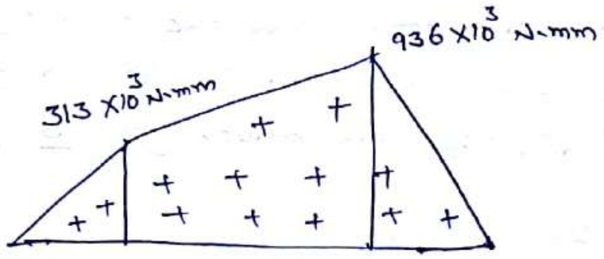
Horizontal Bending moment



Vertical bending moment



Horizontal bending moment



Resultant bending moment

Considering vertical loading:

Taking moments about the end A, we get

$$R_{BV} \times 750 = (10611 \times 650) - (1159 \times 150)$$

$$\therefore \ R_{BV} = \frac{(10611 \times 650) - (1159 \times 150)}{750} = 8964 \text{ N}$$

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$R_{AV} = (8964 + 1159) - 10611 = -488\ N$

i.e $R_{AV} = 488\ N$ (acting upwards).

Considering Horizontal loading:

Taking moments about end A, we get

$R_{BH} \times 750 = (3862 \times 650) - (3183 \times 150)$

$\therefore R_{BH} = 2710\ N.$, $R_{AH} = (2710 + 3183) - 3862$

$\boxed{R_{AH} = 2031\ N}$

Vertical Bending moment:

At A: $M_{AV} = 0$

At G: $M_{GV} = 488 \times 150 = +73200$ N-mm.

At P: $M_{PV} = 8964 \times 100 = +896400$ N-mm

At B: $M_{BV} = 0$

Horizontal Bending moment:

At A: $M_{AH} = 0$

At G: $M_{GH} = (-2031 \times 150) = -304650$ N-mm.

At P: $M_{PH} = (2710 \times 100) = +271000$ N-mm.

At B: $M_{BH} = 0$

Resultant Bending moment: at Gear (G)

At G: $M_G = \sqrt{M_{GV}^2 + M_{GH}^2}$

$= \sqrt{(73200)^2 + (-304650)^2}$

$= 313 \times 10^3$ N-mm

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At P; $M_P = \sqrt{(896400)^2 + (271000)^2} = 936 \times 10^3$ N-mm
(Maximum value)

The equivalent torque, $T_e = \sqrt{M^2 + T^2}$

$$T_e = \sqrt{(936 \times 10^3)^2 + (764 \times 10^3)^2} = 1208 \times 10^3 \text{ N-mm.}$$

for the given factor of safety of 2, the design torque is given by

$$\tau = \frac{16 T_e'}{\pi d^3}$$

for safety, the induced stress should be less than permissible value.

$$\tau = \frac{16 T_e'}{\pi d^3} < 42$$

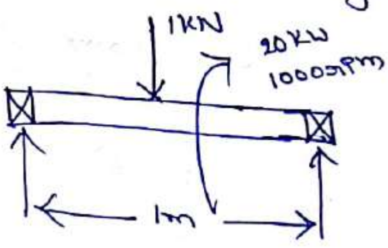
$$d > \left(\frac{16 T_e'}{\pi \times 42}\right)^{1/3} > \left(\frac{16 \times 2416 \times 10^3}{\pi \times 42}\right)^{1/3}$$

$$> 66.4 \text{ mm} = 70 \text{ mm}$$

Take the diameter of shaft = 70 mm. Ans.

(4) A line shaft running at 1000 rpm transmits 20kW. It carries a central load of 1 kN. It is supported in bearings, the distance b/w their centres being 1-m. for permissible shear stress of 45 N/mm² and bearing stress 70 N/mm², determine the diameter of shaft, assuming ~~[illegible]~~ minor shock loading.

sol



Power (P) = 20kW = 20×10^3 W

N = 1000 rpm, W = 1kN = 1000 N.

l = 1m = 1000 mm.

Permissible shear stress (τ) = 45 N/mm²

permissible bending stress (σ_b) = 70 N/mm²

for minor shock loading, i.e. K_b = 2 & K_t = 1.5 (Assumed)
K_m

we know that, torque transmitted

$$T = \frac{60P}{2\pi N} = \frac{60 \times 20 \times 10^3}{2\pi \times 1000} = 191 \text{ N-m} = 191 \times 10^3 \text{ N-mm.}$$

Bending moment developed, $M = \frac{wl}{4} = \frac{1000 \times 1000}{4} = 250 \times 10^3$ N-mm.

since the shaft is subjected to combined bending + twisting moments, the size of the shaft is designed based on equivalent stress induced in the shaft.

The induced equivalent shear stress $\tau = \frac{16 T_e}{\pi d^3}$

$$T_e = \sqrt{(k_b \cdot M)^2 + (k_t \cdot T)^2} = \sqrt{(2 \times 250 \times 10^3)^2 + (1.5 \times 191 \times 10^3)^2}$$

$$= 576 \times 10^3 \text{ N-mm}$$

for safety, the induced shear stress should be less than permissible shear stress.

$$\frac{16T}{\pi d^3} < \tau$$

$$d = 40.2 \text{ mm.}$$

similarly, the induced equivalent bending stress $\sigma_b = \frac{32 M_e}{\pi d^3}$

$$M_e = \frac{1}{2}\left[k_b \cdot M + \sqrt{(k_b \cdot M)^2 + (k_t \cdot T)^2}\right]$$

$$= \frac{1}{2}\left[(2 \times 250 \times 10^3) + \sqrt{(2 \times 250 \times 10^3)^2 + (1.5 \times 191 \times 10^3)^2}\right]$$

$$M_e = 538 \times 10^3 \text{ N-mm}$$

Also, the induced bending stress should be less than permissible value for safe design.

$$\frac{32}{\pi d^3} = \sigma_b$$

$$d = 42.8 \text{ mm}$$

for better safety, take diameter of shaft $= 45$ mm

(5) A shaft is required to transmit a power of 25 kW at 360 rpm. The force analysis due to attached parts results in Bending moment of 830 N-m at a section b/w bearings. if permissible stresses in the shaft are : 60 MPa in bending & 40 MPa in shear. calculate the diameter of the shaft.

sol Given that,

$$P = 25\,kW = 25000\ watt$$

$$N = 360\ rpm.$$

$$M = 830\ N\text{-}m = 830 \times 10^3\ N\text{-}mm$$

$$\sigma_b = 60\ MPa = 60\ N/mm^2, \quad \tau = 40\ MPa = 40\ N/mm^2.$$

$$T = \frac{60P}{2\pi N} = \frac{60 \times 25000}{2 \times \pi \times 360} = 663.14\ N\text{-}m.$$

$$\boxed{T = 663.14 \times 10^3\ N\text{-}mm}$$

Since the shaft is subjected to Bending + twisting moments, the size of the shaft is designed based on equivalent stress induced in the shaft.

The induced equivalent shear stress $\tau = \frac{16 T_e}{\pi d^3}$

$$T_e = \sqrt{(K_b \cdot M)^2 + (K_t \cdot T)^2} = \sqrt{(2 \times 830 \times 10^3)^2 + (1.5 \times 663.14 \times 10^3)^2}$$

$$= \sqrt{2.7556 \times 10^{12} + 9.8944 \times 10^{11}}$$

$$\boxed{T_e = 1935055.555} =$$

$$\frac{16 T_e}{\pi d^3} = \tau = 40$$

$$d = \frac{16 \times 1935055.55}{\pi \times 40}$$

$$d = 62.690\ mm$$

lly. equivalent bending stress $M_e = \frac{1}{2}\left[\left(\sqrt{(K_b M)^2 + (K_t \cdot T)^2}\right) + K_b \cdot M\right]$

$$M_e = \frac{1}{2}\left[(2\times 830\times10^3) + \sqrt{(2\times830\times10^3)^2 + (1.5\times 663.14\times10^3)^2}\right]$$

$$M_e = \frac{1}{2}\left(2\times830\times10^3 + 1935055.55\right)$$

$$M_e = 1797527.775$$

Also, induced bending stress should be less than permissible value for Safe design -

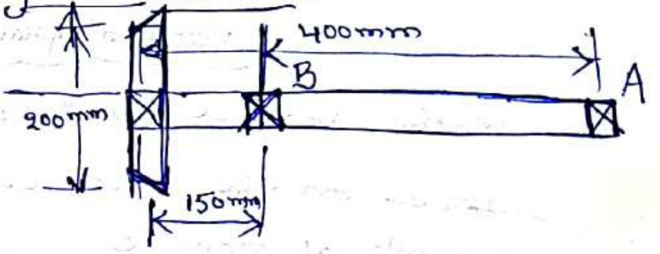
$$\frac{32M}{\pi d^3} = \sigma_b$$

$$\frac{32\times1797527.77}{\pi d^3} = 60\times10^3$$

$$d = 67.324 \text{ mm}$$

for better safety, take diameter of shaft = 67 mm

(6) A shaft is supported by two bearings 400 mm apart and carries a bevel gear of 200 mm pitch diameter at one end that is overhanging beyond the near bearing by 150 mm. The gear produces a radial load of 9800 N and a thrust load of 2940 N & tangential load 28000 N when the speed is 600 r.p.m. Determine the shaft diameter if the shaft is made of steel with allowable shear stress of 40 MPa. Also determine the angle of twist and deflection at the bevel gear location if the modulus of rigidity is 80 GPa and the modulus of elasticity is 210 GPa.



Sol

Tangential load (W_t) (or) $F_t = 28000$

Radial load (W_r) (F_r) = 9000 N

axial (or) Thrust load (W_a) (F_a) = 2940 N

Speed (N) = 600 rpm.

$\tau = 40$ MPa,

dia (d) = ?

(θ) = ?

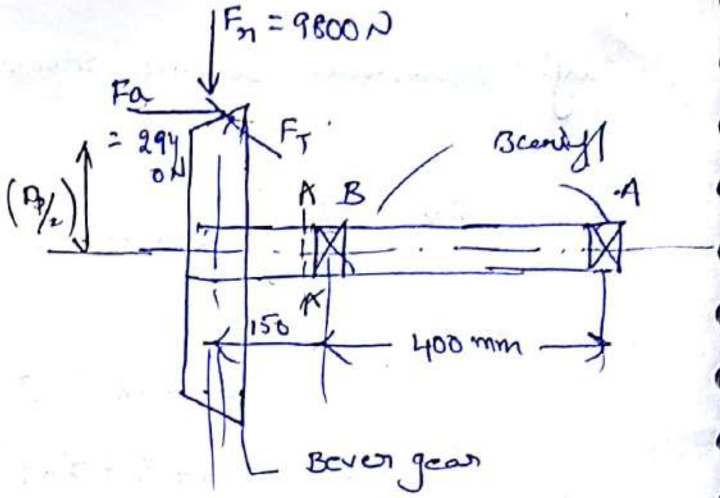
$G = 80$ GPa $= 80\times10^3$ N/mm²

$E = Y = 210$ GPa $= 210\times10^3$ N/mm²

~~for a bevel gear, the axial & radial loads are given,~~

$$\cancel{F_a = F_t \tan\alpha . \sin\delta_1}$$

$$\cancel{F_r = F_t \tan\alpha \cos\delta_1}$$



The permissible shear stress of the material, considering the effect of keys can found by considering the following data:

Torque on the shaft $(T) = F_t \cdot \left(\frac{D_p}{2}\right)$

$$= 28000 \times \left(\frac{200}{2}\right) =$$

$$= 2.8 \times 10^6 \text{ N-mm} = 2.8 \times 10^3 \text{ N-m}$$

Assuming that there are no power losses, the power transmitted by the shaft would be:

$$P = \frac{2\pi N T}{60} = \frac{2 \times \pi \times 600 \times 2.8 \times 10^6}{60 \times 10^3} \text{ KW}$$

$$P = 175.929 \text{ KW}$$

consider the two planes, namely the vertical & horizontal plane

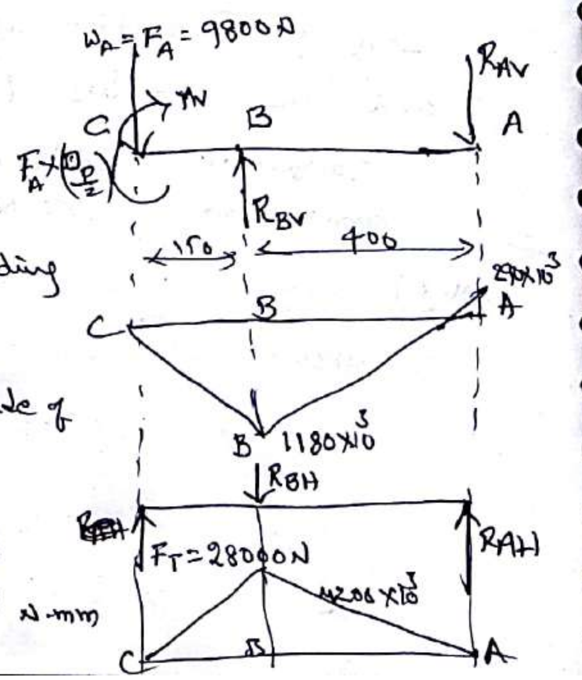
<u>vertical plane</u>

In the vertical plane, the axial force on the bevel causes a bending moment at point <u>C</u>.

~~finding reactions~~ The magnitude of this bending moment will be

$$M_V = F_A \times \frac{D_p}{2} = 2900 \times 100$$

$$= 290 \times 10^3 \text{ N-mm}$$



finding reaction by taking moment about A

$$R_{BV} \times 400 - F_R \times 550 + M_V = 0$$

$$R_{BV} \times 400 - 9800 \times 550 + 290 \times 10^3 = 0$$

$$R_{BV} = 12750 \text{ N.}$$

$$R_{AV} + R_{BV} = F_R. \Rightarrow R_{AV} = 9800 - 12750 = -2950 \text{ N}$$

Bending moment at key point : $M_{CV} = 290 \times 10^3$ N-mm

$M_{BV} = M_{CV} - F_R \times 150 = -1180 \times 10^3$ N-m.

$M_{AV} = 0$

Horizontal plane: finding Reactions by taking moment about A.

$$F_T \times 550 - R_{BH} \times 400 = 0 \Rightarrow 28000 \times 550 - R_{BH} \times 400 = 0$$

$\boxed{R_{BH} = 38500 \text{N}}$, $R_{AH} = R_{BH} - F_T$

$R_{AH} = 10500$ N.

Bending moment at key points :

$M_{CH} = 0$, $M_{BH} = F_T \times 150 = 28000 \times 150$

$= 4200 \times 10^3$ N-mm.

$M_{AH} = 0$

Resultant (or) Equilient bending moment · $M_e = \sqrt{M_V^2 + M_H^2}$

$M_{e_B} = \sqrt{(4200 \times 10^3)^2 + (-1180 \times 10^3)^2}$

$M_{eA} = 0$, $M_{eB} = 4362.614 \times 10^3$ N-mm

$M_T = 2.8 \times 10^6$ N-mm

$$T_e = \sqrt{M^2 + T^2} = \sqrt{(4362.614 \times 10^3)^2 + (2.8 \times 10^6)^2}$$

$$T_e = 5183859.566$$

$$\tau = \frac{16 T_e}{\pi d^3} \Rightarrow 40 = \frac{16 \times 518385.566}{\pi d^3}$$

$\boxed{d = 87.067 \text{ mm}}$

$$\frac{T}{J} = \frac{G\theta}{l}.$$

80×10^3 N

$$\frac{2.8 \times 10^6}{\frac{\pi (87)^4}{32}} = \frac{80 \times 10^3 \times \theta}{550}$$

$\theta = 2$

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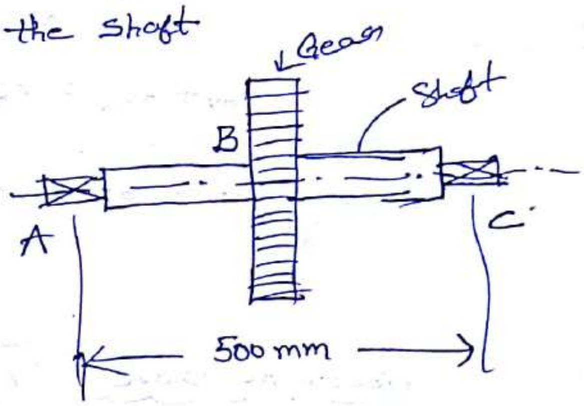
(1) A short shaft made of C45 steel receives 50 kW at 300 rpm via a 300 mm spur gear, the power being delivered to another shaft through a flexible coupling. The gear is keyed midway b/w the bearings. The pressure angle of gear is $\underline{20^\circ}$ and design factor is $\underline{2}$. Bearings are $\underline{500\ mm}$ apart. Find the diameter of the shaft.



Sol

Power $P = 50 \times 10^3$ watts

speed $N = 300$ rpm

Dia of gear $D_g = 300$ mm

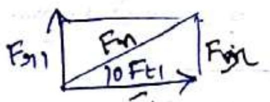
$\alpha = 20^\circ$

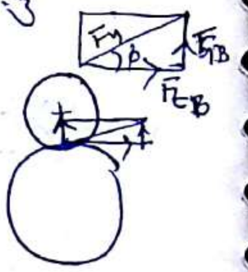
Design of factor $= 2$, shaft length $L = 500$ mm

$$T = \frac{60P}{2\pi N} = \frac{60 \times 50 \times 10^3}{2\pi \times 300} = 1592\ N\text{-}m.$$

Tangential force acting at the gear B, due to this torque is given by

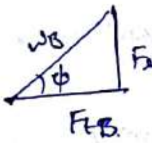
$$F_{tB} = \frac{T}{D_g/2} = \frac{2T}{D_g} = \frac{2 \times 1592}{0.3}$$





$= 10610\ N.$

Normal load acting on the tooth of gear,



$$(F_n)_B \text{ (or) } W_B = \frac{F_{tB}}{\cos 20^\circ} = \frac{10610}{\cos 20} = 11291\ N$$

This normal load acts at $\underline{20^\circ}$ to the vertical. Hence this normal load can be resolved into vertical & horizontal components.

Vertical component $= W_B \cos 20^\circ = 10610\ N$

Horizontal " $= W_B \sin 20 = 11291 \times \sin 20$
$= 3862\ N.$

Vertical B.M at the centre $B = \frac{Wl}{4} = \frac{10610 \times 0.5}{4}$

$B = 1326\ N\text{-}m.$

Horizontal B.M at the centre B, $= \frac{2262 \times 0.5}{4} = 485$ N-m

Resultant B.M at B $= \sqrt{(1326)^2 + (485)^2} = 1411$ N-m.

$\therefore$ Equivalent torque T_e is given by

$$T_e = \sqrt{M^2+T^2} = \sqrt{(1411)^2+(1592)^2} = 2.127 \times 10^3 \text{ N-m}$$

for C45 steel, assume maximum shear stress = 84 N/mm^2.

$\therefore$ Allowable shear stress $\tau = \frac{\text{Maximum stress}}{\text{Design factor}}$

$$= \frac{84}{2} = 42 \text{ N/mm}^2$$

$$\frac{\pi}{16} \tau d^3 = T_e$$

$$\boxed{d = 63.6 \text{ mm}} \simeq 65 \text{ mm}$$

Dia. of shaft required = 65 mm

(2) A solid circular shaft is to transmit 112 kW at 500 rpm with provision for a 20% overload. The angle of twist must not exceed 1° in a length of fifteen diameters. For practical reasons, the diameter must not be more than 85 mm. Design the shaft. Adopt mild steel with a working shear stress of 60 N/mm^2.

Sol $P = 112$ kW $= 112 \times 10^3$ W, $N = 500$ rpm, provision over load is 20%, Diameter of shaft (d) = 85 mm, $\tau = 60$ N/mm^2.

$$T_{mean} = \frac{60P}{2\pi N} = \frac{60 \times 112 \times 10^3}{2\pi \times 500} = 2139 \text{ N-m}$$

Maximum Torque $T_{max} = 2139 + \left(\frac{20}{100} \times 2139\right)$

$$= 2139 + 427.8$$

$$= 2566.8 \text{ N-m}$$

$$= 2567 \times 10^3 \text{ N-mm.}$$

Now this maximum torque is equated to resisting capacity of the shaft, according to strength & rigidity.

According to strength, we know that

$$\frac{T}{J} = \frac{\tau}{r} \qquad T = \frac{\pi}{16} \tau d^3$$

$$\boxed{d = 60.17 \text{ mm}}$$

According to rigidity, $\frac{T}{J} = \frac{G\theta}{L}$

$$\frac{T}{\frac{\pi}{32} d^4} = \frac{G\theta}{15d} \quad \left\{ \begin{array}{l} \text{Assume } G = 8 \times 10^4 \text{ N/mm}^2, \\ L = 15d \text{ (given)} \\ \theta = 1^\circ = \frac{\pi}{180} \end{array} \right]$$

$$\frac{32T}{\pi d^3} = \frac{G\theta}{15}$$

$$\therefore d^3 = \frac{32T \times 15}{\pi \times G \times \theta} = \frac{32 \times 2567 \times 15 \times 180 \times 10^3}{\pi \times \pi \times 8 \times 10^4}$$

$$\boxed{d = 65.5 \text{ mm}}$$

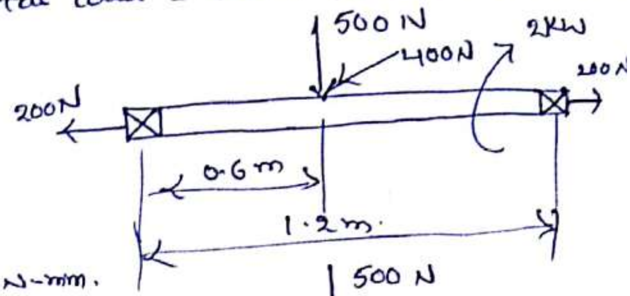
Take the required diameter of the shaft, = 70 mm, which will satisfy the above two specified conditions.

(3) A horizontal shaft 1.2 m long is supported on bearings at its ends and transmits 2KW at 1440 rpm. The critical section of the shaft which is at the mid span is subjected to a vertical load of 500N, a horizontal load

of 400N and an axial load of 200 N. Determine the diameter of the shaft for an allowable shear stress of 50MPa.

Sol
Given that $L = 1.2\,m$, $P = 2000\,W$, $N = 1440$ rpm

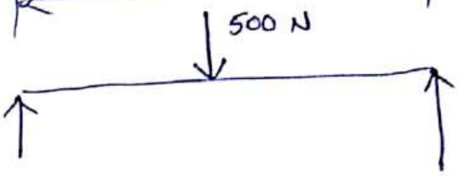
Vertical load = 500 N, Horizontal load = 400N, Axial load (F) = 200N



$\tau = 50\,MPa = 50\,N/mm^2$

Torque transmitted $T = \frac{60P}{2\pi N}$

$T = 13.26\,N\text{-}m = 13.26 \times 10^3\,N\text{-}mm.$

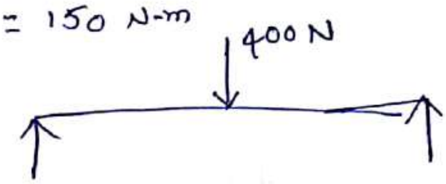


Vertical loading

Bending moment Calculations:

Vertical bending moment at the

centre $(M_V) = \frac{W_1 L}{4} = \frac{500 \times 1.2}{4} = 150\,N\text{-}m$

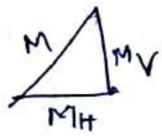


Horizontal loading

Horizontal Bending moment

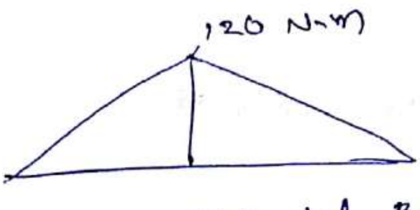
$(M_H) = \frac{400 \times 1.2}{4} = 120\,N\text{-}m.$

Resultant Bending moment at the centre

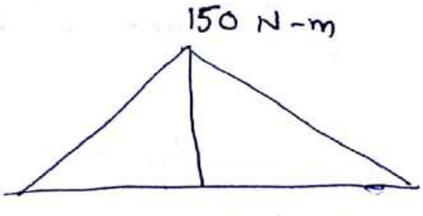


$M = \sqrt{(M_V)^2 + (M_H)^2} = \sqrt{(150)^2 + (120)^2}$

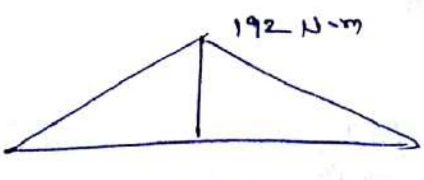
$= 192\,N\text{-}m = 192 \times 10^3\,N\text{-}mm$



Horizontal B.M.



Vertical B.M



R. B.M

Equivalent twisting moment, for solid shaft with axial tensile load is given by $T_e = \sqrt{\left(M + \frac{Fd}{8}\right)^2 + T^2}$

$$\boxed{T_e = \frac{\pi}{16} \tau d^3}$$

$$\sqrt{\left(192\times10^3 + \frac{200\times d}{8}\right)^2 + (13.26\times10^3)^2} = \frac{\pi}{16}\times50\times d^3$$

By trial & error, the diameter of the shaft is found to be

27 mm , take (d) = 30 mm

Now $T_e = \sqrt{[(192\times10^3) + (25\times30)]^2 + (13.26\times10^3)^2}$

$= 193\times10^3$ N-mm $= \frac{\pi}{16}\tau d^3$

$\tau = \frac{193\times10^3\times16}{\pi\times30^3} = 36.4$ N/mm²

$= 36.4\times10^6$ N/m²

$= 36.4 < 50$ MPa.

Hence the design is safe.

(1) Determine the diameter below which the angle of twist of a shaft, and not the maximum stress, is the criterion of design of a solid shaft in torsion. The allowable shear stress is 40 MPa and the maximum allowable twist is 0.5 deg/m. Consider a shaft without keyway. Assume G = 84 GPa

Sol Shear stress τ = 40 MPa = 40 N/mm²

$\theta_{permissible}$ = 0.5 deg/m

$G = 84\times10^3$ N/mm²

By eqn shear strength

$$\tau = \frac{16T}{\pi d^3}$$

The torque that can be transmitted by a shaft coupling considering its shear strength, can be given by the equation:

$$T = \frac{\pi}{16} d^3 \tau \rightarrow (a)$$

Torsional Rigidity eqn $\frac{T}{J} = \frac{G\theta}{l} \Rightarrow \theta = \frac{Tl}{JG}$

$$\cancel{T = \frac{G\theta J}{l}} =$$

$$\cancel{\theta = \frac{Tl}{\frac{\pi d^4}{32} G} = \frac{32 Tl}{\pi d^4}}$$

$$\theta_r = \frac{Tl}{GJ}$$

converting θ_r from radian to degrees (θ)

Angle of twist radian $= \theta_r$

$$\theta^\circ_{Dg} = \frac{180}{\pi} \times \frac{T \cdot l}{GJ} = \frac{180}{\pi} \times \frac{T \cdot l}{G \frac{\pi d^4}{32}}$$

$$\theta = \frac{180 \times 32\, Tl}{\pi G \pi d^4} = \frac{5760\, T \cdot l}{(\pi)^2 G d^4}$$

$$\theta_{Degree} = \frac{584 \cdot T \cdot l}{G d^4} \rightarrow (1) \quad \cancel{T =} \theta_{De}$$

$$\theta_{Degre} = \frac{\theta_{per} \times l}{1000} \rightarrow (2)$$

$$\frac{\theta_{per} \times \cancel{l}}{1000} = \frac{584\, T \cancel{l}}{G d^4}$$

$$\frac{G d^4 \theta_{per}}{584 \times 10^3} = T \rightarrow (b)$$

$$\frac{T}{J} = \frac{G\theta}{l}$$

$$T = \frac{G\theta \cdot J}{l}$$

$$T = \frac{G \pi d^4}{32 \cdot l} \frac{0.5}{100}$$

$$= \frac{0.5^\circ}{1000} \times l$$

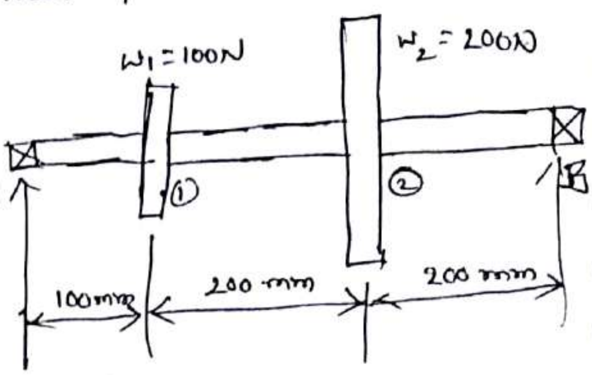
eqn (a) & (b)

$$\frac{G d^4 \theta_{per}}{584 \times 10^3} = \frac{\pi}{16} d^3 \tau$$

$$d = \frac{\pi \times \tau \times 584 \times 10^3}{16 \times G \times \theta_{per}} = \frac{\pi \times 40 \times 584 \times \cancel{10^3}}{16 \times 84 \times \cancel{10^3} \times 0.5}$$

$$\boxed{d = 109.20} \text{ mm}$$

(2) Determine the lowest (i.e., first) critical speed for the shaft of 25 mm diameter



Sol $W_1 = 100$ N, $W_2 = 200$ N

According to Dunkerley eqn, the lowest critical speed is given by

$$\frac{1}{\omega_c^2} = \frac{1}{\omega_{c1}^2} + \frac{1}{\omega_{c2}^2} + \cdots + \frac{1}{\omega_{cn}^2} + \frac{1}{\omega_{cs}^2}$$

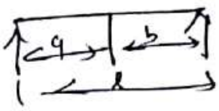
ω_c = first critical speed of shaft with all loads acting together

$\omega_{c1}, \omega_{c2} \cdots$ = Critical speeds of the shaft with each load acting separately

ω_{cs} = Critical speed of shaft under its own weight.

for a shaft negligible weight with single attached disk,

$$\omega_c = \sqrt{\frac{g}{\delta}} \qquad \delta = \frac{Wa^2b^2}{3EIl}$$



W = non central load

Now for the deflection δ_1, $W_1 = 100N$, $a = 100$ mm, $b = 400$ mm

$E = 2\times10^5 N/mm^2$ (Assumed)

$$I = \frac{\pi d^4}{64} = \frac{\pi}{16}\times25^4 = 19175 \text{ mm}^4$$

$\boxed{l = 500 \text{ mm}}$

$$\therefore \delta_1 = \frac{100\times100^2\times400^2}{3\times2\times10^5\times19175\times500} = 0.028$$

$$\omega_{c1} = \sqrt{\frac{g}{\delta_1}} = 592 \text{ rad/s}$$

$\delta_2 = \quad W_2$

$$\omega_{c2} = \sqrt{\frac{g}{\delta_2}} = \sqrt{\frac{9.81\times10^3}{0.125}} = 280 \text{ rad/s}$$

$W = 200N$ $\quad E = 2\times10^5 N/mm^2$

$a = 300$ mm $\quad I = 19175 \text{ mm}^4$

$b = 200$ mm $\quad l = 500$ mm

$\delta_2 = 0.125$ mm

$$\frac{1}{\omega_c^2} = \frac{1}{\omega_{c1}^2} + \frac{1}{\omega_{c2}^2} = \frac{1}{592^2} + \frac{1}{280^2}$$

$$\frac{1}{\omega_c^2} = \frac{(280^2+592^2)}{(592^2+280^2)}$$

$\boxed{\omega_c = 253 \text{ rad/s}}$

$$N_c = \frac{30}{\pi}\omega_c = \frac{30}{\pi}\times253 = 2416 \text{ rpm.}$$

$$N_c = \frac{945.8}{\sqrt{\delta_1+\delta_2}} = \frac{945.8}{\sqrt{0.028+0.125}} = 2418 \text{ rpm}$$

Couplings:- couplings are attachments whereby shafts are connected to each other. for example, a coupling is used to connect the shaft of an electric motor to the line shaft of a machine (e.g. rice mill) or a hydraulic turbine to an electric generator (e.g. hydro electric power) or engine shaft to propeller shaft of an automobile or to join many shafts of standard lengths. (generally 7 meters or 20 feet). to get long shaft.

Some couplings are employed in pipe line system simply to increase the length of pipes and they are known as pipe joints and generally available as threaded couplings.

coupling used in automobile engine are known as clutches.

Depending upon the method of applications, couplings are differentiated into permanent couplings and temporary couplings. Shaft couplings & pipe couplings are permanent couplings whereas the clutches are temporary couplings. The reason for mentioning shaft and pipe couplings as permanent couplings is because their connections would be broken only for repairs and general maintenance. but the clutches are used for periodical disengagement.

Types of couplings:

I. permanent couplings

(a) shaft couplings

1. Rigid couplings

(i) Muff or sleeve couplings

(ii) Clamp or split coupling

(iii) Flange couplings

2. Flange couplings

(i) Flange couplings

(ii) Universal couplings

(iii) oldham couplings.

(b) Pipe coupling (Rigid type only)

(i) Muff or sleeve couplings

(ii) Flange couplings.

(II) Temporary couplings (or) clutches

1) plate or Disc clutches

2) cone clutches

3) Jaw clutches.

Shaft couplings and pipe couplings

Both shaft couplings and pipe couplings are made of same material usually by Cast-Iron and almost same construction with slight difference. Also in both type of couplings, we have muff & flange type models. But the shaft couplings are employed for connecting power transmitting shafts whereas the pipe couplings are employed for connecting pipes in order to transport the liquids like oil or water and some times gases through them to distant places.

Usually the shaft couplings are connected with the shafts by Keys. on other hand, pipe couplings are connected to the pipes through threaded ~~joint~~ format. The design of shaft couplings is mainly based on the shear stress due to power transmission, whereas the design of pipe couplings is decided by the gas or liquid pressures inside the pipes.

Permanent Shaft couplings (Rigid couplings)

They are used to connect two shafts rigidly without allowing any angular misalignment so that their axes will be colinear. this type of couplings are mainly used in connection with line shaft, motor shafts to machine shaft and so on. Some of rigid couplings

(1) sleeve or Muff coupling (2) clamp or split muff coupling
(3) Flange coupling.

Sleeve or Muff coupling: The sleeve or muff coupling is the simplest form of rigid coupling. It is nothing but a hollow shaft made of cast-Iron in which two shafts, which are to be connected, are inserted into it equally and coupled by means of single lengthy gib head key or by two shaft keys arranged in different places in the circumference of the shaft, generally at 90° to each other, to allow a slight misalignment if it is happened. The key & shafts are made of mild steel.

The permissible stresses in shearing for mild steel may be assumed as 40 to 50 MPa and for cast Iron it may be assumed as 14 to 20MPa. Even though the shaft, key & coupling are made of different materials, they should have enough strength to transmit the required torque.

(a) Design of sleeve couplings:

Since this coupling is used to transmit power, i.e. torque, its various dimensions, like diameter of shaft, dia of coupling etc. are determined by comparing the corresponding shear strength and crushing strength to the transmitted torque. Sometimes the designing work may be done in different way. That is by assuming certain dimensions, their induced stresses are calculated and checked with the permissible stresses. By this method, some complicate problems may be simplified. Generally the latter method is preffered. For example if d is the diameter of the shaft, then the dimensions of coupling and key are assumed as follows:

1. outside diameter of sleeve $D = 2d$
2. Length of sleeve $L = 3d$
3. Length of key $l = \frac{L}{2} = 1.5d$
4. No. of keys required $= 2$

For the above assumed values, suppose the induced stresses are more than the permissiable stresses, then the corresponding dimensions are suitably altered.

(II) Steps to be followed in designing sleeve coupling:

1. Determine the torque (T) to be transmitted from the power, using the relation $P = \frac{2\pi NT}{60}$ in S.I units

2. Find out diameter of the shaft (d) by comparing its torsional shear strength to the transmitted torque as

$$\frac{\pi}{16} \tau d^3 = T$$

where τ is shear strength of shaft (usualy 40 to 50 N/mm²)

3. Assume the dimensions of couplings and keys as mentioned earlier.. i.e $D = 2d$, $L = 3d$, $l = 1.5d$, $w = \frac{d}{4}$, $t = \frac{d}{6}$.

4. Determine the induced shear stress for the coupling assuming it as a hollow shaft, by equating the torque transmitting capacity of the coupling to the applied torque

$$\text{i.e } \frac{\pi}{16} \tau D^3 (1 - k^4) = T.$$

where τ = Induced shear stress $\quad K = \frac{d}{D} = \frac{d_i}{d_o}$

5. Evaluate the induced shear stress and crushing stress of the key using $l \times w \times \tau_{s1} \times \frac{d}{2} = T$, and $l \times \frac{t}{2} \times \tau_{c1} \times \frac{d}{2} = T$.

where w = width of key, t = thickness of key.

τ_{s1} & τ_{c1} = Induced shear & crushing stress of key material.

6. Compare the above induced stress values with their allowable stress values and suitably adjust the dimensions.

7. Draw a neat sketch and mention complete specifications.

Clamp or split muff coupling:

The clamp or split muff coupling is the modified type of muff coupling in which the muff (i.e. sleeve) is split into two halves longitudinally and they are held in position by means of mild-steel bolts or studs. The bolts & nuts are placed in the recesses provided in the coupling. The number of bolts used in this coupling may be four to eight. Since the shafts are compressed by split muff due to tightness of bolts, this coupling is also called as compression coupling.

The advantage of this coupling is that the position of the shafts need not be changed for assembling or dismantling of the coupling. The split muff coupling is usually employed for heavy duty and moderate speeds. Here also, the coupling (i.e sleeve) is made of cast-Iron. In this coupling, the power is transmitted from one shaft to another shaft. Only one key may be used in this coupling in contrast with the ordinary muff coupling in which one or two keys may be utilised.

The design of split muff coupling follows the same method as that of ordinary muff coupling. Additionally, the

the Compression bolts are designed based on the frictional force produced between the coupling and the shaft. Since the method of designing the muff & key has already been discussed here, the design of clamping bolts is only explained.

Let T = Torque transmitted by the shaft

d = Diameter of the shaft

d_c = Root or core diameter of bolt

n = Number of bolts

σ_t = Tensile strength of bolt material.

μ = Coefficient of friction b/w the muff & shaft &

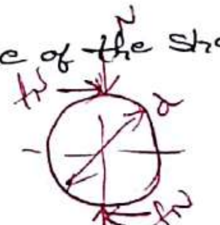
L = Length of muff.

n = number of bolts.

when tightening the nut, the force exerted by each bolt on the shaft,

$$F_{b1} = \frac{\pi}{4} d_c^2 \sigma_t$$

∴ Force exerted by the bolts on each side of the shaft.



Half number of bolts will give pressure on input shaft & Remain half will apply out put shaft

$$F_b = \frac{\pi}{4} d_c^2 \cdot \sigma_t \cdot \frac{n}{2}$$

Assuming p as the bearing pressure on the shaft and the muff-surface due to the above force, then the uniform pressure distribution over the surface, the pressure is given by

$$p = \frac{\text{Force}}{\text{projected Area}} = \frac{\frac{\pi}{4} d_c^2 \cdot \sigma_t \cdot n/2}{\frac{1}{2} \cdot L \cdot d} = \frac{\pi d_c^2 \cdot \sigma_t \cdot n}{4 L \cdot d}$$

Now, the frictional force produced b/w each shaft & muff,

$$F_f = \mu \times \text{pressure} \times \text{Area}$$

$$= \mu \times p \times \frac{1}{2} \times \pi d L$$

$$= \mu \times \frac{\pi d_c^2 \cdot \sigma_t \cdot n}{4 L d} \times \frac{1}{2} \pi d L$$

$$\boxed{F_f = \mu \times \frac{\pi^2}{8} d_c^2 \cdot \sigma_t \cdot n}$$

and the torque that can be transmitted by the coupling due to friction,

$$T_f = F_f \times \frac{d}{2} = \mu \cdot \frac{\pi^2}{8} d_c^2 \cdot \sigma_t \cdot n \cdot d/2$$

i.e $$\boxed{T_f = \frac{\pi^2}{16} \cdot \mu \cdot d_c^2 \cdot \sigma_t \cdot n \cdot d}$$.

By equating this frictional torque to the torque transmitted by the shaft, the bolt diameter (d_c) can be evaluated.

Since some recesses are provided for clamping bolts in the coupling, its dimensions are slightly higher than ordinary muff coupling. The values of various parameters are given as follows.

outside diameter of sleeve $D = 2.5d$

Length of sleeve $L = 3.5d$

Length of key $l = L = 3.5d$

Number of keys $= 1$

Number of bolts $n = 4, 6, or 8$.

The bolts are fitted in the coupling in proper positions based on the sizes of coupling.

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Rigid Flange Coupling:

The flanged-face coupling is also intended to connect strictly co-axial shafts. The coupling consists of two members shaped as flanges. The coupling members are fitted at the ends of the shafts being connected by keys and are drawn together by bolts. For better axial alignment one of the flanges has a circular projection and the other has a corresponding circular recess. Here also the coupling is made of cast iron and other elements like shafts, keys & bolts are made of mild steel.

The coupling of this type are very common. They are employed to connect shafts from 18 mm to 200 mm in diameter and sometimes even more. Torque is transmitted either by the forces of friction b/w the faces of the coupling members drawn together by bolts or by the bolts working in shear. Generally flange coupling transmits more torque than sleeve coupling.

(I) Design of flange coupling:

Here also, the various dimensions of coupling are determined by comparing their shear strength and crushing strength against failure. That is, when the coupling transmits power, the shaft, coupling hub, coupling flange, bolts may fail due to shear and the keys may fail due to shear or crushing. Hence they are designed to overcome these failures. That is, their shear strength and crushing strength should be more than the applied power.

(II) Steps to be followed in designing flange coupling

1. As usual calculate the ~~torque~~ from the given power.
2. Determine the diameter of the shaft (d) using the relation as

$$\frac{\pi}{16} \tau d^3 = T$$

3. Assume the dimensions of coupling and key in terms of shaft diameter, d, as follows.

(I) outside diameter of hub $D_1 = 2d$

(II) pitch circle diameter of bolts $D_2 = 3d$

(III) outside diameter of flange $D_3 = 4d$

(IV) Length of hub $L = 1.5d$

(V) Thickness of flange $t_f = 0.5d$

(VI) Thickness of protected flange $t_p = 0.25d$

(VII) Length of key $l = L = 1.5d$

(VIII) Width of key $w = \frac{d}{4}$

(IX) Thickness of key $t = d/6$ (for rectangular key)

$= d/4$ (for square key).

The width & thickness of key may also be selected from Table J.D.B.

(X) The number of bolts required for joining flanges, as

$n = 3$ for $\underline{d}$ upto 40mm

$= 4$ for $\underline{d}$ upto 100 mm

$= 6$ for $\underline{d}$ upto 180 mm

$= 8$ for $\underline{d}$ above 180mm.

Let τ_1 = Allowable shear stress for shaft, key & bolt materials (ie mild steel)

$= 40$ to $50 N/mm^2$

τ_2 = Allowable shear stress for coupling material (ie cast iron)

$\tau_2 = 14$ to $20 N/mm^2$

τ_c = Allowable crushing stress for key material

= 80 to 120 N/mm^2.

4. Determine the maximum induced torsional shear stress on the hub of the coupling, assuming it as a hollow shaft, by equating its induced turning moment to the applied torque.

$$\text{i.e}\quad \frac{\pi}{16}\tau_2 d_0^3(1-k^4) = T,\quad \text{where}\quad k = \frac{d_i}{d_o}$$

5. obtain the induced pure shear stress i.e plain shear stress on coupling flange by equating its induced turning moment to applied torque (T)

Induced turning moment T_i = Area resisting shearing × Shear stress × Max. radius

$$= \pi d_o t_f \times \tau_2 \times \frac{d_o}{2} = \frac{\pi}{2} d_o^2 t_f \tau_2$$

6. Determine the induced shear stress and crushing stress of key as usual.

7. Calculate the diameter of bolts by comparing their torque transmitting capacity to the applied torque.

Torque transmitting capacity of all bolts,

T_b = Torque transmitting capacity of one bolt × number of bolts

$$= \frac{\pi}{4} d_b^2 \tau_1 \frac{D_2}{2} \times n \quad \text{where } d_b \text{ dia of bolt}$$

for safe design, $T_b \geq$ Applied torque (T)

select the standard dia of bolts with respect to Indian standard system.

8. check the induced crushing stress of the bolt.

9. Draw the neat sketch and present the complete details of couplings.

Note: The standard dimensions for forged end rigid coupling is given

J.D.B. Page No. 7-8, table 7.1

Flexible Shaft Coupling:

Flexible Shaft coupling are used to connect shafts
subject to one or more kinds of misalignment and to reduce
the effect of shock and impact loads that could be transferred
b/w shafts. These couplings are, in general, used in machines
subjected to varying loads and frequent on-off cycles, and in
cases where accurate alignment of the shafts cannot be
guaranteed.

Flexible flange coupling:

The simplest and commonest type of coupling is the
flexible rubber-bushed coupling with respect to design, it
is similar to the flanged-face rigid coupling, but the
bolts in one of the coupling members are replaced by steel
pins on which rubber or leather bushes are provided. Any
Jerks and shocks originated during the transmission of torque
are damped owing to deformation of the rubber rings.
This coupling makes up for the parallel misalignment of
shafts (upto 0.5mm), angular ~~disp~~ misalignment upto 1.5°
and axial displacement. Rubber bushed couplings are
standardised, their sizes should be selected to suit the
torque transmitted.

(I) Design of Rubber bushed flexible flange coupling:

Since the flexible coupling includes the soft material, i.e rubber bush, the bearing capacity of this bush should be considered when designing. Also the bearing pressure on the rubber or leather bush is very low, around 0.5 to 1 N/mm². the pitch circle diameter of pins and sizes of pins are increased to make the coupling efficient.

~~Let~~

Let d_1 = diameter of pin, d_2 = diameter of bush.

l = Length of bush in the flange

p_b = Bearing pressure on the bush

n = Number of pins.

D = Diameter of pitch circle of the pins.

Now T = Torque transmitted by the coupling

= Torque transmitted by all the pins

= Bearing load on one pin × distance of action × number of pins.

$= W \times \frac{D}{2} \times n$

= Bearing pressure × bearing area × distance of action × no. of pins

$= P_b \times d_2 \times l \times \frac{D}{2} \times n.$

Since the pins and bushes are not rigidly held in the coupling there should be a shear stress due to torsional load and a bending stress due to flexibility.

Direct shear stress $\tau_b = \dfrac{W}{\frac{\pi}{4}(d_1)^2}$

Bending stress $\sigma_b = \dfrac{M}{Z} = \dfrac{W\left(\frac{l}{2} + 5\,mm\right)}{\frac{\pi}{32} d_1^3}$

where M = Bending moment, Z = section modulus.

Since the pin is subjected to bending & shear stresses, Rankine's theory or Guest theory may be applied to find out the maximum principal stress or maximum shear stress.

Maximum principal stress $(\sigma_b)_{max} = \frac{1}{2}\left[\sigma_b + \sqrt{\sigma_b^2 + 4\tau^2}\right]$

Maximum shear stress $\tau_{max} = \frac{1}{2}\sqrt{\sigma_b^2 + 4\tau^2}$

Design principal stress ranges from 30 to 50 N/mm².

Design procedure for flexible flange coupling

1. Calculate the torque to be transmitted and the diameter of the shaft as mentioned in rigid flange coupling.

2. Assume some of the coupling dimensions and key dimensions as follows:

(I) Hub diameter $D_1 = 2d.$

(b) Hub length $L = 1.5d$

(c) Flange thickness $t_f = 0.5d$

(d) Length of Key $l_1 = L = 1.5d$.

(3) Determine the dimensions of pins and bush as

(a) Diameter of pin $d_1 = \frac{0.5d}{\sqrt{n}}$ (an empirical formula)

where n = number of pins, and d = diameter of shaft

(Here the number of pins is generally more, comparing to rigid flange coupling).

(c) Check these stresses with allowable stresses.

(b) Then evaluate diameter of bush, load on each pin, direct shear stress, bending stress, maximum principal stress & maximum shear stress etc.

(d) verify the induced stresses on the hub, key with their allowable stresses as mentioned in rigid flange coupling.

(e) if any induced (i.e calculated) stress is more than allowable stress, then alter the corresponding dimensions suitably to overcome failure.

(f) Draw a neat sketch and tabulate the specifications.

Universal or Hooke's coupling:

The universal or hook's couping is employed to connect two power-transmitting shaft whose axes intersect at a small angle. The main application of this coupling is found in automobiles in which the power from gear box at a higher level has to be transmitted to the differential or back axle positioned at a lower level through a long shaft called Propeller shaft. In such a case, two Universal coupling are fitted one at each end of the propeller shaft, connecting gear box at one end and the differential on the other end. A Hook's coupling may also be used for transmi-ssion of power to different spindles of multi-spindle drilling machine and also used in milling machine.

(1) Design and make a neat dimensioned sketch of a muff coupling to transmit a power of ~~100~~ 40 ~~30~~ kW from a shaft running at ~~100~~ 350 r.p.m. Assum ~~suitable material and stresses~~. The material for the shafts and key is plain carbon steel for which allowable shear and crushing stresses may be taken as 45MPa & 85MPa respectively. The material for the muff is cast iron for which the allowable shear stress may be assumed as 15MPa.

Sol

$P = 40\text{ KW} = 40 \times 10^3\text{ W}$, $N = 350\text{ r.p.m}$, $f_{s_1} = \tau_{s_1} = 45\text{ MPa} = 40\text{ N/mm}^2$ (M.S)

$\sigma_c = 85\text{ MPa} = 80\text{ N/mm}^2$ (M.S) $\quad f_{s_2} = \tau_{s_2} = 15\text{ N/mm}^2$ (C.I)

Shaft Diameter : Torque transmitted by the shaft

$$T = \frac{60 \times P}{2\pi N} = \frac{60 \times 40 \times 10^3}{2 \times \pi \times 350} = 1100\text{ N-m}$$

$$= 1100 \times 10^3\text{ N-mm.}$$

we also know that the torque transmitted

$$T = \frac{\pi}{16} (\tau_{s_1})\, d^3$$

$$1100 \times 10^3 = \frac{\pi}{16} \times 40 \times d^3$$

$$\boxed{d = 52}\text{ mm.}$$

Coupling & key dimensions:

Dia of coupling $(D) = 2d = 2\times52 = 104$ mm

Length of " $(L) = 3d = 2\times52 = 156$ mm.

Length of key $(l) = \frac{L}{2} = \frac{156}{2} = 78$ mm.

Since the crushing stress is twise the shear stress, square key may be employed, whose width & thickness are equal to $d/4$

$$w = t = \frac{52}{4} = \underline{13 \text{ mm}}.$$

$$w = t = \underline{\underline{13 \text{ mm}}}$$

Checking for the strength of sleeve (a) coupling

$$T = \frac{\pi}{16} \tau_{s2} D^3 (1 - k^4) \qquad k = \frac{d}{D} = \frac{52}{104} = 0.5$$

∴ Induced shear stress

$$\tau_{s2} = \frac{16T}{\pi D^3 (1-k^4)} = \frac{16\times1100\times10^3}{\pi\times(104)^3(1-0.5^4)}$$

$$= \frac{17600000}{\pi\times 124864\left(\frac{15}{16}\right)} = \frac{17600000}{\cancel{3312997.949}\ 2.9452\times1124864}$$

$$= 5.3124 \text{ N/mm}^2 < \tau_{s2}\,(15 \text{ N/mm}^2)$$

Hence design is safe.

Checking the strength of key in shear & crushing

$$T = l \times w \times \tau_{s1} \times d/2$$

$$1100 \times 10^3 = \cancel{l \times w}\ 78 \times 13 \times \tau_{s_1} \times \frac{52}{2}$$

$$\tau_{s_1} = 41.72\ N/mm^2 \quad < (45\ MPa)\ \text{given}$$

considering crushing

$$T = l \times \frac{t}{2} \times \sigma_c \times \frac{d}{2}$$

$$1100 \times 10^3 = 78 \times \frac{13}{2} \times \sigma_c \times \frac{52}{2}$$

$$\sigma_c = \frac{1100 \times 10^3}{78 \times 6.5 \times 26}$$

$$\sigma_c = 83.44 \quad < (85\ N/mm^2\ \text{given})$$

(2) Design a clamp coupling to transmit 30 KW at 960 rpm. The allowable shear stress for the shaft & key is 40 N/mm² and crushing stress for key is 90 N/mm². The number of bolts connecting the two halves is 6 & the permissible tensile stress for the bolts is 70 N/mm². Assume the coefficient of friction b/w the shaft & the muff as 0.3 and the design shear stress for the coupling as 15 N/mm².

Sol

$P = 30\ KW = 30 \times 10^3\ W$, speed, $N = 960$ r.p.m

$\tau_{s_1} = 40\ N/mm^2$, $\tau_{s_2} = 15\ N/mm^2$, $\sigma_c = \tau_c = 90\ N/mm^2$

$\mu = 0.3$

$\sigma_t = 70\ N/mm^2$, $n = 6$,

$$P = \frac{2\pi N T}{60} \qquad T = 298 \times 10^3\ N\text{-}mm$$

$$\tau = \frac{16T}{\pi d^3} \Rightarrow d = 33.6\ mm$$

$$d = 35\ mm$$

coupling and key dimensions:

Dia of muff $D = 2.5\,d = 2.5 \times 35 = 87.5 = 90\ mm$

Length of coupling $L = 3.5\,d = 3.5 \times 35 = 122.5\ mm = 125\ mm.$

Length of key $l = L = 125\ mm$

width of key $w = 10\ mm$ [from databook page 6.11, table 6.3]

$t = 8$ (taper key.)

checking the induced stress in the muff.

Assuming the muff as hollow shaft, the torque transmitted is given by

$$T = \frac{\pi}{16} \tau_{s_2} D^3 (1 - K^4) \qquad K = \frac{d}{D} = \frac{35}{90} = 0.4$$

$$\tau_{s_2} = 2.14\ N/mm^2 < (\tau_{s_2}\ \text{given})$$

Hence, Design is safe.

checking the induced stress in the key

considering shear, the torque transmitted is given by

$$T = l_1 \cdot w \cdot \tau_{s_1} \cdot d/2$$

$$\tau_{s_1} = \frac{2T}{l_1 \cdot w \cdot d} \qquad \left(\text{Here } l_1 = l/2 = \frac{125}{2} = 62.5\ mm\right)$$

$$\tau_{s_1} = 27.2\ N/mm^2 < (40\ N/mm^2\ \text{given})$$

crushing

$$T = l_1 \cdot t/2 \cdot \sigma_c \cdot d/2$$

$$\sigma_c = 68.1\ N/mm^2 < (\text{give value } 90\ N/mm^2)$$

Hence design is safe.

Size of bolts:

Let d_c = Root or core diameter of bolt. we know that torque transmitted

$$T = \frac{\pi^2}{16} \cdot \mu \cdot d_c^2 \cdot \sigma_t \cdot n \cdot d$$

$$d_c = \left[\frac{16T}{\pi^2 \cdot \mu \cdot \sigma_t \cdot n \cdot d}\right]^{1/2} = \left[\frac{16 \times 298 \times 10^3}{\pi^2 \times 0.3 \times 70 \times 6 \times 35}\right]^{1/2}$$

$$\boxed{d_c = 10.5 \text{ mm}}$$

(3) A Rigid flange coupling is to be designed to transmit 20 KW at 1000 r.p.m. Assuming suitable allowable stresses, design the coupling.

Sol. $P = 20 \times 10^3$ W, $N = 1000$ r.p.m,

mild steel,

Assume $\tau_{s_1} = 40 \text{ N/mm}^2$, $\tau_{s_2} = 15 \text{ N/mm}^2$

$\sigma_c = 80 \text{ N/mm}^2$

Shaft dia (d) $T = \frac{60P}{2\pi N} = 191 \text{ N-m} = 191 \times 10^3 \text{ N-mm}$

$$T = \frac{\pi}{16} \tau_{s_1} d^3$$

$$d = 28.97 \text{ mm} = 30 \text{ mm}$$

Coupling and key dimensions

~~Diameter~~ $D_1 = 2d = 60$ mm, $D_2 = 3d = 90$ mm, $D_3 = 4d = 120$ mm

$L = 1.5d = 45$ mm, $l = L = 45$, $t_f = 0.5d = 15$ mm.

$t_p = 0.25d = 7.5$ mm

Checking the strength of coupling

considering the hub as hollow shaft, the torque transmitted

$$T = \frac{\pi}{16} \tau_{s2} D_1^3 (1-k^4) \qquad k = \frac{d}{D_1}$$

$$\tau_{s2} = 4.8 \ N/mm^2 \leq (\text{given value } 15 N/mm^2)$$

$\therefore$ Design is safe.

considering the flange coupling, we know that

$$T = \pi D_1 t_f S_{s2} \times \frac{D_1}{2}$$

$$\tau_{s2} = 2.25 \ N/mm^2 < (\tau_{s2}) \text{ given value.}$$

$$\sigma_c = 2 \tau_{s1}, \quad w = t = \frac{d}{4} = \frac{30}{4} = 7.5 \ mm$$

$$w = t = 8 \ mm$$

checking the strengths of key:

considering the shear, we know that

$$T = l \times w \times \tau_{s1} \times d/2$$

$$\tau_{s1} = 35.4 \ N/mm^2 < \text{given value.}$$

considering the crushing

$$T = l \times \frac{t}{2} \times \sigma_c \times d/2$$

$$\sigma_c = 70.8 \ N/mm^2 < \text{given value.}$$

Design is safe.

Design of bolts: No of bolts, n = 3, for d = 30 mm

$$T = \frac{\pi}{4} d_b^2 \times \tau_{s1} \times n \times \frac{D_2}{2}$$

$\boxed{d_b = 6.7 \text{ mm}}$ $d_b = 8\text{ mm}$

checking the crushing strengths of bolts

we know that ~~torque~~ $T = d_b \cdot t_f \cdot \sigma_c \cdot n \cdot \frac{D_2}{2}$

$\sigma_c = 11.8 < 80 \text{ N/mm}^2$

(4) Design a flexible flange coupling of bush type to transmit 3 KW power at 960 rpm with a service factor of 1.2. Assume design stresses as,

for shaft, bolt & key in shear $(\tau_{s1}) = 50 \text{ N/mm}^2$

for coupling in shear $(\tau_{s2}) = 20 \text{ N/mm}^2$

for bushes in bearing $(\tau_b) = 2 \text{ N/mm}^2$

for key in crushing $(\sigma_c) = 100 \text{ N/mm}^2$.

Sol

$P = 3\text{KW} = 3000\text{W}$, $N = 960 \text{ rpm}$, service factor $(K_s) = 1.2$.

Shaft diameter: It is known that

Design power = Rated power × Service factor

$= P \times K_s = 3000 \times 1.2 = 3600 \text{ W}$

Torque transmitted $T = \frac{60 \times P}{2\pi N} = 35.8 \text{ N-m}$

$= 35.8 \times 10^3 \text{ N-mm}.$

Dia of shaft $\tau = \frac{16T}{\pi d^3}$

$\boxed{d = 15.4 \text{ mm}}$ take $\boxed{d = 20\text{mm}}$

coupling & key dimensions:

Hub diameter $D_1 = 2d = 40 \text{ mm}$

$= 30 \text{ mm}.$

Flange thickness $t_f = 0.5\,d = 10$ mm.

Length of key $l_1 = l = 30$ mm

Dimensions of bushed pin :

Let d_1 = dia. of the pin. d_2 = dia of bush

Dia. of the pin $d_1 = \frac{0.5d}{\sqrt{n}} = \frac{0.5 \times 20}{\sqrt{3}} = 5.8$ mm.

In order to reduce the bearing pressure, on the projected area of the rubber bush, the diameter of the pin is increased.

i.e Take $d_1 = 8$ mm.

The enlarged portion of the pin size may be taken as 10 mm

on the enlarged portion, ~~of the pin~~ a brass bush of thickness 1 mm is pressed and a rubber bush of 2mm thickness is pressed over the brash bush.

∴ overall diameter of the bush,

$$d_2 = 10 + (2\times1) + (2\times2) = 10+2+4 = 16\text{ mm.}$$

and diameter of the pitch circle of the pins.

$$D_2 = D_1 + \left(2\times\frac{d_2}{2}\right) + (2\times\text{clearance})$$

$$= 40 + 16 + (2\times7)$$

$$= 70\text{ mm}$$

(Assume clearance dia $= 2\times7 = 14$ mm)

The bearing load acting on each pin is given by

$$W = \text{Bearing pressure} \times \text{Area}$$

$$= P_b \times d_2 \times l \quad (l = \text{Length of bush in the flange})$$

$$= 2\times16\times l = 32\times l \text{ N}$$

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Torque transmitted by all the pins.

$$T = W \times n \times \frac{D_2}{2}$$

$$35.8 \times 10^3 = 32\, l \times 3 \times \frac{70}{2}$$

$$\therefore l = 10.65 \text{ mm}, \quad \text{take } \boxed{l = 11 \text{ mm}}$$

and $W = 32 \times l = 32 \times 11 = 352$ N

To verify the correctness of our design, the maximum shear stress of pin due to direct shear stress and bending stress may be checked with allowable shear stress.

Now direct shear stress due to pure torsion

$$\tau_s = \frac{W}{\frac{\pi}{4} d_1^2} = \frac{352 \times 4}{\pi \times 8^2} = 7.0 \text{ N/mm}^2$$

Bending stress due to flexible connection is given by

$$\sigma_b = \frac{M}{Z}$$

where M = Bending moment on the pin.

$$= W\left(\frac{l}{2} + c\right) \qquad (c = \text{clearance b/w two couplings} = 3)$$

Assum.

$$= 352\left(\frac{11}{2} + 3\right) = 2992 \text{ N-mm}$$

and $Z = \frac{\pi}{32}(d_1)^3 = \frac{\pi}{32} \times 8^3 = 50.26 \text{ mm}^3$

$$\boxed{\sigma_b = 59.5 \text{ N/mm}^2}$$

Maximum shear stress $\tau_{max} = \frac{1}{2}\sqrt{(\sigma_b)^2 + 4\tau^2}$

$$\tau_{max} = \frac{1}{2}\sqrt{(59.5)^2 + 4(7.0)^2}$$

$$\tau_{max} = 30.56 \text{ N/mm}^2 < (\text{given value})$$

our design is safe

Now checking for the dimensions of the coupling

Assuming the hub as hollow shaft, the transmitted torque is equated to the strength of coupling.

$$T = \frac{\pi}{16} \tau_{s2} D_1^3 (1-k^4) \qquad k = \frac{d}{D_1} = \frac{d}{2d} = 0.5$$

$$35.8 \times 10^3 = \frac{\pi}{16} \tau_{s2} (40)^3 (1 - 0.5^4)$$

$$\therefore \quad \tau_{s2} = 3.04 \text{ N/mm}^2 < \text{allowable } 20 \text{ N/mm}^2$$

Checking the strength of flange.

$$T = \pi \cdot D_1 \cdot t_f \cdot \tau_{s2} \times \frac{D_1}{2} = \pi D_1^2 t_f \frac{\tau_{s2}}{2}$$

$$\tau_{s2} = \frac{2T}{\pi D_1^2 t_f} = 1.42 \text{ N/mm}^2 < \text{allowable.}$$

Hence coupling design is safe.

Now flange diameter $D_3 = D_2 + 2 \times \frac{d_2}{2} + (2 \times \text{clearance})$

$$= (70) + (16) + (2 \times 7) = 100 \text{ mm}$$

Checking the strength of key

Let w = width of key, t = thickness of key

crushing strength = 2 × shear strength

$$w = t = \frac{d}{4} = \frac{20}{4} = 5 \text{ mm}$$

Shear strength of key $= l_1 \times w \times \tau_{s1} \times \frac{d}{2}$

Equating this to transmitted torque, we get

$$T = l_1 \times w \times \tau_{s1} \times \frac{d}{2}$$

$$35.8 \times 10^3 = 30 \times 5 \times \tau_{s1} \times \frac{20}{2}$$

$$\boxed{\tau_{s1} = 23.9} \text{ N/mm}^2 < \text{allowable}$$

Similarly crushing strength of key is equated to torque

$$T = l_1 \times \frac{t}{2} \times \sigma_c \times \frac{d}{2}$$

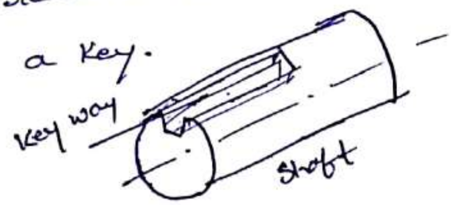
$$35.8 \times 10^3 = 30 \times \frac{5}{2} \times \sigma_c \times \frac{20}{2}$$

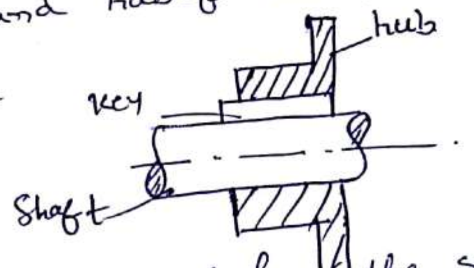
$$\boxed{\sigma_c = 47.8\ N/mm^2} < \sigma_c$$

UNIT – IV

Introduction:

A key is a piece of mild steel inserted b/w the shaft and hub or boss of the pulley to connect these together in order to prevent relative motion b/w them. It is always inserted parallel to the axis of the shaft. Keys are used as temporary fastenings and are subjected to considerable crushing and shearing stresses. A keyway is a slot or recess in a shaft and hub of the pulley to accommodate a key.





Types of keys:

The following types of keys are important from the subject point of view:

1. Sun Keys 2. Saddle Keys 3. Tangent Keys 4. Round Keys and 5. splines.

Sunk Keys: The sun keys are provided half in the keyway of the shaft and half in the keyways of the hub or boss of the pulley. The sunk keys are of the following types:
(the sunk key, made of mild steel)

1. **Rectangular sunk key:** A rectangular sun key is shown below: The usual proportions of this key are:

width of key, $w = d/4$; and thickness of key $t = \frac{2w}{3} = \frac{d}{6}$

where d = Diameter of the shaft or diameter of the hole in the hub.

The key has taper 1 in 100 on the top side only.

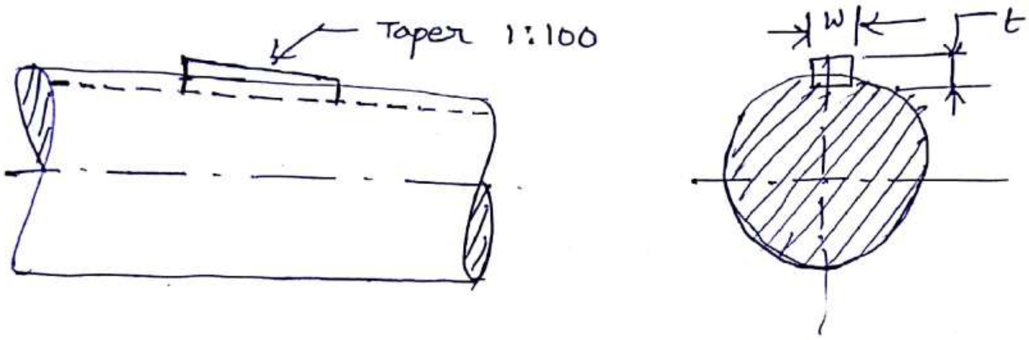


fig: sunk key.

2. Square sunk key: The only difference b/w a rectangular sunk key and a square sunk key is that its width and thickness are equal, i.e. $w = t = d/4$.

3. Parallel sunk key: The Parallel sunk keys may be of rectangular or square section uniform in width and thickness throughout. It may be noted that a parallel key is a taper less and is used where the pulley, gear or other mating piece is required to slide along the shaft.

4. Gib-head key: It is a rectangular sunk key with a head at one end known as gib head. It is usually provided to facilitate the removal of key. A gib head key is shown in fig(a) below and its use is shown in fig (b).

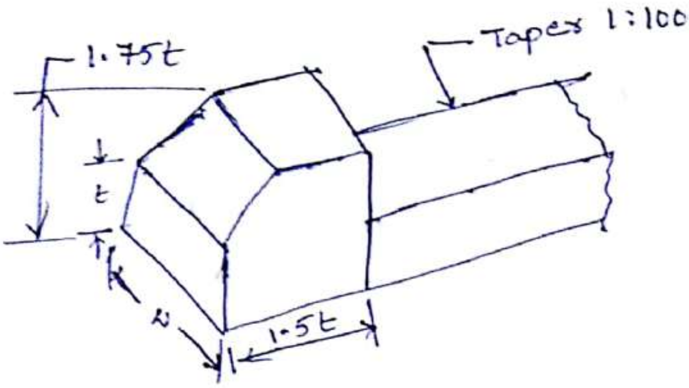


fig (a)

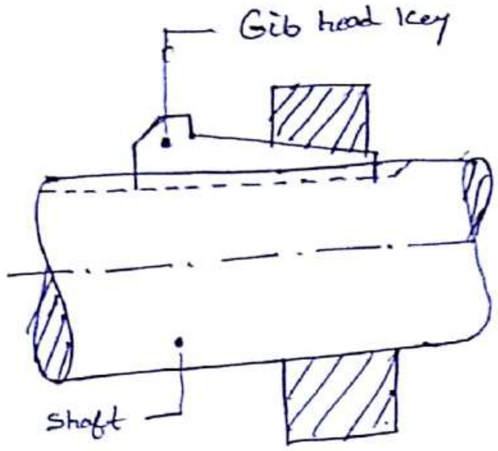


fig (b)

fig: Gib head key and its use.

The usual proportions of the gib head key are:

width, $w = \frac{d}{4}$, and thickness at large end,

$$t = \frac{2w}{3} = \frac{d}{6} \qquad \because w = \frac{d}{4}$$

(5) Feather Key (Kennedy key.): A ~~feather~~ key attached to one member of a pair and which permits relative axial movement is known as feather key. It is a special type of parallel key which transmits a ~~turning~~ moment and also permits axial movement. It is ~~fastened~~ fixed either to the shaft or hub, the key being a sliding fit in the key way of the moving piece.

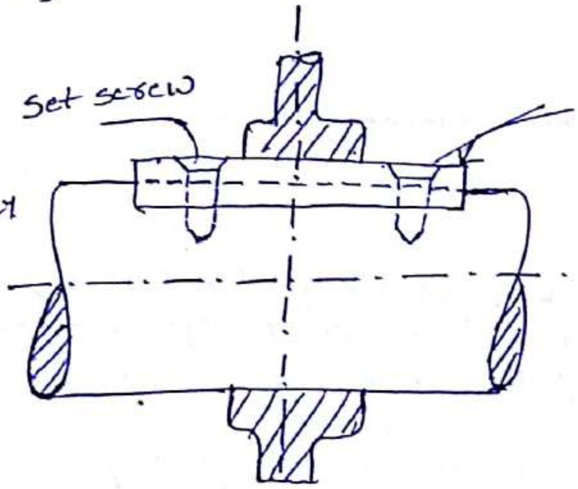


fig (a)

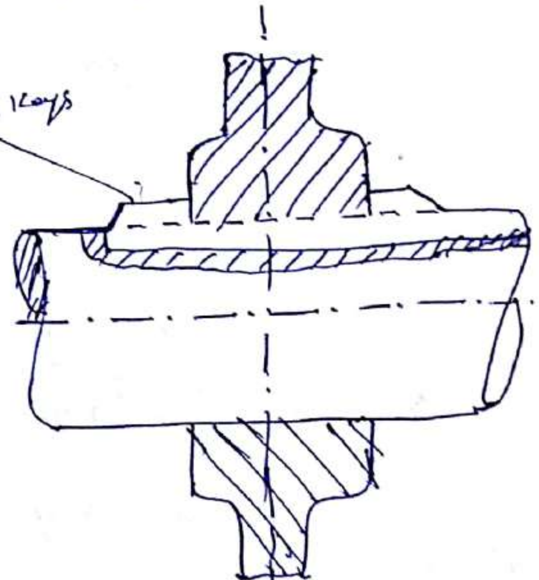


fig (b)

fig (6) Feather keys

(6) Woodruff Key: The woodruff key is an easily adjustable key. It is a piece from a cylindrical disc having segmental cross-section in front view as shown. A woodruff key is capable of tilting in a recess milled out in the shaft by a cutter having the same curvature as the disc from which the key is made. This key is largely used in machine tool and automobile construction.

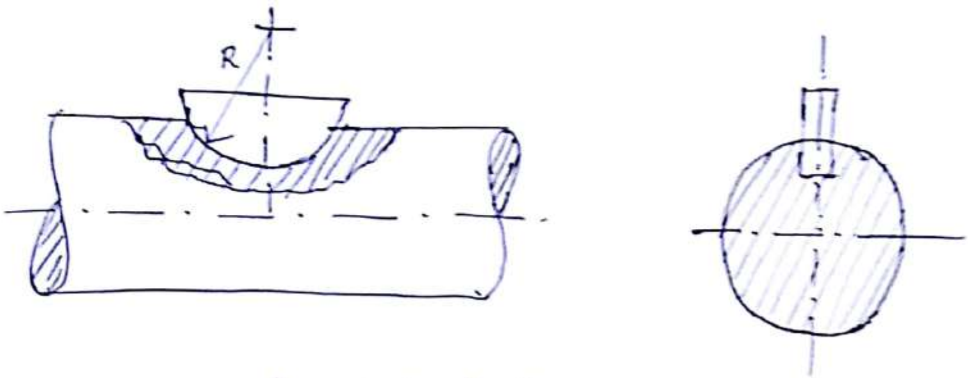


fig: woodruff key

The main advantages of a woodruff key are as follows:

1. It accommodates itself to any taper in the hub or boss of the mating piece.
2. It is useful on tapering shaft ends. Its extra depth in the shaft prevents any tendency to turn over in its keyway.

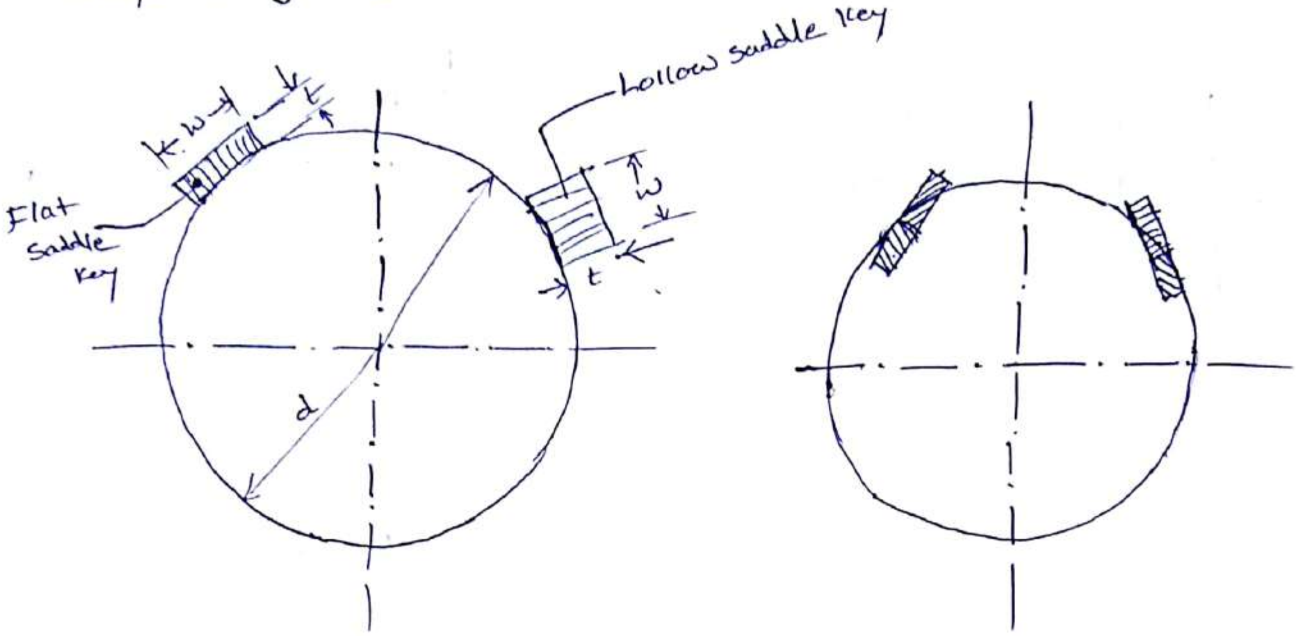
The disadvantages

1. The depth of the key way weakens the shaft.
2. It can not be used as a feather.

Saddle Keys: A saddle key is a key that fits in the keyway of the hub only. In this case there is no keyway on the shaft. The saddle keys are of the following two types:

1. Flat saddle key and 2. Hollow saddle key

A flat saddle key is a taper key which fits in a keyway in the hub and is flat on the shaft as shown. It is likely to slip round the shaft under load. Therefore it is used for comparatively light loads.



$t = \frac{w}{3} = d/12$

fig: Flat saddle key and Tangent keys.

A hollow saddle key is a taper key which fits in a keyway in the hub and the bottom of the key (concave surface at the bottom of the key) is shaped to fit the curved surface of the shaft. Since hollow saddle keys hold on by friction, therefore these are suitable for light loads. It is usually used as a temporary fastening in fixing and setting eccentrics, cams etc.

Tangent keys:- The tangent keys are fitted in pair at right angles as shown in fig. Each key is to withstand torsion in one direction only. These are used in large heavy duty shafts.

Round keys:- The round keys, as shown fig (a) are circular in section and fit into holes drilled partly in the shaft and partly in the hub. They have the advantage that their keyways may be drilled and reamed after the mating parts have been assembled. Round keys are usually considered to be most appropriate for low power drives.

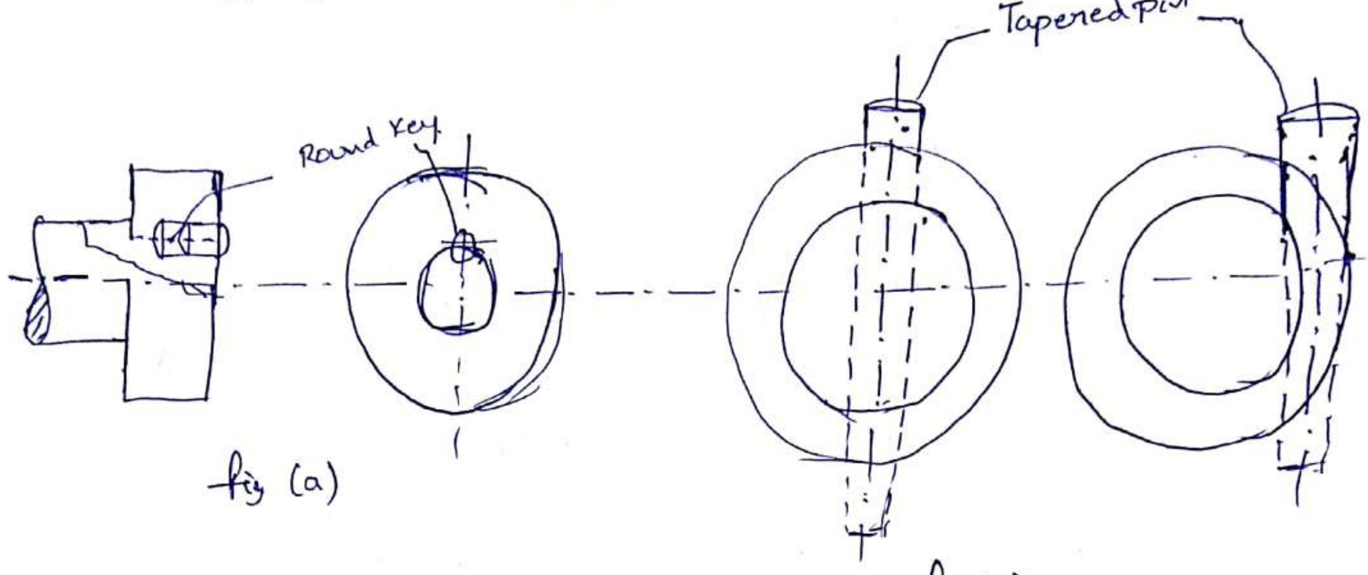
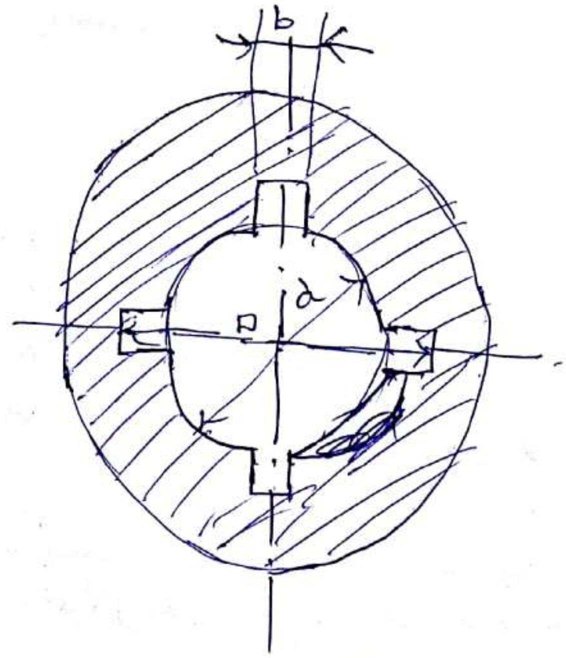


fig (a)

fig (b).

Splines :-

Sometimes, keys are made integral with the shaft which fits the keyways broached in the hub. such shafts are known as splined shafts. as shown in fig. These shafts usually have four, six, ten, or sixteen splines. The splined shafts are relatively stronger than shafts having a single keyway.



$D = 1.25\,d, \quad b = 0.25\,d$

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Stresses in Keys:

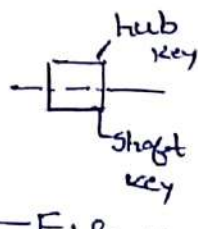
Forces acting on a sunk Key:

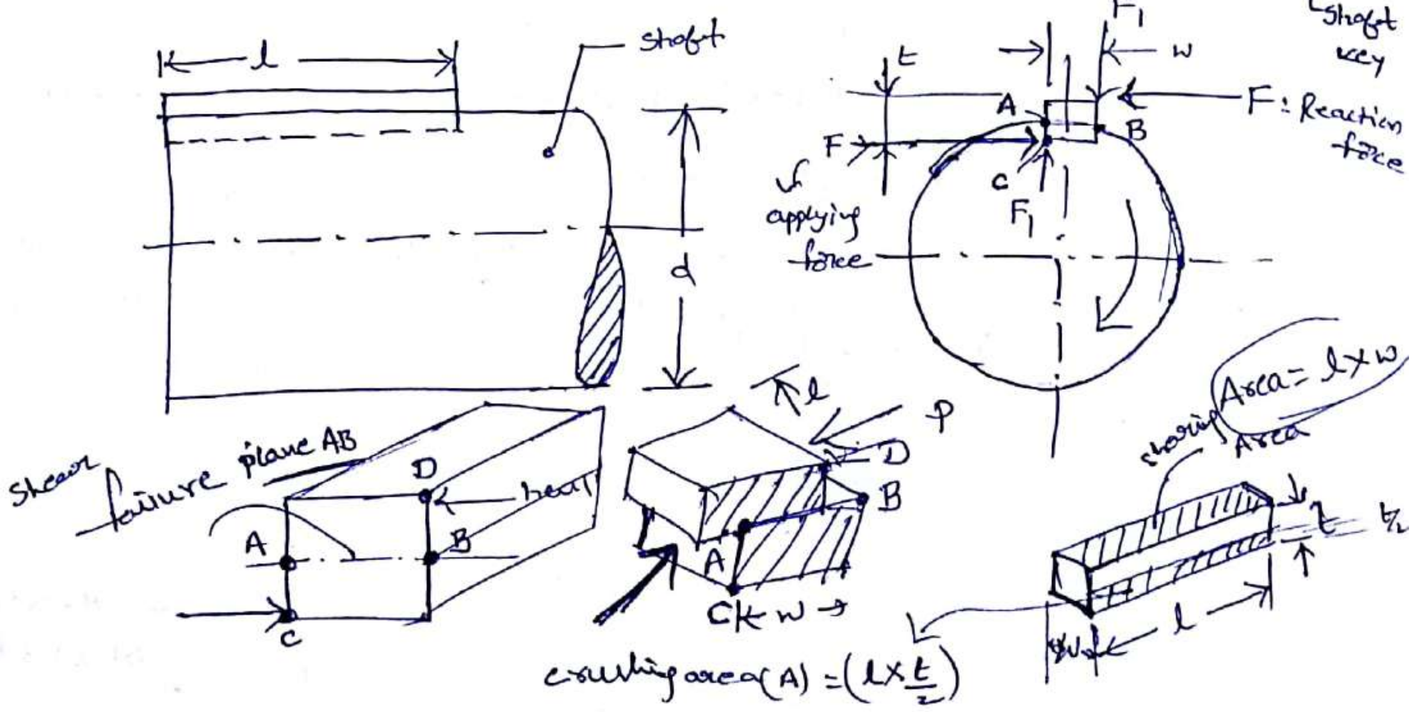
When a key is used in transmitting of torque from a shaft to a rotor or hub, the following two types of forces act on the key. There are many types of key, only square and flat keys are extensively used in practice.

1. Forces (F_1) due to fit of the key in its keyway, as in a tight fitting straight key or in a tapered key driven in place. These forces produce compressive stresses in the key which are difficult to determine in magnitude.

2. Forces (F) due to the torque transmitted by the shaft. These forces produce shearing and compressive (or crushing) stresses in the key.

The forces acting on a key for a clockwise torque being transmitted from a shaft to a hub are shown below. In Designing a key, forces due to fit of the key are neglected and it is assumed that the distribution of forces along the length of key is uniform.





Strength of a sunk key :-

A key connecting the shaft and hub is shown above

Let T = Torque transmitted by the shaft

F = Tangential force acting at the circumference of the shaft,

d = Diameter of shaft,

l = Length of key,

b = w = width of key,

h = t = Thickness (height) of key, and

τ and σ_c = Shear and crushing stresses for the material of key

A little consideration will show that due to the power transmitted by the shaft, the key may fail due to shearing ~~load~~ or crushing. Considering shearing of the key, the tangential shearing force (plane AB) acting at the circumference of the shaft,

F = Area resisting shearing × shear stress

$$F = (l \times w) \times \tau \Rightarrow \boxed{F = lw\tau} \longrightarrow (I)$$

Therefore, Torque transmitted by the shaft,

(force P is tangential to shaft diameter)

$$T = F \times \frac{d}{2} = l \times w \times \tau \times \frac{d}{2} \longrightarrow (I)$$

$$\Rightarrow \tau = \frac{2T}{dlw}$$ (τ – in plane AB)

Considering crushing of the key, the tangential crushing force acting (AC or DB plane) at the circumference of the shaft,

F = Area resisting crushing × crushing stress

$$= \left(l \times \frac{t}{2}\right) \times \sigma_c$$

[Crushing Area b/w shaft & key]

$$\sigma_c = \frac{2F}{tl}$$

Therefore, Torque transmitted by the shaft,

$$T = F \times \frac{d}{2} = \left(l \times \frac{t}{2} \times \sigma_c\right) \times \frac{d}{2} \longrightarrow \text{(II)}$$

The key is equally strong in shearing and crushing, if.

$$\cancel{l} \times w \times \tau \times \frac{\cancel{d}}{\cancel{2}} = \left(\cancel{l} \times \frac{t}{2} \times \sigma_c\right) \times \frac{\cancel{d}}{\cancel{2}}$$

or

$$\frac{w}{t} = \frac{\sigma_c}{2\tau}$$

The permissible crushing stress for the usual key material is at least twise the permissible shearing stress. Therefore from the above eqn, we have $\underline{w = t}$. In other words, a Square key is equally strong in shearing and crushing.

In order to find the length of the key to transmit full power of the shaft, the shearing strength of the key is equal to the torsional shear strength of the shaft. We know that the shearing strength of key,

$$T = l \times w \times \tau \times \frac{d}{2}$$

and torsional shear strength of the shaft,

$$T = \frac{\pi}{16} \times \tau_1 \times d^3$$

from the above

$$l \times w \times \tau \times \frac{d}{2} = \frac{\pi}{16} \times \tau_1 \times d^3$$

$$l = \frac{\pi}{8} \times \frac{\tau_1 d^2}{w \times \tau} = \frac{\pi d}{2} \times \frac{\tau_1}{t} = 1.571 d \times \frac{\tau_1}{t}$$

when the key material is same as that of the shaft, then $\tau = \tau_1$, so, $\underline{l = 1.1571 d}$

Cottered Joints:

A cotter is a flat wedge shaped piece of rectangular cross-section and its width is tapered (either on one side or both sides) from one end to another for an easy adjustment. The taper varies from 1 in 48 (1:48) to 1 on 24 (1:24) and it may be increased up to 1 in 8, (1:8) if a locking device is provided. The locking device may be a taper pin or a set screw used on the lower end of the cotter. The cotter is usually made of mild steel or wrought iron. A cotter Joint is a ~~tempo~~ rary fastening and is used to connect rigidly two co-axial or bars which are subjected to axial tensile or compressive forces. It is usually used in connecting a piston rod to the crosshead of a reciprocating steam engine, a piston rod and its extension as a tail or pump rod, strap end of connecting rod etc.

Types of cotter Joints:

Following are the three commonly used cotter Joints to connect two rods by a cotter:

1. Socket and Spigot cotter Joint
2. Sleeve and cotter Joint
3. Gib and cotter Joint

Socket and spigot cotter Joint:

In a socket and spigot cotter Joint, one end of the rods (say A) is provided with a socket type of end as shown in figure, and the other end of the other rod (say B) is inserted into a ~~socket~~ spigot. The end of the rod which goes into a socket is also called spigot. A rectangular hole is made in the socket and spigot. A cotter is then driven tightly through a hole in order to make the temporary connection b/w the two ~~ends~~. rods. The load is usually acting axially, but it changes its direction and hence the cotter Joint must be designed to carry both the tensile and compressive loads. The compressive load is taken up by the collar on the spigot.

fig: socket and spigot cotter Joint

Design of socket and spigot cotter Joint.

The socket and spigot cotter Joint is shown in figure

Let P = Load carried by the rods,

d = Diameter of the rods,

d_1 = outside diameter of socket,

d_2 = Diameter of spigot or inside diameter of socket

d_3 = outside diameter of spigot collar,

t_1 = Thickness of spigot collar,

d_4 = Diameter of socket collar,

c = Thickness of socket collar,

b = Mean width of cotter,

t = Thickness of cotter,

l = Length of cotter,

a = Distance from the end of the slot to the end of rod,

σ_t = Permissible tensile stress for the rods material,

τ = Permissible shear stress for the cotter material, and

σ_c = Permissible crushing stress for the cotter material

The dimensions for a socket and spigot cotter Joint may be obtained by considering various modes of failure as discussed below:

1. Failure of the rods in tension;

The rods may fail in tension due to the tensile load P,

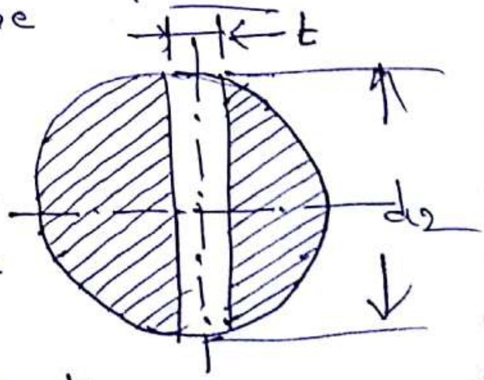
$$P = \frac{\pi}{4} \times d^2 \times \sigma_t$$

From this equation, diameter of the rods (d) may be determined.

2. Failure of spigot in tension across the weakest section

Since the weakest section of the spigot is that section (or slot) which has a slot in it for the cotter, therefore

$$P = \left[\frac{\pi}{4}(d_2)^2 - d_2 \times t\right] \sigma_t$$



from this equation, the diameter of spigot or inside diameter of socket (d_2) may be determined. In actual practice, the thickness of cotter is usually taken as $\frac{d_2}{4}$.

3. Failure of the rod or cotter in crushing

$$P = d_2 \times t \times \sigma_c$$

$d_2 \times t$ = area of resisting crushing.

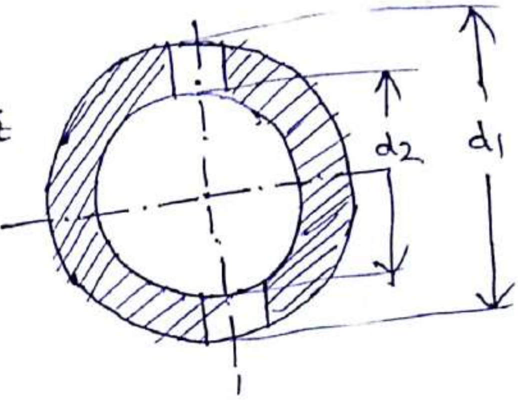
from this equation, the induced crushing stress may be checked.

4. Failure of the socket in tension across the slot:

we know that the resisting area of the socket across the slot

$P = A\sigma_t$

$$P = \left\{ \frac{\pi}{4}\left[(d_1)^2 - (d_2)^2\right] - (d_1 - d_2)t \right\} \sigma_t$$

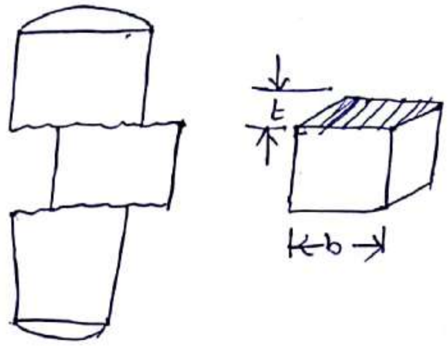


from this equation, outside diameter of socket (d_1) may be determined.

5. Failure of cotter in shear:

considering the failure of cotter in shear, since the cotter is in double shear, shearing area = $2b \times t$

$$\boxed{P = 2b \times t \times \tau}$$

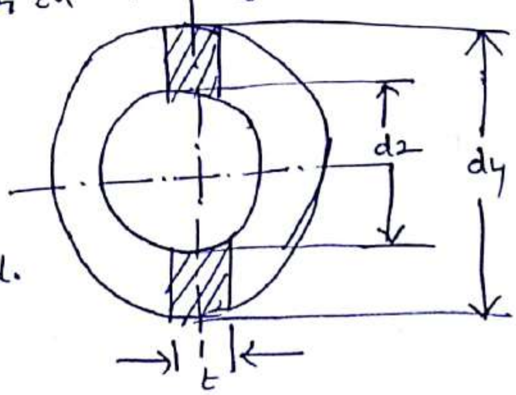


from this equation, width of cotter (b) is determined.

(6) Failure of the socket collar in crushing

considering the failure of socket collar in crushing as shown

$$P = (d_4 - d_2)\, t \times \sigma_c$$



from this equation, the diameter of socket collar (d_4) may be obtained.

(7) Failure of socket end in shearing

$P = 2(d_4 - d_2)\, c \times \tau$, from this equation, the thickness of socket collar (c) may be obtained.

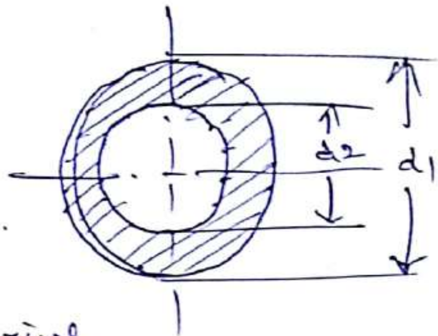
(8.) Failure of rod end in shear:

$$P = 2a \times d_2 \times \tau$$

from this equation, the distance from the end of the slot to the end of the rod (a) may be obtained.

(9.) Failure of spigot collar in crushing

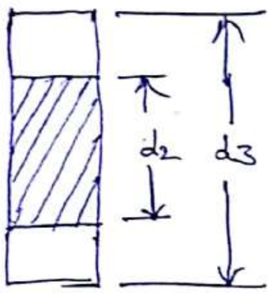
$$P = \frac{\pi}{4}\left[(d_3)^2 - (d_2)^2\right]\sigma_c$$



from this equation, the diameter of the spigot collar (d_3) may be obtained.

(10.) Failure of the spigot collar in shearing

$$P = \pi d_2 \times t_1 \times \tau$$



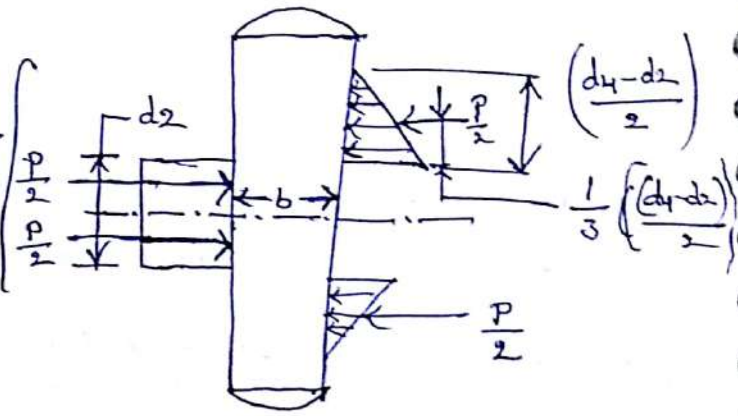
from this equation, the thickness of spigot collar (t_1) may be obtained.

(11.) Failure of cotter in bending.

The maximum bending moment occurs at the centre of the cotter and is given by

$$M_{max} = \frac{P}{2}\left(\frac{1}{3} \times \frac{(d_4-d_2)}{2} + \frac{d_2}{2}\right) - \frac{P}{2} \times \frac{d_2}{4}$$

$$= \frac{P}{2}\left(\frac{d_4-d_2}{6} + \frac{d_2}{2} - \frac{d_2}{4}\right)$$



$$M_{max} = \frac{P}{2}\left(\frac{d_4 - d_2}{6} + \frac{d_2}{4}\right)$$

We know that section modulus of the cotter,

$$Z = \frac{t \times b^2}{6}$$

Bending stress induced in the cotter,

$$\sigma_b = \frac{M_{max}}{Z} = \frac{\frac{P}{2}\left(\frac{d_4 - d_2}{6} + \frac{d_2}{4}\right)}{\left(\frac{t \times b^2}{6}\right)}$$

$$\boxed{\sigma_b = \frac{P(d_4 + 0.5 d_2)}{2t \times b^2}}$$

The bending stress induced in the cotter should be less than the allowable bending stress of the cotter.

(12) The length of cotter (l) is taken as 4d.

(13) The taper in cotter should not exceed 1 in 24 (1:24). In case the greater taper is required, then a locking device must be provided.

(14) The draw of cotter is generally taken as 2 to 3 mm.

Notes: 1. When all the parts of the joint are made of steel, the following proportions in terms of diameter of the rod (d) are generally adopted.

$d_1 = 1.75d$, $d_2 = 1.21d$, $d_3 = 1.5d$, $d_4 = 2.4d$, $a = c = 0.75d$,

$b = 1.3d$, $l = 4d$, $t = 0.31d$, $t_1 = 0.45d$, $e = 1.2d$.

(2) If the rod and cotter are made of steel or wrought iron, then $\tau = 0.8\,\sigma_t$ and $\sigma_c = 2\sigma_t$ may be taken.

(1) Design and draw a cotter joint to support a load varying from 30 KN in compression to 30 KN in tension. The material used is carbon steel for which the following allowable stresses may be used. The load is applied statically. Tensile stress = Compressive stress = 50 MPa, Shear stress = 35 MPa, and crushing stress = 90 MPa.

Sol. Given $P = 30\,KN = 30\times10^3\,N$, $\sigma_t = \sigma_c = 50\,N/mm^2$, $\tau = 35\,MPa = 35\,N/mm^2$

$\sigma_c = 90\,MPa = 90\,N/mm^2$.

1. Diameter of the rods:

Let d = Diameter of the rods.

Considering the failure of the rod in tension, we know that load (P),

$$P = \frac{\pi}{4} d^2 \sigma_t$$

$$30\times10^3 = \frac{\pi}{4} \times d^2 \times 50$$

$$\therefore \quad d = 27.6 \text{ say } 28\text{ mm}$$

(2) Diameter of spigot and thickness of cotter:

Let d_2 = Dia. of spigot or inside dia. of socket, and

t = Thickness of cotter. It may be taken as $\frac{d_2}{4}$.

Considering the failure of spigot in tension the weakest section. We know that the load (P),

$$P = \left[\frac{\pi}{4}(d_2)^2 - d_2\times t\right]\sigma_t$$

$$30\times10^3 = \left[\frac{\pi}{4}(d_2)^2 - d_2 \times \frac{d_2}{4}\right]50 \qquad \because \boxed{t = \frac{d_2}{4}}$$

$$(d_2)^2 = 1119.4$$

$$d_2 = 33.4 \text{ say } 34 \text{ mm}$$

and thickness of cotter, $t = \frac{d_2}{4} = \frac{34}{4} = 8.5$ mm.

Let us now check the induced crushing stress. we know that load (P), $P = d_2 \times t \times \sigma_c \longrightarrow (I)$

$$30\times10^3 = d_2\times t\times\sigma_c = 34\times8.5\times\sigma_c$$

$$\boxed{\sigma_c = 103.8 \text{ N/mm}^2} \quad (\because \text{crushing } \sigma_c)$$

Since this value of σ_c is more than the given value of $\sigma_c = 90$ N/mm^2 therefore the dimensions $d_2 = 34$ mm, & $t = 8.5$ mm. are not safe. Now let us find the values of d_2 and t by substituting the value of $\sigma_c = 90$ N/mm^2 in the above expression, i.e Eqn (I)

$$30\times10^3 = d_2 \times \frac{d_2}{4} \times 90$$

$$d_2 = 36.5 \text{ say } 40 \text{ mm}$$

$$t = \frac{d_2}{4} = \frac{40}{4} = 10 \text{ mm.}$$

(3.) outside diameter of socket:

Let d_1 = outside diameter of socket

considering the failure of the socket in tension across the slot. we know that load (P), $P = \left[\frac{\pi}{4}\{(d_1)^2 - (d_2)^2\} - (d_1 - d_2)t\right]\sigma_t$

$$30 \times 10^3 = \left[\frac{\pi}{4} \left\{ (d_1)^2 - (40)^2 \right\} - (d_1 - 40)\, 10 \right] 50$$

$$\frac{30 \times 10^3}{50} = 0.7854\, (d_1)^2 - 1256.6 - 10 d_1 + 400$$

(or) $$(d_1)^2 - 12.7\, d_1 - 1854.6 = 0$$

$d_1 = 49.9$ say $\underline{50\ mm}$ ($\because$ Taking +ve sign)

4. Width of cotter

Let b = width of cotter

Considering the failure of the cotter in shear. since the cotter is in double shear, therefore load (P),

$$P = 2 b t \tau = 2b \times 10 \times 35$$

$$b = \frac{30 \times 10^3}{700}$$

$\therefore$ $\boxed{b = 43\ mm}$

(5) Diameter of socket collar!

Let d_4 = Diameter of socket collar

Considering the failure of the socket collar and cotter in crushing. we know that load (P),

$$P = (d_4 - d_2)\, t \times \sigma_c = (d_4 - 40)\, 10 \times 90$$

$$P = (d_4 - 40)\, 900$$

$d = 73.3$ say $\underline{75\ mm}$

(6) Thickness of Socket collar:

Let c = Thickness of socket collar

Considering the failure of the socket end in shearing. Since the socket end is in double shear, therefore load (P),

$$P = 2(d_4 - d_2)\, c \times \tau$$

$$= 2(75-40)\, c \times 35$$

$$30 \times 10^3 = 2450\, c$$

$$\therefore \boxed{c = 12\ mm}$$

(7) Distance from the end of the slot to the end of the rod

Let a = Distance from the end of slot to the end of the rod.

Considering the failure of the rod end in shear. Since the rod end is in double shear, therefore load (P),

$$P = 2 a d_2 \tau = 2a \times 40 \times 35 = 2800\, a$$

$$a = 30 \times 10^3 / 2800$$

$$a = 10.7 \text{ say } 11\ mm$$

$$\therefore \boxed{a = 11\ mm}$$

(8) Diameter of spigot collar:

Let d_3 = Diameter of spigot collar.

Considering the failure of spigot collar in crushing. We know that load (P), $P = \frac{\pi}{4}\left[(d_3)^2 - (d_2)^2\right]\sigma_c$

$$30\times10^3 = \frac{\pi}{4}\left[(d_3)^2-(40)^2\right]90$$

$$\boxed{d_3 = 45\text{ mm}}$$

(9) Thickness of spigot collar

Let t_1 = Thickness of spigot collar

considering the failure of spigot collar in shearing. we know that load (P),

$$P = \pi d_2 \times t_1 \times \tau$$

$$30\times10^3 = 4400\,t_1$$

$$t_1 = 6.8 \text{ say } 8\text{ mm}$$

$$\therefore \boxed{t_1 = 8\text{mm}}$$

(10) The length of cotter (l) is taken as 4d.

$$l = 4d = 4\times28 = 112\text{ mm}.$$

(11) The dimension e is taken as 1.2d.

$$e = 1.2\times28$$

$$e = 33.6$$

say

$$\boxed{e = 34\text{ mm}}$$

Gib and cotter Joint:

The Joint is generally used to connect two rods of square or rectangular section. To make the Joint, one end of the rod is formed into a U-fork, into which, the end of the other rod fits-in. when a cotter is driven-in, the friction between the cotter and straps of the U-fork, causes the straps open. This is prevented by the use of a gib.

A gib is also a wedge shaped piece of rectangular cross-section with two rectangular projections, called lugs. one side of the gib is tapered and the other straight. The tapered side of the gib bears against the tapered side of the cotter such that the outer edges of the cotter and gib as a Unit are parallel. This facilitates making of slots with Parallel edges, Unlike the tapered edges in case of ordinary Cotter Joint. The gib also provides larger surface for the cotter to slide on. For making the Joint, the gib is placed in position first, and then the cotter is driven.

Let F be the maximum tensile or compressive force in the connecting rod, and

b = width of the strap, which may be taken as equal to the diameter of the rod (d).

h = height of the rod end

t_1 = thickness of the strap at the thinnest part

t_2 = thickness of the strap at the curved portion

t_3 = thickness of the strap across the slot

l_1 = Length of the rod end, beyond the slot

l_2 = Length of the strap, beyond the slot

B = Width of the cotter and Gib

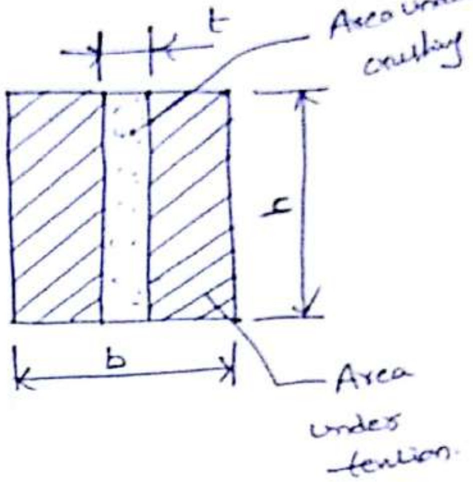
t = Thickness of the cotter.

Let the rod, strap, cotter and gib are made of the same material σ_c', σ_t' and τ: as the permissible stresses. The following are the possible modes of failure, and the corresponding design equations, which may be considered for the design of the Joint:

(1) Tension failure of the rod across the section of diameter (d)

$$F = \frac{\pi d^2}{4} \times \sigma_t$$

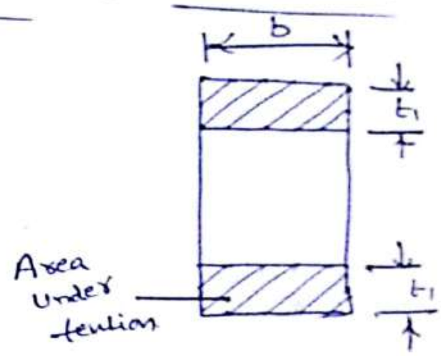
(2) Tension failure of the rod across the slot



$$F = (bh - ht)\,\sigma_t$$

If the rod and strap are made of the ~~small~~ same material, and for equality of strength, $\boxed{h = 2t_3}$

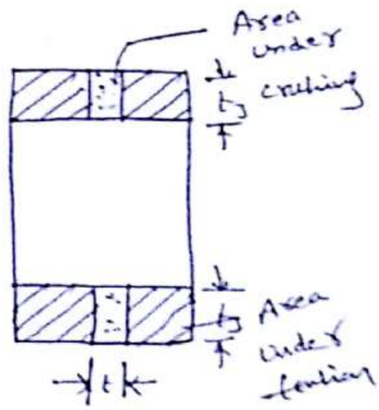
(3) Tension failure of the strap, across the thinnest part



$$F = A \cdot \sigma_t$$

Tearing strength of strap $\boxed{F = (2b\,t_1)\,\sigma_t}$

(4) Tension failure of the strap across the slot



$$F = (2bt_3 - 2tt_3)\,\sigma_t$$

$$F = 2t_3\,(b - t)\,\sigma_t$$

The thickness, t_2 may be taken as $(1.15 \text{ to } 1.5)\,t$, and

Thickness of the cotter $t = b/4$.

(5) crushing between the rod and cotter

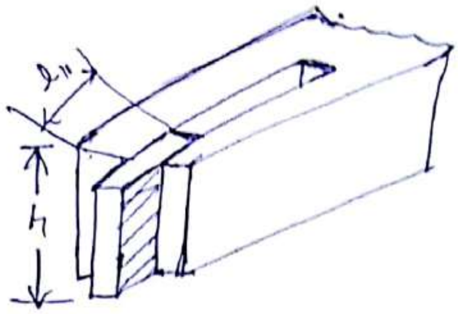
$$F = h \cdot t \cdot \sigma_c \quad \text{and} \quad h = 2t_3 .$$

(6) crushing between the strap and gib

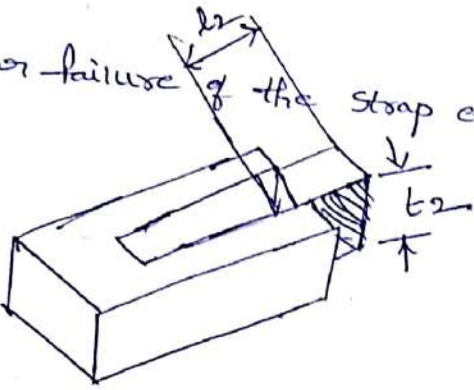
$$F = 2t \cdot t_3 \cdot \sigma_c$$

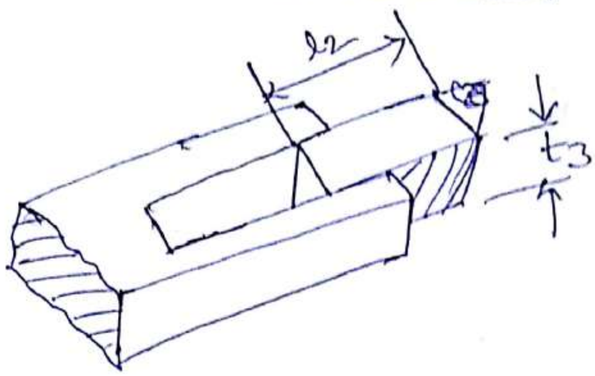
(7) shear failure of the rod end. It is under double shear

$$F = 2 l_1 h \cdot \tau$$



(8) Shear failure of the strap end. It is under double shear.





$$F = 4 \cdot l_2 \cdot t_3 \cdot \tau$$

(9) shear failure of the cotter and gib. It is under double shear.

$$\boxed{F = 2 \cdot B \cdot t \cdot \tau}$$

The following proportions for the widths of the cotter and gib may be followed.

width of the cotter = 0.45B

width of the gib = 0.55B

The above equations may be solved, keeping in mind about the various relations and proportions suggested.

(1) Design a cotter Joint to connect piston rod to the crosshead of a double acting steam engine. The diameter of the cylinder is 300 mm and the steam pressure is 1 N/mm². The allowable stresses for the material of cotter and piston rod are as follows: $\sigma_t = 50$ MPa, $\tau = 40$ MPa & $\sigma_c = 84$ MPa.

sol Given that: $D = 300$ mm, $p = 1$ N/mm², $\sigma_t = 50$ N/mm²

$\sigma_c = 84$ N/mm², $\tau = 40$ N/mm².

we know that maximum load on the piston rod,

$$P = \frac{\pi}{4} \times D^2 \times p = \frac{\pi}{4}(300)^2 \times 1 = 70695 \text{ N}$$

The various dimensions for the cotter Joint are obtained by considering the different modes of failure as discussed below:

1. Diameter of piston rod at cotter:

Let d_2 = Diameter of piston rod at cotter, and

t = Thickness of cotter. It may be taken as $0.3 d_2$.

considering the failure of piston rod in tension at cotter.

we know that load (P),

$$P = \left[\frac{\pi}{4}(d_2)^2 - d_2 \times t\right] \sigma_t = \left[\frac{\pi}{4}(d_2)^2 - 0.3(d_2)^2\right] 56$$

$$70695 = 24.27 (d_2)^2$$

$$d_2 = 53.97 \text{ say } 55 \text{ mm.}$$

and. $t = 0.3\, d_2 = 0.3 \times 55 = 16.5$ mm

2. Width of cotter

Let b = width of cotter

Considering the failure of cotter in shear. since the cotter is in double shear, therefore load (P),

$$P = 2b \times t \times \tau$$

$$70695 = 2b \times 16.5 \times 40$$

$$b = 53.5 \text{ say } 54 \text{ mm}$$

(3) Diameter of socket :-

Let d_3 = Diameter of socket

considering the failure of socket in tension at cotter.

we know that load (P),

$$P = \left\{ \frac{\pi}{4}\left[(d_3)^2 - (d_2)^2\right] - (d_3 - d_2)\, t \right\} \sigma_t$$

$$70695 = \left\{ \frac{\pi}{4}\left[(d_3)^2 - (55)^2\right] - (d_3 - 55)\, 16.5 \right\} 56$$

$$\boxed{d_3 = 72 \text{ mm}}$$ - (Taking +ve sign)

Let us now check the induced crushing stress in the socket

we know that load (p), $P = (d_3 - d_2)\, t \times \sigma_c \rightarrow$ (I)

$$70695 = (d_3 - d_2)\, t \times \sigma_c$$

$$\therefore \boxed{\sigma_c = 252\ N/mm^2}$$

Since the induced crushing stress is more than the Permissible value of $\underline{84\ N/mm^2}$, therefore let us find the value of $\underline{d_3}$ by substituting $\sigma_c = 84\ N/mm^2$ eqn (I)

$$70695 = (d_3 - 55)\, 16.5 \times 84$$

$$d_3 = 55 + 51 = \underline{106\ mm}$$

we know the tapered length of the piston rod,

$$L = 2.2\, d_2 = 2.2 \times 55 = \underline{121\ mm}$$

Assuming the taper of the piston rod as 1 in 20, therefore ~~Assuming~~ the diameter of the parallel part of the piston rod,

$$d = d_2 + \frac{L}{2} \times \frac{1}{20} = 55 + \frac{121}{2} \times \frac{1}{20}$$

$$\boxed{d = 58\ mm}$$

and diameter of the parallel part of the piston rod,

$$d_1 = d_2 - \frac{L}{2} \times \frac{1}{20} = 55 - \frac{121}{2} \times \frac{1}{20}$$

$$\boxed{d_1 = 52\ mm}$$

Design of Knuckle Joint:

A knuckle Joint is used to connect two rods which are under the action of tensile loads. However, if the Joint is guided the rods may support a compressive load. A knuckle Joint may be readily disconnected for adjustments or repairs. Its use may be found in the link of a cycle chain, tie rod joint for roof truss, valve rod Joint with eccentric rod, pump rod Joint, tension link in bridge structure and lever and rod connections of various types.

In knuckle Joint, one end of one of the rods is made into an eye and the end of the other rod is formed into a fork with an eye in each of the fork leg. The knuckle pin passes through both the eye hole and the fork holes ~~or~~ and may be secured by means of a collar and taper pin or split pin. The knuckle pin may be prevented from rotating in the fork by means of a small stop, pin, peg or snug. In order to get a better quality of Joint, the sides of the fork and eye are machined, the hole is accurately drilled and pin turned. The material used for the Joint may be steel or wrought iron.

Dimensions of various parts of the knuckle Joint:

The dimensions of various parts of the knuckle Joint are fixed by empirical relations as given below. It may be noted that all the parts should be made of the material. i.e mild steel or wrought iron.

if d is the diameter of rod, then diameter of pin.

$$\boxed{d_1 = d}$$

outer diameter of eye, $\boxed{d_2 = 2d}$

Diameter of knuckle pin head and collar, $\boxed{d_3 = 1.5d}$

Thickness of single eye or rod end, $t = 1.25\,d$.

Thickness of fork, $t_1 = 0.75d$

Thickness of pin head, $t_2 = 0.5d$

Methods of failure of knuckle Joint:

Consider a knuckle Joint as shown

Let P = Tensile load acting on the rod,

d = Diameter of the rod,

d_1 = Diameter of the pin,

d_2 = outer diameter of eye,

t = thickness of single eye,

t_1 = Thickness of fork.

σ_t, τ and σ_c = Permissible stresses for the Joint material in tension, shear and crushing respectively.

In determining the strength of the Joint for the various methods of failure, it is assumed that

1. There is no stress concentration, and
2. The load is uniformly distributed over each part of the Joint.

Due to these assumptions, the strengths are approximate, however they serve to indicate a well proportioned Joint. Following are the various methods of failure of the Joint:

1. Failure of the solid rod in tension:

Since the rods are subjected to direct tensile load, therefore tensile strength of the rod,

$$= \frac{\pi}{4} \times d^2 \times \sigma_t$$

Equating this to the load (P) acting on the rod, we have

$$\boxed{P = \frac{\pi}{4} \times d^2 \times \sigma_t}$$

from this equation, diameter of the rod (d) is obtained.

(2) Failure of the Knuckle pin in shear

since the pin is in double shear, therefore cross-sectional area of the pin under shearing

$$= 2 \times \frac{\pi}{4} (d_1)^2$$

and the shear strength of the pin

$$= 2 \times \frac{\pi}{4} (d_1)^2 \times \tau$$

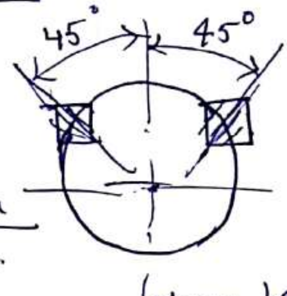
Equating this to the load (P) acting on the rod, we have

$$\boxed{P = 2 \times \frac{\pi}{4} (d_1)^2 \times \tau}$$

from this eqn, diameter of the knuckle pin (d_1) is obtained.

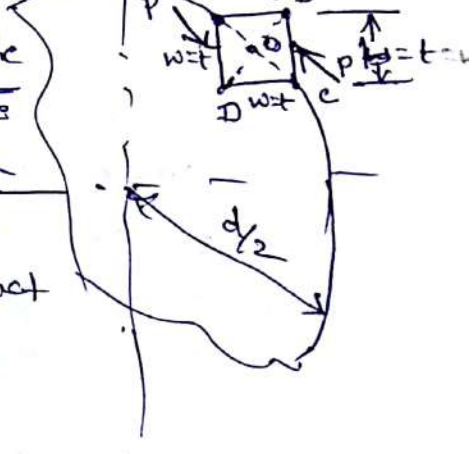
Design of Kennedy Key (Feather key)

The Kennedy key consists of two square keys, as shown. In this case, the hub is bored off the center & the two keys force the hub & the shaft to a concentric position. Kennedy key is used for heavy duty applications.



The analysis of the Kennedy key is similar to that of the flat key. It is based on two criteria, viz. failure due to shear stress and failure due to compressive stresses..

The forces acting on one of the two Kennedy keys are shown below



Since there are two keys, the torque transmitted by each key is one half of the total torque. The two equal & opposite forces P are due to the transmitted torque. The exact location of force P is unknown. It is assumed that to act tangential to the shaft diameter

$$T = F \times \frac{d}{2}$$

$$\boxed{F = \frac{2T}{d}}$$

$$\frac{T}{2} = F\frac{d}{2}$$

$$\boxed{F = \frac{T}{d}}$$

Total torque = T

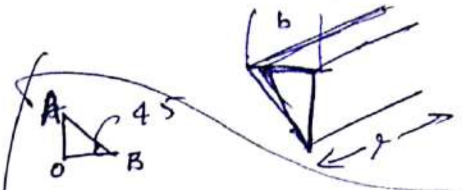
one half $= \frac{T}{2}$

The failure due to shear stresses will occur in plane AC. The area of plane AC is $[\overline{AC} \times l)$ or $[\sqrt{2}bl]$.

The shear stress is given by,

$$\tau = \frac{P \text{ (or) } F}{\sqrt{2}bl}$$

$$\tau = \frac{T}{\sqrt{2}\,dbl} \text{ (or) } \frac{T}{\sqrt{2}\,dw.l}$$



$\tan 45 = \frac{OA}{OB}$ $\tan 45 = \frac{1}{\sqrt{2}}$

$\frac{b}{l} = \frac{1}{\sqrt{2}}$

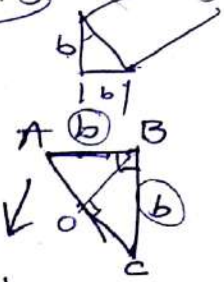
$AC = 0.707\,b$

The compressive stress is given by,

$$\sigma_c = \frac{P \text{ (or) } F}{\text{Projected Area.}} = \frac{P \text{ (or) } F}{[\overline{OB} \times l]}$$

$$= \frac{P \text{ (or) } F}{\left(\frac{b}{\sqrt{2}}\right) l}$$

$$\boxed{\sigma_c = \frac{\sqrt{2}\,T}{dbl}} \text{ (or) } \frac{\sqrt{2}\,T}{dw.l}$$



$\sin 45 = \frac{AB}{AC}$

$\sin 45 = b$

$\frac{\sqrt{2}}{2} = \frac{b}{AC}$

$AC = \frac{2}{\sqrt{2}} b$

$\boxed{AC = \sqrt{2}\,b.l}$ or $\frac{1}{\sqrt{2}}$

$OB = OA \tan 45$

where, l is Length of the key.

$\overline{OB}$

$\sin 45 = \frac{OB}{BC}$

$l \times \frac{\sqrt{2}}{2}(b) = OB$

$\frac{\sqrt{2}}{2} BC = OB$

$\frac{b}{\sqrt{2}} \times l =$

(1) A shaft, 40 mm in diameter, is transmitting 35 kW power at 300 rpm by means of Kennedy Keys of 10 X 10 mm cross-section. The keys are made of steel 45C8 ($S_{yt} = S_{yc} = 380\ N/mm^2$) and the factor of safety is 3. Determine the required length of the keys.

Sol for the key material.

$$\sigma_c = \frac{S_{yc}}{f_s} = \frac{380}{3} = 126.67\ N/mm^2.$$

According to distortion energy theory of failure,

$$S_{sy} = 0.577\ S_{yt} = 0.577(380) = 219.26\ N/mm^2$$

$$\tau = \frac{S_{sy}}{f.s} = \frac{219.26}{3} = 73.09\ N/mm^2.$$

The torque transmitted by the shaft is given by,

$$T = \frac{60\times10^6 (kW)}{2\pi n} = \frac{60\times10^6 \times 35}{2\pi n}$$

$$P = \frac{2\pi NT}{60\times10^6} kW.$$

$$= 1114084.6\ N\text{-}mm$$

Shear stress

$$l = \frac{T}{\sqrt{2}\ d.w\ \tau} = \frac{1114084.6}{\sqrt{2}\times40\times10\times73.09} = 26.95\ mm$$

Compressive

$$l = \frac{\sqrt{2}\ T}{d w.d \sigma_c} = \frac{\sqrt{2}\times1114084.6}{40\times10\times126.67} = 31.10\ mm.$$

~~The dimensions of a woodruff key for~~ a

) It is required to design a square key for fixing a gear on a shaft of 25 mm diameter. The shaft is transmitting 15 kW power at 720 rpm to the gear. The key is made of steel 50C4 (S_{yt} = 460 N/mm²) and f.s is 3. for key material, the yield strength in compression can be assumed to be equal to the yield strength in tension. Determine the dimensions of the key.

key material. $S_{yc} = S_{yt} = 460\ N/mm^2$)

$$\sigma_c = \frac{S_{yc}}{fs} = \frac{460}{3} = 153.33\ N/mm^2$$

According maximum shear stress theory of failure.

$$S_{sy} = 0.5\ S_{yt} = 0.5(460) = 230\ N/mm^2$$

$$\tau = \frac{S_{sy}}{fs} = \frac{230}{3} = 76.67\ N/mm^2.$$

$$T = \frac{60\times10^6\times P}{2\pi n}(kW) = \frac{60\times10^6\times15}{2\pi\, 720} = 198943.6\ N\text{-}mm$$

The industrial practice is to use a square key with sides equal to one-quarter of the shaft diameter.

$$\therefore \; w = b = \frac{d}{4} = \frac{25}{4} = \underline{6.25} \text{ (or) } 6\,mm$$

$$l = \frac{2T}{\tau \cdot d \cdot w} = \frac{2 \times 198943.68}{(76.67)(25)(6)} = 34.\underline{60\,mm}$$

$$l = \frac{4T}{\sigma_c \cdot d \cdot t} = \frac{4 \times 198943.68}{153.33 \times 25 \times 6} = \underline{34.60\,mm}$$

∴ the length of the key should be 35 mm. The dimensions of the key are $6 \times 6 \times \underline{35}$ mm
$(w \times t \times l)$

(3) The standard cross-section for a flat key, which is fitted on a 50 mm diameter shaft, is 16×10 mm. The key is transmitting 475 N-m torque from the shaft to the hub. The key is made of commercial steel ($S_{yt} = S_{yc} = 230\ N/mm^2$). Determine the length of the key, if the factor of safety 3.

Sol For the key material,

$$\sigma_c = \frac{S_{yc}}{f_s} = \frac{230}{3} = 76.67\ N/mm^2.$$

According to maximum shear stress theory of failure

$$S_{sy} = 0.5\,S_{yt} = 0.5(230) = 115\ N/mm^2.$$

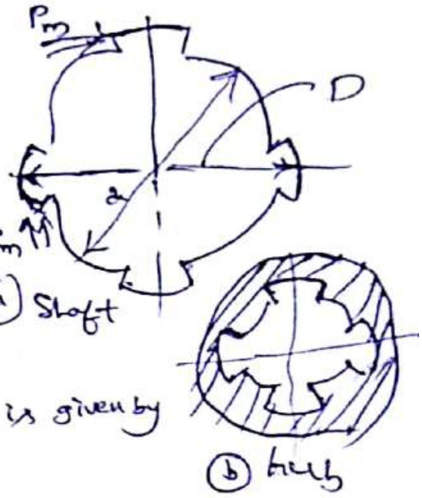
$$\tau = \frac{S_{sy}}{f_s} = \frac{115}{3} = 38.33\ N/mm^2.$$

$$l = \frac{2T}{\tau \cdot w \cdot d} = \frac{2 \times 475 \times 10^3}{38.33 \times 50 \times 16} = 30.98\ mm.$$

$$l = \frac{4T}{\sigma_c \cdot d \cdot t} = \frac{2 \times 475 \times 10^3}{76.67 \times 50 \times 10} = \underline{49.56\ mm}$$

The length of the key should be $\underline{50\ mm}$

splines :- splines are keys that are made integral with the shaft. They are used when there is a relative axial motion b/w the shaft & hub. The gear shifting mechanism in automobile gearboxes requires such type of construction. Splines are cut on the shaft by milling and on the hub by broaching. A splined connection with straight splines, is shown. following notations are used.



(a) Shaft   (b) hub

D = major diameter of splines (mm)

d = minor diameter of splines (mm)

l = Length of hub (mm)

n = no of splines

The torque transmitting capacity of splines is given by

$$T = P_m \, A \, R_m$$

where

T = transmitted torque (N-mm)

P_m = permissible pressure on spline

A = Total area of splines

R_m = mean Radius of splines

$$A = \frac{1}{2}(D-d)\, l \cdot n \quad , \quad R_m = \frac{D+d}{4}$$

$$\boxed{T = \frac{1}{8} P_m \cdot l \cdot n \cdot (D^2 - d^2)} \quad \text{(straight splines)}$$

The permissible pressure on the splines is limited to $6.5 \, N/mm^2$ [(other type is involute, serrations) splines]

A standard splined connection $8 \times 52 \times 60$ mm is used for the gear and the shaft assembly of a gearbox. The splines transmit 20 kW power at 300 rpm. The dimensions of the splines are as follows!

Major dia 60 mm, minor dia = 52 mm, number of splines = 8, permissible normal pressure on splines is 6.5 N/mm². The coefficient of friction is 0.06. calculate:

(I) The length of hub of the gear

(II) The force required for shifting the gear.

Sol. The torque transmitted by the shaft is given by,

$$T = \frac{60 \times 10^6 \times (kW)}{2\pi n} = \frac{60 \times 10^6 \times 20}{2\pi \times 300} = 636619.76 \text{ N-mm.}$$

$$l = \frac{8T}{P_m \cdot n (D^2 - d^2)} = \frac{8 \times 636619.76}{6.5 \times 8 (60^2 - 52^2)} = 109.31$$

$$\simeq 110 \text{ mm}$$

Due to torque T, a normal force (F) acts on the splines. It is assumed that the force F acts at the mean radius of the splines.

$$\therefore \quad T = P \cdot R_m$$

$$R_m = \frac{D+d}{4} = \frac{60+52}{4} = 28 \text{ mm,}$$

$$P = F = \frac{T}{R_m} = \frac{636619.76}{28} = 22736.42 \text{ N}$$

$$\text{Friction force} = \mu \cdot P = 0.06 (22736.42)$$

$$= 1364.19 \text{ N}$$

The force required to shift the gear is equal and opposite of the friction force.

$\therefore$ force required to shift gear is 1364.19 N

① Design a knuckle Joint for transmitting an axial load of 60 KN, for the following stresses. in tension 60 MPa, in compression 75 MPa, in shear 40 MPa., sketch the Joint.

Sol Given that: $P = 60\ KN = 60,000\ N$

$\sigma_t = 60\ MPa = 60\ N/mm^2$, $\sigma_c = 75\ MPa = 75\ N/mm^2$

$\tau_s = 40\ MPa = 40\ N/mm^2$

(I) Diameter of rod (d) :- compare the tensile strength of rod to the applied load.

$$\frac{\pi}{4} d^2 \cdot \sigma_t \geq P$$

$$d \geq \left[\frac{4P}{\pi \sigma_t}\right]^{1/2} = \left[\frac{4 \times 60,000}{\pi \times 60}\right]^{1/2}$$

$$d \geq 35.7\ mm$$

Take $\boxed{d \simeq 40\ mm}$

By empirical relations, the dimensions of double eye, single eye & knuckle pin are fixed as follows:

Dia of knuckle pin $d_1 = d = 40\ mm$

outside dia. of double (or) single eye $d_2 = 2d = 80\ mm$,

outside dia. of knuckle pin head & collar $d_3 = 1.5d = 60\ mm$

Thickness of single eye $t_2 = 1.25\ d = 1.25 \times 40 = 50\ mm$.

Thickness of double eye $t_1 = 0.75d = 0.75 \times 40 = 30\ mm$

Thickness of knuckle pin head (or) collar

$t_2 = 0.5\ d = 0.5 \times 40 = 20\ mm$.

Due to above adopted values, stresses are induced in various parts which may be tensile, compressive & shear. These induced stresses should be less than the allowable stresses otherwise failure will occur.

(2) Consider knuckle pin:

we know that the knuckle pin may fail due to induced shear stress.

The induced shear stress is determined using,

$$2 \times \frac{\pi}{4} d_1^2 \times \tau_s = P$$

$$2 \times \frac{\pi}{4} \times 40^2 \times \tau_s = 60{,}000$$

$$\tau_s = \frac{60000 \times 4}{2 \times \pi \times 40^2} = 23.87 \ N/mm^2$$

$$\tau_s = 23.87 \ N/mm^2 < (\text{allowable } \tau_s = 40 \ N/mm^2)$$

Consider single eye rod!

The single eye rod may fail due to induced tensile stress or shear stress or crushing stress.

3. Induced tensile stress is calculated by equating the tensile strength to the applied load.

$$(d_2 - d_1) t \sigma_t = P$$

$$(80 - 40) \ 50 \times \sigma_t = 60{,}000$$

$$\sigma_t = 30 \ N/mm^2 < [\sigma_t \ (\text{allowable}) = 60 \ N/mm^2]$$

4. Induced shear stress is calculated by equating the shear strength to the applied load,

$$(d_2 - d_1) t \cdot \tau_s = P$$

$$(80-40) \ 50 \times \tau_s = 60{,}000$$

$$\tau_s = 30 \ N/mm^2 < \tau_s \text{ allowable} = 40 \ N/mm^2$$

(5) Induced crushing stress is calculated by equating the crushing strength to the applied load.

$$d_1 \cdot t \cdot \sigma_c = P$$

$$\sigma_c = \frac{60{,}000}{40 \times 50} = 30\ N/mm^2 < \left(75\ N/mm^2 \text{ allowable} \right)$$

Consider double eye rod:

similar to single eye rod, this also may fail due to induced tensile, shear and crushing stresses.

for Induced tensile stress:

$$(d_2 - d_1)\ 2t_1\ \sigma_t = P$$

$$\sigma_t \times (80 - 40)\ 2 \times 30 = 60{,}000$$

$$\sigma_t = 25\ N/mm^2 < (\sigma_t \text{ allowable} = 60\ N/mm^2)$$

for induced shear stress.

$$(d_2 - d_1)\ 2t_1 \cdot \tau_s = P$$

$$(80 - 40)\ 2 \times 30 \times \tau_s = 60{,}000$$

$$\tau_s = 25\ N/mm^2 < \tau_s \text{ allowable} = 40\ N/mm^2$$

for induced crushing stress

$$d_1 \cdot 2t_1 \cdot \sigma_c = P$$

$$40 \times 2 \times 30 \times \sigma_c = 60{,}000$$

$$\sigma_c = 25\ N/mm^2 < \text{allowable } 75\ N/mm^2.$$

since the induced stresses are less than the allowable stresses, our design is satisfactory.

UNIT- V

Springs

A spring is defined as an elastic machine element that deflects under the action of the load and returns to its original shape when the load is removed.

~~Apd~~ **Applications**

① springs are used to absorb shocks and vibrations.

② springs are used to store energy.

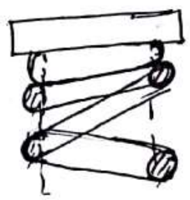
③ springs are used to measure the force.

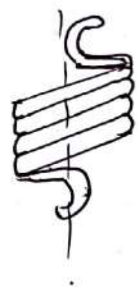
④ springs are used to apply force and control motion.

⑤ Measuring divicese.

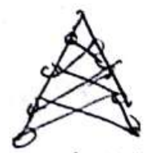
Types of springs:

Helical springs: The Helical springs are made up of a wire coiled in the form of a helix & is primarilly intnended for compressive or tensile load.





conical and Volute springs: The conical and volute springs, are used in special applications where a telescoping spring or a spring with a spring rate that increases with the load is desired.



Torsion springs: The springs may be helical or spiral type as The helical type may be used only in applications where the load tends to wind up the spring and used ~~where~~ in various electrical mechanisms.

Laminated or leaf springs :- The laminated or leaf spring (flat spring or carriage spring) consists of a number of flat plates (known as leaves) of varying lengths held together by means of clamps and bolts.

Disc or belleville spring : These springs consist of a number of conical discs held together against slipping by a central bolt or tube.

Special purpose springs : These springs are air or liquid springs, rubber springs, ring springs etc. The fluids (air or liquid) can behave as a compression spring. These springs are used for special types of application only.

Spring Materials: Materials for springs should have high elastic properties which should be stable with time. Required properties of spring materials are such as high fatigue ~~strength~~ Resistance, good thermal and corrosion resistance, high shear strength.

The chief materials for springs are medium & high - carbon steels, manganese steels, chromium steels, chromium-vanadium steels etc. usually the above steels are oil tempered. Another high quality carbon steel called as music wire is adopted for small size springs. To avoid corrosion, the springs are cadmium plated or coated with neoprene film. sometimes, the non-ferrous metals like phosphor bronze, monel metal, beryllium copper and non metals like fibre glass reinforce plastics can also be used for making springs for specific applications.

Selection of materials is sometimes based on the service requirements such as severe service, average service and light service.

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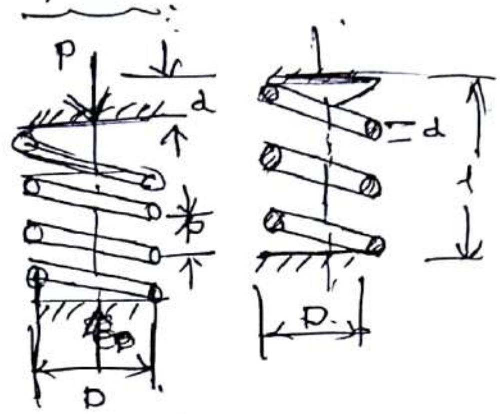
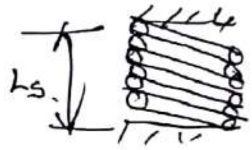
Terminology of a Helical Compression spring:



consider a helical compression spring whose diameter is 'd' and mean diameter of the coil is D is subjected to an axial compressive load P. Due to this load, the spring may be deflected to a distance of δ.

Spring Index: (C): It is defined as the ratio of mean diameter of the coil to wire diameter.



$$\boxed{C = \frac{D}{d}}$$

Spring Rate (k): It is defined as the load required per unit deflection of the spring.

$$\boxed{k = \frac{P}{\delta}}$$

The spring rate (k) may also be called as stiffness of spring.

Solid Length (L_s): It is total length of spring when it is fully compressed so that the coils are touching each other and there is no gap or clearance b/w the adjacent coils. mathematically this is equal to the product of wire dia of the total number of coils.

$$\boxed{L_s = d \cdot N}$$

where N is the total number of coils.

free Length (L_f): It is the length of spring when it is free from load. i.e in unloaded conditions.

for compression spring:

$$L_f = L_s + \delta_{max} + \text{clearance}$$

the total clearance may be adopted as 15% of maximum deflection,

$$L_f = L_s + \delta_{max} + 0.15\,\delta_{max}$$

$$= Nd + \delta_{max} + 0.15\,\delta_{max}$$

for tension spring,

$L_f = nd + (n-1)\ 1mm$, where n is the number of active coils which has been discussed ~~in the~~.

clearance b/w coils is taken as 1mm for very big spring the clearance may be increased.

Pitch (p): It is defined as the axial distance b/w the corresponding points of the adjacent coils when the spring is in the free state, or unloaded conditions.

Mathamatically

For compression spring, pitch $(p) = \frac{\text{Free length } (L_f)}{N-1}$

For tension spring, $p = \frac{L_f}{n-1}$

Note: In the helical compression spring, usualy the spring will not be allowed to compress until reaching solid strength. if it is made so, then the spring may lose its spring action and hence certain gap is maintained b/w the adjacent coils in order to make the spring effective.

Different forms of spring coil Ends

~~When th~~

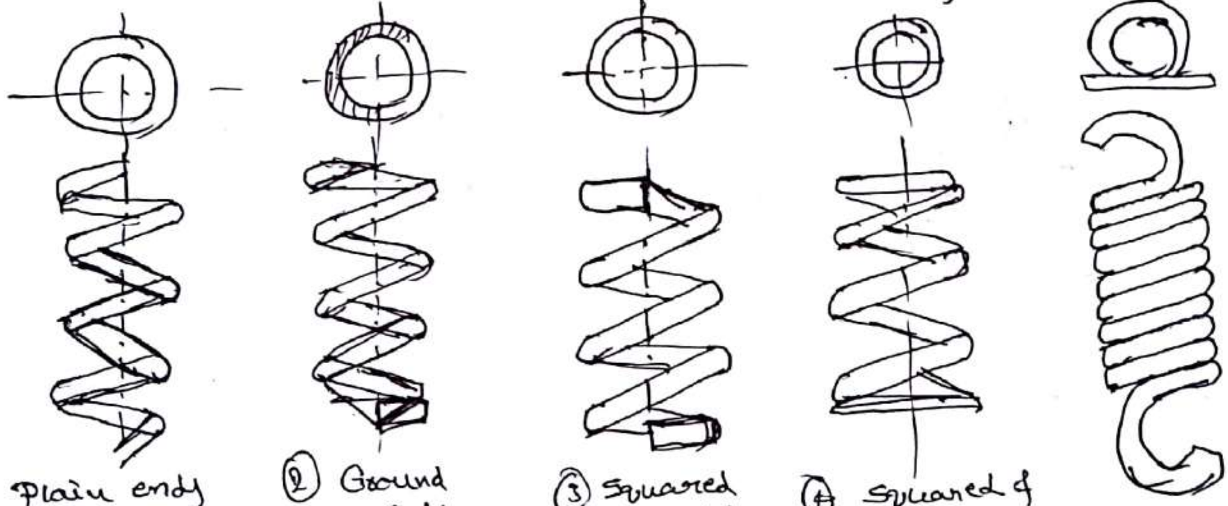
For helical Compression springs:

when the spring is cut from a lengthy piece and is used as it is, the supplied compressive loads (force) will not act along the axis of the spring due to the point of contact of the spring. Because of this eccentric application of force, one side of the spring is stressed more than the other side and the spring may buckle (i.e bend one side).

To make all the sides of spring to be equally loaded. & making the load to act along the axis, the ends of spring are made flat and they are ground. By doing so, the contact surfaces may be increased. But at the same time the contact surfaces which are made flat and ground lose their spring action & hence these end coils are called as inactive coils and the remaining coils are called as active coils because of their spring action.

Usually the ends of spring are made into four forms as,

1. Plain ends. (here, no mechanical work is done on the ends)
2. Ground ends (here, the ends are ground without making flat)
3. Squared ends (here, the ends are made flat without grinding)
4. Squared and ground ends (here the ends are ground after making them flat,)



① Plain ends ② Ground ends ③ Squared ends ④ Squared & ground ends

Ends Condition for compression springs.

Ends of a tension spring

for helical tension springs:

In the case of helical tension springs, the ends are made into loops in order to connect the spring with other machine elements. Here the total number of turns is equal to the number of active turns which are in b/w the starting points of loops in both ends plus half turn for each loop and hence

$N = n+1$ for tension spring

Design of circular wire helical Spring for static loading:

for most of the applications, the circular section spring wires are employed due to their easy manufacturing and high flexibility. In general, the design of helical spring (i.e determination of Various parameters of spring) is based mainly on two concepts,

(a) Amount of stress induced in the spring

(b) Amount of deflection produced, by the applied load.

(a) Amount of stress induced:

Consider a compression spring is loaded axially by the load

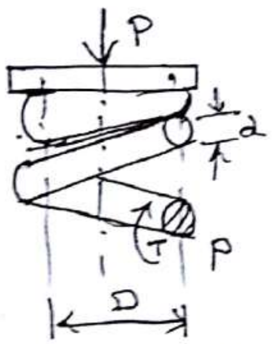


fig Loaded on helical spring

P as shown in fig. Due to this loading, the spring wires try to twist and hence a twisting moment is produced. Because of this twisting moment, a torsional shear stress is induced in the wires, which is given by

$$\tau_{s_1} = S_{s_1} = \frac{16T}{\pi d^3}$$

where T = Twisting moment produced $= P \times \frac{D}{2}$

d = Dia. of spring wire.

Substituting $T = P \times (D/2)$ in the above relation

we get

$$\tau_{s_1} = \frac{16PD}{\pi d^3 (2)} = \frac{8PD}{\pi d^3}$$

where D = mean dia of spring coil.

A part from this torsional shear stress, other types of stresses like (a) direct shear stress due to axial load, P and (b) stress due to effect of curvature of spring wire are also induced in the spring.

$$\text{Direct shear stress } \tau_{s_2} = \frac{\text{Axial load}}{\text{cross-sectional area of spring wire}}$$

$$S_{s_2} = \tau_{s_2} = \frac{P}{\frac{\pi}{4} d^2} = \frac{4P}{\pi d^2}$$

Combining these two shear stresses, τ_{s_1} & τ_{s_2}, we get the resultant stress as,

$$\tau_s = \frac{8PD}{\pi d^3} + \frac{4P}{\pi d^2} = \frac{8PD}{\pi d^3}\left[1 + \frac{d}{2D}\right]$$

$$= \frac{8PD}{\pi d^3}\left[1 + \frac{1}{2C}\right] = \frac{8PD}{\pi d^3} K_s = \frac{8PC}{\pi d^2} K_s.$$

where K_s is known as shear stress factor and $\rightarrow$ (a) is equal to $\left[1 + \frac{1}{2C}\right]$.

usually the stress due to curvature may be neglected for static loading, but considering the wire curvature will give better result. Hence by considering all effect the three stresses, Mr. A.M Wahl introduced a factor called Wahl's shear stress factor and derived an expression for maximum induced shear stress as

$$\tau_s = K \times \frac{8PD}{\pi d^3} \text{ or } \frac{8KPC}{\pi d^2}$$

where K = Wahl's shear stress factor

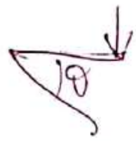
$$= \frac{4C-1}{4C-4} + \frac{0.615}{C}.$$

The value of K for different spring index values may be obtained from graph. (J.D.B),

(b) Deflection produced in helical spring:

When a helical compression spring is subjected to an axial compressive load P, its free length is reduced. This reduction of length is called as deflection δ, which is the total deformation of all the coils.

If θ is the angular deflection of spring wire due to torque, T, then the linear (i.e axial) deflection is given by



$$\delta = \theta \times \frac{D}{2}$$

from the known relation-ship $\frac{T}{J} = \frac{G\theta}{l}$

we get $\theta = \frac{Tl}{JG}$.

where T = Twisting moment produced on the wire $= P \times \frac{D}{2}$

J = polar moment of Inertia $= \frac{\pi}{32} d^4$

G = Modulus of rigidity of spring material

l = Length of spring (i.e the active length of spring)

= Length of one coil × number of active coils

$= \pi D \times n$

Substituting all the values

$$\delta = \theta \times D/2 = \frac{Tl}{JG} \times \frac{D}{2}.$$

$$= \frac{P\,(D/2) \times \pi D n \times D}{\left(\frac{\pi}{32} d^4\right) \times G \times 2}$$

$$\delta = \frac{8PD^3 n}{Gd^4} = \frac{8PC^3 n}{Gd}$$

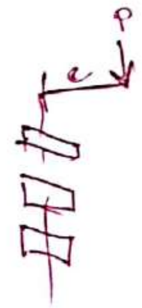
$\because C = \frac{D}{d}$

Spring rate (or) stiffness of spring $k = \frac{P}{\delta} = \frac{Gd^4}{8D^3 n} = \frac{Gd}{8C^3 n}$

Eccentric Loading of spring

In certain applications, the loading line does not coincide with the axis of the spring. In such occasions the spring is said to be concentric loaded and this load increases the stresses on one side of the spring and decreases on the other side and due to this action the stiffness of spring is reduced. if the load is to be acted on a spring eccentrically at a distance of 'e' from the spring axis, then the safe value of that load on the spring may be calculated as

$$\text{safe (or design) Load} = \text{Axial load} \times \frac{D}{(2e+D)}$$



where e = Eccentric distance

D = Mean dia of spring coil.

In order to make the load to act axially, the ends of spring should be squared and ground.

Buckling of Compression spring

$L_f = 4D$

When designing the compression spring, we should remember this point that the free length of spring should not be very long. It is found that when the free length (L_f) is more than four times the mean diameter of coil (D), the spring will buckle by the applied load which is undesirable due to poor performance. Hence when $L_f > 4D$, a safe load (which should be less than buckling load) may be applied on the spring so as to avoid buckling. The critical axial load (P_{cr}) that causes buckling may be calculated using the relation as,

$$P_{cr} = K \times K_b \times L_f$$

where K = Spring rate or stiffness $= \frac{P}{\delta}$

K_b = Buckling factor depending upon ratio (L_f/D)

It may be mentioned that a hinged end spring is one supported on pivots at both ends as in the case of plain ends whereas a built-in end spring is one in which a squared and ground end spring is compressed b/w two rigid & parallel flat plates.

To avoid the buckling of the spring, the spring may be mounted either on a central load or located on a tube with sufficient clearance b/w the tube & the spring.

Stiffness properties Based on spring Arrangements:

In order to improve the stiffness properties (i.e load carrying capacity per unit deflection) of the springs, two options may be followed. one is by selecting the stronger material the stiffness can be improved. The next is by arranging the available spring in different forms, the stiffness can be changed. The various arrangement of springs are as follows:

(a) Spring in series.

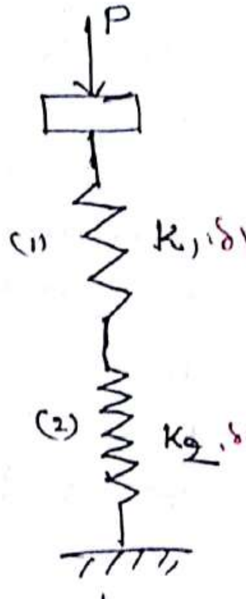


fig (v)

consider two springs connected in series

Let P = Load applied on the springs

δ_1 = Deflection of spring 1.

δ_2 = Deflection of spring 2.

k_1 = Stiffness of spring 1 = $\frac{P_1}{\delta_1}$

and k_2 = Stiffness of spring 2 = $\frac{P_2}{\delta_2}$

In this series connection of the springs, the total deflection produced by the combined springs is equal to the sum of the deflections of the individual springs.

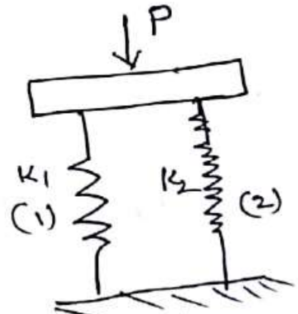
i.e Total deflection of the springs,

$$\delta = \delta_1 + \delta_2.$$

$$\frac{P}{k} = \frac{P_1}{k_1} + \frac{P_2}{k_2}$$

$$\boxed{\frac{1}{k} = \frac{1}{k_1} + \frac{1}{k_2}} \qquad \left(\because \boxed{P_1 = P_2 = P}\right)$$

where k is the equivalent stiffness of combined springs.



(b) springs in Parallel:

consider two springs connected in Parallel

In this arrangement, the applied load P is shared by the two springs and at the same time the deflections produced by the individual springs as well as the combined spring are the same.

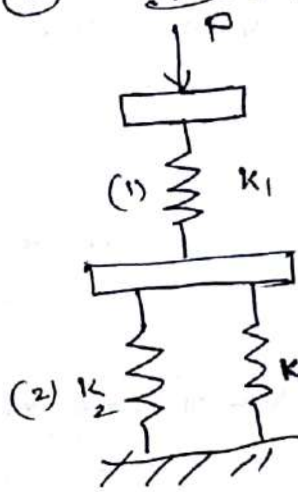
i.e $P = P_1 + P_2$ and $\delta = \delta_1 + \delta_2$

Now $P = K\delta$, $P_1 = k_1\delta_1$, $P_2 = k_2\delta_2$

substituting we get, $K\delta = k_1\delta_1 + k_2\delta_2$

$$\boxed{k = k_1 + k_2} \qquad (\because \delta = \delta_1 + \delta_2)$$

(c) springs in series - parallel combination.



consider three springs connected in combined series parallel arrangement as shown.

Let P = Total load applied

δ = Total deflection produced

P_1, k_1, δ_1 = Load, deflection, & stiffness for spring 1.

P_2, k_2, δ_2 = Load, deflection & stiffness for spring 2.

P_3, δ_3, K_3 = Load, deflection & stiffness for spring 3

Now $P = P_1 + P_2 + P_3$

$\delta = \delta_1 + \delta_{23}$ (where $\delta_{23} = \delta_2 = \delta_3$)

$$\frac{1}{k} = \frac{1}{k_1} + \frac{1}{k_{23}}$$

i.e $\frac{1}{k} = \frac{1}{k_1} + \frac{1}{(k_2+k_3)}$ $(\because k_{23} = k_2 + k_3)$

where k = Equivalent stiffness of combined springs in the whole set up.

k_{23} = stiffness of equivalent spring in parallel connection

δ_{23} = Deflection produced by the equivalent spring in parallel connection.

Design of concentric (OR) Composite Helical springs:

Concentric springs is also another form of spring arrangement to improve the spring force when the space for installation is limited. This kind of spring set-up insures the operation of the mechanism in the event of failure of the one of the springs In concentric spring the coils of inner spring are wound in the opposite direction with that of outer spring as shown above. The concentric springs are mostly employed in automobile clutches, Valve springs in air-craft, heavy duty diesel engines and rail-road car sus-pension systems.

The design of concentric spring is carried out based on certain assumptions. They are

1. The inner and outer springs should be made of same materials so as to have equal strength.

2. Their free lengths, solid lengths and deflection produced should be same.

3. The spring index ($C = D/d$) should be same for both the springs.

Let P = Applied axial load

$P_1, d_1, D_1, \delta_1, n_1$ & $P_2, d_2, D_2, \delta_2, n_2$ are load, wire diameter, coil diameter, deflection, active number of turns respectively for outer spring.

Since both the springs are made of same material, the shear stress induced in both the springs is approximately same.

i.e. $\tau_{s1} = \tau_{s2}$

or $$\frac{8K_1P_1D_1}{\pi d_1^3} = \frac{8K_2P_2D_2}{\pi d_2^3} \rightarrow (1)$$

Since both the springs produce same deflection.

$$\delta_1 = \delta_2$$

$$\frac{8P_1D_1^3n_1}{Gd_1^4} = \frac{8P_2D_2^3n_2}{Gd_2^4} \rightarrow (2)$$

Also since the free lengths, solid lengths and spring indices are same for both the springs, we have

(I) $C_1 = C_2 = C$

$$\frac{D_1}{d_1} = \frac{D_2}{d_2} = C \Rightarrow D_1 = Cd_1 \text{ \& } D_2 = Cd_2.$$

(II) $L_{s1} = L_{s2}$ (or) $n_1d_1 = n_2d_2$ $(\because L_s = nd)$

(III) $K_1 = K_2$ $\left(\because K = \frac{4C-1}{4C-4} + \frac{0.615}{C}\right.$ for same $\underline{C}$ values, $\underline{K}$ values are equal)

from eqn (1) by cancelling the terms $\frac{D}{d}$, K, and π, we get

$$\frac{P_1}{d_1^2} = \frac{P_2}{d_2^2}$$

$$\frac{P_1}{P_2} = \left(\frac{d_1}{d_2}\right)^2 \longrightarrow (3)$$

The above relation may be obtained from eqn (2) also - That is, by cancelling the common terms $\frac{D}{d}$, G etc. ~~the~~ eqn (2) gives

$$\frac{P_1 n_1}{d_1} = \frac{P_2 n_2}{d_2}$$

$$\frac{P_1}{P_2} = \frac{d_1}{d_2} \cdot \frac{n_2}{n_1} = \frac{d_1}{d_2} \cdot \frac{n_2 d_2}{d_2} \times \frac{d_1}{n_1 d_1}$$

$$= \left(\frac{d_1}{d_2}\right)^2 \qquad (\because n_2 d_2 = n_1 d_1 \text{ assumed})$$

The radial clearance 'x' b/w the two springs.

$$x = \left(\frac{D_1}{2} - \frac{D_2}{2}\right) - \left(\frac{d_1}{2} + \frac{d_2}{2}\right) \qquad \text{from fig}$$

In general x may be taken as $x = \frac{d_1 - d_2}{2}$

Hence $\left(\frac{D_1}{2} - \frac{D_2}{2}\right) - \left(\frac{d_1}{2} + \frac{d_2}{2}\right) = \frac{d_1 - d_2}{2}$

$$D_1 - D_2 - d_1 - d_2 = d_1 - d_2$$

$$\text{i.e } \boxed{D_1 - D_2 = 2d_1}$$

substituting as $D_1 = cd_1$ and $D_2 = cd_2$ where c is the spring index, we get

$$cd_1 - cd_2 = 2d_1$$

$$cd_1 - 2d_1 = cd_2$$

$$d_1(c-2) = cd_2$$

$$\boxed{\frac{d_1}{d_2} = \frac{c}{c-2}} \rightarrow (4)$$

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using the above relations suitably, the parameters of concentric spring can be determined.

Design steps for Helical Springs in Static Loading

1. From the given problem, note the type of spring needed, the required spring material, the amount of supplied load and the other working conditions like space available, place of application etc.

2. Select the design shear stress for the spring material from the table

3. Designing the spring means that the principal dimensions like wire diameter (d), mean coil diameter (D), the number of turns (N), free length of spring (L_f), pitch (p) etc. are to be found out. First using the expression

$$\tau_s = \frac{8KPD}{\pi d^3} \text{ or } \frac{8KPC}{\pi d^2}$$, determine the dia of wire (d)

In the above equation, K = wahl's shear stress factor,

$$= \frac{4C-1}{4C-4} + \frac{0.615}{C}$$, where C = spring index $= \frac{D}{d}$

4. Select the next standard diameter of wire from table and then find out the mean coil diameter (D).

5. Then using the relationship $\delta = \frac{8PD^3 n}{Gd^4}$ or $\frac{8PC^3 n}{Gd}$, find out the number of active turns (n). In the above eqn G = Modulus of rigidity which may be selected from table

6. For helical compression spring, based on the end conditions decide the total number of turns (N). Usually, the squared & ground end is preferred. for compression spring for squared and ground end condition

$$\boxed{N = n+2}$$

7.) Calculate the free length of spring, (L_f) as

$$L_f = L_s + \delta_{max} + 0.15\,\delta_{max}$$

δ_{max} = Maximum Permissible compression.

8). Find out the pitch of the spring using.

$$\text{pitch} = \frac{\text{Free length } (L_f)}{(N-1)}$$

9). For helical tension spring, since the ends are made into loops, calculate the total number of turns as

$$N = n+1$$

and its free length $L_f = nd + (n-1)c$, where c = clearance b/w the adjacent coils which may vary from 1mm to several mm depending upon the size of springs.

Determine the pitch as $p = \dfrac{\text{Free length } (L_f)}{(n-1)}$

10) Draw a neat sketch of the corresponding spring.

Design of Helical springs for Fatigue Loading

For certain applications, springs are employed used in fatigue loaded conditions when the operating load varies from one lower to another higher load continuosly. Some of such fatigue loaded springs are padlock springs, toggle-switch spring and valve spring of an automobile etc. Among these three, for the first two springs, the required number of cycles of operation in their life are about a few thousands whereas for the value springs, they should operate for millions of cycles without failure.

The springs subjected to fatigue shear loads are designed based on Soderberg criterian. if the fatigue (variable) loads are completely reversible, then we can make use of general Soderberg equation. But in the case of springs, they are operated in one type of loading only, it may be fully in compression side or in tension side. Usually helical springs can never be used for both compressive and tensile loading. Since the springs are subjected to oneway shear load, a modified Soderberg procedure is adopted. The stress-time diagram expresses the usual condition of working for helical springs. The worst condition will occur when one side of the shear stress is zero. Let the maximum & minimum stresses are shear endurance stress, (S_{es}) and zero stress.

for such conditions, both mean stress and variable stress amplitude are equal to $\left(S_{es}/2\right)$.

Designing procedure for spring subjected to fatigue loads:

1. From the given data, note the range of working loads (P_{max}, P_{min}) and the stress details like shear endurance stress (τ_{es}) (S_{es}) and shear yield stress (S_{ys}) etc.

$$\text{usually} \quad S_{ys} = 0.6 S_y, \quad S_{es} = 0.6 S_e \quad \text{and} \quad S_{us} = 0.6 S_u$$

where
S_y = yield stress in axial loading
S_e = Endurance stress in variable bending loading
S_u = ultimate tensile stress

2. Determine the mean shear load (P_m), variable shear load (P_v) and corresponding shear stresses (S_{ms}, S_{vs})

3. Using the soderberg eqn as

$$\frac{1}{f_s} = \frac{S_{ms} - S_{vs}}{S_{ys}} + \frac{2 S_{vs}}{S_{es}}$$

, calculate the diameter of spring and select the next standard diameter from table

4. Then using the same formulas mentioned in designing procedure for static loading, as

$$\delta = \frac{8PD^3 n}{Gd^4} \quad \text{or} \quad \frac{8PC^3 n}{Gd}$$

find out number of active turns.

5. Similarly determine the other parameters like free length, pitch etc., and draw the neat sketch of the spring.

Non circular wire Helical compression springs

Eventhough for most of the occasions the springs with ~~the~~ circular wire are employed, some times helical compression springs made of rectangular or square wire are utilized in order to provide greater resilience in a given space and to provide for pre-determined altering of the stiffness of the spring. Such springs are costlier to manufacture.

For such springs made of rectangular wire having ~~of~~ width of wire (b) and thickness (t) with $t > b$ (i.e (when the longer side of wire is parallel to the spring axis), the maximum induced shear stress is given by

$$S_s = \frac{KPD(1.5t + 0.9b)}{b^2 t^2} = \frac{KPC(1.5t + 0.9b)}{bt^2}$$

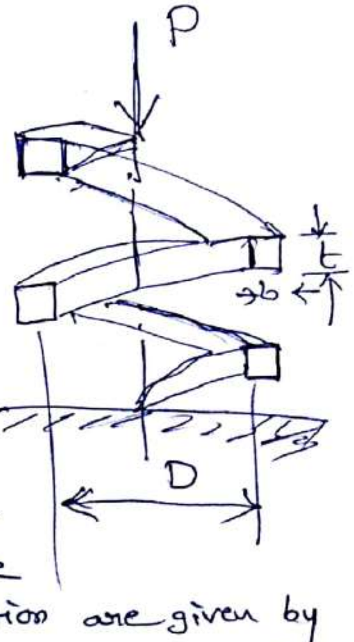
and when $t < b$ (i.e when shorter side is parallel to spring axis) $\left(\because C = \frac{D}{b}\right)$

$$S_s = \frac{KPD(1.5b + 0.9t)}{b^2 t^2} = \frac{KPC(1.5b + 0.9b)}{bt^2}$$

The deflection of the spring is given by

$$\delta = \frac{2.83\, PD^3 n\,(b^2 + t^2)}{G b^3 t^3}$$

$$= \frac{2.83\, PC^3 n\,(b^2 + t^2)}{G t^3}$$



for springs made of square wire, since the width (b) and thickness (t) are equal, the maximum shear stress and the deflection are given by

$$S_s = \frac{2.4\,KPD}{b^3} = \frac{2.4\,KPC}{b^2} \quad \text{and} \quad \delta = \frac{5.66\,PD^3}{Gb^4} = \frac{5.66\,PC^3 n}{Gb}$$

where

$$K = \frac{4C-1}{4C-4} + \frac{0.615}{C} \quad \text{(Whal's factor)}$$

$$C = \frac{D}{b} \quad \text{(spring index)}.$$

free length of spring for both rectangular & square spring.

$$\boxed{L_f = Nt + 1.15\,\delta_{max}}$$

The above type of springs have some disadvantages which are,

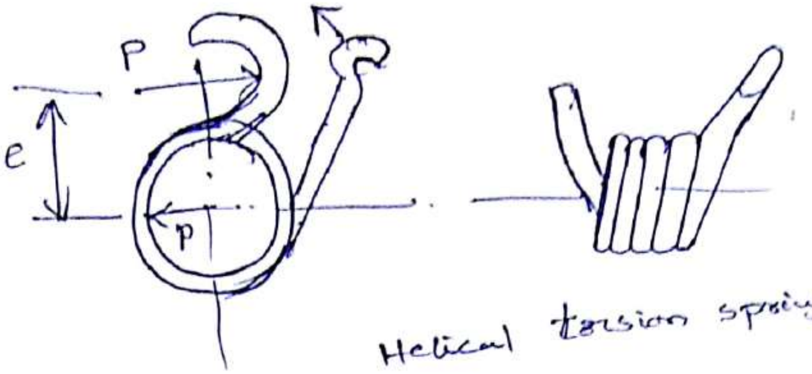
1. The shape of wire does not remain rectangular or square while forming helix, resulting in trapezoidal cross-sections and hence energy absorbing capacity may be reduced.

2. the stress distribution is not as favourable as for circular wires which result the reduction of spring life.

Helical. Torsion springs:

Helical torsion springs are made similar to helical compression springs except that their ends are formed in such a shape so as to transmit torque to a particular component of a machine or mechanism as shown. These springs are mostly made by circular wire and sometimes by rectangular (or) square wire depending upon the requirement. They are widely used in door-hinges, automobile starters, levers, writing pad clips, paper punching machine etc.

The primary stress developed in this type of spring is the bending stress in contrast with the helical compression or tension spring where torsional shear stress is the resulting stress.



Helical torsion spring.

When the torsion spring is loaded, the radius of curvature of each coil is changed and also because of this eccentric loading, bending moment is developed which induces the bending stress in the coil. According to A.M. Wahl, the bending stress in a helical torsion spring made of round wire is given by

$$S_b = \sigma_b = K \frac{32M}{\pi d^3} = K \frac{32 \cdot P.e}{\pi d^3}$$

where M = Bending moment = $P.e$

P = Applied load on the spring

e = Eccentric (or offset) distance from axis

K = Wahl's stress factor = $\frac{4C^2 - C - 1}{4C(C-1)}$

C = spring index = $\frac{D}{d}$

D = coil diameter

d = wire diameter.

It is known that $\frac{M}{I} = \frac{E}{R} \Rightarrow R = \frac{EI}{M}$

The angular deflection or total angle of twist,

$$\theta = \frac{l}{R} = \frac{Ml}{EI} = \frac{M \times \pi D n}{E \times \frac{\pi d^4}{64}} = \frac{64 MDn}{Ed^4}$$

where l = Total length of coil = $\pi D n$

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R = Radius of curvature $= \frac{l}{\theta}$

E = young's modulus

I = Moment of inertia $= \frac{\pi d^4}{64}$

n = Number of coils.

The stiffness of helical ~~torsion~~ spring is given by

$$k = \frac{\text{Bending moment}}{\text{Angular deflection}} = \frac{M}{\theta} = \frac{Ed^4}{64Dn}.$$

1. Design a helical compression spring to carry a load of 1.5 KN with a deflection of 40 mm. Spring index is 5. Allowable shear stress is 400 N/mm². Modulus of rigidity is 8×10^{10} N/m².

Sol. Load $P = 1.5\,KN = 1500\,N$, Deflection $\delta = 40\,mm$

Spring index $C = 5$, Allowable stress (τ_s) & $(s_s) = 400\ N/mm^2$

Modulus of rigidity, $G = 8\times10^{10}\ N/m^2 = 8\times10^4\ N/mm^2$

We know that the shear stress induced due to the load, P is given by the relation as

$$\tau_s = s_s = \frac{K \cdot 8PC}{\pi d^2}$$ which should be less than allowable stress for safe design.

$$K = \frac{4C-1}{4C-4} + \frac{0.615}{C} = \frac{4\times5-1}{4\times5-4} = \frac{19}{16} + \frac{0.615}{4}$$

$$= 1.31$$

$$\frac{8\times1.31\times1500\times5}{\pi d^2} = 400$$

$$\boxed{d = 7.91\ mm}$$

from J.D.B page No. 13.18, table no. 13.5, next higher diameter is selected as 8.23 mm, which is ~~mentioned~~ as SWG (Standard wire gauge) ①.

Now, $d = 8.23$ mm (i.e the wire diameter)

The mean diameter of coil, $D = Cd = 5 \times 8.23 = 41.15$

for the helical spring, the deflection produced is given by the expression as $\delta = \frac{8PC^3 n}{Gd}$. from which the number of active coils (n)

$$n = \frac{\delta \cdot Gd}{8PC^3} = \frac{40 \times 80000 \times 8.23}{8 \times 1500 \times 5^3} = 17.6.$$

Let $n = 18$ turns.

Assuming the ends of spring are squared & ground, the total number of turns, $N = n+2 = 18+2 = 20.$

free length, L_f = Solid length + Maximum deflection + Total clearance b/w the coils

$$L_f = Nd + \delta_{max} + 0.15\,\delta_{max}$$

$$= (20 \times 8.23) + 40 + (40 \times 0.15)$$

$$= 211 \text{ mm}$$

Pitch of the spring, $p = \frac{\text{Free length}}{(N-1)} = \frac{211}{(20-1)} = 11$ mm

(2) Design a close coiled helical compression spring for a service load ranging from 2.5 KN to 3 KN. The deflection for this load range is 6mm. use a spring index of 5. Take the shear yield strength as 700 N/mm². and modulus of rigidity a 8×10^4 N/mm². factor of safety is not to be less than 1.3. Also check the spring for buckling.

Sol

Minimum load $P_1 = 2.5\,KN = 2500\,N$

Maximum load $P_2 = 3KN = 3000\,N$

Deflection $\delta = 6mm$ for the load range 2500 N to 3000 N

(i.e for the load of 500 N)

Spring index $C = 5$.

Shear yield strength $(S_{sy}) = 700\,N/mm^2$

Modulus of rigidity $G = 8\times10^4\,N/mm^2$.

$f_s = > 1.3,$ $1.3 < f_s$

Let $f_s = 1.4$.

Then allowable shear yield strength (S_s) $= \frac{S_{sy}}{f_s} = \frac{700}{1.4} = 500\,N/mm^2$.

Let d = Diameter of spring wire

D = Diameter of spring coil.

$$C = \frac{D}{d} = 5 \text{ (given)}$$

$$S_s = \tau_s = \frac{8KPC}{\pi d^2}, \quad K = \frac{4C-1}{4C-4} + \frac{0.615}{C}$$

$$= \frac{4\times5-1}{4\times5-4} + \frac{0.615}{5} = \underline{1.31}.$$

It should be noted that the induced shear stress for the maximum load should be less than the allowable shear stress for safe condition.

$$\frac{8KP_2C}{\pi d^2} = S_s$$

$$d^2 = \frac{8\times1.31\times3000\times5}{\pi\times500}$$

$$\boxed{d = 10.00}\ mm$$

next standard dia. is $\underline{10.16}$ mm from J.D.B . page 13.18

table 13.5

Mean diameter of spring coil $D = cd = 5 \times 10.16 = 50.80\,mm$.

Let n = Total number of active turns of spring.

N = Total number of turns, containing active & inactive turns

$$\delta = \frac{8PC^3 n}{Gd}$$

$\because$ $\delta = 6\,mm$ for the load value of 500 N, we get

$$6 = \frac{8 \times 500 \times 5^3 \times n}{8 \times 10^4 \times 10.16}$$

$$\boxed{n = 9.8 \text{ turns}}$$

Let $\boxed{n = 10}$

Assuming the ends of spring are squared & ground

we get $N = n + 2 = 10 + 2 = 12$ turns.

L_f = solid length + δ_{max} + $0.15\,\delta_{max}$

$L_f = L_s + \delta_{max} + 0.15\,\delta_{max}$

~~$L_f = N$~~ δ_{max} = Maximum deflection for the maximum load.

$$\delta_{max} = \frac{6}{500} \times 3000 = 36\,mm$$

3000/500

$$L_f = Nd + 36 + (0.15 \times 36)$$

$$= (12 \times 10.16) + (0.15 \times 36)$$

$$= 163.32\,mm.$$

6
500 = 3000
6 x
x = 6x3000/500

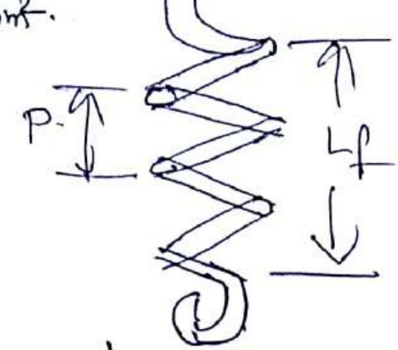
6m 500
500 6m
3000 (x)

~~$L_f =$~~ $P = \frac{L_f}{N-1} = \frac{163.32}{(12-1)} = 14.85\,mm$

Now $\frac{L_f}{D} = \frac{163.32}{50.80} = 3.2 < 4$ (the critical value for buckling).
K_b

Hence the design is safe against buckling.

(3) A spring for a spring balance is to elongate 100 mm, when subjected to a load of 20 kgf. Assume that the mean diameter of the coil is to be 6 times the diameter of the wire and the maximum stress to be induced is limited to $40\ kgf/mm^2$. Determine the diameter for the wire, for the coil and the number of coils required and length of spring. Modulus of rigidity $G = 0.8 \times 10^4\ kgf/mm^2$.



Sol spring is tensile spring : Type

Load $P = 20$ kgf

Deflection $\delta = 100$ mm

Spring index $C = \frac{D}{d} = 6.$ $D = 6d$, $D/d = 6$

τ_s (or) $S_s = 40\ kgf/mm^2$, $G = 0.8 \times 10^4\ kgf/mm^2$

τ_s (or) $S_s = \frac{8KPC}{\pi d^2} = \frac{8 \times 1.1977 \times 20 \times 6}{}$ $K = 1.1977$ $c = \frac{D}{d}$

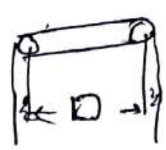
$K = \frac{4C-1}{4C-4} + \frac{0.615}{C} = \frac{24-1}{24-4} + \frac{0.615}{6}$

$\boxed{d = 3.09\ mm}$

~~$K = \frac{24d-1}{24d-4} + 0.615$~~ $\frac{23}{21} + \frac{0.615}{6}$

next (std) value is 3.25 mm (i.e SWG 10 wire).

$D = Cd \Rightarrow D = 6 \times 3.25 = 19.5$ mm



outer dia of coil $D_o = D + d = 19.5 + 3.25 = 22.75$ mm

Inner dia of coil $D_i = D - d = 19.5 - 3.25 = 16.25$ mm.

$\delta = \frac{8PC^3 n}{Gd}$

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No. of active turns $n = \frac{100 \times 0.8 \times 10^{4} \times 3.25}{8 \times 20 \times 6^{3}}$

$$\boxed{n = 75.2}$$

Let $n = 76$ turns.

$N = n+1 = 76+1 = 77$.

Assuming the spring is having loops on both ends.

free length of spring $L_f = nd + (n-1)\, 1\,mm$

(Assuming the gap b/w the adjacent coils as 1 mm)

$L_f = (76 \times 3.25) + (76-1)1 = 322\ mm.$

$\text{pitch} = \frac{\text{Free length}}{n-1} = \frac{L_f}{n-1} = \frac{322}{76-1} = 4.29\ mm$

(4) A helical compression spring made of oil tempered carbon steel, is subjected to a load which varies from 600 N to 1600 N. The spring index is 6 and the design factor of safety is 1.5 N. if the yield shear stress is 700 N/mm² and the endurance stress in shear is 350 N/mm². The compression at the maximum load is 40 mm. Assume G = 80 GPa. Find the size of the spring wire and mean diameter of the spring coil, pitch of the spring and its free length.

Sol

Load is carrying from 600 N to 1600 N

spring index $C = 6$.

factor of safety $(F_s) = 1.5$

yield shear stress $(S_{ys}) = 700\ N/mm^2$

Endurance shear stress $(S_{es}) = 350\ N/mm^2$

Deflection $\delta = 40\,mm$, $G = 80\,MPa = 80 \times 10^{3}\,N/mm^2$

∵ The spring is subjected to variable load, we can make use of soderberg eqn to find out the size of the spring wire.

According to soderberg eqn $\frac{1}{f_s} = \frac{S_{ms} - S_{vs}}{S_{ys}} + \frac{2S_{vs}}{S_{es}}$

where S_{ms} = Mean shear stress

S_{vs} = Variable shear stress

S_{ys} = Yield stress in shear

S_{es} = Endurance stress in shear.

$$S_{ms} = \frac{8KP_mC}{\pi d^2}$$

where P_m = Mean shear load (i.e axial load.)

$$P_m = \frac{P_{max} + P_{min}}{2} = \frac{1600+600}{2} = 1100\,N$$

$$K = \frac{4C-1}{4C-4} + \frac{0.615}{C} = \frac{4\times6-1}{4\times6-4} + \frac{0.615}{4} = 1.25.$$

$$S_{ms} = \frac{8 \times 1.25 \times 1100 \times 6}{\pi d^2} = \frac{21010}{d^2}\,N/mm^2$$

$\tau_{sn} = \frac{8KP_mC}{\pi d^2}$

similarly, $S_{vs} = \frac{8KP_vC}{\pi d^2}$

$$P_v = \text{Variable shear load} = \frac{P_{max} - P_{min}}{2} = \frac{1600-600}{2}$$

$P_v = 500\,N$

$$S_{vs} = \frac{8 \times 1.25 \times 500 \times 6}{\pi d^2} = \frac{9550}{d^2}\,N/mm^2$$

Now using soderberg criterion

$$\frac{1}{f_s} = \frac{S_{ms} - S_{vs}}{S_{ys}} + \frac{2S_{vs}}{S_{es}}$$

i.e $$\frac{1}{1.5} = \frac{\frac{21010}{d^2} - \frac{9550}{d^2}}{700} + \frac{2 \times \frac{9550}{d^2}}{350}$$

$$= \frac{16.4}{d^2} + \frac{54.6}{d^2} = \frac{71}{d^2}$$

$$\therefore\ d = \sqrt{71 \times 1.5} = \underline{10.32\ mm}$$

Let us select the next standard diameter for the spring wire as, $d = 10.97\ mm$ (i.e SWG 5/8) wire.

$\therefore$ Mean diameter of spring coil $D = 6 \times 10.97 = 65.82\ mm$

free length of spring $L_f = Nd + \delta_{max} + 0.15\ \delta_{max}$

$N = n + 2$, $\delta = \frac{8PC^3 n}{Gd}$

for maximum load,

$$40 = \frac{8 \times 1600 \times 6^3 \times n}{80 \times 10^3 \times 10.97}$$

$40 = 3.15\,n$ $\boxed{n = 13 \text{ turns}}$

$N = 13 + 2 = \underline{15}$ turns

free length $L_f = (15 \times 10.97) + (1.15 \times 40) = 211\ mm$

$$\text{Pitch} = \frac{L_f}{N-1} = \frac{211}{15-1} = \underline{15\ mm}.$$

(5) Design for a window shade a helical torsion spring made of cold drawn steel wire yield strength is 1300 N/mm². and the

factor of safety is 2. Due to space limitations, the mean dia- of the coil is kept as 28 mm. The maximum bending moment acting on the spring is 300 N-mm. The modulus of elasticity of the spring material is 2.05×10^5 N/mm². The stiffness of the spring should be 12 N-mm/rad.

Sol Yield strength $S_y = 1300$ N/mm², factor of safety $f_s = 2$.

Mean dia of coil $D = 28$ mm, Bending moment $M = 300$ N-mm.

$E = 2.05 \times 10^5$ N/mm², stiffness of spring $k = 12$ N-mm/rad.

Allowable ~~shear~~ stress $(S_b) = \frac{S_y}{f_s} = \frac{1300}{2} = 650$ N/mm²

when the torsional spring subjected to a bending moment, the induced bending stress is given by

$$\sigma_b = K \times \frac{32M}{\pi d^3}$$

$$K = \frac{4c^2 - c - 1}{4c(c-1)} \quad \text{and} \quad c = \frac{D}{d}$$

Since the spring index is not known, the wire diameter can be determined by trial & error method.

Let us select SWG 16 wire whose dia $d = 1.63$ mm

$$c = \frac{D}{d} = \frac{28}{1.63} = 17.18 \text{ mm.}$$

$$K = \frac{4 \times (17.18)^2 - 17.18 - 1}{(4 \times 17.18)\ (17.18 - 1)} = 1.045$$

The induced stress $S_b = K \frac{32M}{\pi d^3} = \frac{1.045 \times 32 \times 300}{\pi \times 1.63^3}$

$= 737$ N/mm² $> (S_b) = 650$ N/mm²

∴ The induced stress is more than the allowable value, let us select slightly larger diameter wire.

NOW, select SWG 15 wire.

Hence $d = 1.83$ and $C = \frac{28}{1.83} = 15.3$

$$K = \frac{4\times(15.3)^2 - 15.3 - 1}{(4\times 15.3)(15.3-1)} = 1.051$$

$$S_b = \frac{1.051 \times 32 \times 300}{\pi \times 1.83^3} = 524\ N/mm^2 < S_b = 650\ N/mm^2.$$

Hence design is safe. wire dia $= 1.83$ mm,

The number of active coil may be obtained from the eqn.

$$n = \frac{Ed^4}{64Dk} = \frac{2.05\times10^5 \times 1.83^4}{64\times28\times12} = 107 \text{ coils.}$$

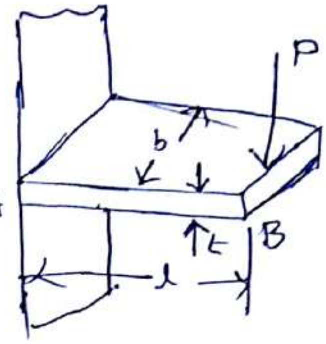
Leaf springs,

Leaf springs (flat spring or laminated spring) mainly used in automobile vehicle like car, bus etc: are made of flat rolled steel plates and they are arranged in a laminated structure. Their lengths are gradually reduced from first plate to last plate and all the plates are held together to act as a single spring by means of central band, containing two U clips and bolted at the centre. Before connecting with the main body of automobile, these springs are formed into a semi-elliptical shape by providing an initial curvature (camber) which is nothing but a pre-deflection and this chamber may be straightened during operation under the applied load.

Analysis of Leaf spring Characteristics

The function of leaf spring may be analysed by means of some simple types of beams and their characteristics.

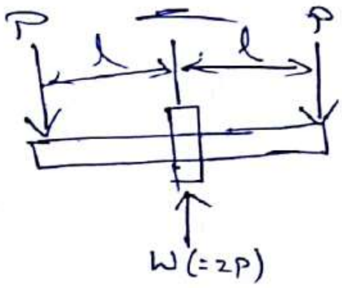
Consider a cantilever beam of rectangular cross-section whose width is b, thickness t and length l, is subjected to a load P at its free end. Due to this load, the beam tries to bend and the maximum bending stress at the fixed end A is given by



$$\sigma_b = \frac{M}{Z} = \frac{My}{I} = \frac{P.l \times t/2}{\frac{bt^3}{12}}$$

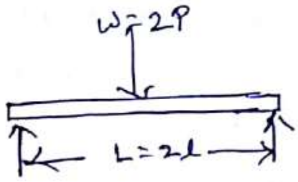
$$= \frac{6Pl}{bt^2}$$

$$\delta = \frac{Pl^3}{3EI} = \frac{Pl^3}{3E\frac{bt^3}{12}} = \frac{4Pl^3}{Ebt^3} = \frac{2}{3}\sigma_b \frac{l^2}{Et}.$$



Simply supported Beam

with b, thickness t



$$\sigma_b = \frac{My}{I} = \frac{\frac{Wl}{4} \times \frac{t}{2}}{\frac{bt^3}{12}} = \frac{\frac{2P \times 2l}{4} \times \frac{t}{2}}{\frac{bt^3}{12}} = \frac{6Pl}{bt^2}$$

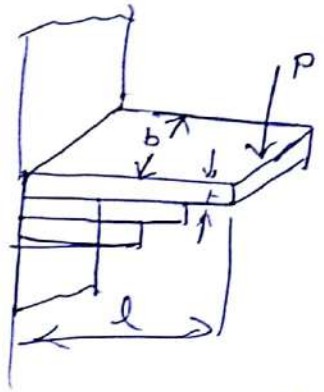
$$\delta = \frac{Wl^3}{48EI} = \frac{2P \times (2l)^3}{48 \times E \times \frac{bt^3}{12}} = \frac{4Pl^3}{Ebt^3}$$

From the above two analysis we can come to conclusion that the simply supported beam of length (L=2l) ; subject to a load of W(2P) may be treated as double cantilever beam fitted side by side and loaded at its ends.

Design of Leaf spring:

Since the leaf spring is constructed by certain number of full length leaves and others by graduated length leaves, it is designed on the basis of combined strength and deflection characteristics of both full length and graduated leaves.

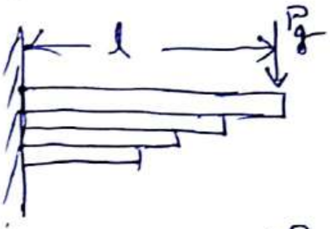
For analysis, the leaves are divided into two groups such as the master leaf along with graduated leaves forming one group and the extra full length leaves forming the other. Now, assume the following parameters.



P = Load applied at the end of the spring

P_f = Load shared by the full length leaves

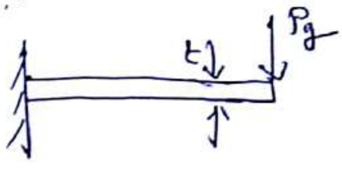
P_g = Load shared by the graduated leaves along with master leaf



n = Total number of leaves

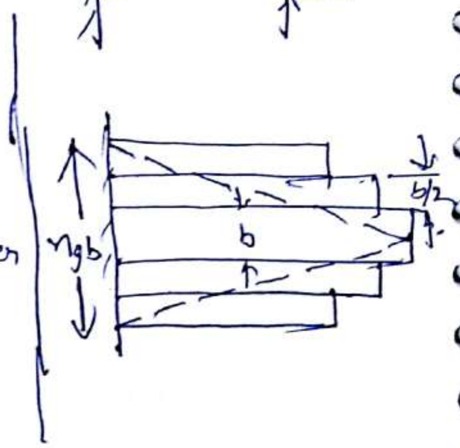
n_f = Number of extra full length leaves

n_g = Number of graduated leaves including master leaf



b = Width of each leaf

t = Thickness of each leaf.



l = ~~Half Length~~ Half length of spring or Length of cantilever

W = ~~Half~~ Total load transmitted to the axle of vehicle

Also $W = 2P$, $P = P_f + P_g$, $n = n_f + n_g$

First consider the half portion of the first group of the of the leaf spring which looks like a cantilever beam containing n_g number of graduated leaves including master leaf. Since the load is applied at the free end of the master leaf, in order to find out the induced stress and deflection, the leaves are assumed to be arranged in such a way that they can be treated as a triangular plate. for this, the individual leaves are separated and the master leaf is placed at the centre. Then the second leaf is cut logitudinally into two halves, each of width $(b/2)$ and placed on each side of the master leaf. The same method is adopted for other leaves. The resultant shape is approximately a triangular plate thickness (t) and a maximum width at the support as $(n_g \cdot b)$

for the set up, the maximum bending stress produced at the fixed end is given by

$$S_{bg} = \frac{My}{I} = \frac{(P_g \cdot l)(t/2)}{\frac{(n_g b)\, t^3}{12}} = \frac{6 P_g \cdot l}{n_g \cdot b \cdot t^2}$$

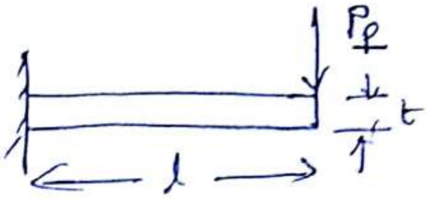
The maximum deflection produced at the free end of the ~~trang~~ triangular plate (i.e., at the load point) is given by

$$\delta_g = \frac{P_g \cdot l^3}{2EI} = \frac{P_g \cdot l^3}{2E\left[\frac{(n_g \cdot b)\, t^3}{12}\right]} = \frac{6 P_g \cdot l^3}{E\, n_g \cdot b \cdot t^3}$$

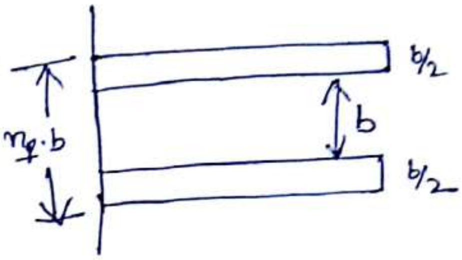
$$\boxed{\delta_g = \frac{S_{bg} \cdot l^2}{E t}}$$

Similarly the second group of leaves. i.e the extra full length leaves can be treated as a rectangular plate of thickness (t) and uniform width $(n_f \cdot b)$

for this case, the maximum induced bending stress at the fixed end,



$$S_{bf} = \frac{My}{I} = \frac{(P_f \cdot l)(t/2)}{\frac{1}{12}(n_f b)t^3} = \frac{6 P_f \cdot l}{n_f \cdot b \cdot t^2}$$



The deflection at the free end (i.e point load) is given by

$$\delta_f = \frac{P_f \cdot l^3}{3EI} = \frac{P_f \cdot l^3}{3E\left[\frac{1}{12}(n_f b)t^3\right]} = \frac{4 P_f l^3}{E n_f \cdot b t^3}$$

$$= \frac{2}{3} S_{bf} \frac{l^2}{Et}$$

Since the leaf spring is the combined form of full length leaves and graduated leaves, the deflection produced in both types of leaves are same.

$$\delta_f = \delta_g \qquad \frac{2}{3} S_{bf} \frac{l^2}{Et} = S_{bg} \frac{l^2}{Et}$$

$$\boxed{S_{bf} = \frac{3}{2} S_{bg}}$$